Commodity Trade and the Carry Trade: A Tale of Two Countries

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Abstract

Persistent differences in interest rates across countries account for much of the profitability of currency carry trade strategies. We provide a general equilibrium model of commodity trade and currency pricing that implies such heterogeneity due to specialization in goods production and frictions in the shipping sector. The model predicts that commodity-producing countries are insulated from global productivity shocks by the limited shipping capacity, which forces the final goods producers to absorb the shocks. As a result, a commodity currency is risky as it tends to depreciate in bad times, yet has higher interest rates on average due to lower precautionary demand, compared to the final good producer. The model's predictions are strongly supported in the data. We show that countries that primarily export basic commodities exhibit systematically high real interest rates while countries that specialize in exporting finished consumption goods typically have lower rates. The resulting commodity-currency carry trade explains roughly half of the carry-trade risk premia, and exhibits pro-cyclical conditional expected returns, as predicted by the model.

Keywords: carry trade, currency risk premia, exchange rates, international risk sharing, commodity trade

JEL codes: G15, G12, F31

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1 Introduction

A currency carry trade is long high interest rate currencies and short low interest rate currencies. A typical carry trade involves buying the Australian dollar, which for much of the last three decades earned a high interest rate, and funding the position with borrowing in Japanese yen, and thus paying extremely low rate on the short leg. Such a strategy earns positive expected returns on average, and despite substantial volatility and risk of large losses, such as ones incurred during the global financial crisis, exhibits high Sharpe ratios. In the absence of arbitrage this implies that the marginal utility of an investor whose consumption basket is denominated in yen is more volatile than that of an Australian consumer. Are there fundamental economic differences between countries that could give rise to such a heterogeneity in risk? One source of differences across countries is the composition of their trade. Countries that specialize in exporting basic commodities, such as Australia or New Zealand, tend to have high interest rates. Conversely, countries that import most of the basic input goods and export finished consumption goods, such as Japan or Switzerland, have low interest rates on average. These differences in interest rates do not translate into the depreciation of "commodity currencies" on average; rather, they constitute positive average returns, giving rise to a carry trade-type strategy. We document new evidence that this pattern is systematic and robust over the recent time period and is by no means limited to the Australia-Japan pair.

The fact that carry trade strategies typically earn positive average returns is a manifestation of the failure of the Uncovered Interest Parity (UIP) hypothesis, which is one of the major longstanding puzzles in international finance. It is commonly recognized that time-varying risk premia are a major driver of carry trade profits. In fact, a longstanding consensus in the international finance literature attributed all of the carry trade average returns to conditional risk premia, with no evidence of non-zero unconditional risk premia on individual currencies throughout most of the twentieth century (e.g. see Lewis (1995)). Consequently, much of the literature has focused on explaining the conditional currency risk premia by ruling out asymmetries (e.g., Verdelhan (2010), Bansal and Shaliastovich (2012), Colacito and Croce (2012)). However, Lustig, Roussanov, and Verdelhan (2011) show that unconditional cur-

rency risk premia are in fact substantial; indeed, they account for between a third and a half of the profitability of carry trade strategies.¹ Lustig, Roussanov, and Verdelhan (2011) argue that these returns are compensation for global risk, and the presence of unconditional risk premia implies that there is persistent heterogeneity across countries' exposures to common shocks. In this paper we uncover a potential source of such heterogeneity.

We show that the differences in average interest rates and risk exposures between countries that are net importers of basic commodities and commodity-exporting countries can be explained by appealing to a natural economic mechanism: trade costs. We model trade costs by considering a simple model of the shipping industry. At any time the cost of transporting a unit of good from one country to the other depends on the aggregate shipping capacity available. While the capacity of the shipping sector adjusts over time to match the demand for transporting goods between countries, it does so slowly, due to gestation lags in the shipbuilding industry. In order to capture this intuition we assume marginal costs of shipping an extra unit of good is increasing - i.e., trade costs in our model are convex. Convex shipping costs imply that the sensitivity of the commodity country to world productivity shocks is lower than that of the country that specializes in producing the final consumption good, simply because it is costlier to deliver an extra unit of the consumption good to the commodity country in good times, but cheaper in bad times. Therefore, under complete financial markets, the commodity country's consumption is smoother than it would be in the absence of trade frictions, and, conversely, the commodity importer's consumption is riskier. Since the commodity country faces less consumption risk, it has a lower precautionary saving demand and, consequently, a higher interest rate on average, compared to the country producing manufactured goods. Since the commodity currency is risky - it depreciates in bad times - it commands a risk premium. Therefore, the interest rate differential is not offset on average by exchange rate movements, giving rise to a carry trade.

We show empirically that sorting currencies into portfolios based on net exports of finished (manufactured) goods or basic commodities generates a substantial spread in average excess returns, which subsumes the unconditional (but not conditional) carry trade documented by

¹See also Campbell, Medeiros, and Viceira (2010) and Lustig, Roussanov, and Verdelhan (2013) for additional empirical evidence. Theoretical models of Hassan (2010) and Martin (2011) relate currency risk premia to country size.

Lustig, Roussanov, and Verdelhan (2011). Further, we show that aggregate consumption of commodity countries is less risky than that of finished goods producers, as our model predicts.

The model makes a number of additional predictions that are consistent with salient features of the data. Commodity-currency carry trade returns are positively correlated with commodity price changes, both in the model and in the data (we provide evidence using an aggregate commodity index, which complements the result obtained by Ferraro, Rossi, and Rogoff (2011) who use individual currency and commodity price data). Moreover, the model predicts that conditional expected returns on the commodity-currency carry trade are especially high when global goods markets are most segmented, i.e. when trade costs are particularly high. We show that a popular measure of shipping costs known as the Baltic Dry Index (BDI) forecasts unconditional carry trade returns (but not their conditional component). Our model also rationalizes the evidence of carry trade predictability with a commodity price index documented by Bakshi and Panayotov (2012), since commodity prices are typically high in the model during booms, when trade costs are also high.

2 Model

2.1 Setup

There are two countries, each populated by a representative consumer endowed with CRRA preferences over the same consumption good, with identical coefficients of relative risk aversion γ and rates of time preference ρ . The countries differ in their production technologies, each specializing in the production of a single good. The "commodity" country produces a basic input good using a simple production technology

$$y_c = z_c l_c^{\alpha};$$

assuming one unit of commodity country labor l_c is supplied inelastically, so that this is equivalent to an exogenous endowment of basic commodity equal to the productivity shock

$$z_c \ (y_c = z_c).$$

The "producer" country only produces a final consumption good using basic commodity input b and labor:

$$y_p = z_p b^{1-\beta} l_p^{\beta},$$

which is subject to a productivity shock z_p , with one unit of producer country labor also supplied inelastically.

The countries are spatially separated so that transporting goods from one country to the other incurs shipping costs. Our model of shipping costs extends variable iceberg cost of Backus, Kehoe, and Kydland (1992), where each unit of good shipped in either direction loses a fraction

$$\tau(x, z_k) = \kappa_0 + \kappa_1 \frac{x}{z_k},$$

which depends on the total amount of goods shipped in the same direction, x, and the shipping capacity available at time t, z_k . For simplicity we assume that this shipping capacity (or, equivalently, shipping sector productivity) is exogenous (although a model with investment in shipping capacity yields similar implications).

Since the commodity country has no alternative use for its good, in equilibrium all of its supply is shipped to the producer country. Total output of the final consumption good is therefore

$$y_p = z_p[z_c(1 - \tau(z_c, z_k))]^{1-\beta}l_p^{\beta}.$$

In the producer country, the representative competitive firm solves

$$\max_{l_p \in [0,1]} \pi_p = z_p (z_c (1 - \tau(z_c, z_k)))^{1-\beta} l_p^{\beta} - w_p l_p - P z_c (1 - \tau(z_c, z_k)),$$

where w_p is the wage paid to labor and P is the price of one unit of basic commodity. From the first-order conditions and zero profits, the price of the basic commodity is given by

$$P = \frac{(1-\beta)y_p}{(1-\tau(z_c, z_k))y_c} = (1-\beta)z_p[z_c(1-\tau(z_c, z_k))]^{-\beta}$$

Consumption allocations for the commodity country and the producer country, c_c and

 c_p , are determined by the output of the producer country y_p and the amount X of final consumption good exported to the commodity country. In (financial) autarky, whereby trade is balanced in every period, the producer country consumption equals to its labor income (the total wage bill being equal to $\beta z_p[z_c(1-\tau(z_c,z_k))]^{1-\beta})$, while the remainder of the output is exported to the commodity country in the form of payment for the basic commodity $(X=(1-\tau(z_c,z_k)))$, which implies that after trade cost the commodity country income/consumption would equal $X(1-\tau(X,z_k))$.

