Short-sale Constraints, Information Acquisition, and Asset Prices

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JEL Codes: *G11, G14*
Keywords: Short-Sale constraints, Information acquisition

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Abstract

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1 Introduction

It is well recognized in the literature that short-sale constraints affect investors' use of information in financial markets. Investors who face short-sale constraints may not be able to trade based on their beliefs or private information, so asset prices will not fully reflect their views. For example, Miller (1977) argues that, under binding short-sale constraints, a stock's price will reflect the valuations that optimists attach to it, but not the valuations of pessimists, since they are sidelined by the constraints. Theoretical models on short-sale constraints (see, e.g., Jarrow, 1980; Diamond and Verrecchia, 1987; and Hong and Stein, 2003) examine the effects of these constraints on information use by market participants and study the implications for investment decisions and equilibrium prices.

Short-sale constraints not only can affect investors' use of information in their investment decisions, but also can affect their incentives to acquire information. For example, one important type of short-sale constraint is that some investors, such as mutual fund and pension fund managers, are explicitly prohibited from short-selling.[1] For these "long-only" investors who face restrictions on short-selling, if information acquisition is costly, their information acquisition decisions can differ from those without such restrictions. The differences in information acquisition can have important implications for investment decisions and asset prices.

In this paper, we develop a model of information acquisition and portfolio choice under short-sale constraints. Our model follows the recent theoretical literature on endogenous information acquisition in financial markets and explicitly incorporates the information acquisition decision in investors' overall investment decision.[2] In the model, investors take short-sale constraints into consideration in their information acquisition decisions before they acquire the information. Short-sale constraints and the information acquisition decisions then jointly determine the investment decisions.

[1] Almazan et al. (2004) provide evidence on such restrictions in the mutual fund industry. They also show that only a small portion of the mutual fund managers that are allowed to short stocks have actually done so.

[2] See, e.g., Van Nieuwerburgh and Veldkamp (2009), Van Nieuwerburgh and Veldkamp (2010), and Mackowiak and Wiederholt (2012), for models of information acquisition in financial markets.

In the baseline model, two assets — a risk-free asset and a risky asset — are traded in the market. An investor decides how much information to acquire prior to (and in anticipation of) his investment decision. Acquiring information is costly; however, doing so reduces the uncertainty that the investor is facing regarding the return of the risky asset. With short-sale constraints, the acquired information may be "wasted" if the investor is not allowed to sell an asset short. In the baseline model, we specify short-sale constraints as prohibition on short-selling. We later extend short-sale constraints to include costly short-selling and further extend the model to include multiple risky assets.

We first examine and compare investors' information acquisition and investment decisions with and without short-sale constraints. Without short-sale constraints, investors acquire more information when information is more valuable, i.e., when the (unconditional) expected return is low and/or the return variance is high. We find that short-sale constraints have a significant impact on the information acquisition decision: (1) short-sale constraints reduce equilibrium information acquisition; and more important, (2) short-sale constraints reduce equilibrium information acquisition more strongly when information acquisition is more valuable. Because the investor decides optimally on information acquisition in anticipation of the potential short-sale constraints in investment decisions, the investor acquires less information when the expected return of the risky asset is low or the return variance is high, i.e., when the likelihood of short-selling is high.

In the model, short-sale constraints affect investment decisions both directly through restricting short positions and indirectly through the effects on information acquisition. The reduced information acquisition decreases the investor's expected investment in the risky asset, but the binding of the short-sale constraints increases the investor's expected investment by eliminating negative positions in the risky asset. In numerical results, we show that the expected investment is higher than without short-sale constraints when short-sale constraints are more likely to be binding, however, the reverse is true when short-sale constraints are less likely to be binding.

We next evaluate the overall effects of short-sale constraints by examining how our model differs from "traditional" models of short-sale constraints, i.e., those without endogenous information acquisition. In the traditional models, information acquisition is exogenous, so short-sale constraints do not play any role in the information acquisition decision. The effects of short-sale constraints on investment decisions are very different in the two types of models. In our model, the first-order effect of short-sale constraints is through information acquisition — investors acquire less information, even less when the risky asset is likely to be subject to short-sale constraints. The investment decisions are then affected by the reduced information acquired, the cross-sectional differences in information acquisition, and the binding short-sale constraints. For the traditional models, short-sale constraints affect investment decisions, but not information acquisition. Thus, investors have "too much" information, and more so when are more short-sale constraints likely to be binding.

The different investment decisions from our model and the traditional models lead to different predictions on the asset-pricing implications of short-sale constraints. In the traditional models, short-sale constraints affect investment, but not information acquisition. So, compared with the case of no short-sale constraints, investors hold more of the risky asset with short-sale constraints, and much more when the short-sale constraints are binding, leading to the overvaluation results in the traditional models. In our model, with reduced information acquisition, investors reduce holdings of the risky asset. This information acquisition effect can partly offset the effects of short-sale constraints when the constraints are likely to be binding, and can dominate the effects of short-sale constraints when the constraints are less likely. Consequently, compared with our model, the traditional models *overestimate* the pricing effects of short-sale constraints on assets that are likely to be subject to short-sale constraints and *underestimate* (in the opposite direction) the pricing effects of short-sale constraints on assets that are less likely to be subject to such constraints.

We extend our baseline model along three important dimensions. First, we examine the second form of short-sale constraints, costly short-selling, in which investors can sell the risky asset short, but short positions incur additional costs relative to long positions. In practice, although only a subset of investors face explicit short-selling prohibition, almost all investors face higher costs for establishing and maintaining short positions. The basic results of our model hold with costly short-selling, though the effects on information acquisition and investment decisions are generally weaker than outright short-prohibition.

Second, we extend our model of information acquisition to the case of short-sellers and consider their information acquisition decisions and how their information acquisition affect their investment (shorting) decisions. Many short-sellers are specialists, and these short-sellers typically do not hold long positions or typically maintain net short positions.[3] Consequently, their information acquisition decisions can be very different from those investors who do not face any holding constraints and are willing to take either long or short positions subsequent to information acquisition. Examining the decisions of short-sellers, we find that a dedicated short-seller acquires much less information, even when the expected return of the risky asset is low and short-selling is likely, than an investor who does not face any holding constraints.

Third, we extend the baseline one risky asset model to include multiple risky assets. Using the case of two risky assets, we show that the effects of short-sale constraints on information acquisition decision and portfolio choice in the one risky asset model are robust. With short-sale constraints, the investor on aggregate acquires less information in the two risky assets than without short-sale constraints. More important, with two risky assets, the effects of short-sale constraints on information acquisition and investment at the individual asset level are stronger than in the one risky asset case. Because of the substitution effect between the two risky assets, i.e., the investor can invest more in one risky asset when the other is likely to be subject to short-sale constraints, the reduction of information acquisition on the second risky asset is greater.

