The Marginal Value of Cash and Corporate Savings*

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Abstract

This paper provides a non-parametric empirical analysis of the structural model of corporate savings and investment by Bolton, Chen and Wang (2011). The key variable of that model is the firm's cash-to-capital ratio w. Firm value is increasing and concave in w, and the firm's marginal value of cash is decreasing in w. As a result, investment is increasing in w. These predictions and other less obvious results of the model are confirmed by our empirical analysis. A major finding of our analysis is that on average, the value of capital for a firm is increasing and concave in the corporation's cash-to-capital ratio. Moreover, the slope of this value function, which measures the corporation's marginal value of cash, is decreasing in the cash-to-capital ratio w as predicted by the model. Together, these findings provide a parsimonious and powerful explanation of US corporate savings and investment behavior.

^{*}We thank.

1 Introduction

Corporate savings represent a substantial share in total savings in the largest economies in the world. As Karabarbounis and Neiman (2012) have documented, 80% of total savings in the US in 2005 came from corporate savings. Similarly, corporate savings in 2005 add up to 45% of total savings in China, 55% in Germany, and 80% in Japan. Despite the aggregate significance of corporate savings, there has been relatively little theoretical and empirical research on the determinants of corporate savings. To be sure, although there is a large and rapidly growing literature on corporate investment by financially constrained firms (see the recent survey by Strebulaev and Whited, 2012), for the most part this literature does not concentrate on the determinants of corporate savings.

Generally, financial constraints are taken as given when exploring the effects of these constraints on corporate investment. But in practice firms can dynamically relax their financial constraints by saving cash and holding liquid securities. Thus, through their savings decisions firms effectively face endogenous financial constraints and the strength of the constraint they face at any moment can be measured by the firm's marginal value of cash. In other words, the dynamics of corporate investment are driven by marginal q and the marginal value of cash, both of which are affected by the corporation's savings policy.¹

But how do firms manage their cash holdings, how do they decide how much cash to accumulate, and by how much do they choose to scale back their capital expenditures in order to preserve cash? These are the main questions addressed in Bolton, Chen, and Wang (2011), who develop a simple dynamic model of a firm facing external financing costs, and determine the firm's optimal dynamic cash balances policy in terms of a tradeoff between the gains from investment and the opportunity cost of spending cash. Bolton, Chen and Wang (2011), henceforth BCW, incorporate external financing costs for the firm into the neoclassical q theory of investment à la Hayashi (1982). They also make a number of simplifying assumptions, which allow them to provide a parsimonious characterization of the

¹Here, we assume that capital adjustment costs are convex. This makes it straightforward to see the relation between our model and the neoclassical q theory of investment (Hayashi, 1982).

firm's dynamic corporate financial policy as a function of a liquidity measure w: the ratio of the firm's cash stock W and its capital stock K, i.e. w = W/K.

The BCW model is homogeneous of degree one in the firm's capital stock K, which implies that firm value P(K,W) can be expressed as the scaled value per unit of capital p(w) multiplied by the firm's capital stock K: P(W,K) = p(w)K. Under this formulation the firm's marginal value of cash takes a particularly simple expression: $P_W(K,W) = p'(w)$. It is given by the derivative of the scaled value of the firm with respect to the cash-to-capital ratio. This analytical simplicity of the BCW model brings out powerful intuitions on the above questions and provides a rich set of empirical implications. As always, however, the concern with simplifying assumptions is that the model may turn out to be too stylized to be empirically relevant.

Therefore, in this paper we seek to find out how the predictions of the BCW model are borne out in the data. In the process we also seek to uncover new broad empirical regularities about how firm value and corporate policies vary with the firm's cash-to-capital ratio w. Our sample covers publicly traded US corporations in industries where firms typically have significant tangible assets over the period from 1990 to 2009. Taking a simple non-parametric approach, we explore how firm value and the marginal value of cash vary with the firm's cash-to-capital ratio w, and how in turn corporate policies, such as investment, asset sales, payout, and seasoned equity offerings are affected by the firm's marginal value of cash. We further explore how the main comparative statics predictions from the BCW model hold up by verifying how the firm's scaled value and corporate policies change with the firms' investment opportunities or idiosyncratic risk.

The first fundamental prediction of the BCW model involves the relation between a firm's value and its cash holdings. In the standard neoclassical model where capital markets are perfect and the Modigliani-Miller (MM) irrelevance theorem holds, the marginal value of cash is one, and the corporate financial policy is indeterminate. Firm value P(K, W) increases one-for-one with cash holdings and the scaled value function p(w) has a slope of one in the firm's cash-capital ratio w. The BCW model makes a fundamentally different prediction.

Due to external financing costs the marginal value of cash p'(w) is typically greater than one, as corporate cash holdings allow the firm to buffer negative earnings shocks without having to turn to costly external funding. For firms with low cash balances, the buffer-stock value of cash is greater and hence the marginal value of cash is higher. Perhaps the most important result of our empirical analysis is that this prediction is accurately borne out in the data, as is shown in Figure 1. This finding is also robust in both the time-series and the cross-section.²

A second basic prediction of the BCW model involves the relation between investment and cash holdings. In the standard neoclassical model, investment is entirely determined by the firm's marginal product of capital (or equivalently marginal q), and is entirely independent of the amount of cash that a firm has in hand.³ As a result, the neoclassical model predicts that the investment-capital ratio i = I/K does not change with the firm's cash holdings; in other words, the function i(w) has a slope of zero. In contrast, in the BCW model investment depends on the ratio of the firm's marginal q and the marginal value of cash, p'(w). Thus the neoclassical model is a special case of the BCW model, in which the marginal value of cash is always equal to 1, so that investment only depends on q. In general, since the marginal value of cash is higher for a firm with lower cash holdings, the BCW model predicts that i(w) will have a positive slope. As we show, this is also the case in the data: i(w) has a positive slope in our sample.

Third, we examine several important comparative static predictions from the BCW model. For example, the positive effect on corporate savings of an increase in idiosyncratic risk has the predicted sign. BCW also show that higher idiosyncratic volatility decreases firm value, but that there is a subtle non-monotonic relation between idiosyncratic volatility and the marginal value of cash. For most values of w, higher idiosyncratic volatility increases the marginal value of cash. The intuition is as follows: Higher idiosyncratic volatility in-

²We verify this by calculating p(w) for the first and second halves of our sample and for large and small firms, respectively.

 $^{^{3}}$ Note however that in the presence of fixed adjustment costs for the capital stock, inaction (or no change in the capital stock) is often optimal and marginal q alone is no longer sufficient to characterize corporate investment behavior (see Caballero and Leahy, 1996). Even for this optimal inaction case in the neoclassical framework, the marginal value of cash still equals one.

creases the probability that an unexpected shock will leave the firm with so little cash that it must either liquidate or raise costly external finance. This increases the firm's precautionary savings demand and hence causing the firm's marginal value of cash to increase with its idiosyncratic volatility. However, when w is close to 0, another effect comes into play: Low idiosyncratic volatility increases the probability that the firm will survive (without the need to issue costly external finance), increasing the marginal value of cash. For w sufficiently close to 0, the second effect dominates in the BCW model, so the low idiosyncratic volatility firm has a higher marginal value of cash. Remarkably, these subtle predictions are borne out in the data, as we find that for higher values of w the marginal value of cash tends to be higher when idiosyncratic volatility is high, while when w is close to 0 it is higher when idiosyncratic volatility is low.

We also find confirmation of other subtle predictions from the BCW model. For example, we find that firms with better investment opportunities or higher idiosyncratic volatility tend to delay pay-outs to shareholders until they have accumulated larger cash stocks; and that firms facing higher external financing costs tend to raise larger amounts of cash when they undertake a seasoned equity offering. Finally, another prediction of the BCW model is that the marginal value of cash is always greater than one and approaches one as w nears the optimal payout boundary \overline{w} . We find that this is broadly the case in our sample.

All in all, our findings suggest that BCW provide a more reasonable theory of corporate investment than the dominant neoclassical q-theory of investment. What is more, their model offers sharp predictions about corporate savings behavior and the marginal value of cash for corporations that are largely borne out in the data.

Our non-parametric methodology allows us to examine comparative static questions where the BCW model suggests that there are opposing forces and the comparative static result depends on parameter values. For example, BCW show that when a firm faces better investment opportunities the firm's marginal value of cash is higher, which tends to make the firm want to hold more cash. On the other hand, the firm's capital expenditure is also higher when it faces better investment opportunities. It is, thus, not immediately clear

what the overall effect of a change in the firm's investment opportunities is on corporate savings. BCW show that for plausible parameter values the net effect on corporate savings of improved investment opportunities is positive, and we find that this is also the case in our sample. Interestingly, we also find that when investment opportunities are better, firms' value of cash is higher along with their investment expenditures. Still, on net their savings are higher, suggesting that the mechanism described in the BCW model drives the direction of the change in corporate savings.

A criticism sometimes voiced against dynamic structural modeling and estimation approaches is that the models arbitrarily leave out too many relevant variables, which makes it difficult to identify the models precisely (see Welch, 2011). Our analysis demonstrates that highly simplified, parsimonious, structural models such as BCW can provide powerful insights into the data and specifically into corporate financial behavior, even if they leave out many plausible relevant forces. Our analysis also highlights that our simple non-parametric methodology can be a fruitful complementary approach to existing structural estimation methods, especially for assessing structural models that do not admit closed-form solutions. What is more, our non-parametric methodology, which does not rely directly on the estimation of structural first-order conditions can overcome some of the concerns raised in Welch (2011) with respect to identification of the full structural model.