2.2 Dynamics

We assume that the shocks to productivity experienced by the final good producer are permanent, so that its evolution (in logs) follows a jump-diffusion process:

$$d\log z_{pt} = (\mu - \mu_Z \eta) dt + \sigma_p dB_{pt} + dQ_t.$$

Let N(t) be a Poisson process with intensity η , and let Z_1, Z_2, \ldots be a sequence of identically and negatively distributed power-law random variables with minimum jump \underline{x} and shape parameter α . Denote this distribution's mean as μ_Z . Define the compound Poisson process:

$$Q(t) = \sum_{j=1}^{N(t)} Z_j = \int_0^t Z_s dN_s, \ t \ge 0.$$

$$\Rightarrow dQ(t) = Z_{N(t)} dN(t),$$

so that μ is the uncompensated drift of the jump-diffusion, and the growth rate of the productivity shock process can be written as

$$\frac{dz_{pt}}{z_{pt-}} = \left(\mu - \mu_Z \eta + \frac{1}{2} \sigma_p^2\right) dt + \sigma_p dB_{pt} + (e^{Z_{N(t)}} - 1) dN(t)$$

$$\stackrel{\circ}{=} \mu_p dt + \sigma_p dB_{pt} + (e^{Z_{N(t)}} - 1) dN(t)$$

In order to ensure stationarity of the model economy, we further assume that commodity country productivity shock are cointegrated with the producer country shocks. Specifically, we assume that their cointegrating residual

$$q_t = \log z_{pt} - \beta \log z_{ct}$$

is stationary, following a mean-reverting jump-diffusion process

$$dq_t = \left[(1 - \beta)(\mu - \mu_z \eta) - \beta \psi q_t \right] dt + \sigma_p dB_{pt} - \beta \sigma_c dB_{ct} + dQ_t,$$

so that the commodity country productivity shock process (in logs) follows

$$d \log z_{ct} = (\mu + \psi q_t) dt + \sigma_c dB_{ct}$$

and therefore we can write

$$\frac{dz_{ct}}{z_{ct-}} = \left(\mu + \psi q_t + \frac{1}{2}\sigma_c^2\right)dt + \sigma_c dB_{ct}$$
$$\equiv \mu_c dt + \sigma_c dB_{ct}.$$

This cointegrated relationship can be interpreted as a reduced form representation of an economy where supply of the commodity is inelastic in the short run (based on the currently explored oil fields, say) but adjusts in the long run to meet the demand by the final good producers (e.g., as new fields are explored more aggressively when oil prices are high).

Similarly, we assume that shipping sector productivity is cointegrated with the commodity supply, with the cointegrating residual defined

$$q_{kt} = \log z_{ct} - \log z_{kt},$$

which follows a mean-reverting process

$$dq_{kt} = -\psi_k q_{kt} dt + \sigma_c dB_{ct} - \sigma_k dB_{kt}$$

so that the shipping shock process follows

$$d \log z_{kt} = (\mu + \psi q_t + \psi_k q_{kt}) dt + \sigma_k dB_{kt}$$

$$\Rightarrow \frac{dz_{kt}}{z_{kt-}} = \left(\mu + \psi q_t + \psi q_{kt} + \frac{1}{2}\sigma_k^2\right) dt + \sigma_k dB_{kt}$$

$$\equiv \mu_k dt + \sigma_k dB_{kt},$$

where the Brownian motions B_{pt} , B_{ct} , and dB_{kt} are independent. The latter assumption captures the idea that shipping capacity cannot be adjusted quickly in response to shocks (e.g., Kalouptsidi (2011)), and therefore shipping costs are very sensitive to demand shocks in the short run. Our modeling of cointegrated jump-diffusion processes is similar to the model of cointegrated consumption and dividend dynamics in Longstaff and Piazzesi (2004). We can solve for output and commodity price dynamics by application of Ito's lemma (see Appendix).

2.3 Complete Markets

In order to emphasize that our mechanism does not rely on any financial market imperfections, we consider consumption allocations under complete markets. Under complete markets, the equilibrium allocation is identical to that chosen by a central planner for a suitable choice of a (relative) Pareto weight λ .

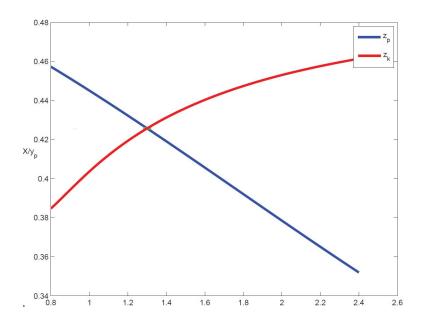
The planner's problem is therefore

$$V(z_{ct}, z_{pt}, z_{kt}) = \max_{\{X_t\}} \mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)} \left(\frac{c_{cs}^{1-\gamma} - 1}{1-\gamma} + \lambda \frac{c_{ps}^{1-\gamma} - 1}{1-\gamma}\right) ds \middle| \mathcal{F}_t\right],$$

where X_s is exports of final good to the commodity country, the commodity country consumption is $c_c = X_s(1 - \tau(X_s, z_k))$, and the producer country consumption is $c_p = y_{ps} - X_s$. The first-order condition implies that

$$g(X_t, z_{ct}, z_{pt}, z_{kt}) \equiv \left[X_t (1 - \kappa_0 - \kappa_1 \frac{X_t}{z_{kt}}) \right]^{-\gamma} \left(1 - \kappa_0 - 2\kappa_1 \frac{X_t}{z_{kt}} \right) - \lambda (y_{pt} - X_t)^{-\gamma} = 0$$

Figure 1: Effect of Shocks on Output Sharing



must hold state by state for all t. In general, this nonlinear equation must be solved numerically. In the special case of log utility ($\gamma = 1$) it simplifies to

$$\kappa_1(2+\lambda)X_t^2 - [z_{kt}(1-\kappa_0)(1+\lambda) + 2\kappa_1 y_{pt}]X_t + (1-\kappa_0)y_{pt}z_{kt} = 0.$$

Solving this equation yields

$$X_{t} = \frac{z_{kt}(1-\kappa_{0})(1+\lambda) + 2\kappa_{1}y_{pt} - \sqrt{[z_{kt}(1-\kappa_{0})(1+\lambda) + 2\kappa_{1}y_{pt}]^{2} - 4(1-\kappa_{0})y_{pt}z_{kt}\kappa_{1}(2+\lambda)}}{2\kappa_{1}(2+\lambda)}$$

which is the only root that allows positive producer-country consumption.

For a reasonable range of parameter values, the fraction of final good output exported to the commodity country, $x_t = \frac{X_t}{y_{pt}}$ is decreasing in total output y_{pt} , as shipping becomes increasingly costly. The effect of individual state variables on this ratio are displayed in Figure 1. The figure plots the effect on a function by changing one shock while holding all other shocks constant. They are intuitive: greater shipping capacity increases the share allocated to the commodity country, but higher productivity of the final goods producer has a negative effect, as it increases trade costs by raising output.

2.4 Asset pricing

Stochastic discount factors for the two countries are given by

$$\pi_{pt} = e^{-\rho t} c_{pt}^{-\gamma}$$

$$\Rightarrow \frac{d\pi_{pt}}{\pi_{pt}} = -\rho dt - \gamma \frac{dc_{pt}^c}{c_{pt}} + \frac{1}{2} \gamma (1+\gamma) \frac{(dc_{pt}^c)^2}{c_{pt}^2} + \frac{1}{c_{pt}^{-\gamma}} d\left(\sum_{0 < s \le t} (c_{ps}^{-\gamma} - c_{ps}^{-\gamma}) \right)$$

$$= -\left\{ \rho + \gamma \frac{\mu_{cp}}{c_{pt}} dt - \frac{1}{2} \gamma (1+\gamma) \frac{\sigma_{cp}^T \sigma_{cp}}{c_{pt}^2} \right\} dt - \gamma \frac{\sigma_{cp}^T}{c_{pt}} dB_t + \frac{1}{c_{pt}^{-\gamma}} d\left(\sum_{0 < s \le t} (c_{ps}^{-\gamma} - c_{ps}^{-\gamma}) \right)$$

for the final good producer and

$$\pi_{ct} = e^{-\rho t} c_{ct}^{-\gamma}$$

$$\Rightarrow \frac{d\pi_{ct}}{\pi_{ct}} = -\left\{\rho + \gamma \frac{\mu_{cc}}{c_{ct}} - \frac{1}{2}\gamma(1+\gamma) \frac{\sigma_{cc}^T \sigma_{cc}}{c_{ct}^2}\right\} dt - \gamma \frac{\sigma_{cc}^T}{c_{ct}} dB_t + \frac{1}{c_{ct}^{-\gamma}} d\left(\sum_{0 < s \le t} (c_{cs}^{-\gamma} - c_{cs}^{-\gamma})\right)$$

for the commodity producer.

Risk-free rates are the (negative) drifts of the stochastic discount factors:

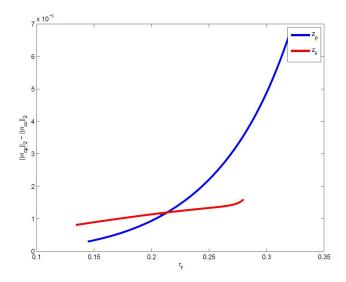
$$r_{pt}^{f} = \rho + \gamma \frac{\mu_{cp}}{c_{pt}} - \frac{1}{2}\gamma(1+\gamma)\frac{\sigma_{cp}^{T}\sigma_{cp}}{c_{pt}^{2}} - \eta \mathbb{E}_{Z} \left[\frac{1}{c_{pt}^{-\gamma}} d \left(\sum_{0 < s \le t} (c_{ps}^{-\gamma} - c_{ps-}^{-\gamma}) \right) \right]$$

and

$$r_{ct}^{f} = \rho + \gamma \frac{\mu_{cc}}{c_{ct}} - \frac{1}{2}\gamma(1+\gamma)\frac{\sigma_{cc}^{T}\sigma_{cc}}{c_{ct}^{2}} - \eta \mathbb{E}_{Z} \left[\frac{1}{c_{ct}^{-\gamma}} d\left(\sum_{0 \le s \le t} (c_{cs}^{-\gamma} - c_{cs}^{-\gamma}) \right) \right]$$

for the final goods and commodity producer, respectively. The first two terms of the interest rate expressions above are equal between the two countries on average, as long-run consumption growth rates are equalized by the social planner. However, the last terms – the precautionary saving demands – differ. Since the final goods producer absorbs the bulk of productivity shocks to output, consuming a greater share in good times and a lower share in bad times, it experiences greater consumption volatility. Consequently, it has a greater precautionary demand and a lower interest rate on average. Figure 2 illustrates this effect for the case of logarithmic utility and no jumps: the difference between conditional volatilities

Figure 2: Trade Costs and Endogenous Segmentation in Risk



increases following good productivity shocks (or bad shipping sector shocks), which increase trade costs and consequently the degree of goods markets segmentation, reducing risk-sharing opportunities.