The combined results from our baseline model and its extensions are able to explain much of the time series and cross-sectional evidence of the observed short-selling activities, the relation

[3] For example, "short only" hedge funds typically do not hold long position, and "dedicated-short" hedge funds maintain net short positions.

between short-selling activities and stock returns, and provide new predictions on the asset pricing implications of short-sale constraints. The results show that short-sale constraints, through their effects on information acquisition, can have a more profound impact on investor portfolio choice than suggested by models without endogenous information acquisition. Because the decisions of information acquisition and investment are jointly determined, the effects of short-sale constraints vary across characteristics of the risky assets, the costs of information acquisition, and in the case of costly short-selling, the costs of short-selling. For example, costly shorting together with costly information acquisition helps to explain why few mutual fund managers, even when they are allowed to sell stocks short, would actually do so (Almazan et al., 2004). The results from the baseline model, along with the results on short-sellers, can help to explain why short-sellers target a small number of stocks and why the distribution of short interest across stocks is highly skewed (Asquith, Pathak, and Ritter, 2005). The asset-pricing implications of the model, which are different from those suggested by the traditional models, can explain the "puzzling" evidence that high short interest is weakly related to negative returns, but low short interest is strongly associated with positive returns (Boehmer, Huszar, and Jordan, 2010).

This paper offers a new approach to studying the effects of short-sale constraints. Perhaps the most important insight from the results is that the first-order effect from short-sale constraints is on the acquisition of information, not the use of information. Intuitively, if the information is unlikely to be used, there is little incentive for the investor to acquire such information. With short-sale constraints, investors acquire less information, and there is less divergence in opinions. Consequently, asset prices reflect less information, both positive and negative. Because of the information acquisition effects, short-sale constraints affect the prices of risky assets that are likely to be subject to short-sale constraints, as well as those that are less likely. One policy implication is that with restrictions on short-selling, if investors can choose the types of information they acquire, they may choose not to acquire negative information. As a result, short-sale constraints could diminish the market's role in monitoring and disciplining managerial misbehavior.

The rest of the paper is organized as follows. Section 2 discusses the related literature. The baseline model is presented in Section 3. The properties and implications of the model are studied in Section 4. Section 5 discusses extensions and robustness of the model. Section 6 discusses the empirical predictions of the model and the relation to the existing empirical literature. Section 7 offers concluding remarks.

2 Related Literature

2.1 Short-Sale Constraints

Short-sale constraints, which include legal and institutional restrictions on short-selling as well as the various costs and risks of short-selling, are important frictions in financial markets. Diamond and Verrecchia (1987) classified two forms of constraints on short-selling: (i) short-prohibition and (ii) costly short-selling. To start, some investors are explicitly prohibited from engaging in short-selling activities. Almazan et al. (2004) document that roughly 70% of mutual funds are not permitted to short stocks as stipulated in the mutual fund advisory contracts. In addition, they find that only 10% of mutual funds that were allowed to short-sell actually did so. The contractual restrictions on short-selling can be related to agency considerations in delegated portfolio management (see Almazan et al., 2004; and He and Xiong, 2013).

More generally, aside from the explicit restrictions on short-selling, short-selling is costly. Extensive studies have been conducted on the costs and risks associated with short-selling (see, e.g., D'Avolio, 2002; and Lamont and Thaler, 2003). These studies show that investors face additional costs for shorting stocks (relative to holding long positions), which can severely affect investors' ability to establish and maintain short positions. These costs include the availability of loanable shares, the costs of searching and borrowing shares, the direct costs of holding short positions such as short rebates and other cash payments to the lenders, the risks of share recall, and many others.[4] For the two forms of short-sale constraints, short-prohibition is more restrictive than costly

[4] See D'Avolio (2002), Geczy, Musto, and Reed (2002), and Mitchell, Pulvino, and Stafford (2002), for detailed discussions.

short-selling. While short-prohibition may only apply to a subset of investors, costly short-selling applies to almost all investors. We consider both forms of short-sale constraints in our model.

2.2 Theoretical Models on Short-Sale Constraints

A large theoretical literature exists that studies the effects of short-sale constraints on investment decisions and asset prices. Different from our approach, the existing literature focuses on the use of information by investors in the presence of short-sale constraints after they acquired the information. These models generally find that short-sale constraints affect asset prices, though the effects can be ambiguous (increasing or decreasing the prices) depending on the specific model assumptions.

Miller (1977) argues that short-sale constraints keep more pessimistic investors out of the market and therefore cause overpricing, since positive information, but not negative information is fully incorporated in asset prices. Jarrow (1980) examines Miller's arguments in a single-period equilibrium model and shows that the prices of risky assets rise only with additional assumptions on homogeneous belief regarding the covariance of asset returns. Harrison and Kreps (1978) show that with short-sale constraints, the prices of risky assets can be higher than the valuations of the optimists in a multi-period model. Duffie, Garleanu, and Pedersen (2002) studies the effects of asset lending and lending fees on short-selling and asset prices and finds that with costly shorting, asset prices can be even higher than when shorting is not allowed.

Diamond and Verrecchia (1987) study the effects of short-sale constraints on asset prices and on the speed of adjustment of asset prices to private information. They find that short-sale constraints eliminate some informative trades but do not bias prices upward. Their results depend on the existence of a perfectly rational market maker with unlimited resources who can instantaneously estimate the unbiased stock value, conditional on all publicly available information. Bai, Chang, and Wang (2006) and Cao, Zhang, and Zhou (2007) relax some of the assumptions in Diamond and Verrecchia (1987) and find ambiguous effects of short-sale constraints on asset prices.

We group these models together and classify them as "traditional models," since they are mainly concerned with the incorporation of information in asset prices. We compare the results from our model with endogenous information acquisition and those from the traditional models.

2.3 Empirical Evidence

A large empirical literature studies the effects of short-sale constraints on trading activities and asset prices. We leave the more detailed discussions of the empirical literature to Section 6, where we examine our model predictions and their link to the empirical evidence. In general, the nature and the significance of the impact of short-sale constraints on asset prices remain inconclusive. First, on the effects of short-sale constraints on trading activities, studies find that short-selling activities differ significantly across stock characteristics and over time (see, e.g., Asquith and Meulbroek, 1996; Dechow et al., 2001; and Lamont and Stein, 2004), but the overall short-selling activities are low (see Asquith, Pathak, and Ritter, 2005). Furthermore, the low short-selling activities and the cross-sectional differences in short-selling are not explained by the availability of loanable stocks or the direct costs of borrowing shares (D'Avolio, 2002; Kaplan, Moskowitz, and Sensoy, forthcoming).