While the implications of our findings for corporate finance are clear, the relevance of our results on corporate savings, and how they affect corporate investment policies, extends more widely to the economy as a whole. Before the financial crisis of 2007-08, macroeconomists focused mostly on how interest rates affect aggregate investment and household savings behavior, neglecting other aspects of the link between the financial state of corporations, the financial sector, and the real economy. However, the financial crisis has brought home the aggregate importance of financial frictions and how much aggregate investment and savings can vary with these frictions. Riddick and Whited (2009), henceforth RW, also develop a dynamic model of corporate savings.⁴ Although the empirical applications are somewhat

⁴We provide a more detailed discussion of RW in the section on related literature.

easier to see in BCW, the key elements of the RW model are similar, so our results provide strong support for the RW model. The RW and BCW models provide the first elements for the development of dynamic structural models of the economy with financial constraints that incorporate a role for corporate savings. As our results in this paper reveal, corporate savings are a quantitatively important aspect of economies in which corporations face external financing costs. The absence of corporate savings from current dynamic macroecomic models—whether in the neo-classical q-theory based models, or in the financial-constraint based models in the tradition of Bernanke and Gertler (1989), Kyiotaki and Moore (1997, 2008) and Bernanke, Gertler and Gilchrist (1999)—is an important omission, which future dynamic macroeconomic models will need to remedy. The RW and BCW models point one empirically plausible direction in which current macroeconomic models could be amended to include a role for corporate savings.

2 Theory

We first review the main idea and key results of BCW, and then sketch out the empirically testable predictions.

2.1 Main idea

BCW propose an integrated dynamic framework to determine a firm's optimal investment, asset sales, cash, external finance, and payout policies. At the center of the BCW model is a firm's liquidity management policy and costly external financing decisions embedded into an otherwise neoclassical q theory of investment a la Hayashi (1982). Cash, more broadly financial slack, is valuable because it relaxes financial constraints and reduces the need for external financing.

To have a tractable dynamic model of cash management and investment, BCW make two simplifying assumptions: (1) productivity shocks are independently and identically distributed (i.i.d.); (2) investment involves only convex adjustment costs. The first assumption shuts down stochastic investment opportunities and hence allows us to focus on the role of financing frictions on investment. The second assumption induces the firm to smooth investment over time but the level and the slope of investment respond to the degree of financial constraints.

Let K and W denote capital stock and cash holding, respectively. Let C(I, K) denote the total cost of investing I including investment good purchase and capital adjustment costs (that is, C(I, K) = I + G(I, K)). They show that the firm's optimal investment decision is then characterized by the following first-order condition (FOC):

$$C_I(I,K) = \frac{P_K(K,W)}{P_W(K,W)},\tag{1}$$

where $P_K(K, W)$ is the marginal value of an additional unit of capital, and $P_W(K, W)$ is the marginal value of cash. In the macroeconomics and investment literature, the marginal value of capital is also known as marginal q.

In general, due to the presence of external financing costs, the marginal value of cash $P_W(K, W)$ exceeds one and the firm chooses to hoard cash rather than to pay it out. When the marginal value of cash declines to one, the firm optimally pays out some of retained earnings to shareholders. Only when the firm optimally pays out retained earnings to its shareholders, the firm's marginal value of cash equals one.

Homogeneity assumptions. To further simplify the analysis and derive intuitive and operational analytical formulas for investment and firm value, BCW make additional homogeneity assumption which implies that the firm's value function is homogeneous of degree one in W and K. That is, if the firm doubles both items (cash W and capital K) on its asset side of the balance sheet, the firm's value doubles and its investment also doubles. Whenever applicable, the lower case letter refers to the variable in the corresponding upper case scaled by capital. For example, W is the cash-capital ratio, W and W is the investment-capital ratio, W is the cash-capital ratio, W is the investment-capital ratio, W is the cash-capital ratio, W is the

Specifically, BCW's homogeneity property⁵ refers to the following set of assumptions:

- 1. the firm's revenue from operations is given by KdA_t , where dA_t is the iid productivity shock with mean μ and volatility σ per unit of time;
- 2. the firm's capital adjustment cost G(I, K) is homogeneous of degree one in I and K, G(I, K) = g(i)K where i = I/K;
- 3. the firm's fixed cost of external equity financing is proportional to its capital stock, $\Phi = \phi K$ and additionally, the firm incurs a constant marginal cost of issuance γ ;
- 4. the firm's cash carry cost is λ per unit of cash balance per unit of time;
- 5. the firm's liquidation value (excluding cash) is also proportional to its capital stock, $L = \ell K$.

2.2 Summary of the BCW solution

The solution of the BCW model is such that depending on the size of the firm's cash-to-capital ratio w, it can be in one of three possible regions: (1) the internal financing region; (2) the payout region; and (3) the external financing/liquidation region. In the internal financing region, the firm is seeking to relax its external financing constraints by amassing a precautionary cash buffer; its marginal value of cash is greater than one in this region implying that the firm underinvests relative to the neoclassical benchmark with no financial constraints. In the payout region, the firm's cash buffer w is greater than its optimally chosen payout boundary \overline{w} ; the firm has excess cash and no precautionary savings motive in this reguion. It then pays out all excess cash $(w - \overline{w})$ to shareholders. In the third region, when the firm runs too low on cash, w hits the lower boundary when it has to either raise costly external funds or liquidate its capital stock.

⁵See Hayashi (1982) for the homogeneity property and implications on the equality between marginal and average q in the Modigliani-Miller (MM) framework.

Firm value, enterprise value, and average q. As we have already mentioned above, BCW show that the value function of the firm P(K, W) can be expressed as the firm's market value of one unit of capital–p(w)–multiplied by the firm's capital stock K:

$$P(K, W) = p(w) \cdot K, \tag{2}$$

where w = W/K is the firm's cash-to-capital ratio. Therefore, the extent to which the firm is financially constrained is measured by the *liquidity ratio* w.

The firm's enterprise value is generally defined as firm value P(K, W) in excess of its cash balances W. It represents the present value of the firm's cash flow from capital. Intuitively, by valuing cash at its face value, the firm's enterprise value is meant to measure the value added from its productive technology. In the BCW model, however, cash is worth more than its face value, as it allows the firm to invest more and hence enhances enterprise value.

By dividing the firm's enterprise value by the book value of capital K one obtains the following expression for the firm's average q, denoted by $q_a(w)$,

$$q_a(w) = \frac{P(K, W) - W}{K} = p(w) - w.$$
 (3)

While average q is an informative measure for the value of capital, under the neoclassical q-theory of investment with convex adjustment costs it is the marginal q that determines investment. Unlike in this neoclassical theory, BCW show that for a financially constrained firm, investment depends not only on marginal q, but also on the marginal value of cash. We next describe how the firm's marginal value of capital and marginal value of cash are derived.

Marginal q and marginal value of cash. In BCW, marginal q is given by

$$q_m(w) = \frac{d(P(K, W) - W)}{dK} = p(w) - wp'(w).$$
(4)

They show that for a financially constrained firm neither marginal q nor average q determine investment, even in a model with constant returns to scale as in Hayashi (1982). Although marginal q does not measure the firm's investment opportunities in BCW, it is still useful for us to understand the firm's behavior and to design our empirical strategy.

In the absence of any external financing constraints, as in Hayashi (1982), the marginal value of cash is one, and that average q and marginal q are equal due to the homogeneity of the production function. However, for a financially constrained firm, cash is generally worth more than one and its value in the BCW model is given by $P_W(K, W) = p'(w)$.

Wedge between marginal q and average q. In BCW, this wedge is given by

$$q_m(w) - q_a(w) = -(p'(w) - 1) w. (5)$$

This wedge disappears when the firm runs out of cash (w = 0) or when w reaches the optimal payout boundary \overline{w} , where $p'(\overline{w}) = 1$. In the internal financing region $(w \ge 0)$, the marginal value of cash p'(w) > 1; it follows from (5) that marginal q is always lower than average q in this region.

Investment, marginal q, and marginal value of cash p'(w). Using the homogeneity property, BCW simplify the firm's FOC for investment (1) as follows:

$$1 + g'(i) = \frac{q_m(w)}{p'(w)}, \tag{6}$$

where g'(i) is the marginal capital adjustment cost. With the additional auxiliary assumption that $g(i) = \theta i^2/2$, the optimal investment-capital ratio solves

$$i(w) = \frac{1}{\theta} \left(\frac{q_m(w)}{p'(w)} - 1 \right). \tag{7}$$

As in the neoclassical q theory of investment, corporate investment depends on marginal productivity of capital, often referred to as the marginal q. However, unlike the neoclassical

q theory, investment in BCW also depends on the marginal value of cash, $P_W(K, W) = p'(w)$. Moreover, marginal q also depends on financial slack w. Therefore, the neoclassical q theory of investment not only misses the importance of the marginal value of cash in determining investment, but also the relation between marginal q and the marginal value of cash that flow from the fact that both depend on the firm's cash balances (i.e., on w).

3 Empirical Analysis

The BCW model does not admit a simple closed-form solution and the non-linear solution for p(w) is computed numerically. As is illustrated in Figure 3B in BCW, the marginal value of cash p'(w) has a highly non-linear shape, with a steep downward slope near w=0, which quickly flattens as w moves beyond the immediate vicinity of w=0, and then very gradually converges to one as w approaches the payout boundary. Standard parametric approaches (using a quadratic or low-order Taylor series approximation) may do a poor job of fitting the slope and curvature of p'(w). This is an important reason why we use a non-parametric approach that allows the data to reveal whether the function has the shape predicted by the BCW model, rather than trying to force the data into a potentially ill-fitting functional form. As will become clear, we take the simplest possible non-parametric approach to test the predictions of the BCW model, by looking at how the marginal value of cash p'(w) and corporate policies vary with the firms' cash-to-capital ratio w in the sample. This approach has the advantage of not imposing any functional form restrictions on the data generating process. In addition, we verify the robustness of our results by looking at different subperiods of our sample and sorting firms by size.

3.1 Data

The sample consists of U.S. firms for the period 1990-2009, with data drawn from COMPU-STAT, CRSP, and various sources of industry and aggregate data. Firm-year observations with missing data are deleted, as are those for which total assets, the gross capital stock,

cash, or sales are either zero or negative. Regulated, financial, and public service firms (primary SIC codes 4900-4999, 6000-6999, and greater than 9000) are omitted.⁶ For similar reasons, we exclude eight industries where the ratio of market capitalization to PPEGT (property, plant, and equipment) is greater than 4, since these are industries where PPEGT is likely to provide a poor measure of the firm's total capital stock.⁷ Because of concerns about data quality, we exclude observations with assets of less than \$10 million. In order to reduce noise in the data due to mergers, acquisitions, or other corporate events that lead to significant accounting changes, we trim the 3% tails of the following variables: $Sales_t/K_{t-1}$, $Cost_t/K_{t-1}$, I_t/K_{t-1} , and the firm-specific variance of returns, where Sales is Net Sales (Compustat variable SALE), Cost is the sum of the Cost of Goods Sold (Compustat variable COGS) and Selling, General, and Administrative Expense (Compustat variable XSGA; when this item is not reported, it is set to zero), and the firm-specific variance of returns is calculated using CRSP monthly returns up to and including year t-1.