2.5 Exchange rates and the carry trade

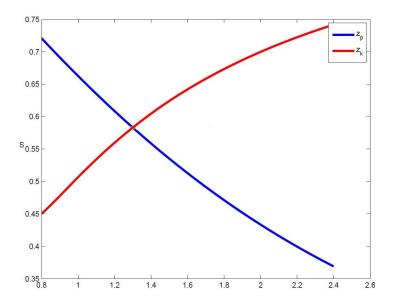
The spot exchange rate in the absence of arbitrage is proportional to the ratio of the pricing kernels,

$$S_t = \lambda \frac{\pi_{pt}}{\pi_{ct}} = \lambda \left(\frac{c_{ct}}{c_{pt}}\right)^{\gamma} = \lambda \left(\frac{X_t(1 - \tau(X_t, z_{kt}))}{y_{pt} - X_t}\right)^{\gamma}$$
$$= \lambda \left(\frac{1 - \tau(X_t, z_{kt})}{1 - x_t}\right)^{\gamma} = \left(1 - \kappa_0 - 2\kappa_1 \frac{X_t}{z_{kt}}\right)$$

where the last equality follows from (2.3), implying that the real exchange rate is proportional to the marginal value to the commodity country consumer of a unit of the consumption good shipped from the country where it is produced (e.g., see Dumas (1992)).

Since the real exchange rate is monotonic in the ratio of the two countries' consumption levels, it is not surprising that it is closely related to the share of final good output exported to the commodity country, as well as the trade costs. These effects are displayed in Figure

Figure 3: Shocks and Exchange Rates



3, which plots the exchange rate S (in units of commodity currency per one unit of final good producer currency), as a function of the three state variables. The commodity currency appreciates following good shocks to the final good production technology (as its good becomes more highly demanded). This is reminiscent of the terms-of-trade effect, which is present even in the absence of complete financial markets, as emphasized by Cole and Obstfeld (1991). The commodity currency depreciates when shipping capacity z_k increases, as improved risk-sharing opportunities imply decreased disparity between the marginal utilities of the two countries.

We can define the instantaneous excess return process for the currency trading strategy that is long the commodity currency (and short the producer currency) as

$$dRet_t = (r_{ct}^f - r_{pt}^f)dt - \frac{dS_t}{S_t}.$$

This return can be earned by a final-good producing country investor directly, by shipping a unit of consumption good (borrowed at rate r_{pt}^f) and purchasing S_t units of the commodity-country risk free bonds, earning interest r_{ct}^f on these bonds, and converting it back into its

own consumption good by shipping fewer units of the consumption good to the commodity country. It can also be obtained indirectly, by trading a state-contingent claim that replicates the payoff on this strategy, given complete financial markets. A commodity country investor can obtain a similar return, adjusted for the exchange rate.

This commodity currency carry-trade is profitable, on average, since the commodity currency is risky: it tends to appreciate in good times (when final good output is high) and depreciate in bad times, so that $E\left[dRet_t\right] = E\left[\frac{dS_t}{S_t}\frac{d\pi_{pt}}{\pi_{pt}}\right] > 0$. As long as exchange rates are persistent and close to random walks, the bulk of average carry excess return comes from the interest rate differentials. These effects are demonstrated in Figure 4, which plots sample paths of the key variables simulated from the model. While both interest rates fluctuate, with the commodity country interest rate being more volatile, and sometimes falling below that of the final good producer, on average the latter is lower. Therefore, a long position in the commodity currency and a short position in the "safe" currency of the final good producer is indeed a carry trade strategy, at least unconditionally.

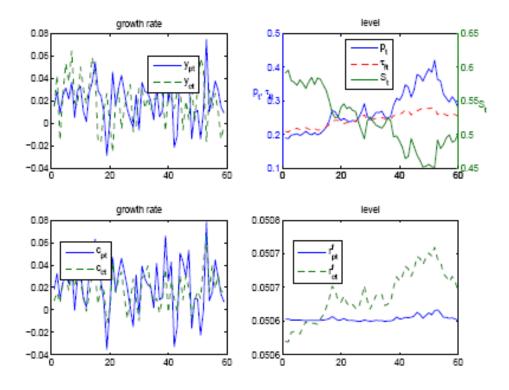
Consistent with intuition, commodity currency exchange rate comoves with the commodity price P as well as realized shipping costs measured by $\tau(X, z_k)$ (for S the relationship is inverse). Interestingly, while carry trade returns are positively correlated with these variables, so are *expected* returns on the carry trade. This is due to the fact that the degree of dispersion between the conditional volatilities of consumption growths (and therefore the risk premium) is pro-cyclical, as trade costs are high in good times (especially is shipping capacity is lagging behind). This effect of trade costs on conditional volatilities is displayed in Figure 2 above.

2.6 Summary of implications

The model makes a set of predictions for the risk and return properties of exchange rates.

 The final good-producing country bears more aggregate consumption risk. Therefore, it has a larger precautionary demand and lower interest rates, on average, than the commodity-producing country.

Figure 4: Model Dynamics: Example



- 2. The commodity country currency is risky, as it appreciates in good times and depreciates in bad times. Therefore, it earns a risk premium, giving rise to a carry trade.
- 3. The commodity currency exchange rate (and therefore the carry trade) is positively correlated with the commodity price as well as the realized shipping costs, since they both increase in good times.
- 4. As high shipping costs imply lower degree of international risk sharing and therefore greater dispersion between conditional volatilities in consumption, conditional expected carry trade returns are positively correlated with trade costs.

Our model of exchange rate determination is deliberately simple and meant to highlight the mechanism leading to a carry trade: specialization combined with non-linear shipping costs. The model nevertheless makes a rich set of qualitative predictions, which we evaluate empirically before proceeding to analyzing its quantitative implications.

3 Empirical evidence

3.1 Data

Following Lustig, Roussanov, and Verdelhan (2011) we use forward and spot exchange rates to construct forward discounts and excess returns on currencies. We use the same set of currencies. Data is provided by Barclays and Reuters and is available via Datastream. We use monthly series from November 1983 to June 2010.

We use two samples in our analysis. The sample of all 35 developed and emerging countries includes: Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Euro area, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Kuwait, Malaysia, Mexico, Netherlands, New Zealand, Norway, Philippines, Poland, Portugal, Singapore, South Africa, South Korea, Spain, Sweden, Switzerland, Taiwan, Thailand, United Kingdom. The sub-sample of 21 developed-country currencies includes: Australia, Austria, Belgium, Canada, Denmark, Euro, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom.

Table 1 shows average returns over our portfolio holding period (post-1995) in addition to average forward discounts over the formation period (pre-1995) of 1-month currency forward contracts for the 8 developed countries outside of the Eurozone for which data is available prior to 1995. Also included is the forward discount for the German Deutschmark paired with the return to investing in Deutschmark forward contracts prior to 1999 and Euro forwards post-1999. The table is sorted from low average returns to high average returns. What is immediately apparent is that the high return countries tended to have a high discount over the formation period, consistent with the unconditional carry trade strategy documented in Lustig, Roussanov, and Verdelhan (2011).

Table 1: Average FX Discounts and Returns

Country	Forward Discount	Excess Return
	(Pre 1995)	(Post 1995)
Switzerland	-1.64	-1.00
Japan	-2.78	-0.91
Sweden	3.62	-0.31
United Kingdom	3.34	0.38
Germany - Euro	-2.99	0.80
Canada	2.17	0.93
Norway	4.89	1.23
Australia	7.09	2.53
New Zealand	10.39	3.56

Interestingly, this relation between average forward discounts and excess returns is not a perfectly monotonic one, in that some low return countries have high discounts over the formation periods. This is not necessarily surprising since other factors than expected returns (e.g. expected inflation) can have an effect on nominal interest rates, and therefore forward discounts. It is clear, however, that the countries with low returns tend to be countries with advanced manufacturing economies which are also relatively resource poor. Indeed, the entire top half of the table: Japan, Sweden, the UK and Germany all fit this description to some degree. In contrast, the high return countries on the bottom half of the table tend to be large exporters of either oil (Canada and Norway) or other base agricultural or mineral

commodities (New Zealand and Australia). This simple observation suggests a potential unconditional carry trade strategy based on the trade characteristics of each country.

In order to classify countries based on their exports we utilize the U.N. COMTRADE database of international trade flows. We use the NBER extract version of this data, available for years 1980-2000. The two goods in the model are a basic good, which is used as an input in production, and a final good, which is used in consumption. While this suggests a potential classification of goods as either "input" or "final" goods, there are many goods for which this classification struggles to conform to the intuition of the model. The important mechanism in the model hinges on the extra trade costs associated with shipping complex produced goods back to the commodity exporter rather than the specific use of the goods as consumption or input. For instance, New Zealand is a large exporter of many agricultural commodities, some of which (such as butter) are in their final consumable form. Likewise, New Zealand imports a large amount of sophisticated construction equipment which is produced using basic commodities (e.g., metals, energy) as an input. However, in the context of the model, the complex piece of construction equipment seems more closely related to the final good rather than the basic good, while butter is a better representation of the basic good. Therefore to be consistent with the model mechanism we classify goods as a basic good (ie. a commodity) or a complex good. This is accomplished using their SITC codes, and the full classification is available upon request.

Using this classification of goods we create two different country-specific measures, the first is the ratio of each countries' net *exports* in basic goods to its total trade in basic goods in each year of the formation period, and the second is the ratio of net *imports* in complex goods to its total trade in complex goods. Both of these measures by construction take a value between -1 and 1. The first sort captures the extent to which a country specializes in the production of basic commodities, and the second variable captures the extent to which a country imports complex goods. Intuitively, for a given country a high ratio of commodity exports tends to be accompanied by a high ratio of complex imports.