Second, on the effects of short-sale constraints on asset-prices, although studies generally find that short-selling activities are informative, the effects of short-sale constraints on stock prices are less clear. Results based on short-interest, arguably a noisy proxy for short-sale constraints, show at best a weak relation between short interest and subsequent stock returns (see, e.g., Desai et al., 2002; Asquith, Pathak, and Ritter, 2005; Boehmer, Huszar, and Jordan, 2010). Studies that utilize variations in short-selling regulations across countries and the short-selling ban during the financial crisis of 2007-2009 tend to find a significant relation between short-selling constraints and market efficiency, but find little evidence on the valuation effects (see Beber and Pagano, 2013; Boehmer, Jones, and Zhang, forthcoming; Bris, Goetzmann, and Zhu, 2007; and Saffi and Sigurdsson, 2011).

2.4 Information Acquisition Models

Our paper is closely related to the growing literature that studies the interactions of information acquisition and investment decisions. The literature shows that the information acquisition decision is a critical component of the overall investment decision process and that studying the information acquisition decision can offer new insights on understanding investment decision making. Van Nieuwerburgh and Veldkamp (2010), for example, study the interactions of costly information acquisition and portfolio choice for an investor who can invest in multiple risky assets. They find that when an investor can choose what information to acquire prior to his portfolio choice, he may optimize information acquisition and hold an under-diversified portfolio. Van Nieuwerburgh and Veldkamp (2009) use a two-country general equilibrium model to explain the home bias puzzle based on costly information acquisition in domestic and foreign markets. Mackowiak and Wiederholt (2012) study the effects of limited liability on the incentive of agents to acquire information prior to their investment decision. They find that agents who face limited liability acquire less information than agents with unlimited liability, particularly when the downside risk is greater and when information acquisition is more costly.[5]

3 Model

3.1 Model Setup

We start with the portfolio choice problem of a single investor who decides how much information to acquire and process prior to his portfolio choice. The model is static and two assets are traded in the market: a risk-free asset and a risky asset. The return on the risk-free asset is denoted by r_f. The return on the risky asset is denoted by r, and the investor has the prior belief that $r \sim N(\mu_r, \sigma_r^2)$, where μ_r and σ_r are the (unconditional) mean and standard deviation of the return of the risky asset. We break the time period into three subperiods. Figure 1 illustrates the sequence of the events in the model. In the second subperiod, the investor receives a signal, denoted by s, about the return of the risky asset. The investor updates his belief using Bayes's law and chooses his portfolio holdings, subject to a budget constraint that is determined by his wealth, w. In the third subperiod, the portfolio return and the investor's utility are realized.

[5] Though less directly related, a strand of finance literature also examines the effects of security design on information acquisition in financial markets. See, for example, Cao (1999), and Massa (2002).

The signal the investor receives in the second subperiod is noisy but unbiased, and it has the following form:

$$s = r + \epsilon,$$

where the noise ϵ is independent of r and $\epsilon \sim N(0, \sigma_\epsilon^2)$, σ_ϵ is the standard deviation of the noise. In the first subperiod, the investor chooses the precision of the signal, i.e., $(\sigma_\epsilon^2)^{-1}$, subject to an increasing cost for more precise information. Following Sims (2003), we quantify acquiring information as a reduction in uncertainty, where uncertainty is measured by entropy. In particular, we assume that the amount of information contained in the signal about the return of the risky asset is

$$I(r; s) = \frac{1}{2} \log_2 \left(\frac{\sigma_r^2}{\sigma_{r|s}^2} \right),$$

where $\sigma_{r|s}$ denotes the conditional (on the observed signal s) standard deviation of the return of the risky asset.[6] The cost of acquiring information is assumed to be linear in the amount of information contained in the signal about the return of the risky asset. Let $\gamma > 0$ denote the marginal cost of acquiring information; then, the cost of acquiring information is $\gamma I(r; s)$.

Let $E_1[.]$ denote the expectation conditional on the prior belief and $E_2[.]$ the expectation conditional on the prior and the observed signal s. The investor's problem can be written as

$$\max_{\sigma_\epsilon^{-1} \geq 0} E_1 \left[\max_a E_2[u(wr_f + (r - r_f)a)] \right] \quad - \quad \gamma I(r; s)$$

s.t.

$$\begin{aligned} s &= r + \epsilon \\ I(r; s) &= \frac{1}{2} \log_2 \left(\frac{\sigma_r^2}{\sigma_{r|s}^2} \right), \end{aligned} \tag{1}$$

where a is the amount of investment in the risky asset, w is the investor's initial wealth, and the utility function $u(.)$ represents the preference of a risk averse investor over his wealth in the third subperiod, which is $wr_f + (r - r_f)a$. In problem (1), the inner maximization problem solves the

[6] Entropy provides an informativeness ordering that is independent of investor preferences, the initial wealth level, or the investment problem. See Cabrales, Gossner, and Serrano (2013) for a discussion of the properties of this measure.

investor's portfolio choice problem, i.e., a, after the investor observes the signal and updates his belief regarding the return of the risky asset using Bayes's law. The outer maximization problem solves the investor's information acquisition problem, i.e., σ_ϵ^{-1}.

For the utility function in (1), we assume that the investor has exponential utility over his wealth in the third subperiod. In particular, the investor's utility is given by

$$u(w_f) = -exp(-\rho w_f),$$

where w_f denotes the end of period wealth of the investor, and ρ is a constant that represents the degree of risk aversion.[7]

3.2 Solution without Short-Sale Constraints

We first solve the model assuming that the investor does not face short-sale constraints. To solve the investor's problem, we first solve his portfolio choice in the second subperiod. Given that $r|s \sim N(\mu_{r|s}, \sigma_{r|s})$, it is straightforward to show that the investor's choice of risky asset, a^*, can be written as

$$a^* = \frac{\mu_{r|s} - r_f}{\rho\sigma_{r|s}^2}, \tag{2}$$

where $\mu_{r|s}$ and $\sigma_{r|s}^2$ are the mean and variance of the return on the risky asset conditional on the observed signal s in the second subperiod, which can be written as

$$\mu_{r|s} = \frac{\sigma_r^2\sigma_\epsilon^2}{\sigma_r^2+\sigma_\epsilon^2}\left(\frac{\mu_r}{\sigma_r^2}+\frac{s}{\sigma_\epsilon^2}\right) \tag{3}$$

$$\sigma_{r|s}^2 = \frac{\sigma_r^2\sigma_\epsilon^2}{\sigma_r^2+\sigma_\epsilon^2}. \tag{4}$$

Equation (2) shows that optimal investment in the risky asset depends on the posterior mean and variance of the return of the risky asset. Moreover, equations (3) and (4) show that the optimal investment in the risky asset depends not only on the precision of the signal, i.e., $(\sigma_\epsilon^2)^{-1}$, but also on the realized signal. In other words, in the first subperiod, following the information acquisition decision (but prior to observing the signal), the portfolio choice of the investor, a, is a random

[7] We obtain qualitatively similar results with quadratic utility function. The results are available upon request.

variable. Therefore, investors with the same information precision may hold different portfolios depending on their observed signals.