- 1. Cash W is defined as the sum of cash (Compustat variable CH) and short-term investments (Compustat variable IVST), both measured at the end of the year.
- 2. Capital stock K is defined as the property, plant, and equipment of the firm (Compustat variable PPEGT) at the beginning of the year.
- 3. **Investment** I is defined as capital expenditures (Compustat variable CAPX).
- 4. Value P = ME/AF, where ME is defined as the number of common shares outstanding (Compustat variable CSHO) multiplied by the share price at the end of the year

⁶Hennessy, Levy, and Whited (2007) use similar criteria.

⁷The results are qualitatively similar if we include these industries, except that their inclusion leads to many observations with very high values of w (because the denominator of w is understated in the firm's accounts) and corresponding distortions in p(w) and p'(w) at the right tail of the distribution of w. The eight industries are computer and electronic products, publishing industries (includes software), information and data processing services, legal services, computer systems design and related services, educational services, transit and ground passenger transportation, and miscellaneous manufacturing.

(Compustat variable PRCC_F) and

$$AF = \sum_{i=1}^{\#A} a_i$$

is an observation-specific adjustment factor, where a_i is the amount of asset (divided by K) and #A is the number of assets. The adjustment factor AF accounts for the fact that, in the data, market value reflects assets other than the two assets in the model (K and W). The summation in the correction factor essentially includes all assets other than W.⁸ Specifically, the non-cash assets are receivables, inventories, current assets –other, PPE, intangibles, investments and advances, and assets– other.

As for the two main parameters of the BCW model, the mean productivity shock μ and the idiosyncratic volatility σ , they are measured as follows:

- Drift rate of the revenue shock μ is measured by the firms real sales growth over the two prior years. Real sales is defined as sales (Compustat variable SALE) divided by the price of output (as provided by the BEA Chain-Type Price Indexes for Value Added by Industry).
- Idiosyncratic volatility σ . Our measure of idiosyncratic volatility is based on CAPM, which is estimated over 1962 to 2009 using monthly excess returns by firm from CRSP and excess market returns (EMR) as provided on Ken French's website. Volatility σ is defined as the standard deviation of the CAPM residuals for the firm, calculated over all available monthly observations from 1962 through the prior year.

3.2 Methodology

We begin by briefly describing our empirical strategy before stating and testing the main predictions of the BCW model. The starting point of our analysis, which is consistent with

⁸The only asset that is not included in the summation is the depreciation account. Since we follow the literature in defining K as PPE, the depreciation account should not be counted as an asset.

the model, is to treat the data as if it were generated by a representative firm subject to i.i.d. productivity shocks.

Accordingly, we calculate the scaled value function of the firm for each firm-year observation and sort observations into deciles by the cash-to-capital ratio w. (Recall that w is the key state variable in BCW: It measures the extent to which firms are dynamically financially constrained and determines their cash balances, investment, equity issuance, and dividend payout.) For each w decile, we calculate the median value of scaled firm value p(w). When we plot the resulting values of p(w) (the medians for each w decile) against w, we trace out an empirical proxy for p(w). Based on this proxy, we use a discrete approximation to calculate the derivative p'(w). We use a similar approach to calculate i(w) and i'(w). We then assess whether the model's predictions about p(w), p'(w), i(w), i'(w), and other variables are borne out in the data.

Note that we are not using common nonparametric estimators, such as the Nadaraya-Watson estimator for the following reason: Let $\widetilde{m}(x)$ refer to the general class of nonparametric estimators, let y be the variable of interest, and let m be some function of x. The Nadaraya-Watson estimator sets $\widetilde{m}(x) = \widehat{m}(x)$, where $\widehat{m}(x)$ is the sample mean of the corresponding y_i . However, "the mean is only an efficient estimator of m if the errors u_i are normally distributed." This is not the case in the BCW model. Moreover, the mean can be strongly influenced by outliers, and the BCW model implies that there will be outliers (because the distribution of p(w) in the model is both strongly skewed and kurtotic). Under these circumstances, setting $\widetilde{m}(x)$ equal to the sample median of the corresponding y_i has some appealing properties, as discussed in more detail in Pagan and Ullah (1999, Section 3.7). Our procedure is motivated by these considerations.

While we treat the data as if it were generated by a representative firm, it is possible to study the way different firms would behave by analyzing the effect of changes in the model's parameters (e.g., μ , which captures investment opportunities, or σ , which reflects idiosyncratic risk). The tractability of the BCW model allows in many cases to obtain analytical

⁹See Pagan and Ullah (1999, page 122).

results on the effect of a parameter change. It is thus possible to allow for differences in parameters across firms. Where the model makes a clear prediction, this allows us to see whether the data corresponds to the model. Where the model delineates opposing effects without predicting which effect will dominate, this allows us to determine which effect is dominant in the data.

To construct the empirical counterpart of the scaled firms value p(w), we calculate the median value-to-capital ratio for each w decile (we replicate this procedure for the investment-to-capital ratio i(w)). We then plot these empirical counterparts of the main endogenous variables of the model and address two questions: first, do the plots reveal the presence of dynamic financial constraints driven by external financing costs?; and second, if they do, how do these constraints affect corporate behavior?

If the plot for p(w) has approximately a constant slope close to one, so that firm value is basically independent of the firm's cash holdings W, then $prima\ faciae$ firms would appear to face negligible external financing costs. If on the other hand, the plot of p(w) is strictly concave with a slope strictly higher than one for w close to zero, and a gradually decreasing slope as firms move up to higher w deciles, then firms would appear to face high external financing costs as the BCW model posits.

3.3 Corporate Savings and the Value of Cash

A central prediction of BCW is that a risk-neutral financially constrained firm behaves in a risk-averse manner. Therefore, cash has an additional role as the buffer stock and firm value P(K, W) should be concave in cash W, which implies that the marginal value of cash p'(w) should be decreasing in w and strictly greater than one, except at the payout boundary \overline{w} where $p'(\overline{w}) = 1$. Intuitively, cash is most valuable when the firm is about to run out of cash and needs to tap costly external funding sources, and least valuable when the firm has accumulated enough cash and no longer has a precautionary savings motive.

Figure 1 illustrates that the empirical counterpart of the scaled firm value p(w) has the shape predicted by BCW: it is increasing and concave in w far from having a constant slope

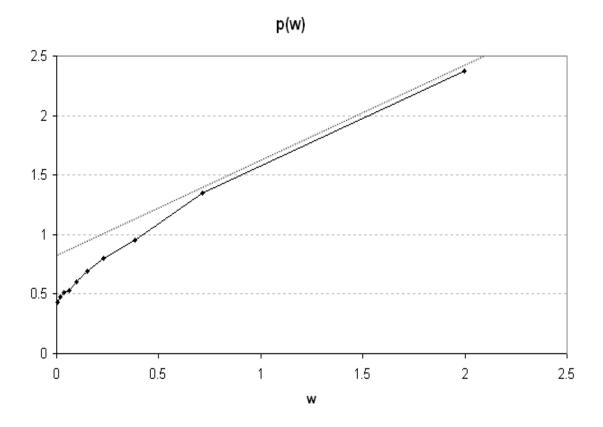


Figure 1: Firm-year observations are divided into deciles by w (cash/capital ratio). The vertical axis shows the median p(w) (value/capital ratio) for each decile. The straight line is a visual aid to make it straightforward to see how much the estimated p(w) function deviates from having a constant slope, as predicted by the neoclassical model without external financing frictions.

of one, as should be the case in the neoclassical model with no external financing costs.

Moreover, as is illustrated in Figure 2, which graphs the slope of p(w) based on the results in Figure 1, the marginal value of cash p'(w) is greater than one and decreases with w. We obtain the results displayed in Figure 2 using the discrete approximation

$$p'(w_i) = \frac{p(w_i) - p(w_{i-1})}{w_i - w_{i-1}},$$

where $i = 1, \dots, 10$ denotes the w-decile number. When a firm is close to running out of cash (w is close to 0), the market value of the firm increases by more than one dollar for

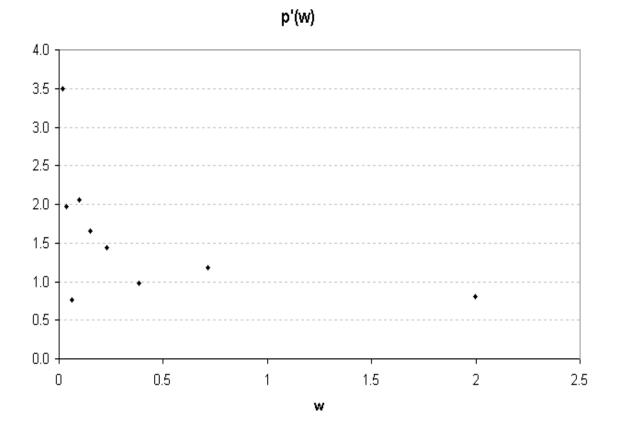


Figure 2: Firm-year observations are divided into deciles by w. The vertical axis plots p'(w), the slope defined as the change in the median of p(w) for the change in the median w from one decile to the next.

each additional dollar of cash. As a firm's cash stock increases, the marginal value of cash p'(w) declines.