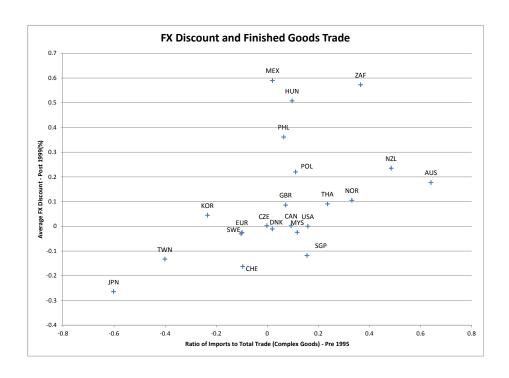


Figure 5: Average Forward Discounts and Import Ratios

3.2 Currency Portfolios Sorted on Import/Export Data

The main prediction of the model is that countries which export basic goods and import complex goods should have lower loadings on the technology shock, and therefore their currencies should have higher average discounts and earn higher returns. Figure 5 plots the average forward discounts on individual currencies over the time period following the creation of the euro (post-1999) against the average ratio of the final good *imports* to total trade over the first half of our sample (pre-1995). The two variables appear to line up well, with higher levels of the import ratio typically corresponding to high average forward discounts (e.g., this includes the so-called "commodity countries" - Australia, New Zealand, Norway - as well as Greece), where as low values of final good import ratio correspond to low average forward discounts (Japan is the most salient extreme case). The exceptions to the pattern tend be countries experiencing high inflation over the sample period (Mexico, Poland, and Hungary).

In order to examine the patterns of average excess returns predicted by the model, we sort all of the countries in our sample into 6 portfolios (5 for the subsample of developed countries) using the rolling five-year average of the export ratio of input goods. We then repeat this strategy using the import ratio of complex goods. We compute the average forward discounts and average log excess returns for each of the portfolios.

Average forward discounts and average returns are computed from 1988-2010² The construction of these portfolios represents an implementable trading strategy, relying only on trade data from available at the time of portfolio formation. Furthermore, since the composition of countries' imports and exports is generally stable over time, the strategy is essentially an unconditional carry-trade strategy, similar to the unconditional interest rate strategy described by Lustig, Roussanov, and Verdelhan (2011).

Using the individual currency forward discounts $f_t - s_t$ and log excess returns approximated as

$$rx_{t+1} = f_{t+1} - s_{t+1},$$

we compute the log currency excess return rx_{t+1}^j for each portfolio $j=1,2\ldots,6$ by averaging:

$$rx_{t+1}^j = \frac{1}{N_j} \sum_{i \in P_j} rx_{t+1}^i.$$

We do not take into account bid/ask spreads in the construction of these portfolios. Since our portfolios do require very little rebalancing, transaction costs are likely to be small (returns based on long-horizon, e.g. one-year, forward contracts are typically similar to those obtained by rolling over shorter-horizon contracts)³

The results are reported in Tables 2 and 3. The results using both sorts are very similar: portfolios representing high complex good export ratios and those with high basic good export ratios have low average forward discounts, suggesting that they capture countries whose interest rates are typically low relative to the U.S. Conversely, portfolios with high values

²Currency forward data is available from 1985. In order to construct the portion of the standard currency carry-trade unrelated to the commodity-currency carry-trade constructed using import and exports we rely on three year rolling regressions, resulting in a post 1988 sample period. Details are in Section 3.3.

³The portfolio is rebalanced to handle the introduction of the Euro. Prior to 1999 breakpoints are calculated including the component countries of the Euro as separate entities. Post 1999 the breakpoints are recalculated counting the Eurozone as a single country.

Table 2: Currency Portfolios Sorted on Final Good Exports

Portfolio	1	2	3	4	5	6	1	2	3	4	5
		Pane	el I: All	l Coun	tries		Pan	el 2: De	velope	d Cour	ntries
		Forwar	d Disc	ount: .	$f^j - s^j$			٠	$f^j - s^j$		
Mean	-0.85	1.19	1.52	2.14	1.78	4.21	-1.08	0.24	0.43	1.39	2.83
Std	0.65	0.79	0.94	0.64	0.66	0.56	0.70	0.81	0.76	0.60	0.51
		Exc	cess Re	turn:	rx^j				rx^j		
Mean	-0.71	1.13	2.08	2.47	1.97	3.42	-0.12	1.17	1.22	1.68	4.27
Std	7.93	8.86	9.33	7.09	8.15	9.82	9.11	10.37	9.12	8.64	10.26
SR	-0.09	0.13	0.22	0.35	0.24	0.35	-0.01	0.11	0.13	0.19	0.42

This table reports average forward discounts and average log excess returns on currency portfolios sorted on the ratio of the countries' net exports of finished goods relative to total trade in such goods, in descending order. Each year's ranking is computed using the average ratio for the prior three years. Trade data are annual, from UN Comtrade (available via NBER extracts). Forward and spot exchange rate data are monthly, from Barclays and Reuters (available via Datastream). The returns do not take into account bid-ask spreads. The sample period is 1/1988 to 6/2010.

of the commodity exports ratio and low values of final good exports exhibit high average forward discount, indicating high average interest rates. The pattern is virtually monotonic across portfolios for both sorts, especially for developed countries subsample, with differences between the highest and the lowest portfolios' average forward discounts of around 4% per annum for the basic good sort over 5% per annum for the complex good sort.

Importantly, portfolio average excess returns follow the pattern of the average forward discounts, being negative for the low portfolios and positive for the high portfolios, with the spreads in average returns between extreme portfolios close to 4% per year for both the basic good sort and the complex good sort. Thus, the differences in the average forward discounts translate almost fully into average excess returns, contrary to the UIP hypothesis. Since the sorting variables are very persistent, these differences are likely to capture unconditional rather than conditional risk premia.

Table 3: Currency Portfolios Sorted on Basic Good Exports

Portfolio	1	2	3	4	5	6	1	2	3	4	5
		Pane	el I: All	l Coun	tries		Pan	el 2: De	veloped	Count	ries
		Forwar	d Disc	ount: .	$f^j - s^j$			٠	$f^j - s^j$		
Mean	-1.73	0.68	1.59	3.27	3.17	2.57	-0.97	0.24	0.99	1.57	2.11
Std	0.47	0.76	0.67	0.92	0.58	0.49	0.77	0.63	0.73	0.66	0.60
		Exc	cess Re	turn:	rx^j				rx^j		
Mean	-0.60	0.62	2.04	3.06	1.93	3.27	0.06	0.33	1.26	3.39	3.08
Std	7.00	9.26	9.15	8.58	7.46	9.03	9.06	10.05	10.15	8.80	8.97
SR	-0.09	0.07	0.22	0.36	0.26	0.36	0.01	0.03	0.12	0.39	0.34

This table reports average forward discounts and average log excess returns on currency portfolios sorted on the ratio of the countries' net exports of basic input goods relative to total trade in such goods, in ascending order. Each year's ranking is computed using the average ratio for the prior three years. Trade data are annual, from UN Comtrade (available via NBER extracts). Forward and spot exchange rate data are monthly, from Barclays and Reuters (available via Datastream). The returns do not take into account bid-ask spreads. The sample period is 1/1988 to 6/2010.

3.3 Comparison with Traditional Carry Trade Strategies

To facilitate comparison with traditional carry-trade strategies, we sort countries based on a measure of the extent to which the country both exports basic goods and imports complex goods, constructed as the sum of net exports of basic goods and net imports of complex goods, divided by the total trade in all goods. Average forward discounts and excess returns for these portfolios are shown in Table 4. We then consider returns on a portfolio which is long the portfolio with the highest ratio and short the lowest among all countries over the prior five years. We refer to this strategy as IMX (Importers minus eXporters of finished goods). We then construct two additional carry-trade strategies. The first uses the traditional method of sorting currencies based on the interest rate. Following Lustig, Roussanov, and Verdelhan (2011) we follow a strategy forming portfolios based on the current interest rate in each month, and label this strategy HML_{FX} . In addition, in order to construct a strategy which is related to the part of the standard carry trade not related to the IMX strategy, we construct a tradeable strategy that is long HML_{FX} and short a number of units of IMX equal to its contribution to HML_{FX} . This strategy (which we refer to as $CHML_{FX}$) is calculated as

Table 4: Currency Portfolios Sorted on Combined Imports/Exports Measure

Portfolio	1	2	3	4	5	6	1	2	3	4	5
		Pane	el I: Al	l Coun	tries		Pane	el 2: De	velope	d Cour	ntries
	-	Forwar	d Disc	ount:	$f^j - s^j$				$f^j - s^j$		
Mean	-0.45	0.94	0.76	2.30	3.28	2.96	-0.65	-0.19	0.73	1.24	2.83
Std	0.69	0.67	0.66	0.69	0.78	0.55	0.74	0.73	0.65	0.73	0.51
		Exc	cess Re	eturn:	rx^j				rx^j		
Mean	-0.75	1.67	2.28	1.81	1.58	3.88	-0.15	0.82	1.06	2.20	4.27
Std	8.11	8.63	9.36	7.52	8.12	9.46	9.07	10.52	9.14	8.60	10.26
SR	-0.09	0.19	0.24	0.24	0.20	0.41	-0.02	0.08	0.12	0.26	0.42

This table reports average forward discounts and average log excess returns on currency portfolios sorted on a ratio designed to capture the extent to which each country exports basic goods and imports finished goods. The ratio is constructed by adding the level of net exports in basic goods to the level of net imports in finished goods, and then dividing by the level of total trade in all goods. Each year's ranking is computed using the average ratio for the prior three years. Trade data are annual, from UN Comtrade (available via NBER extracts). Forward and spot exchange rate data are monthly, from Barclays and Reuters (available via Datastream). The returns do not take into account bid-ask spreads. The sample period is 1/1988 to 4/2010.