Given the optimal choice of the risky asset, the information choice problem for the investor can be written as

$$\begin{aligned} \max_{\sigma_\epsilon^{-1}\geq 0} \; & E_1[E_2[u(wr_f + (r - r_f)a^*)]] \quad - \quad \gamma I(r;s) \\ \text{s.t.} \; & \\ & a^* = \frac{\mu_{r|s} - r_f}{\rho\sigma_{r|s}^2} \\ & s = r + \epsilon \\ & I(r;s) = \frac{1}{2}\log_2\left(\frac{\sigma_r^2}{\sigma_{r|s}^2}\right). \end{aligned} \tag{5}$$

The solution to problem (5), i.e., σ_ϵ^{-1}, is the optimal precision of the signal or, equivalently, the optimal information acquisition decision of the investor.

Proposition 1: The optimal precision of the signal for an investor who does not face short-sale constraints and acquires information about the return of the risky asset is characterized by the following equation:

$$\sigma_\epsilon = \frac{\sigma_r}{\sqrt{\chi^2 - 1}}, \tag{6}$$

where χ is a constant and can be written as $\chi = \frac{\log(2)}{\gamma} exp\left(-\rho w r_f - \frac{(\mu_r - r_f)^2}{2\sigma_r^2}\right)$.

Proofs of all propositions are in Appendix A.

The following corollary summarizes the relationship between the choice of information and the characteristics of the risky asset, the investors risk aversion, and the cost of information acquisition.

Corollary 1: (1) If the risk premium is positive, the acquired information is a decreasing function of expected return of the risky asset. (2) The acquired information is an increasing function of the standard deviation of the risky asset. (3) The acquired information is a decreasing function of information acquisition cost. (4) The acquired information is a decreasing function of risk aversion.

As can be seen from equation (6), the information acquisition decision of the investor is determined by both the return and risk characteristics of the risky asset and the information acquisition cost. There is a trade-off between the benefit of information precision and the cost of information acquisition. As the investor acquires more information, he can rely more on his signal in his portfolio choice, but he also pays a higher cost for acquiring the more precise information. When the expected return is low, more precise information can drastically affect the investment decision and the benefit of acquiring information is high. So information acquisition is more valuable when the expected return is low. Similarly, if the unconditional standard deviation of the return is high, the benefit of acquiring information is also high. Overall, a higher expected return is related to less information acquisition, but a higher standard deviation of the risky asset return is related to more information acquisition.

Without short-sale constraints, the expected investment and the standard deviation of the investment in the risky asset for an investor with optimal information acquisition are, respectively,

$$E_1[a^*] = \frac{\mu_r - r_f}{\rho\sigma_r^2}\chi^2, \quad \sigma_1[a^*] = \frac{1}{\rho\sigma_{r|s}}\frac{\sigma_r}{\sigma_\epsilon}.$$

The expected investment can be interpreted as the average investment across homogeneous investors who solve their information acquisition problems and receive independent signals regarding the return of the risky asset. The standard deviation refers to the standard deviation of the investments of these investors. The following corollary summarizes the relationship between expected investment and expected return of the risky asset:

Corollary 2: The expected investment in the risky asset is not a monotone function of its expected return for an investor who acquires information about the risky asset. In particular if $\frac{\mu_r - r_f}{\sigma_r} \leq \frac{\sqrt{2}}{2}$ then expected investment is an increasing function of expected return. If $\frac{\mu_r - r_f}{\sigma_r} \geq \frac{\sqrt{2}}{2}$ then expected investment is an decreasing function of expected return of the risky asset.

Corollary 2 highlights the effects of information acquisition on investment decisions. Without endogenous information acquisition, the investment in the risky asset is a monotone function of the expected return of the asset. With endogenous information acquisition, reduced information acquisition can lead to lower investment in the risky asset even when the expected return is higher. We illustrate the optimal information acquisition decision and investment decision and their relation with the characteristics of the risky asset in Section 4.

3.3 Solution with Short-Sale Constraints

We now introduce short-sale constraints into the model. For the baseline model of short-sale constraints, we assume that the investor is not allowed to short-sell the risky asset (i.e., short-prohibition). So, in problem (1) if the investor faces short-sale constraints, then the investor faces the constraint that $a \geq 0$.

Equation (2) shows that the investor shorts the risky asset if and only if $\mu_{r|s} < r_f$. Therefore, the investor is more likely to short-sell the risky asset if the unconditional return of the risky asset is low and/or he receives a highly negative signal about the return on the risky asset. If the investor faces short-sale constraints, then the optimal choice of the risky asset a^* is

$$a^* = \max\left\{0, \frac{\mu_{r|s} - r_f}{\rho\sigma^2_{r|s}}\right\}.$$

Given the optimal investment in the risky asset, the following proposition characterizes the optimal information acquisition decision of the investor.

Proposition 2: The optimal precision of the signal for an investor who faces short-sale constraints and acquires information about the return of the risky asset is characterized by the following equation:

$$\Phi\left(\frac{(\mu_r - r_f)\sigma_\epsilon}{\sigma_r^2}\right) - \frac{\gamma}{\log(2)}\exp\left(\rho w r_f + \frac{(\mu_r - r_f)^2}{2\sigma_r^2}\right)\sqrt{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}} = 0, \tag{7}$$

where $\Phi(.)$ is the standard normal cumulative distribution function.

Comparing with Proposition 1, we obtain the following corollary:

Corollary 3: An investor who faces short-sale constraints acquires less or equal information than an investor who does not face such constraints.

The optimal signal precision, along with the signal the investor receives in the second subperiod, would determine the optimal portfolio choice for the investor. The relation between asset return and risk characteristics and information acquisition with and without short-sale constraints has important implications for the effects of short sale constraints on investment decisions. We solve equation (7) and the associated portfolio choice problems numerically and discuss their properties in Section 4.

3.4 Traditional Models of Short-Sale Constraints

Traditional models of short-sale constraints examine the effects of the constraints on investment decisions without taking into consideration the effects on information acquisition decision. How does our model of short-sale constraints with endogenous information acquisition differ from the traditional models? Here, we present the traditional models within our baseline model and derive the results to assess the differences.