To see how heterogeneity in the time series or in the cross section affects the shape of p(w), we perform a number of robustness exercises. Figure 3 plots p(w) separately for small and large firms and reveals that the two plots are similar.¹⁰

Figure 4 which plots p(w) in an early subperiod of our sample (up to and including the year 2000) and in a later subperiod, shows that the function p(w) has a similar shape in the

¹⁰The empirical literature suggests that small firms may have a higher fixed cost of external financing ϕ . The BCW model predicts that higher ϕ will lead to a higher marginal value of cash, especially for lower values of w. While Figure 4 suggests that the shape of the p(w) function is fairly similar for small and large firms, the figure is consistent with a higher ϕ for small firms, since the slope of p(w) is greater for small firms for lower values of w.

p(w): Small and Large Firms

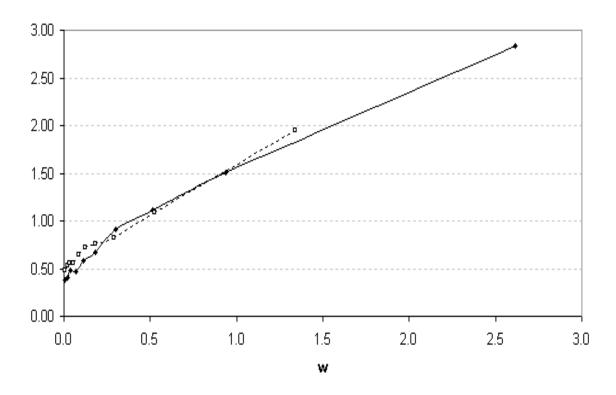


Figure 3: Firm-year observations are divided into deciles by w (cash/capital ratio). The vertical axis shows the median p(w) for each decile. Large firms are firms with capital stock above the median, and small firms are firms with capital stock below the median. The black and white dots correspond to small and large firms, respectively.

first and second halves of our sample.¹¹

One major episode that has surely affected firms' investment opportunities, external financing costs, and idiosyncratic volatility is the financial crisis of 2007-2008. Most of the changes – the deterioration in investment opportunities, the increase in idiosyncratic volatility, and the increase in external financing costs – worked to push down firm value. Figure 5 shows p(w) before the crisis and during the crisis years of 2008 and 2009.

¹¹The BCW model predicts that the payout boundary \overline{w} will increase with higher idiosyncratic volatility. The right-most point can be considered to be a proxy for \overline{w} and shows an increase in the payout boundary that is consistent with the existing empirical literature, which points to an increase in idiosyncratic volatility over time (see e.g. Bates et al., 2009). Figure 5 does not, however, suggest that there was a large enough change in idiosyncratic volatility between the earlier and later periods to cause a major change in the shape of the p(w) function.

p(w): Earlier and Later Time Periods

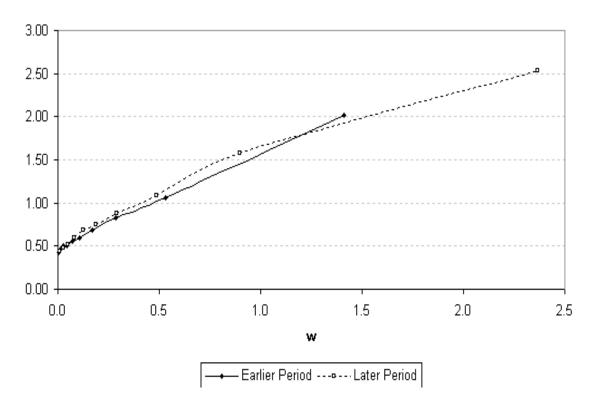


Figure 4: Firm-year observations are divided into deciles by w (cash/capital ratio). The vertical axis shows the median p(w) (value/capital ratio) for each decile. The earlier time period is all years up to and including 2000 (the middle year of the sample), while the later time period is all years after 2000.

The shape of the function p(w) during the crisis years is similar to that before the financial crisis. However, our estimate of p(w) is clearly noisier during the crisis years. One reason may be the smaller sample size. Another potential explanation is that stock prices may have provided a noisier measure of firm value during the crisis.

3.3.1 The Effect of Investment Opportunities

Another important form of heterogeneity is differences in investment opportunities. How do corporate savings and firm value change as firms' investment opportunities change? In the neoclassical model with no external financing costs the answer is straightforward: an

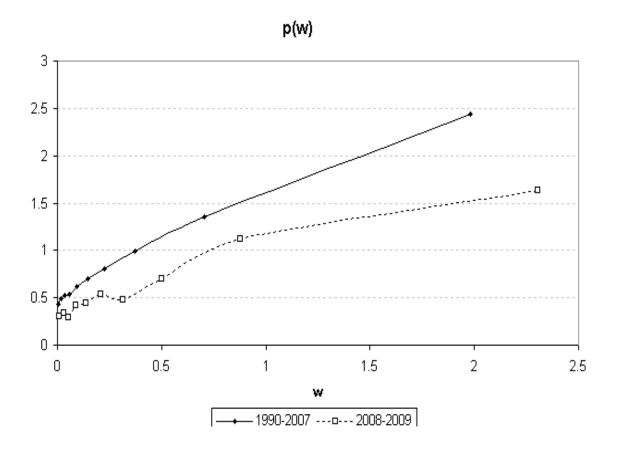


Figure 5: Firm value before and after financial crisis.

increase in μ implies an upward shift in the value function p(w), which always has a slope of one, and has no predictable effect on corporate savings. In contrast, the BCW model predicts that firm value P(K, W) and the value-to-capital ratio p(w) is (uniformly) higher when the firm has better investment opportunities μ . Thus, if we sort firms into different μ -quintiles we should see $p(w \mid \mu_i)$ lie above $p(w \mid \mu_j)$ if $\mu_i > \mu_j$ for all w deciles.

Predictions on how the firm's cash holdings vary with μ , while less clearcut a priori, can be obtained based on simulations of the model for plausible parameter values. In the neoclassical model, when the firm is more profitable (μ is higher), the firm will spend more on investment. Although the neoclassical model makes no predictions about the effect on cash holdings, it might be natural to imagine that more spending on investment would reduce w. However, there is a second effect in the BCW model. By increasing the return on

investment, higher w increases the marginal value of cash, which induces the firm to hold more cash. The BCW model therefore leaves open the net effect of a change in μ on cash holdings; it depends on the parameter values. However, for the baseline parameter values in BCW, the simulations establish that the net effect of an increase in μ on firm cash holding is indeed positive. That is, the cumulative distribution of w for firms with a higher μ first-order stochastically dominates the cumulative distribution of w for firms with a lower μ .

To examine how changes in investment opportunities μ_i affect $p(w | \mu_i)$, $p'(w | \mu_i)$, and corporate savings w empirically, we first divide firm-year observations into quintiles by μ_i . In a second step, we divide the observations in each quintile into deciles by w. We then compute the median $p(w | \mu_i)$ for each combination of w and μ_i . The results are shown in Figure 6, where high μ represents the top quintile and low μ the bottom quintile.

In the neoclassical model, the marginal value of cash is one, so $p(w | \mu_i)$ always has a slope of 1. Better investment opportunities therefore imply an exactly parallel upward shift in $p(w | \mu_i)$. In the BCW model, the marginal value of cash is greater than 1 and is especially high when the firm doesn't have much cash. As a result, $p(w | \mu_i)$ is concave in w and better investment opportunities imply a non-parallel upward shift in $p(w | \mu_i)$. Furthermore, the BCW model predicts that better investment opportunities increase the marginal value of cash $p'(w | \mu_i)$. In the data, $p(w | \mu_i)$ is concave in w for both high and low μ , the upward shift in $p(w | \mu_i)$ is non-parallel, and the marginal value of cash tends to be higher for higher μ , as illustrated in Figure 7. For w < 0.5, the mean marginal value of cash is 1.90 for high μ and 1.05 for low μ (where these means are calculated over the relevant points in Figure 7).

Finally, we also compute the empirical cumulative distribution function (CDF) of w for each quintile. Figure 8 plots the empirical CDF for the highest and lowest quintiles. As can be seen, the effect of higher μ_i on the marginal value of cash is stronger than its effect on spending cash on investment, so that the empirical CDF for high μ_i first-order stochastically dominates that for low μ_i as predicted by the BCW model simulations.

In the BCW model, firms pay out dividends when they have a sufficiently high level of cash. More precisely, there is an endogenous upper barrier – the payout boundary \overline{w} . When

p(w): High and Low Mu

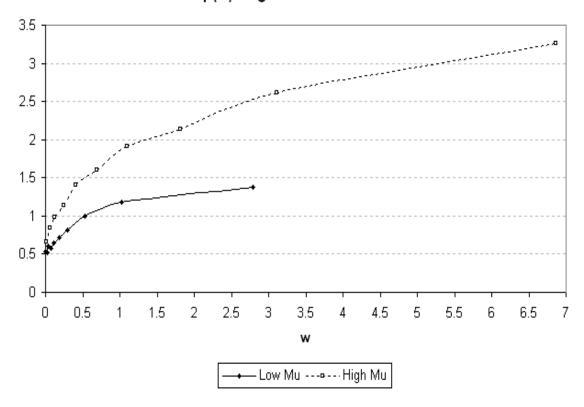


Figure 6: Firm-year observations are divided into quintiles by μ . Each quintile is then divided into deciles by w. The vertical axis shows the median p(w) for each combination of quintile μ and w decile. High μ represents the top quintile and low μ the bottom quintile.

shocks cause w to hit \overline{w} , the firm pays out a discrete amount of dividends to shareholders. The payout boundary is a function of several parameters in the model, including investment opportunities μ and idiosyncratic volatility σ .

Since higher μ implies a higher marginal value of cash, it makes the firm more reluctant to pay out dividends, so the model predicts a higher payout boundary \overline{w} . Although \overline{w} is not directly observable, we calculate a proxy as follows. We divide the firm-year observations into μ deciles and divide each μ decile into quintiles of w. Our proxy for \overline{w} for each μ decile is then the median w in the highest w quintile in that μ decile. When we regress this proxy for \overline{w} on the median μ for the corresponding decile, the coefficient on μ is positive and significant. (The t-statistic is greater than 4.) The R^2 of the regression is 0.68.

p'(w): High and Low Mu

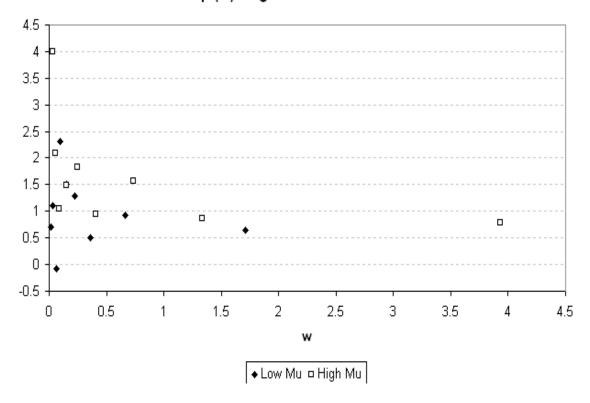


Figure 7: Firm-year observations are divided into quintiles by μ . Each quintile is then divided into deciles by w. The vertical axis shows p'(w), defined as the change in median p(w) for the change in median w from one decile to the next.