 $CHML_{FX,t+1} = HML_{FX,t+1} - \beta_t^{HML,IMX} IMX_{t+1}$, where $\beta_t^{HML,IMX}$ is estimated using a 3-year rolling regression up to time t.

Table 5 reports the returns and standard deviations of the portfolios for each of these strategies. By construction $CHML_{FX}$ and IMX have very low correlation, while both strategies are positively correlated with HML_{FX} . While the import-based strategy underperforms the traditional carry trade strategy, it does have a significantly positively return. Brunnermeier, Nagel, and Pedersen (2008) suggest that crash-risk is important for understanding carry-trade risk, interestingly this table shows that the portion of the traditional carry-trade related to IMX seems to account for nearly all of the negative skewness in traditional carry trade strategy.

Table 5: Carry-Trade Strategies

Strategy	Mean	St. Dev.	SR	Skewness	Correla	ation Matrix
IMX	4.63 (2.00)	9.35 (0.58)	0.50 (0.21)	-0.59 (0.34)		
HML_{FX}	10.36 (2.03)	9.55 (0.51)	1.08 (0.24)	-0.59 (0.20)	0.26 (0.08)	
$CHML_{FX}$	9.35 (1.91)	9.07 (0.46)	1.03 (0.23)	-0.10 (0.18)	-0.06 (0.07)	0.85 (0.03)

Summary characteristics of returns on different carry-trade strategies. IMX is the return on a strategy long the currencies of complex good importers and short exporters, based on the combined imports/exports measure. Imports and exports are the average over a rolling window of the three prior years. HML_{FX} is the return on a strategy which is long high-interest rate countries and short low interest rate countries which is rebalanced each month. $CHML_{FX}$ is the return of a strategy which is long HML_{FX} and short a proportional amount of IMX where the proportion is determined using a 3-year rolling regression of HML_{FX} on IMX. The returns do not take into account bid-ask spreads. Bootstrap standard errors are in the parentheses.

3.4 Explaining the Carry Trade with IMX Factor

While the high return of $CHML_{FX}$ shows that the IMX factor does not completely subsume the traditional carry trade, there appears to be a portion of carry-trade returns that is related to the characteristics of countries' trade, which are very stable over time. Again the magnitude of the return differential is similar to the unconditional interest rate carry-trade Lustig, Roussanov, and Verdelhan (2011), who show that roughly half of the carry-trade premium can be explained as an unconditional premium on countries with a high average interest rate compared to those with a low average interest rate. To test if the import/export sort is capturing the same effects, we follow Lustig, Roussanov, and Verdelhan (2011) and construct an unconditional sort based on the average interest rates of countries over a preformation period from 1984 - 1995, and then examine portfolio returns over a period from 1995 to 2009. The return to this strategy we term $UHML_{FX}$. We then test whether the constructed IMX factor can explain the positive returns to the traditional interest rate carry trade strategies, $UHML_{FX}$ and HML_{FX} .

Table 6 reports regressions of the form

$$R_t^i = \alpha_i + \beta_i IM X_t.$$

The test assets i in the regression are the component portfolios (rebalanced according to interest rate each period) of both the standard HML_{FX} factor as well as the component portfolios (sorted based on the average interest rate over the pre-1995 formation period) of the $UHML_{FX}$ strategy, in addition to the long-short strategies HML_{FX} and $UHML_{FX}$. The results show that the IMX strategy fully explains the returns to the $UHML_{FX}$ strategy, with monotonically increasing betas, insignificant alphas, and a high R^2 , while explaining only some of the returns to the traditional HML_{FX} . These results emphasize that the mechanism in this model is most useful in understanding the returns to the unconditional portion of the carry trade, due the fact that the composition of traded goods for each country is highly stable through time. This is consistent with the evidence in Lustig, Roussanov, and Verdelhan (2013) who show that two separate global factors are needed to explain the unconditional and the conditional currency risk premia.

To further shed light on the underlying mechanism, we now turn to the relation between these strategies and the salient variables of the model.

3.5 Differences in consumption risk

The model's key prediction is that commodity country consumption is less risky than that of the final good-producing country. While our two-country model is too stylized to be taken to the data directly, we provide evidence by grouping countries that more closely resemble the two types. We form two baskets of OECD countries for which we have currency forward contract data: one with countries falling in the top third of the import ratio ranking (commodity countries) and the other of those falling into the bottom third (final good producer countries). Table 7 displays the standard deviation of quarterly consumption growth rates for the two baskets over the period 1993-2009. As the model predicts, aggregate consumption growth of final goods producers is more volatile than that of commodity producers (1.25% per annum vs. 0.88%).

Table 6: Carry Trade Alphas and IMX

	HML_{FX} Portfolios											
1 2 3 4 5 6 HML_{FX}												
IMX Beta	0.05	0.27**	0.33**	0.35**	0.48**	0.53**	0.48**					
	(0.09)	(0.07)	(0.06)	(0.06)	(0.09)	(0.14)	(0.09)					
IMX Alpha	-4.02*	-3.40*	-2.14	0.57	-2.40	5.68	9.70**					
	(2.00)	(1.66)	(1.56)	(1.51)	(2.08)	(2.89)	(2.51)					
R^2							0.18					

	$UHML_{FX}$ Portfolios											
	1	2	3	4	5	6	$UHML_{FX}$					
IMX Beta	-0.16*	0.36**	0.38**	0.44**	0.75**	0.88**	1.05**					
	(0.06)	(0.09)	(0.07)	(0.09)	(0.09)	(0.09)	(0.06)					
IMX Alpha	-0.51	-2.97	-1.40	-2.31	-1.19	-1.43	-0.92					
	(1.95)	(2.64)	(1.70)	(2.06)	(2.13)	(2.61)	(2.06)					
R^2							0.58					

This table reports regressions of the form

$$R_t^i = \alpha_i + \beta_i IM X_t$$

where portfolios i are the six component portfolios of both the HML_{FX} and $UHML_{FX}$, where $UHML_{FX}$ is formed using a sort on an unconditional average forward discount over the period 1984 - 1995. Returns are monthly from 1995 - 2010. Standard errors are White (1980).

The model predicts that producer country consumption is more sensitive to the global productivity shocks that are transmitted into the carry trade, rising faster in good times (when carry strategy does well) and declining in bad times (when carry trade does poorly). We can evaluate this prediction by computing the consumption betas for the commodity-currency carry trade factor IMX using both baskets. As indicated in Table 7, producer country consumption is almost twice as sensitive to the carry returns, compared to the commodity-country consumption, with IMX betas of 3.05 for the producer basket and 1.67

for commodity countries. The short sample makes for imprecise estimates, but the final goods producers' consumption beta is significant at the 10% level using OLS standard errors (though not significantly different from the commodity countries betas).

Table 7: Riskiness of aggregate consumption baskets, data

Portfolio	σ	β_{IMX}
Commodity Producers	0.88	1.67
	(0.09)	(1.21)
Final Goods Producers	1.25	3.05
	(0.16)	(1.62)

This table reports summary statistics from consumption portfolios formed on a country's commodity-making or final-good-producing status. The data are quarterly and taken from the OECD. The countries for which data for consumption and forward contracts are available are ranked according to the average import export measure used in constructing IMX. The commodity and final goods producers are the top and bottom third respectively. Consumption growth is calculated as the average growth rate of consumption weighted by the GDP of each country. Annualized standard deviations are estimated using quarterly growth rates for the time period from fourth-quarter 1993 until fourth-quarter 2009. Consumption betas are with respect to the quarterly IMX return. Standard errors are bootstrapped for the standard deviations and OLS for the IMX Betas.

3.6 Currency Carry-Trades, Commodity Prices, and Shipping Costs

In this section we examine the contemporaneous relation between the different carry-trade strategies and two of the important variables in our model: commodity prices and shipping costs. According to the model, if the returns to IMX are compensation for exposure to global economic activity, we should expect returns to this strategy to be positively correlated with changes in commodity prices. Since convex shipping costs in the model are the key drivers of the carry trade excess returns, we also expect that positive shocks to global productivity should increase trade costs while also generating a positive return to IMX. Therefore, we expect realizations of IMX to be positively correlated with changes in trade costs. To test the first hypothesis we construct an equal weighted portfolio of 32 agricultural, industrial, and energy commodities. In order to proxy for levels of trade cost we use the Baltic Dry Index (BDI).

Table 8 reports contemporaneous regressions of the three currency carry-trade strategies on returns to both of the commodity indices and innovations to the log of the BDI. The IMX strategy loads heavily on these two variables, with contemporaneous R^2 near 15%. The traditional carry-trade loads on them as well, but the relation ship is much weaker, and the residual component $CHML_{FX}$ has very little relation with these two variables with negligible R^2 .

This is again consistent with the mechanism in the model. Since the composition of exports for a given country is very stable through time, we would expect the predictions of the model to explain an unconditional carry trade strategy but to be less likely to explain a strategy relying upon a continuous rebalancing of portfolios. We interpret the fact that these predictions of the model are only present in the unconditional portion of the carry trade strategy as evidence for this explanation.

3.7 Predicting Carry-Trade Returns

In addition to contemporaneous correlations between carry returns, commodity prices, and the Baltic Dry Index, another important implication of the convex adjustment costs in the model is that the difference in exposure to the aggregate shock of the two countries is more severe during times when it is costly to ship goods, this leads a predictive relation between the level of shipping costs and expected return on the carry trade. Since shipping costs tend to be high in good times, this expected return is pro-cyclical.