Because traditional models of short-sale constraints do not explicitly consider the information acquisition decision, the amount of information investors observe is exogenous. In order to compare the traditional models with our model, we assume that the exogenous amount of information that an investor observes is equal to the amount an investor acquires without short-sale constraints. Thus, within our modeling framework, the traditional models work in two steps. In the first step, an investor who faces short-sale constraints solves his information acquisition problem without taking into account the short-sale constraints at the investment stage. In particular, his choice of information precision is

$$\sigma_\epsilon = \frac{\sigma_r}{\sqrt{\chi^2 - 1}},$$

which is identical to the case of information acquisition without short-sale constraints (equation 6). In the second step, for the investment decision, the investor maximizes expected utility with short-sale constraints, given the level of information precision determined above and the acquired information. In particular, his optimal investment in the risky asset is

$$a^* = \max\left\{0, \frac{\mu_{r|s} - r_f}{\rho\sigma^2_{r|s}}\right\},$$

where $\mu_{r|s}$ and $\sigma_{r|s}$ are determined using $\sigma_\epsilon = \frac{\sigma_r}{\sqrt{\chi^2-1}}$.

Therefore, the main distinction between our model and the traditional approach is as follows. In our model, the investor decides on information acquisition and investments jointly, and short-sale constraints affect the information acquisition decisions. In the traditional models, the two decisions are separate: the information acquisition decision comes first and is not affected by the potential short-sale constraints. Because the information acquisition decisions are different in the two models and are further affected by the return and risk characteristics of the risky asset, the overall effects of short-sale constraints on the investment decisions will be different and are also affected by the return and risk characteristics of the risky asset. We examine the implications of this distinction for the information acquisition decision and portfolio choice in Subsection 4.3.

4 Main Results

In this section we solve the models numerically and discuss their properties. We compare the results and predictions of the models with and without short-sale constraints and also compare our model with traditional models of short-sale constraints.

For the numerical results, we choose the following benchmark parameter values. We set $w = \frac{1}{\rho r_f}$ and the coefficient of the risk aversion to $\rho = 10$. The parameter r_f is set to $r_f = 1.02$, which corresponds to an annual risk-free rate of 2%. The benchmark μ_r is set to $\mu_r = 1.07$, which corresponds to an annual risk premium of 5%. The standard deviation of the risky asset return is set to $\sigma_r = 15\%$. To study how the information acquisition and investment decisions differ across stock characteristics and information acquisition costs, we choose different combinations of the values of the expected returns μ_r, standard deviations σ_r, and marginal costs of information acquisition γ.

4.1 Results without Short-Sale Constraints

We start with the model in which the investor does not face short-sale constraints. The results from this model serve as a benchmark for comparison with those from models with short-sale constraints.

Table 1 presents the results on joint information acquisition and investment decisions with endogenous information acquisition. Panel A presents the results with the information acquisition cost, $\gamma = 0.1$. Panels B and C present results with lower and higher information acquisition costs ($\gamma = 0.05 and 0.15$, respectively). In the table, the first six columns present comparative statics with respect to the (unconditional) expected return of the risky asset, μ_r, and the second six columns present comparative statics with respect to the (unconditional) standard deviation of the return of the risky asset, σ_r. We report the optimal choice of information acquisition, I, the conditional standard deviation of the return of the risky asset, $\sigma_{r|s}$, and ex ante statistics on the amount of investments in the risky asset, $E_1[a]$ and $\sigma_1[a]$. For comparison, we also report the investment decision without information acquisition, $\bar{a}$.

Table 1 shows that the return and risk characteristics of the asset affect the information acquisition decision, and the information acquisition decision has a direct impact on the investment decision. Consistent with Corollary 1, information acquisition decreases with expected return and increases with standard deviation of the risky asset return. The results hold for all three levels of information acquisition costs.

To assess the effects of the information acquisition decision on the investment decision, we can compare the expected investment with information acquisition and the level of investment without information acquisition ($\bar{a}$). Not surprisingly, with information acquisition, the investor's expected investment in the risky asset is higher than without information acquisition. But the information acquisition effects vary greatly across different levels of expected returns. In Panel A, with a high expected return, say 13%, the ratio of expected investment with information acquisition relative to investment without information acquisition is 3.8. With a low expected return of 0.05%, the ratio is 6.5. The effects of the information acquisition on investment decisions are greater with low expected returns.[8] The effects of the information acquisition on investment decisions are greater with high standard deviations. With a low standard deviation (5%), the ratio of the investments is 2.4. With a high standard deviation (27%), the ratio is 6.3.

[8]If information acquisition, i.e., I, remains constant across different levels of expected returns, the ratio of expected investment to unconditional investment is constant in the model.

Figure 2 plots optimal information acquisition as a function of the expected return and the standard deviation of the risky asset. The figure shows that the relation between optimal information acquisition and the expected return and the relation between optimal information acquisition and standard deviation of the risky asset reported in Table 1 hold in general. Information acquisition is high when the expected return is low and the standard deviation is high. In unreported results, we also confirm that the results in Table 1 on the effects of information acquisition on investment decisions hold across different levels of expected returns and standard deviations.

4.2 Results with Short-Sale Constraints

We next present the results from the model with short-sale constraints. As discussed earlier, here we take the short-sale constraints as restrictions on short-selling. For ease of comparison, all the benchmark parameter values are the same as in the case of no short-sale constraints, unless specified otherwise.

Table 2 presents the numerical results. As in Table 1, Panel A presents the results with the information acquisition cost, $\gamma = 0.1$. Compared with the results in Table 1, several differences on the information acquisition decision stand out. First, consistent with Corollary 3, when the investor faces short-sale constraints, he acquires less information about the risky asset than without such constraints. The results hold across all levels of expected returns and standard deviations. Second, the effect of short-sale constraints on information acquisition is more prominent when the expected return is low and when the standard deviation is high. When the expected return is low or standard deviation is high, the investor who faces short-sale constraints acquires less information than without short-sale constraints. In comparison, when the expected return is high or standard deviation is low, the difference between information acquisition with and without short-sale constraints is small.

Third, different from the monotonic negative relation between expected returns and information acquisition without short-sale constraints, there is an inverse 'U-shape' relation with short-sale constraints. Although information acquisition intensifies with low expected returns without short-sale constraints, the relation is reversed with short-sale constraints as the constraints render the information acquisition less valuable when the expected return is low. Starting from highly positive expected returns, information acquisition initially increases as expected return declines, exhibiting the same pattern as in the case without short-sale constraints. However, as the expected return drops further, the probability of binding short-selling constraints becomes higher, and the amount of information acquired declines. There is also an inverse 'U-shape' relation between standard deviation and information acquisition with short-sale constraints. Overall, the investor acquires less information with short-sale constraints, and the effects of short-sale constraints on information acquisition are greater when information acquisition is more valuable and the probability of binding short-sale constraints is high.