3.3.2 The Effect of Idiosyncratic volatility

Just as firms differ in their access to investment opportunities, they also differ in idiosyncratic volatility σ . According to the neoclassical model with no external financing costs, firm value should be independent of σ and changes in σ also have no predictable effect on corporate savings. In contrast, the BCW model predicts that $p(w \mid \sigma_i)$ is lower for higher idiosyncratic volatility σ_i , as higher idiosyncratic volatility increases the risk that the firm will run low on cash and be forced either to liquidate or to make a costly equity issuance. Moreover, the BCW model predicts that higher idiosyncratic volatility should shift the payout boundary to the right. Note that this is a relatively clean test on the importance of financing constraints. Simulations in BCW also establish that the cumulative distribution of w for firms with a

Cumulative Distribution Function for w, High and Low Mu

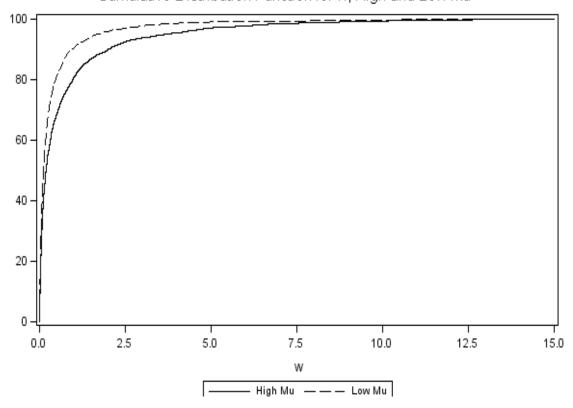


Figure 8: High μ represents the top quintile of firm-year observations by μ and low μ the bottom quintile.

higher σ_i first-order stochastically dominate the cumulative distribution of w for firms with a lower σ_i . In other words, firms with higher volatility σ_i also tend to hoard more cash.

To examine how changes in σ_i affect $p(w \mid \sigma_i)$, $p'(w \mid \sigma_i)$, and corporate savings w empirically, we again divide firm-year observations into quintiles by idiosyncratic volatility, and within each quintile divide the observations into deciles by w. We then again calculate the median $p(w \mid \sigma_i)$ for each combination of w and idiosyncratic volatility σ_i . The results are shown in Figure 9. As predicted by the model, $p(w \mid \sigma_i)$ is shifted downward for high idiosyncratic volatility observations, and this is true for all values of w.

The BCW model generates subtle predictions on how idiosyncratic volatility affects the marginal value of cash p'(w) (See Figure 7 in the NBER working paper version of BCW). For most values of w, higher idiosyncratic volatility increases the marginal value of cash.

p(w): High and Low Idiosyncratic Volatility

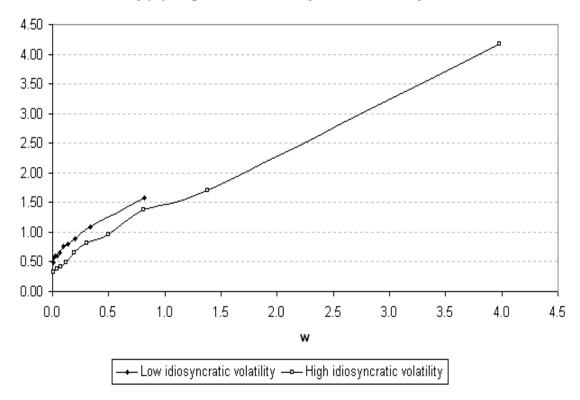


Figure 9: Firm-year observations are divided into quintiles by idiosyncratic volatility. Each idiosyncratic volatility quintile is then divided into deciles by w. The vertical axis shows the median p(w) for each combination of idiosyncratic volatility quintile and w decile. High idiosyncratic volatility represents the top idiosyncratic volatility quintile, low idiosyncratic volatility the bottom quintile.

However, when w is close to 0, low idiosyncratic volatility increases the probability that the firm will survive (without the need to issue costly external finance), increasing the marginal value of cash. For w sufficiently close to 0, the survival effect dominates in the BCW model, so the low idiosyncratic volatility firm has a higher marginal value of cash. Remarkably, the results shown in Figure 10 reveal that these rather subtle predictions are borne out in the data: when the firm has enough cash ($w \ge 0.1$), the marginal value of cash is higher for high idiosyncratic volatility (p'(w) = 1.27) than for low idiosyncratic volatility (p'(w) = 1.17). But, as predicted by the BCW model, when the firm is close to running out of cash ($w \le 0.1$), the marginal value of cash is substantially higher when idiosyncratic

p'(w): High and Low Idiosyncratic Volatility

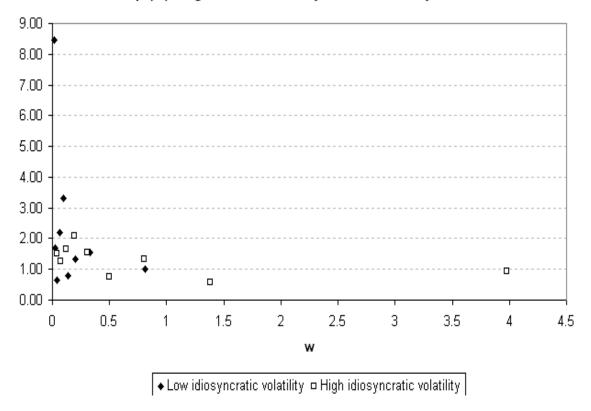


Figure 10: Firm-year observations are divided into quintiles by idiosyncratic volatility. Each idiosyncratic volatility quintile is then divided into deciles by w. The vertical axis shows the slope of p(w), defined as the change in median p(w) for the change in median w from one decile to the next.

volatility is low (p'(w) = 3.26) than when idiosyncratic volatility is high (p'(w) = 1.37). 12

In addition, the BCW predictions that higher idiosyncratic volatility increases corporate savings and shifts the payout boundary to the right are also confirmed in the data. First, as Figure 11 shows, higher idiosyncratic volatility shifts the empirical CDF to the right, consistent with the theory. Second, when we regress our proxy for \overline{w} (constructed in the same way as discussed above) on idiosyncratic volatility, the coefficient on idiosyncratic volatility is positive and highly significant (with a t-statistic of about 14 for the coefficient and an R^2 of 0.96 for the regression). Thus, in the data, there is also a strong relation

¹²These results are obtained by taking the mean over the relevant points in the figure.

CDF for w: Low and High Idiosyncratic Volatility

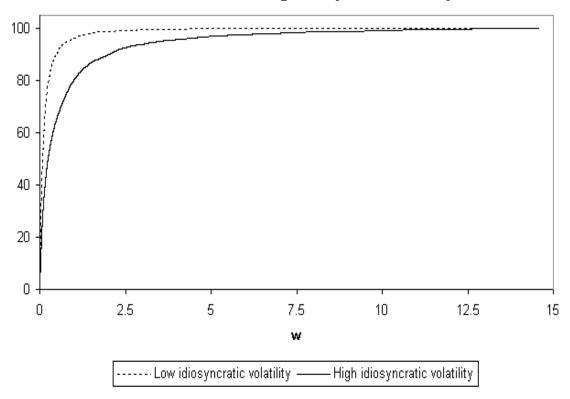


Figure 11: High idiosyncratic volatility represents the top quintile of firm-year observations by idiosyncratic volatility, low idiosyncratic volatility the bottom quintile.

between idiosyncratic volatility and this proxy for payout, which is again consistent with the BCW model predictions.

3.4 Corporate Cash Holdings, Investment and Asset Sales

The evidence reported so far highlights the presence of external financing costs and the importance of dynamic financial constraints for firms. How do these constraints affect corporate behavior and real decisions such as investment and asset sales? As equation (7) illustrates, the BCW model provides a simple and intuitive characterization of corporate investment behavior for financially constrained firms. Relative to the neoclassical benchmark, where investment is driven by marginal q, the key modification for a financially constrained firm is

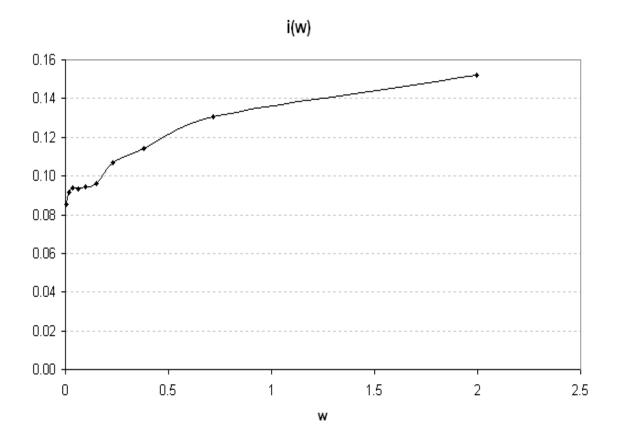


Figure 12: Firm-year observations are divided into deciles by w. The vertical axis shows the median i(w) (investment/capital ratio) for each decile.

that marginal q must be divided by the firm's marginal value of cash p'(w). Given that the marginal value of cash is a decreasing function of w, a key prediction of the BCW model for corporate investment is that i(w) is an increasing function of w. In other words, less financially constrained firms (with a lower marginal value of cash p'(w)) invest more, other things equal. As for p(w), we can calculate an empirical counterpart to i(w) by again dividing firmyear observations into deciles by w and then calculating the median investment-to-capital ratio for each decile. Figure 12 plots this empirical counterpart of i(w), and as can be seen i(w) is an increasing function of w, just as the BCW model predicts.