This mechanism consistent with recent results from Bakshi, Panayotov, and Skoulakis (2010) documenting the fact that high levels of the BDI predict high returns in many different asset classes, including commodities. We document a similar predictive relation between the BDI and the traditional currency carry trade, but find that all of the predictability is concentrated in the unconditional portion of the strategy, as captured by IMX.

To test for predictability we perform univariate predictive regressions analogous to Bakshi, Panayotov, and Skoulakis (2010)

$$R_{i,t} = \alpha_i + \beta_i \Delta b di_{t-4,t-1} \tag{1}$$

Where i represents the four carry trade strategies, R is monthly return, and the predictive variable, $\Delta b di_{t-4,t-1}$, is the innovation in the log of the BDI over the prior three months. Table 9 shows the results. We find a strong predictive relation in IMX, with R^2 of 4%. This relation is still significant but with lower R^2 for the standard carry-trade HML_{FX} factor. Most interestingly, the relation completely disappears when considering $CHML_{FX}$, which captures the portion of HML_{FX} that is orthogonal to IMX. Again, the predictions of the model match nicely with the observed behavior of the unconditional strategy.

4 Quantitative example

So far we have only explored the qualitative implications of our model. We now turn to a quantitative analysis. Ideally, we would like to calibrate the model parameters to closely match empirical moments. The fact that the model features only two countries (each completely specialized in producing one kind of good) makes such a moment-matching exercise challenging. In order to circumvent this challenge we make an assumptions that countries that are ranked at the top of the final good exporter measure are representative of a finalgood producer country in the model, while countries that rank at the bottom (i.e., the final good importers) are representative of the commodity country. Our empirical results above appear to corroborate this distinction, even though the difference between the two types of countries is much less stark in reality than our model assumes. We form two baskets of OECD countries for which we have currency forward contract data: one with countries falling in the top third of the import ratio ranking (commodity countries) and the other of those falling into the bottom third (final good producer countries). We average macroeconomic and financial variables across countries within each basket and compare their properties to those implied by the model. We average macroeconomic and financial variables across countries within each basket and compare their properties to those implied by the model. Table 10 summarizes these moments while Table 11 lists the parameter values used in the calibration.

We present the summary statistics from the model-generated simulated data in three ways: we simulate the model 1000 times and sample periods of approximately the same length as those in our data (approximately 30 years). Besides reporting average statistics across the

simulations we also report averages conditional on no jumps occurring in the sample, as well as conditional on no "disasters" (jumps that imply an annual consumption drop in the final good producer country that is greater than 5%). Such large jumps occur with probability of 4.91% in the simulated samples given that the jump intensity η is such that a jump occurs on average every 8 years, the smallest jump size is 2.5%, and the power law distribution of jump sizes has a tail exponent of 4.5. Since the probability of such jumps is sufficiently small, these conditional statistics capture the sense in which rare disasters contribute to the observed risk premia. There is some debate in the literature about the extent to which rare disasters and peso problems contribute to currency risk premia⁴. While the economic mechanism of our model does not rely on rare disasters, the simulation results reveal that the possibility of such disasters that may occur but are not observed in sample substantially improves the model's ability to quantitatively account for the carry trade risk premium that is generated by the spread between the dispersion in marginal utilities of the two countries.

The modest degree of relative risk aversion $\gamma=4$ ensures that the model does not overshoot the exchange rate volatility observed in the data too much in the absence of jumps (although jumps make it more than twice the volatility observed in the data), with the levels of the risk-free rates somewhat higher than the corresponding (real) interest rates in the data, which matching the spread between the rates closely at about 2% per annum. Consequently, the Sharpe ratio is almost is high as in the data on average in no disaster samples (just under 0.3) but drops substantially if disasters do occur. However, the model does not completely rely on the peso-problem explanation of the carry trade predictability, as even in the samples including disasters the average carry trade return is positive (albeit smaller), at 1.5% per annum.

The trade cost coefficients combined with the shipping sector dynamics imply that the fraction of total output of the final good that is lost to transportation frictions is substantial, at close to 40%. These costs appear large but are in fact well within the range of values estimated by Anderson and van Wincoop (2004). The dynamics of the trade costs produced

⁴Models such as Farhi and Gabaix (2008) and Gourio, Siemer, and Verdelhan (2013) rely on rare disasters for explanations of the forward premium puzzle. Empirical evidence in Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2009), Jurek (2009), and Burnside, Eichenbaum, Kleshchelski, and Rebelo (2008) points to the importance of crash risk in explaining jointly the carry trade risk premia and prices of currency options.

by the model are much less volatile (and somewhat more persistent) than those observed in the data (we use the Baltic Dry Shipping index as our empirical proxy). Nevertheless, the model does feature predictability of carry trade returns with trade costs (as well as interest rate differentials, and past exchange rate appreciation, suggestive of momentum effects) as long as no disasters happen in sample: these results are reported in Table 12.

5 Conclusion

We present new evidence on the relation of the currency carry trade profits to the patterns in international trade: countries that specialize in exporting basic goods such as raw commodities tend to exhibit high interest rates where as countries primarily exporting finished goods have lower interest rates on average. These interest rate differences translate almost entirely into average returns on currency carry trade strategies. We propose a novel mechanism that helps rationalize these findings: convex shipping costs combined with time-varying capacity of the shipping industry. Nonlinearity of the shipping costs implies that the consumption - and therefore the SDF - of the country producing the consumption good is more sensitive to productivity shocks, and is thus riskier.

Table 8: Carry Trade Contemporaneous Relations

	Panel I: Impor	rt Ratio Sort (I	(MX)
	(1)	(2)	(3)
	IMX	IMX	IMX
Δbdi_t	0.043**		0.032*
	(0.016)		(0.014)
R_t^{Com}		0.056**	0.050**
		(0.013)	(0.011)
Cons.	3.905*	5.046**	4.459*
	(1.959)	(1.833)	(1.814)
Obs	271	271	271
R-sq	0.062	0.122	0.155

Panel 1			Sort (HML_{FX})
	(1)	(2)	(3)
	HML_{FX}	HML_{FX}	HML_{FX}
Δbdi_t	0.029*		0.027*
	(0.013)		(0.012)
R_t^{Com}		0.015	0.010
-		(0.014)	(0.012)
Cons.	9.864**	10.469**	9.969**
	(2.038)	(1.977)	(1.996)
Obs	271	271	271
R-sq	0.028	0.008	0.031
Panel III	$: HML_{FX}$ net	of position in I	$MX (CHML_{FX})$
	(1)	(2)	(3)
	$CHML_{FX}$	$CHML_{FX}$	$CHML_{FX}$
Δbdi_t	0.003		0.007
	(0.010)		(0.010)
R_t^{Com}	, ,	-0.014	-0.016
-		(0.010)	(0.010)
Cons.	9.298**	9.249**	9.126**
	(1.917)	(1.887)	(1.892)
Obs	271	271	271
$\begin{array}{c} \text{Obs} \\ \text{R-sq} \end{array}$	$271 \\ 0.000$	$\frac{271}{0.008}$	$271 \\ 0.010$

Standard errors in parentheses ** p < 0.01, * p < 0.05

Regressions of currency carry-trade strategy returns on contemporaneous innovations in the Baltic Dry Index (BDI) and contemporaneous returns of a basket of commodity price. IMX, HML_{FX} , and $CHML_{FX}$ are as defined in Table 5. R_t^{Com} is the return to a basket of the nearest maturity futures of 32 commodities. Δbdi_t is the change in the log of the BDI. All data is monthly from 1/1988 to 6/2010. Standard errors are White (1980).

Table 9: Predicting the Carry-Trade with the BDI

	Pan		Ratio Sort (II)		
		(1)	(2)	(3)	(4)
		IMX	IMX	IMX	IMX
	Horizon:	1-month	3-month	6-month	12-month
$\Delta bdi_{t-4,t-1}$		0.181**	0.070	-0.030	-0.018
		(0.055)	(0.036)	(0.034)	(0.027)
Observations		271	269	266	260
R-squared		0.041	0.016	0.006	0.004
	Panel II: Co	nditional Inte	erest Rate Sor	$\frac{1}{\mathrm{t} (HML_{FX})}$	
		(1)	(2)	(3)	(4)
		HML_{FX}	$\stackrel{\circ}{HML_{FX}}$	HML_{FX}	$\stackrel{\circ}{HML_{FX}}$
	Horizon:	1-month	3-month	6-month	12-month
$\Delta bdi_{t-4,t-1}$		0.164**	0.082*	-0.016	-0.002
,		(0.052)	(0.035)	(0.027)	(0.020)
Observations		271	269	266	260
R-squared		0.027	0.016	0.001	0.000
Pai	nel III: HM	L_{FX} net of p	osition in IM	$X (CHML_{FX})$	- /
		(1)	(2)	(3)	(4)
		$CHML_{FX}$	$CHML_{FX}$	$CHML_{FX}$	$CHML_{FX}$
	Horizon:	1-month	3-month	6-month	12-month
$\Delta bdi_{t-4,t-1}$		0.055	0.039	0.006	0.019
		(0.033)	(0.029)	(0.030)	(0.019)
Observations		271	269	266	260
R-squared		0.003	0.004	0.000	0.003
1 1 1					

Standard errors in parentheses ** p < 0.01, * p < 0.05

Regressions of currency carry-trade strategy returns on the lag of the innovation to the BDI. IMX, HML_{FX} , and $CHML_{FX}$ are as defined in Table 5. $\Delta bdi_{t-4,t-1}$ is the change in the log of the BDI over the three months prior to the current period. All data is monthly. Standard errors are Newey-West with the number of lags equal to the horizon. For the 1 month horizon 3 lags are used.