Figures 3 and 4 confirm the above results on the effects of short-sale constraints on information acquisition. Figure 3 plots optimal information acquisition under short-sale constraints as a function of the expected return and the standard deviation of the risky asset. Compared with Figure 2, this figure illustrates the general inverse 'U-shape' relation between expected return and information acquisition as well as between standard deviation and information acquisition. Figure 4 plots the differences in optimal information acquisition with and without short-sale constraints. The figure confirms that the effects of short-sale constraints on information acquisition are greater when the expected return is low and the standard deviation is high. If the expected return is high, higher standard deviation leads to greater reduction in information acquisition. However, when the expected return is low, there is already significant reduction of information acquisition, the effect of higher standard deviation is much smaller.

We now study the effects of short-sale constraints on investment decisions. Note that because short-sale constraints affect information acquisition, the information acquisition decision and the binding short-sale constraints jointly affect the investment decisions in the model. So the different investment results in the first two tables reveal the combined effects of information acquisition and the binding short-sale constraints. Because of the reduced information acquisition with short-sale constraints, expected investment in the risky asset can be lower than without short-sale constraints. However, since the constraints are more likely to be binding with low expected returns, the expected investment can also be higher with short-sale constraints than without short-sale constraints. The reason is that, due to restrictions on short-selling, investors can only hold long positions; thus, the average holding, reflecting only no-negative positions, can be higher even with reduced information acquisition.

In the next subsection, we evaluate separately the effects of binding short-sale constraints and information acquisition on the investment decisions by comparing the results from three models. It should be noted here that because of the effects of short-sale constraints on information acquisition decisions, standard deviations of investment are much lower with short-sale constraints than without short-sale constraints. Both channels — the reduction of the optimal acquired information and the binding of the short-sale constraints — play a role in the reduction of the ex ante standard deviation of the investment in the risky asset. With short-sale constraints, the lower information acquisition leads to less divergent opinions, and such effects are reflected in the lower standard deviations of investment in the risky asset. Again, the effects are greater with low expected return or high standard deviation.

Panels B and C of Table 2 consider cases of lower and higher information acquisition costs. The overall effects of short-sale constraints persist across different levels of information acquisition costs. If the information acquisition cost is lower, the effects of short-sale constraints on information acquisition and investment decisions are smaller. The drastic difference between the higher and lower information acquisition cost cases, however, suggests that the effects of short-sale constraints on investment decisions are largely driven by the effects on information acquisition.

4.3 Traditional Models of Short-Sale Constraints

We next study how our model of short-sale constraints with information acquisition differs from the traditional models discussed in Subsection 3.4. We compare the results from three models: the model without short-sale constraints, our model of information acquisition with short-sale constraints, and the traditional models. These comparisons allow us to identify the key differences between the two types of models of short-sale constraints, to evaluate separately the effects of information acquisition and the direct short-sale constraints on the investment decisions, and to assess the asset-pricing implications of short-sale constraints with endogenous information acquisition.

Table 3 presents the results of the traditional models. To start, we first compare the results from the traditional models and those from the model without short-sale constraints (Table 1). Not surprisingly, the information acquisition decisions in the two cases are identical, since both are derived from the same assumption that the investor does not face short-sale constraints. The comparison of the two tables reveals that short-sale constraints affect investment decisions directly. The investor's holding of the risky asset is always larger with short-sale constraints, and more so when the expected returns are low and the standard deviations are high. Because the investor can only hold long positions after acquiring information, the short-sale constraints are binding in the *ex post* sense, i.e., when the investor uncovers unfavorable information. When the (unconditional) expected return of the risky asset is low or the standard deviation is high, short-sale constraints are more likely to be binding. Consequently, the average holding of the risky asset is significantly higher with short-sale constraints than without short-sale constraints. The same effects also explain the lower standard deviations of the investments in the risky asset in the presence of short-sale constraints.

We now turn to the comparison between the traditional models on short-sale constraints and our model. The comparison between Tables 2 and 3 shows that there is always less information acquisition in our model and the differences are greater when the expected return of the risky asset is low or the standard deviation is high, i.e., when information acquisition is more valuable and short-sale constraints are more likely to be binding.

The differences in information acquisition lead to differences in the investments in the risky asset. The comparison between Table 2 and Table 3 captures the effects of information acquisition on the investment decisions as short-sale constraints are present in both models. We plot the ratios of the expected investments from the traditional models and those from our model across different levels of expected returns and standard deviations in Figure 5 to further highlight the differences. As shown in the tables and the figure, the investor's holding of the risky asset in the traditional models is almost always larger than his holding in our model. In the figure, when the expected return is low and standard deviation is high, expected investments in the traditional models are 5 to 6 times as high as those in our model. The differences in investment decisions are greater when the effects of short-sale constraints on information acquisition are stronger. From the tables, the standard deviation of the investments in the risky asset is also lower in our model than in the traditional models, further revealing reduced information acquisition in our model.

How exactly do the effects of short-sale constraints differ between our model of endogenous information acquisition and the traditional models? Because short-sale constraints have no bearing on information acquisition in the traditional models, investors have "too much" information, more so when short-sale constraints are more likely to be binding. Investors who have negative information will not be able to fully use the information because of binding short-sale constraints, while investors who have positive information can use the information. As a result, when short-sale constraints are likely to be binding, the expected investment in the risky asset is much higher in the traditional models than in our model, and much higher than in the model without short-sale constraints. In our model, information acquisition decisions and investment decisions are determined jointly. Investors optimally choose to acquire less information if the short-sale constraints are more likely to be binding. Consequently, investors invest much less in the risky asset in our model than suggested by the traditional models, though still more than in the model without short-sale constraints. A comparison of Tables 1, 2 and 3 on the expected investment reveals the differences. For example, when the expected return is 0.25% with a standard deviation of 15%, the expected investment is 0.144 without constraints, 1.662 in the traditional models, and 0.317 in our model.

The two models also differ significantly when short-sale constraints are less likely to be binding. Because investors reduce information acquisition with short-sale constraints and on average hold less of the risky asset even when short-sale constraints are less likely to be binding, the expected investment in the risky asset is lower in our model than in the traditional models and is also lower than in the model without short-sale constraints. Indeed, the comparison of the three models shows that investors can hold significantly less of the risky asset with short-sale constraints in our model than without short-sale constraints. For example, when the expected return is 13%, the expected investment is 1.857 without constraints, but is 1.139 in our model constraints. In Table 3, the expected investment is 2.094 when the expected return is 13%.

The different results on the investment decisions suggest that the asset-pricing implications can be different between the two models of short-sale constraints. First, stock prices from our model can be less biased than predicted by the traditional models when short-sale constraints are binding. In the traditional models, with exogenous information acquisition, stock price is biased because it reflects the positive information but does not fully reflect the negative information. As our model results show, with reduced information acquisition, investors reduce holdings of the risky asset, and this reduction mitigates the effects of binding short-sale constraints. The traditional models thus *overestimate* the pricing effects of short-sale constraints on assets that are more likely to be subject to short-sale constraints.