This is not a very surprising result and the BCW model is not the only model of financially constrained firms that makes this general prediction. Having said this, there is some confusion in the corporate investment literature on this point as several studies have identified financial constraints only indirectly by taking cash flow as a proxy for financial constraints instead of measuring financial constraints directly by the firm's marginal value of cash p'(w). However, as Kaplan and Zingales (1997) have pointed out, cash flow could be a poor measure of a firm's financial constraints, as a lower cash flow may simply reflect lower productivity, which could be the reason why the firm invests less.

Besides the general prediction that investment is increasing in w, are there other finer predictions which are more closely connected to the BCW model? Interestingly, the BCW model makes rather subtle predictions on the cash-sensitivity of investment i'(w). Namely, BCW show that i'(w) depends on the third derivative of p(w), and that i'(w) is generally a non-monotonic function in w. Importantly, the BCW model makes a further, highly intuitive, prediction: as w increases and the firm is no longer financially constrained, the cash-sensitivity of investment tends towards 0. As Figure 13 illustrates, this prediction is also borne out in the data.

Financial constraints not only affect investment but also asset sales. In the BCW model, a financially constrained firm (with low w) may sell assets in order to avoid liquidation or costly refinancing. More precisely, when the firm's marginal value of cash p'(w) is higher than the firm's marginal $q_m(w)$ the financially constrained firm is better off selling assets (even at fire-sale prices) in order to keep up cash reserves and avoid the higher external financing costs, ϕK (see equation 7). To explore this prediction of the model, we look at the asset sales-to-capital ratio as a function of w in Figure 14. Asset sales are particularly high when w is close to 0, and at higher w asset sales fall off sharply.

3.4.1 Changes in Investment Opportunities

In the neoclassical model a higher μ unambiguously leads to higher investment. In the BCW model, simulations under the baseline parameter values show that the cumulative distributions of i(w) for firms with higher μ first-order stochastically dominate the cumulative distributions of i(w) for firms with a lower μ . There are two effects on investment of an increase in μ . First, investment is more profitable and the firm therefore raises investment.

Investment-Cash Sensitivity: i'(w)

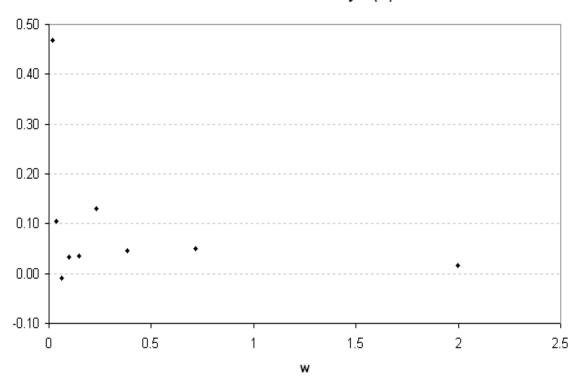


Figure 13: Firm-year observations are divided into deciles by w. The vertical axis shows the slope of i(w), defined as the change in median i(w) for the change in median w from one decile to the next.

Second, however, a higher μ also increases the marginal value of cash p'(w) and therefore induces the firm to preserve cash by reducing its investment outlays. Not surprisingly, the BCW simulations show that on net the first effect dominates the second, so that investment does increase as μ increases, but the effect is stronger for higher w. In Figure 15, we plot the empirical counterpart of i(w) for two groups of firms, those with respectively a high and a low μ .

3.4.2 Changes in Idiosyncratic volatility

Since investment for a financially constrained firm is determined by the ratio of marginal q to the marginal value of cash p'(w), and since the marginal value of cash p'(w) is affected

Asset Sales/K

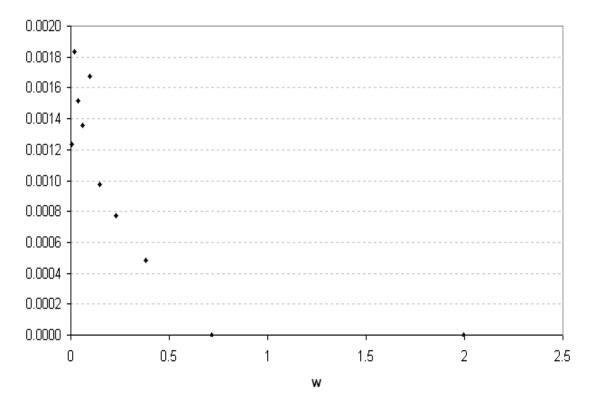


Figure 14: The vertical axis shows the median asset sales/capital ratio for each w decile.

by idiosyncratic risk, changes in idiosyncratic risk also affect investment. Having said this, the BCW model has rather subtle predictions on the relationship between idiosyncratic volatility and investment. For most values of w, higher idiosyncratic volatility increases the risk that the firm will run low on cash and therefore generally increases p'(w) and, holding q_m constant, reduces i(w). However, when w is close to 0, this effect is dominated by the effect of idiosyncratic risk on the probability that the firm will survive without raising costly external finance. In other words, when w is close to 0, high idiosyncratic risk increases the chance that the firm will be liquidated (or needs to raise costly external finance); for w sufficiently close to 0, this means that p'(w) will be higher for a low idiosyncratic volatility firm. (As shown in Figure 10, the data confirms this subtle prediction of the BCW model.) If high and low idiosyncratic volatility firms have the same q(w), the BCW model predicts

i(w): High and Low μ

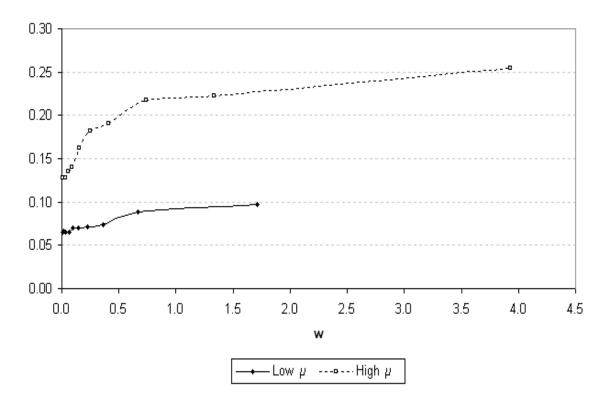


Figure 15: Firm-year observations are divided into quintiles by μ . Each quintile is then divided into deciles by w. The vertical axis shows the median i(w) for each combination of quintile μ and w decile. High μ represents the top quintile, low μ the bottom quintile.

that the low idiosyncratic volatility firm will have lower i(w) for w close to 0, because it has a higher marginal value of cash. (Recall that in the BCW model i(w) depends on q(w)/p'(w).)

Accordingly, we begin by evaluating the effect of idiosyncratic volatility on i(w) by first dividing firm-year observations into quintiles by idiosyncratic volatility σ_i and then dividing the observations into deciles by w within each idiosyncratic volatility quintile. We then calculate median i(w) for each combination of w and idiosyncratic volatility σ_i . The results are summarized in Figure 16. The model predicts that for low w we should see lower investment for low σ_i , while for high w we should see the opposite ranking. Figure 16 confirms one of the subtle predictions of the BCW model: for w close to 0, i(w) is lower for low idiosyncratic volatility so as to increase the likelihood that the firm preserves a higher

i(w): High and Low Idiosyncratic Volatility for w <= 0.2

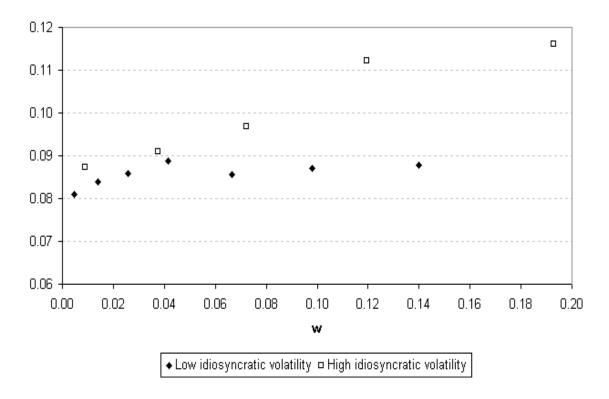
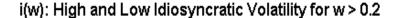


Figure 16: Each cell reports mean i(w) over w decide medians that fall within the indicated range for w. Low idiosyncratic volatility is the lowest idiosyncratic volatility quintile of firm-year observations; high idiosyncratic volatility is the highest quintile; and w decides. Here, we focus on good investment opportunities (specifically, obsrevations in the highest investment opportunity quintile.

going-concern value.

Figure 17 shows that, for higher values of w (specifically, $w \ge 0.2$), investment is higher when volatility is higher. When a firm is not too close to running out of cash, the high idiosyncratic volatility firm will have a higher marginal value of cash and therefore lower i(w), ceteris paribus, in the BCW model. However, both empirically and theoretically, high and low idiosyncratic volatility firms may not have the same q(w). To explore the effect of differences in investment opportunities, we control for variations in μ by first sorting firmyear observations into quintiles by μ and focusing on the top quintile.¹³ We then divide

 $^{^{13}}$ There are two reasons for focusing on the top quintile: 1) Values of q should be fairly similar for firms



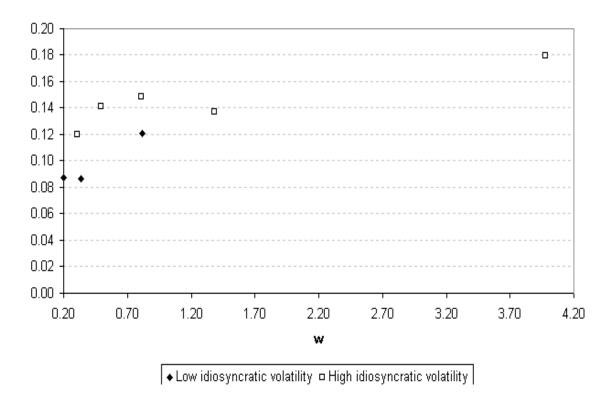


Figure 17:

the firm-year observations in this quintile into idiosyncratic volatility quintiles. Within each of the idiosyncratic volatility quintiles we further divide the observations into deciles of w and then calculate the median i(w) for each combination of idiosyncratic volatility quintile and w decile. The results are reported in Table 1. When $w \leq 0.2$, i(w) is higher for high idiosyncratic volatility (i(w) = 0.148) than low idiosyncratic volatility (i(w) = 0.127). When 0.2 < w < 2.0, i(w) is lower for high idiosyncratic volatility (i(w) = 0.193) than low idiosyncratic volatility (i(w) = 0.217). Thus, when we examine the more precise version of the BCW prediction by controlling for investment opportunities, the data confirm both of the predictions about the relation between idiosyncratic volatility and investment.

in the same investment opportunities quintile; and 2) The BCW prediction should emerge most clearly for firms that have a higher marginal value of cash, and the highest μ quintile should have the highest marginal value of cash according to the BCW model (a prediction that is empirically confirmed in Figure 7).