Table 10: Calibration moments

This table reports summary statistics generated by the model and compares them to data analogues. The macroeconomic variables (consumption, output, exports) are time-aggregated quarterly. All of the financial variables are sampled monthly (monthly carry trade returns are based on continuously rolled-over positions in the model and one-month forward contract returns in the data). "AC" is the sample autocorrelation. All means and standard deviations are annualized, in percentage points. The model moments are averages across 1000 simulated paths of 30 year length, reported for three subsamples: conditional on no jumps occurring over the sample, conditional on no jumps generating producer-country annual consumption declines greater than 5% (these occur with the probability 4.91%), and all paths. The bottom panel reports the key summary statistics of the commodity-currency carry trade return: mean, standard deviation, skewness, and the Sharpe ratio, which are annualized using monthly observations.

	No ju	mps sa	mples	No o	drops >	> 5%	All	samp	les		Data	
	Mean	Std	AC	Mean	Std	AC	Mean	Std	AC	Mean	Std	AC
Δy_{pt}	1.97	1.39	0.21	2.13	2.01	0.22	2.01	2.43	0.21	1.23	1.83	0.31
Δy_{ct}	1.80	1.64	0.22	1.82	1.60	0.26	1.82	1.61	0.25	2.84	0.97	0.43
Δc_{pt}	1.97	1.53	0.22	2.18	2.29	0.23	2.03	2.70	0.21	0.26	1.25	-0.21
Δc_{ct}	1.83	0.63	0.31	1.92	0.78	0.31	1.87	0.93	0.29	0.82	0.88	0.31
ΔX_t	1.96	1.24	0.20	2.10	1.80	0.22	1.98	2.17	0.21	X	X	X
r_{pt}^f	4.18	3.31	0.95	5.91	3.33	0.95	6.21	3.43	0.95	1.97	0.81	0.89
$r_{pt}^f \ r_{ct}^f$	6.58	1.18	0.95	7.22	1.30	0.96	7.16	1.31	0.96	3.74	0.83	0.94
$dRet_t$	2.75	9.79	0.12	1.74	14.93	0.04	1.53	17.17	0.05	3.00	7.84	0.05
dS_t	-0.37	9.22	0.00	-0.76	14.08	-0.01	-0.46	15.97	0.01	1.04	9.45	0.04
dP_t	0.14	26.34	-0.02	0.27	25.60	-0.03	0.20	27.39	-0.02	1.26	10.74	0.19
$d\tau_f(X_t, z_{kt})$	0.11	2.69	0.00	0.21	3.75	-0.01	0.13	4.47	0.01	6.34	55.25	0.30
					IMX (Carry T	rade Re	turns				
Mean $(\%)$		2.75			1.74			1.53			3.00	
Std. dev. $(\%)$		9.79			14.93			17.17			7.84	
Skewness		-0.22			-4.24			-4.94			-0.58	
Sharpe ratio		0.281			0.116			0.089			0.381	

Table 11: Parameter values

Parameter	Value	Description		
λ	1	Relative Pareto weight		
β	0.9	Cobb-Douglas producer-country labor share		
γ	4	Relative risk aversion		
ho	0.001	Rate of time preference		
κ_0	0.001	Fixed trade cost		
κ_1	0.9	Variable trade cost		
σ_p,σ_k	0.0025	TFP and shipping shock volatility		
σ_c	0.01	Commodity shock volatility		
μ	0.018	Uncompensated TFP growth rate		
ψ	0.025	Mean reversion of commodity supply $(z_c \text{ to } z_p)$		
$\psi_{m{k}}$	0.5	Mean reversion of shipping capacity $(z_k \text{ to } z_c)$		
η	1 per 8yrs	TFP jump frequency		
α	4.5	Power tail of jump		
<u>x</u>	2.5%	Minimum jump size		

Table 12: Model predictive regressions

This table reports regression statistics generated by the model. All regressions include an intercept (not reported) and are run on monthly simulated data, excluding all jumps. All quantities and prices are in annualized units. The return horizon lengths denote cumulative horizon returns: dRet(t, t+x), for example, denotes the x-month cumulative return. The first four regressors all are three-month log differences: for example, $\Delta \log S_t = \log(S_t) - \log(S_{t-3})$; IRD_t is the level of the interest-rate differential. Newey-West (1987) standard errors use a lag length that coincides with the cumulative return horizon.

Panel I: Exchange rate								
		(1)	(2)	(3)	(4)	(5)		
		$dRet_t$	$dRet_t$	$dRet_t$	$dRet_t$	$dRet_t$		
	Horizon:	1-month	3-month	6-month	12-month	24-month		
$\Delta \log S_t$		-0.684	-0.647*	-0.585**	-0.487**	-0.366*		
		(0.411)	(0.305)	(0.243)	(0.196)	(0.161)		
R-squared		0.008	0.018	0.025	0.028	0.024		
Panel II: Final-good trade cost								
		(1)	(2)	(3)	(4)	(5)		
		$dRet_t$	$dRet_t$	$dRet_t$	$dRet_t$	$dRet_t$		
	Horizon:	1-month	3-month		12-month	24-month		
$\Delta \log \tau_{ft}$		2.373	2.226*	2.044**	1.664**	1.266**		
		(1.424)	(1.045)	(0.835)	(0.676)	(0.553)		
R-squared		0.008	0.018	0.025	0.028	0.025		
Panel III: Interest-rate differential								
		(1)	(2)	(3)	(4)	(5)		
		$dRet_t$	$dRet_t$	$dRet_t$	$dRet_t$	$dRet_t$		
	Horizon:	1-month	3-month	6-month	12-month	24-month		
$\overline{IRD_t}$		1.463**	1.007**	0.831**	0.672**	0.447**		
-		(0.243)	(0.197)	(0.155)	(0.123)	(0.447)		
R-squared		0.099	0.113	0.128	0.131	0.094		

Standard errors in parentheses ** p < 0.01, * p < 0.05

References

- Anderson, James E., and Eric van Wincoop, 2004, Trade costs, *Journal of Economic Liter-ature* 42, 691–751.
- Backus, David, Patrick Kehoe, and Finn Kydland, 1992, International real business cycles, Journal of Political Economy 100, 745–75.
- Bakshi, Gurdip, and George Panayotov, 2012, Predictability of currency carry trades and asset pricing implications, *Journal of Financial Economics, forthcoming*.
- Bakshi, Gurdip S., George Panayotov, and Georgios Skoulakis, 2010, The baltic dry index as a predictor of global stock returns, commodity returns, and global economic activity, *SSRN eLibrary*.
- Bansal, Ravi, and Ivan Shaliastovich, 2012, A long-run risks explanation of predictability puzzles in bond and currency markets, *Review of Financial Studies*.
- Brunnermeier, Markus K., Stefan Nagel, and Lasse H. Pedersen, 2008, Carry trades and currency crashes, in Daron Acemoglu, Kenneth Rogoff, and Michael Woodford, ed.: *NBER Macroeconomics Annual* (University of Chicago Press).
- Burnside, Craig, Martin Eichenbaum, Isaac Kleshchelski, and Sergio Rebelo, 2008, Can peso problems explain the returns to the carry trade?, NBER Working Paper 14054.
- Campbell, John Y., Karine Serfaty De Medeiros, and Luis M. Viceira, 2010, Global currency hedging, *Journal of Finance* 65, 87–121.
- Colacito, R., and M. Croce, 2012, International asset pricing with recursive preferences, Journal of Finance, forthcoming.
- Cole, Harold L., and Maurice Obstfeld, 1991, Commodity trade and international risk sharing: How much do financial markets matter?, *Journal of Monetary Economics* 28, 3–24.
- Dumas, Bernard, 1992, Dynamic equilibrium and the real exchange rate in a spatially separated world, *Review of Financial Studies* 5, 153–80.

- Farhi, Emmanuel, Samuel P. Fraiberger, Xavier Gabaix, Romain Ranciere, and Adrien Verdelhan, 2009, Crash risk in currency markets, Working Paper.
- Farhi, Emmanuel, and Xavier Gabaix, 2008, Rare disasters and exchange rates: A theory of the forward premium puzzle, Working Paper Harvard University.
- Ferraro, Domenico, Barbara Rossi, and Kenneth S. Rogoff, 2011, Can Oil Prices Forecast Exchange Rates?, *Duke University Working Paper*.
- Gourio, Francois, Michael Siemer, and Adrien Verdelhan, 2013, International risk cycles, Journal of International Economics 89, 471 – 484.
- Hassan, Tarek A., 2010, Country Size, Currency Unions, and International Asset Returns, SSRN eLibrary.
- Jurek, Jakub W., 2009, Crash-Neutral Currency Carry Trades, Working paper.
- Kalouptsidi, Myrto, 2011, Time to build and shipping prices, .
- Lewis, Karen K., 1995, Puzzles in international financial markets, in G. Grossman, and K. Rogoff, ed.: *Handbook of International Economics, Vol.III* (Elsevier: Amsterdam).
- Longstaff, Francis A., and Monika Piazzesi, 2004, Corporate earnings and the equity premium, *Journal of Financial Economics* 74, 401–421.
- Lustig, Hanno, Nikolai Roussanov, and Adrien Verdelhan, 2011, Common risk factors in currency returns, *Review of Financial Studies* 24, 3731–3777.
- ———, 2013, Countercyclical currency risk premia, Journal of Financial Economics, forth-coming.
- Martin, Ian, 2011, The Forward Premium Puzzle in a Two-Country World, *NBER Working Paper no. 17564*.
- Verdelhan, Adrien, 2010, A habit-based explanation of the exchange rate risk premium, Journal of Finance 65, 123–145.

White, Halbert, 1980, A heteroskedasticity-consistent covariance estimator and direct test for heteroskedasticity, *Econometrica* 48, 817–838.