Second, short-sale constraints can also affect the prices of the assets that are less likely to be subject to short-sale constraints. The expected investment in traditional models of short-sale constraints is always higher than in the model without short-sale constraints, resulting an overall overvaluation effect, though the effect is stronger when short-sale constraints are more likely to be binding. In our model, because of the information acquisition effects, investors may hold less of the risky assets with short-sale constraints than without short-sale constraints. So assets that are less likely to be constrained can be undervalued. Thus, the traditional models not only *underestimate*, but also *misestimate* (i.e., in the opposite direction) the pricing effects of short-sale constraints on assets that are less likely to be subject to the constraints.

5 Extensions and Robustness

In this section we study three extensions of the model. In the first extension we study the information acquisition and investment decision with costly short-selling. In the second extension we study the information acquisition and investment decision of a short seller. Lastly, we consider the information acquisition and investment decisions under short-sale constraints with multiple risky assets. These extensions allow us to assess the robustness of the results we presented in the previous sections, and provide further insights on the effects of short-sale constraints.

5.1 Costly Short-Selling

Our earlier analyses focus on one form of short-sale constraints in which investors are prohibited from short-selling. Now we examine the more general case of costly short-selling, where short-selling is allowed, but the cost of shorting a stock and holding a short position is higher than buying a stock and holding a long position. These higher costs include, for example, the availability of borrowed shares, the fees and costs associated with borrowing shares, the constraints on the use of the proceeds, or even the risks of share recall.

We assume that the investor can short the risky asset with additional cost. In particular $\delta \geq 0$ denotes the cost per dollar of shorting the risky asset. Let a denote the investor's optimal investment in the risky asset. The investor's wealth in the third subperiod, denoted by w_f, can be written as

$$w_f = \begin{cases} (w-a)r_f + ar & \text{if} \quad a \geq 0 \\ (w-a)r_f + ar + a\delta & \text{if} \quad a \leq 0. \end{cases}$$

It is straightforward to show that the investor's optimal choice of risky asset is

$$a^* = \begin{cases} \frac{\mu_{r|s} - r_f}{\rho \sigma^2_{r|s}} & \text{if} \quad \mu_{r|s} - r_f \geq 0 \\ 0 & \text{if} \quad \mu_{r|s} - r_f \leq 0 \ \& \ \mu_{r|s} - r_f + \delta \geq 0 \\ \frac{\mu_{r|s} - r_f + \delta}{\rho \sigma^2_{r|s}} & \text{if} \quad \mu_{r|s} - r_f + \delta \leq 0, \end{cases}$$

where $\mu_{r|s}$ and $\sigma^2_{r|s}$ are the mean and variance of the return on the risky asset conditional on the observed signal s in the second subperiod.

Given the optimal portfolio decision on the risky asset, the following proposition characterizes the information problem of the investor.

Proposition 3: The optimal precision of the signal for an investor that faces costly short-selling and acquires information about the return of the risky asset is characterized by the following equation:

$$\Phi\left(\frac{(\mu_r - r_f)\sigma_\epsilon}{\sigma_r^2}\right) + \exp\left(-\frac{\delta(2\mu_r - 2r_f + \delta)}{2\sigma_r^2}\right)\Phi\left(-\frac{(\mu_r - r_f + \delta)\sigma_\epsilon}{\sigma_r^2}\right) = \frac{\gamma}{\log(2)}\exp\left(\rho w r_f + \frac{(\mu_r - r_f)^2}{2\sigma_r^2}\right)\sqrt{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}}. \quad (8)$$

Note that if $\delta = 0$, equation (8) simplifies to

$$\chi^2\sigma_\epsilon^2 = \sigma_\epsilon^2 + \sigma_r^2,$$

which is the same as the solution of the model with no short-sale constraints in equation (6). Also note that as $\delta \to \infty$, equation (8) simplifies to

$$\Phi\left(\frac{(\mu_r - r_f)\sigma_\epsilon}{\sigma_r^2}\right) = \frac{2\gamma}{\log(2)}\exp\left(\rho w r_f + \frac{(\mu_r - r_f)^2}{2\sigma_r^2}\right)\sqrt{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}},$$

which is the same as equation (7) that characterizes the problem with short-prohibition.

We solve problem (8) numerically and present the solution in Table 4. We present results for three levels of short-selling costs ($\delta = 0.01, 0.02, and 0.05$), which represents the cost of shorting the shares and holding the short position. In our discussion, we focus on the case of an annual short-selling cost of 2%.[9] Panels A through C present results with an information acquisition cost of $\gamma = 0.1$, and Panels D through F present results with an information cost of $\gamma = 0.15$.

Comparing the results from Panel A in Table 4 with Panel A in Table 1, we find that information acquisition reduces by one-half to two-thirds with costly short-selling (based on the entropy measure of I). Compared with Table 2, Panel A in Table 4 shows that information acquisition with costly

[9]Short-selling costs differ across investor types and vary across the types of assets (stocks). A conservative estimate of the cost associated with short-rebate and other direct costs is 2%. See D'Avolio (2002), Geczy, Musto, and Reed (2002), and Mitchell, Pulvino, and Stafford (2002), for discussions on the types and levels of costs related to short-selling.

selling ranges from 1.5 to 3 times the information acquisition with short-prohibition. Similar to Table 1 and different from Table 2, Panel A shows that the investor still acquires more information when the expected return is low. The reason is that investors can still sell the asset short, thus benefiting from the acquired information, albeit with additional cost.

The effects of the short-selling cost on information acquisition as well as the cost itself jointly determine the investment decision. With costly short-selling, if the unconditional expected return is low, the investor's investment in the risky asset is higher than in the case of no short-selling constraints (Table 1) and short-prohibition (Table 2). When the unconditional expected return is high, the investment in the risky asset is lower than in the case of no short-selling constraints but higher than short-prohibition. One implication from such a comparison is that the asset prices can be affected more greatly with costly short-selling than with short-prohibition when short-sale constraints are likely to be binding.[10]

Comparing the results in Panels A and C helps us to assess the effects of different levels of short-selling cost. With a higher short-selling cost, the investor's investment in the risky asset on average is higher than with a lower short-selling cost if the unconditional expected return is low. If the expected return is high, the investor's investment in the risky asset on average is lower. Results from Panels D through F with a higher cost of information acquisition confirm the results from the three preceding panels. In the case of costly short-selling, the level of information acquisition cost affects both the information acquisition decision and the investment decision. A higher information acquisition cost thus amplifies the effects of costly short-selling on information acquisition.