Table 1 i(w): High and Low Idiosyncratic Volatility Good Investment Opportunities

	Mean $i(w)$
High volatility, $w < 0.2$	0.148
Low volatility, $w < 0.2$	0.127
High volatility, $0.2 \le w < 2.0$	0.193
Low volatility, $0.2 \le w < 2.0$	0.217

3.5 External Financing Costs and Firms' Equity Issuance Policy

The BCW model has a number of predictions on how firm value p(w) and firms' cash, investment, payout and external financing policies vary with external financing costs ϕK . Specifically, when the firm runs out of cash and seeks to raise new funds through a new equity issue, BCW show that the firm optimally issues equity in lumps to economize on the fixed issuance costs ($\phi > 0$). The total funds raised through the equity issue amount to $m \cdot K$, where m > 0 is optimally determined so that the marginal value of cash with m in cash per unit of capital is equal to the marginal cost, $p'(m) = 1 + \gamma$. (Recall that γ is the marginal cost of issuance, as described in Section 2.)

To be able to examine how changes in external financing costs affect firms' external financing policies empirically, we ideally need both a measure of the fixed cost of issuance ϕK and the marginal cost of an additional dollar raised γ . However, direct measures of fixed issuance costs are typically not available. Still, two general predictions of the BCW model are testable. First, when the fixed cost of issuing equity is higher the firm raises more funds m when it raises new equity capital, and when the marginal cost is higher it raises less funds. Second, given that one condition for optimality in BCW is that $p'(m) = 1 + \gamma$, when a firm has greater investment opportunities, as measured by μ , it will raise more funds when it issues new shares.

To test these two predictions, we look at m (equity issuance/ K_{t-1}).¹⁴ We then divide these offerings into terciles by the fixed costs incurred by the issuers, and further divide the observations into high- μ and low- μ bins (above and below the median of μ for the ϕ category, respectively) and compute the median m in each bin.¹⁵ Figure 18 reports the results. As can be seen, the size of issues m is increasing in the size of the fixed cost of issue. Moreover, for each tercile in ϕ the size of the issue is larger for the firms with higher investment opportunities μ . The two predictions of the model are thus borne out by this data.

3.6 Summary

The BCW model brings out the role of p'(w) in a transparent and tractable way by making some key simplifying assumptions, which serve the role of preserving homogeneity of the model in the firm's capital stock K: i) long-run constant returns to scale; ii) i.i.d. productivity shocks; iii) smooth investment adjustment costs, and iv) fixed costs of issuing equity that are proportional to the size of the firm's capital stock. With the possible exception of the first assumption, these assumptions are unlikely to be true in practice, which raises the concern that the BCW model provides a distorted approximation of the true data generating process. What types of biases are likely to be introduced by these simplifications?

If the first assumption is violated then we should expect firm size to matter. This is why we have divided firms into different size buckets. As our results show, the constant returns to scale assumption is unlikely to introduce major distortions. The second assumption is almost certainly not valid, based on what we know from other studies. However, since we do not attempt to estimate the change in value over time, $p_{t+1}(w_{t+1}) - p_t(w_t)$, our estimation procedure is robust to autoregressive productivity shock processes. If firms face

 $^{^{14}}$ To focus on seasoned equity offerings (rather than small equity issues in connection with, e.g., managerial compensation), we follow the literature [see, e.g., Huang and Ritter (2009), Table 1] in focusing on observations where equity issuance, relative to the firm's assets, is above some threshold. In our case, the threshold is $NSI_t/Assets_{t-1} > 0.02$, where NSI is Sale of Common And Preferred Stock (Compustat variable SSTK).

¹⁵In keeping with the empirical literature, which suggests that smaller firms face higher fixed costs of external finance, we use firm size (specifically, $Assets_{t-1}$) as the measure of ϕ .

m -- Robustness Check

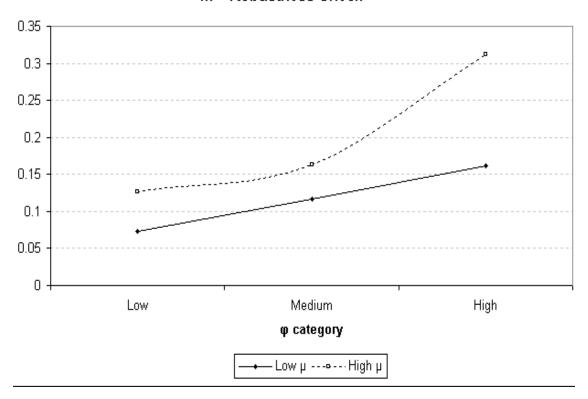


Figure 18: Firm-year observations are divided into three equal categories by external financing costs θ . Each θ tercile is then divided into high and low μ categories (above and below median μ for the ϕ category). Each dot represents the median issuance m for a given ϕ and μ category.

fixed investment adjustment costs, as is most likely the case in practice, then we would expect to see investment (or asset sales) to be more lumpy in the data than what the BCW model predicts. But the general qualitative predictions of the BCW model should be robust to the introduction of fixed investment costs. Finally, if, as is likely to be the case in practice, the fixed costs of issuing equity are an increasing and concave function of firm size, the BCW model would tend to overstate the cost of equity issuance for large firms and understate it for small firms. It is possible to explore the importance of this bias by controlling for firm size in our analysis of seasoned corporate issues.

4 Related Literature

It has long been recognized that firms' holdings of cash and marketable securities, and firms' realized cash flow affects their investment. Broadly speaking, empirical studies on corporate investment can be categorized into two main approaches. The most commonly used one is the 'proxy-based' method, which seeks to test predictions based on static agency-based theories in the tradition of Jensen and Meckling (1976), Myers (1977), and Myers and Majluf (1984), by linking proxies of financial constraints to corporate investment. An influential first study in this vein by Fazzari, Hubbard, and Petersen (1988) shows that in panel regressions the corporate investment-to-capital ratio is correlated with the firm's cash-flow-to-capital ratio. 16 This finding has become known as the cash-flow sensitivity of corporate investment. Although these authors interpret their finding as evidence that firms face financing constraints, and suggest that lower cash flow reduces investment due to a tightening of firms' external financing constraints, an alternative explanation consistent with their findings is that cash-flow itself is correlated with the firm's investment opportunities and that investment responds to cash-flow simply because changes in cash-flow reflect changed investment opportunities. In fact, Kaplan and Zingales (1997) have shown that greater investmentcash flow sensitivity is more likely to be evidence of weaker rather than tighter financial constraints: By first assessing financial constraints directly from financial statements and subsequently relating variations in financial constraints with investment-cash flow sensitivity, they strikingly find that firms that are less financially constrained (such as Microsoft) exhibit greater investment-cash flow sensitivity.

The other approach, which builds the empirical test directly on predictions derived from dynamic structural models of financially constrained firms (generally referred to as "structural approach"), uses various methods of moments (GMM, SMM) to test the firm's optimality condition assuming that the specific structural form of the model is correct.

Our approach lies between these two methodologies. Its starting point is a simple dynamic

¹⁶They control for Tobin's q, which they take to be a proxy for investment opportunities. There are, however, both theoretical and empirical issues with using average q as a proxy for investment opportunities.

structural model of a financially constrained firm, but instead of seeking to estimate the parameters of this model, we pursue a similar strategy to the proxy-based approaches and test whether broad facts in the data correspond to robust predictions of the model. We thus build a bridge between these two broad empirical approaches.

Proxy-based Literature. An early study using a proxy-based approach by Schaller (1993) is explicitly premised on the notion that a firm is likely to face tighter external financing constraints the worse the information asymmetry between management and investors. He considers three different observable attributes of firms with lower information asymmetry and therefore lower external financing costs: the firm's age, ownership concentration, and asset tangibility. He finds evidence that firms facing lower information asymmetries have lower cash-flow sensitivity of investment. Similarly, Gilchrist and Himmelberg (1995) single out firms that do not have access to public debt markets as more financially constrained firms and show that these firms exhibit greater cash-flow sensitivity of investment.

Kim, Mauer, and Sherman's (1998) study focuses more directly on the implications of the presence of external financing costs for firms' cash holding decisions. Based on a static model of corporate earnings retention decisions they test the prediction that higher external financing costs are associated with higher cash holdings in the cross-section. They proxy higher external financing costs by firm size and market-to-book ratio and find evidence that cash holdings are inversely related to firm size and positively related to the market-to-book ratio. If we take external financing costs to vary with firm size, with smaller firms having a higher ϕ , this corresponds directly with the BCW model prediction that firms with higher external financing costs will have a higher marginal value of cash and therefore hold more cash. To the extent that the market-to-book ratio is a measure of investment opportunities, the BCW model involves two opposing forces: 1) Better investment opportunities induce the firm to spend more on investment, reducing its cash holdings (the standard neoclassical effect); and 2) Better investment opportunities increase the marginal value of cash, inducing the firm to hold more cash (the distinctive BCW effect). In the data, we find that the second force dominates, which is consistent with Kim, Mauer, and Sherman's finding that

cash holdings are positively related to the market-to-book ratio.

Opler, Pinkowitz, Stulz, and Williamson (1999) also study the observable factors determining corporate cash holdings and obtain similar findings to Kim, Mauer, and Sherman (1998), in particular that firms with higher credit ratings hold less cash. This corresponds with the BCW model prediction if a higher credit rating is taken to reduce the external financing cost ϕ .