Appendix

Note the compound process's mean is

$$\mathbb{E}[Q(t)] = \sum_{k=0}^{\infty} \mathbb{E}\left[\sum_{j=1}^{k} Z_{j} \middle| N(t) = k\right] \mathbb{P}\{N(t) = k\}$$

$$= \sum_{k=0}^{\infty} \mu_{Z} k \cdot \frac{(\eta t)^{k}}{k!} e^{-\eta t} = \mu_{Z} \eta t e^{-\eta t} \sum_{k=1}^{\infty} \frac{(\eta t)^{k-1}}{(k-1)!} = \mu_{Z} \eta t$$

Output of the final good follows Commodity output y_{ct} equals the level of z_{ct} , so that the final good output dynamics are given by

$$\begin{split} y_{pt} &= z_{pt}[z_{ct}(1-\tau_c(z_{ct},z_{kt}))]^{1-\beta} \\ &= z_{pt}I(z_{ct},z_{kt})^{1-\beta} \\ dy_{pt} &= dz_{pt}^c I_t^{1-\beta} \\ &+ z_{pt}(1-\beta)I_t^{-\beta}I_c dz_{ct}^c + \frac{1}{2}z_{pt}(1-\beta) \left(I_t^{-\beta}I_{cc} - \beta I_t^{-\beta-1}I_c^2\right) dz_{ct}^c dz_{ct}^c \\ &+ z_{pt}(1-\beta)I_t^{-\beta}I_k dz_{kt}^c + \frac{1}{2}z_{pt}(1-\beta) \left(I_t^{-\beta}I_{kk} - \beta I_t^{-\beta-1}I_k^2\right) dz_{kt}^c dz_{kt}^c \\ &+ d \left(\sum_{0 \le s \le t} (y_{ps} - y_{ps-})\right) \\ &= z_{pt}\mu_p I_t^{1-\beta} dt + z_{pt}\sigma_p I_t^{1-\beta} dB_{pt} \\ &+ \left(z_{pt}(1-\beta)I_t^{-\beta}I_c z_{ct}\mu_c + \frac{1}{2}z_{pt}(1-\beta) \left(I_t^{-\beta}I_{cc} - \beta I_t^{-\beta-1}I_c^2\right) z_{ct}^2 \sigma_c^2\right) dt + z_{pt}(1-\beta)I_t^{-\beta}I_c z_{ct}\sigma_c dB_{ct} \\ &+ \left(z_{pt}(1-\beta)I_t^{-\beta}I_k z_{kt}\mu_k + \frac{1}{2}z_{pt}(1-\beta) \left(I_t^{-\beta}I_{kk} - \beta I_t^{-\beta-1}I_k^2\right) z_{kt}^2 \sigma_k^2\right) dt + z_{pt}(1-\beta)I_t^{-\beta}I_k z_{kt}\sigma_k dB_{kt} \\ &+ z_{pt}I_t^{1-\beta} (e^{Z_{N(t)}} - 1) dN_t \\ &= \mu_y dt + \sigma_y^T dB_t + z_{pt}I_t^{1-\beta} (e^{Z_{N(t)}} - 1) dN_t, \end{split}$$

where $I(z_{ct}, z_{kt})$ and its derivatives are defined as follows:

$$I_{t} = I(z_{ct}, z_{kt}) = z_{ct}(1 - \tau_{c}(z_{ct}, z_{kt}))$$

$$I_{c} = (1 - c_{0}) - 2c_{1}\frac{z_{ct}}{z_{kt}}$$

$$I_{cc} = -2c_{1}/z_{kt}$$

$$I_{k} = c_{1}\frac{z_{ct}^{2}}{z_{kt}^{2}}$$

$$I_{kk} = -2c_{1}\frac{z_{ct}^{2}}{z_{kt}^{3}}$$

Commodity price dynamics are given by

$$P_{t} = (1 - \beta)z_{pt} \left[z_{ct}(1 - \tau(z_{ct}, z_{kt}))\right]^{-\beta}$$

$$= \frac{(1 - \beta)y_{pt}}{(1 - \tau(z_{ct}, z_{kt}))z_{ct}}$$

$$\Rightarrow dP_{t} = (1 - \beta)dz_{pt}I_{t}^{-\beta}$$

$$- \beta(1 - \beta)z_{pt}I_{t}^{-\beta-1}(I_{c}dz_{ct} + I_{k}dz_{kt})$$

$$+ \frac{1}{2}\beta(1 - \beta)z_{pt}I_{t}^{-\beta-1}(I_{t}^{-1}I_{c}^{2}(\beta + 1) - I_{cc})dz_{ct}^{2}$$

$$+ \frac{1}{2}\beta(1 - \beta)z_{pt}I_{t}^{-\beta-1}(I_{t}^{-1}I_{k}^{2}(\beta + 1) - I_{kk})dz_{kt}^{2}$$

5.1 Exports of final consumption good

In the special case of log utility we can write

$$X_{t} = \frac{h(z_{ct}, z_{pt}, z_{kt}) - \sqrt{g(z_{ct}, z_{pt}, z_{kt})}}{2\kappa_{1}(2 + \lambda)},$$

where

$$h(z_{ct}, z_{pt}, z_{kt}) = z_{kt}(1 - \kappa_0)(1 + \lambda) + 2\kappa_1 z_{pt} I_t^{1-\beta},$$

$$g(z_{ct}, z_{pt}, z_{kt}) = h(z_{ct}, z_{pt}, z_{kt})^2 - 4(1 - \kappa_0)\kappa_1(2 + \lambda)z_{pt} I_t^{1-\beta} z_{kt}.$$

The derivatives of the export function and its components follow:

$$X_{i} = \frac{h_{i} - \frac{1}{2}g^{-1/2}g_{i}}{2\kappa_{1}(2+\lambda)}, \ \forall i = \{c, p, k\}$$
$$X_{ii} = \frac{h_{ii} + \frac{1}{4}g^{-3/2}g_{i}^{2} - \frac{1}{2}g^{-1/2}g_{ii}}{2\kappa_{1}(2+\lambda)}.$$

In the general CRRA case the derivatives of the export function can be found by implicit differentiation:

$$\frac{dX}{dz_i} = -\frac{g_{z_i}}{g_X} \text{ for } i \in c, p, k$$

$$\frac{d^2X}{(dz_i)^2} = -\left(\frac{g_X\left(g_{z_i,X}\frac{dX}{dz_i} + g_{z_i,z_i}\right) - g_{z_i}\left(g_{X,X}\frac{dX}{dz_i} + g_{X,z_i}\right)}{(g_X)^2}\right)$$

By Ito's lemma,

$$dX_{t}(z_{ct}, z_{pt}, z_{kt}) = X_{c}dz_{ct}^{c} + X_{p}dz_{pt}^{c} + X_{k}dz_{kt}^{c} + \frac{1}{2}X_{cc}dz_{ct}^{c}dz_{ct}^{c} + \frac{1}{2}X_{pp}dz_{pt}^{c}dz_{pt}^{c} + \frac{1}{2}X_{kk}dz_{kt}^{c}dz_{kt}^{c}$$

$$+ d\left(\sum_{0 < s \le t} (X_{s} - X_{s-})\right)$$

$$= \left\{X_{c}\mu_{c}z_{ct} + X_{p}\mu_{p}z_{pt} + X_{k}\mu_{k}z_{kt} + \frac{1}{2}X_{cc}\sigma_{c}^{2}z_{ct}^{2} + \frac{1}{2}X_{pp}\sigma_{p}^{2}z_{pt}^{2} + \frac{1}{2}X_{kk}\sigma_{k}^{2}z_{kt}^{2}\right\}dt$$

$$+ X_{c}\sigma_{c}z_{ct}dB_{ct} + X_{p}\sigma_{p}z_{pt}dB_{pt} + X_{k}\sigma_{k}z_{kt}dB_{kt} + d\left(\sum_{0 < s \le t} (X_{s} - X_{s-})\right)$$

$$= \mu_{X}dt + \sigma_{X}^{T}dB_{t} + d\left(\sum_{0 < s \le t} (X_{s} - X_{s-})\right)$$

5.2 Consumption

For the consumption allocations we have

$$c_{pt} = y_{pt} - X_t$$

$$\Rightarrow dc_{pt} = dy_{pt}^c - dX_t^c + d\left(\sum_{0 < s \le t} (c_{ps} - c_{ps-})\right)$$

$$= (\mu_y - \mu_X) dt + (\sigma_y^T - \sigma_X^T) dB_t + d\left(\sum_{0 < s \le t} (c_{ps} - c_{ps-})\right)$$

$$= \mu_{cp} dt + \sigma_{cp}^T dB_t + d\left(\sum_{0 < s \le t} (c_{ps} - c_{ps-})\right)$$

for the final good producer, and

$$c_{ct} = X_t \left(1 - f_0 - f_1 \frac{X_t}{z_{kt}} \right)$$

$$dc_{ct} = (1 - f_0) dX_t^c - f_1 d \left(\frac{(X_t^2)^c}{z_{kt}^c} \right) + d \left(\sum_{0 < s \le t} (c_{cs} - c_{cs-}) \right)$$

$$= \left\{ \mu_X (1 - f_0) - f_1 \left[\frac{1}{z_{kt}} (2X_t \mu_X + \sigma_X^T \sigma_X) - \frac{X_t^2}{z_{kt}} (\mu_k - \sigma_k^2) - 2X_t X_k \sigma_k^2 \right] \right\} dt$$

$$+ (1 - f_0) \sigma_X^T dB_t - f_1 \frac{2X_t}{z_{kt}} \sigma_X^T dB_t - f_1 \frac{X_t^2}{z_{kt}} \sigma_k dB_{kt}$$

$$+ d \left(\sum_{0 < s \le t} (c_{cs} - c_{cs-}) \right)$$

$$= \mu_{cc} dt + \sigma_{cc}^T dB_t + d \left(\sum_{0 < s \le t} (c_{cs} - c_{cs-}) \right)$$

for the commodity producer.