Almeida, Campello, and Weisbach (2004) develop the analysis of Kim, Mauer, and Sherman (1998) further by examining the sensitivity of corporate cash holdings to changes in cash flow. In a static model they propose the notion of cash flow sensitivity of cash (how much of its cash flow a firm chooses to retain) and show that financially constrained firms have a positive cash flow sensitivity of cash. They then test their main theoretical prediction by sorting firms into more contrained and less constrained deciles (in terms of various criteria such as their payout ratio, their average real asset size, their credit ratings, etc.) and show that constrained firms have a positive propensity to save cash. In the BCW model, this has a natural interpretation if cash flow contains information about investment opportunities, so that higher cash flow implies a higher μ . If the firm is financially constrained its marginal value of cash will be greater than 1; better investment opportunities will then raise the marginal value of cash and induce the firm to hold more cash.¹⁷ If a firm is not financially constrained, its marginal value of cash will be arbitrarily close to 1; better investment opportunities will then have virtually no effect on the marginal value of cash, and therefore no effect on cash balances. 18 Thus the BCW model predicts the Almeida, Campello, and Weisbach result even in the situation emphasized by Kaplan and Zingales, where cash flow provides information about future investment opportunities. This is an example of the type of subtle insights that the BCW model provides on issues that tend to be obscure both in static models-where financial constraints are exogenous-and in dynamic models, that are

 $^{^{17}}$ Strictly speaking, this will hold in the empirically relevant case where the BCW effect (that better investment opportunities increase the marginal value of cash and thus cash balances) dominates the standard neoclassical effect (that higher investment spending reduces cash balances).

¹⁸This analysis holds locally, for a reasonably small change in μ . Of course, if the change in μ is large enough, the marginal value of cash will increase appreciably, the firm will go from being unconstrained to financially constrained, and its cash balances will tend to increase.

not as parsimonious and transparent.

If firms choose to hold cash as a way of relaxing their financial constraints, what is the value of firms' cash assets? This is the question addressed in Faulkender and Wang (2006), who regress excess yearly stock returns on (among other variables) the ratio of unexpected changes in the firm's cash holdings over the firm's lagged equity capitalization for both financially constrained and unconstrained firms (in terms of the same criteria as in Almeida et al. 2004). They find that the marginal value of cash is close to one on average and that constrained firms value cash more than unconstrained firms. In the notation of the BCW model, Faulkender and Wang are measuring

$$\frac{(P_t - P_{t-1})}{P_{t-1}} / \frac{(W_t - W_{t-1})}{P_{t-1}},$$

a discrete analog to dP/dW, which, under the BCW homogeneity assumptions, is equal to the marginal value of cash p'(w). In the BCW model, the marginal value of cash is one for unconstrained firms and greater than one for financially constrained firms, which is in essence what Faulkender and Wang find.

If shareholders value cash assets, how does this value vary with debt? Or, as Acharya, Almeida, and Campello (2007) have framed the question: is cash negative debt? To address this question they derive predictions on firm leverage and cash holdings from a static model and test these predictions in the cross-section. As in Almeida et al. (2004), they sort firms by the severity of their financial constraints and show that constrained firms tend to save cash flow increments, while unconstrained firms tend to use cash flow increments to lower their debt obligations.

While cash can be held as a buffer to relax the firm's future financial constraints, earnings may also be retained inefficiently by managers as a way of loosening the discipline imposed by financial markets. Consistent with this latter hypothesis, Pinkowitz, Stulz, and Williamson (2006) find evidence in cross-country regressions that cash is less valuable in countries with weaker investor protections. Corporate governance problems are represented by the firm's

cash carry cost λ in the BCW model.¹⁹ In the BCW model, a larger value of λ reduces the value of cash, which is consistent with the Pinkowitz, Stulz, and Williamson results, if weaker investor protections lead to more corporate governance problems.

Also, Dittmar and Mahrt-Smith (2007) estimate a similar regression as Faulkender and Wang (2006) but with an additional term interacting the cash variable with a governance dummy (based on the governance indices of Gompers, Ishii and Metrick, 2003, and Bebchuk, Cohen, and Ferrell, 2005) and find that the value of cash is significantly lower for poorly governed firms.²⁰ In light of these latter findings, Denis and Sibilkov (2009) revisit the analysis in Faulkender and Wang (2006) and investigate whether the observed differences in value of cash across firms is driven by their investment choices and conclude that cash holdings of financially constrained firms are an efficient response by firms to relax future external financing constraints.

As Chava and Roberts (2008) note, one general weakness of the above proxy-based approaches is that it is not possible to *identify* the precise mechanism through which financial constraints affect investment decisions. In other words, although the tested predictions are derived from theoretical models, it is not possible to conclusively identify the model used for deriving these predictions. They address this limitation by undertaking a regression-discontinuity analysis around shifts in control from shareholders to creditors triggered by the violation of debt-covenants and find that financial constraints through such changes in control do lower investment. To the extent that Chava and Roberts are successful in identifying an increase in ϕ (i.e., a tightening of financing constraints), the BCW model predicts that the increase in ϕ will increase the marginal value of cash and therefore decrease investment.

More generally, the fact that the tested predictions are derived from static models raises concerns about robustness and whether these predictions remain valid in a fully dynamic

¹⁹Chirinko and Schaller (2004) provide a simple model in which corporate governance problems lead managers to use a discount rate that is lower than the market rate and provide empirical evidence that firms with corporate governance problems have an economically and statistically significant cash carry cost.

²⁰Note that since the Faulkender and Wang regression essentially gives the marginal value of cash, the Dittmar and Mahrt-Smith results correspond with the BCW model, in which weaker corporate governance implies a higher λ and thus a lower marginal value of cash.

model. To address these latter concerns it is essential to derive the corporate behavior to be tested in the data from a fully-fledged dynamic model, which is what the structural approach discussed below attempts to do.

Dynamic Corporate Finance Literature. The first study taking a structural approach to determine the effects of a financial constraints on corporate investment by Whited (1992) considers an Euler equation for a financially constrained firm and estimates the values of the Lagrange multiplier on a constraint restricting a firm's debt issues. Bond and Meghir (1994) take a similar estimation approach using GMM methods but estimate a somewhat different model with debt financing costs that are increasing in leverage to reflect the greater expected deadweight bankruptcy costs.

More recently, Toni Whited and her co-authors have used a simulated methods of moments (SMM) approach to estimate the parameters of increasingly rich structural models of financially constrained firms, with general autoregressive cash-flow processes, general adjustment cost functions, and with firms optimizing along multiple margins such as capital structure, cash and investment (see, in particular, Hennessy and Whited, 2005, 2007, Riddick and Whited, 2009, DeAngelo, DeAngelo, and Whited, 2011, and Nikolov and Whited, 2011).²¹

Notably, BCW and our paper are most closely related to Riddick and Whited (2009), henceforth RW. We both develop dynamic structural models of corporate savings (and investment) for financially constrained firms, and not surprisingly much of the intuition from the BCW model carries over to the RW model.

RW assume that the firm's production function exhibits (long-run) decreasing returns to scale and that productivity shocks are serially correlated. They also allow for non-convex capital adjustment costs, and parametrically estimate their model using the simulated method of moments. This opens up the appealing possibility of simulated comparative static exercises, based on counterfactual parameter settings.²² In contrast, BCW assume that the firm's

 $[\]overline{^{21}}$ See also Morellec, Nikolov, and Schuerhoff (2009) for a structural model of a managerial firm. .

²²As in any exercise of this kind, the results are, of course, conditional on the functional form assumptions

production function exhibits long-run constant returns to scale and that productivity shocks are i.i.d. so as to preserve the model's homogeneity property and be able to provide a more tractable and intuitive analysis. The BCW model focuses on the implications of external financing costs and provides simple economic intuitions on the effects of financing frictions, which is often a challenge in more general, multidimensional, dynamic structural models. Second, the BCW model also generates a host of empirical predictions on the financing frictions. Of course, a potential disadvantage of the strong BCW assumptions (specifically, the i.i.d. productivity shocks and the long-run constant returns to scale) is that BCW might abstract from important features of the real world and therefore do a poor job of fitting the data.²³ The objective of our paper is therefore to see whether the predictions of dynamic structural models with financing constraints (even under the stronger BCW assumptions) are broadly consistent with the data.

Eisfeldt and Muir (2011) study the joint dynamics of internal and external finance, and find that firms tend to simultaneously raise external financing and accumulate liquid assets. They show that a simple corporate investment model with cash hoarding performs well, providing support to parsimonious corporate investment models with financial slack.

More broadly, our paper also relates to the contingent-claim-based dynamic capital structure literature. Strebulaev (2007) estimates a dynamic capital structure model based on a generalized capital-structure tradeoff model of Leland (1994). That literature often ignores the role of cash and financial slack.

and parameter estimates.

²³For example, RW allow for non-convex costs of adjusting the capital stock, which is consistent with empirical evidence of lumpiness in investment expenditures at the plant level, an issue from which the BCW model abstracts. On the other hand, the empirical literature suggests that much of this lumpiness is smoothed out at the firm level, so the BCW model has a chance of providing a reasonable fit to firm-level data.

5 Conclusions

The evidence uncovered in our analysis points to the significance of external financial costs for corporations and to the fundamental importance of the marginal value of cash p'(w)as an indicator of dynamic financial constraints. Our findings are relevant for corporate finance, but also have important implications for macroeconomics. For some time before the recent financial crisis, macroeconomists focused largely on "the interest rate", ignoring other aspects of the relations between the financial sector and the real economy. In the wake of the financial crisis, however, macroeconomic research has moved in the direction of modeling corporate financial constraints and the financial sector, but less attention has been paid to corporate savings. This general area is in its infancy, both theoretically and empirically.²⁴ The RW and BCW models are among recent attempts to use dynamic structural models to study corporate savings. Our paper provides empirical evidence on the importance of corporate savings, which is the focus of analysis of these new dynamic structural models with financing constraints. Our findings show that these models of corporate savings provide a better description of corporate investment and financing than either the classical dynamic structural models that do not include external financing constraints (q-theory of investment) or the 'credit channel' models in the vein of Bernanke and Gertler (1989).²⁵

²⁴Bolton, Chen, and Wang (2012) develop a dynamic model for a financially constrained firm facing stochastic financing conditions.

²⁵Building on Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1999) is an example of a prominent and influential exception that is closer to the RW and BCW models, although these models do not allow for corporate savings.

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