5.2 Short-Sellers

Our model of endogenous information acquisition and short-sale constraints can naturally be extended to study another type of constraint that is essential to understanding short-selling. Though some investors are prohibited from short-selling, many investors can and do short. The most im-

[10]The reason is that the investor acquires more information than in the case of short-prohibition and can use the positive information, whereas the cost of short-selling prevents the full use of the negative information. Duffie, Garleanu, and Pedersen (2002) obtained a similar result in their search and bargaining model of short-selling constraints.

portant type of short-sellers, however, tends to be specialists who do not hold long positions. For example, two major categories of hedge funds are the "short-only" and "dedicated short", whose managers specialize in identifying overvalued stocks and taking short positions. The information acquisition and portfolio decisions of the "short-only" investors can be different from investors who take both long and short positions. In this subsection, we analyze the decisions of the specialist short-sellers within our modeling framework.

Because of the cost associated with information acquisition, the information acquisition and portfolio decisions of the short-only investors are also determined jointly. For simplicity, we assume that the short specialist is not allowed to hold long positions. From problem (1), we know that if the investor faces short-only constraints, then the investor faces the constraint that $a \leq 0$. The investor's optimal choice of risky asset then is

$$a^* = \min \left\{ 0, \frac{\mu_{r|s} - r_f}{\rho \sigma^2_{r|s}} \right\}.$$

Given the optimal investment in the risky asset, the following proposition characterizes the optimal information acquisition decision of the investor.

Proposition 4: The optimal precision of the signal for a short-only investor who acquires information about the return of the risky asset is the solution to the following equation:

$$\Phi\left(\frac{-(\mu_r - r_f)\sigma_e}{\sigma_r^2}\right) - \frac{\gamma}{\log(2)} \exp\left(\rho w r_f + \frac{(\mu - r_f)^2}{2\sigma_r^2}\right)\sqrt{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}} = 0 \tag{9}$$

We solve problem (9) numerically and present the solution for a short-only investor in Table 5. Again, we consider three levels of information acquisition cost. The model yields rather surprising results with respect to short-selling activities. Panel A shows that with information acquisition cost $\delta = 0.1$, the short-only investor acquires information only when the expected return is 2.5%. The investor will not acquire any information when the expected return is equal to or above 3%. From the second six columns of the same panel, we can see again that the investor will not acquire any information within the range of the standard deviation we specify, 5% to 27% when the expected return is 7%. With a low information acquisition cost (Panel B), the short-only investor acquires more information if the expected return is low and/or the standard deviation is high. With a higher information acquisition cost (Panel C), there is no information acquisition at any level of expected returns or any level of standard deviations in the table.

Comparing the results in Table 5 with those in Table 1 shows that the information acquisition decisions of an investor who specializes in shorting and an investor who does not face any holding constraints are very different, even when the expected return of the asset is low and short is likely. Costly information acquisition thus has a significant impact on the information acquisition and investment decisions of the professional short-sellers. As shown in the results, short-sellers acquire information only in a small number of assets and only when the likelihood of short-selling is high.

5.3 Multiple Risky Assets

In this subsection, we consider the information acquisition and investment decisions of an investor who can invest in multiple risky assets. We consider the case of two risky assets. The return on the risky assets is denoted by $r = (r_1, r_2)$, and the investor has the prior belief that $r \sim N(\mu_r, \Sigma_r)$, where $\mu_r = (\mu_1, \mu_2)$ and Σ_r are the (unconditional) mean and covariance matrix of the return of the risky assets. The return on the risk-free asset is denoted by r_f. The timing of the model is the same as in the one-risky asset model presented in Section 3. The investor receives signals, denoted by $s = (s_1, s_2)$, about the return of the risky assets. The signal is noisy but unbiased and has the following form:

$$\begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \end{pmatrix} + \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix},$$

where the noise $\epsilon = (\epsilon_1, \epsilon_2)$ is independent of r and $\epsilon \sim N(0, \Sigma_\epsilon)$, where Σ_ϵ is the covariance matrix of the noise. We assume that the amount of information contained in the signals about the return of the risky assets is

$$I(r; s) = \frac{1}{2} \log_2 \left(\frac{\det \Sigma_r}{\det \Sigma_{r|s}} \right),$$

where "det" denotes the determinant and $\Sigma_{r|s}$ denotes the conditional (on the observed signal s) covariance matrix of the return of the risky assets. The cost of acquiring information is assumed to be linear in the amount of information contained in the signal about the return of the risky assets. Let $\gamma > 0$ denote the marginal cost of acquiring information; then, the cost of acquiring information is $\gamma I(r;s)$.

The investor's problem can be written as

$$\max_{\Sigma_\epsilon} E_1\left[\max_{a_1,a_2} E_2[u(wr_f + a_1(r_1 - r_f) + a_2(r_2 - r_f))]\right] \quad - \quad \gamma I(r;s)$$

s.t.

$$s = r + \epsilon$$
$$I(r;s) = \frac{1}{2}log_2\left(\frac{\det \Sigma_r}{\det \Sigma_{r|s}}\right), \tag{10}$$

where a_1 is the amount of investment in the first risky asset, a_2 is the amount of investment in the second risky asset, and w is the investor's initial wealth. In problem (10) if the investor faces short-sale constraints, then the investor also faces the constraint that $a_1 \geq 0$ and $a_2 \geq 0$.

Following Van Nieuwerburgh and Veldkamp (2010), we make two simplifying assumptions. The first assumption is that the returns on the risky assets are independent. The second assumption is that the signals are independent.

5.3.1 Solution without Short-Sale Constraints

We first solve the model assuming that the investor does not face short-sale constraints. We assume that the investor has exponential utility over his wealth in the third subperiod. Given that $r|s \sim N(\mu_{r|s}, \Sigma_{r|s})$, the investor's choice of risky assets, a_1^* and a_2^*, can be written as

$$a_1^* = \frac{\mu_{1|s_1} - r_f}{\rho\sigma^2_{1|s_1}},\ a_2^* = \frac{\mu_{2|s_2} - r_f}{\rho\sigma^2_{2|s_2}}, \tag{11}$$

where $\mu_{i|s_i}$ and $\sigma^2_{i|s_i}$ are the mean and variance of the return on risky asset i conditional on the observed signal s_i in the second subperiod.

Proposition 5: The optimal precision of the signals for an investor that does not face short-sale constraints is indeterminate. The total information acquired is characterized by

$$\frac{\sigma_1\sigma_2}{\sigma_{1|s_1}\sigma_{2|s_2}} = \frac{\log(2)}{\gamma}\exp\left(-\rho wr_f - \frac{(\mu_1 - r_f)^2}{2\sigma_1^2} - \frac{(\mu_2 - r_f)^2}{2\sigma_2^2}\right). \tag{12}$$

Given that the precision of the signals is indeterminate, the choice of investments is also indeterminate. Why is the optimal precision of the signals indeterminate? We show in appendix A that the expected utility of the agent is a function of $(\det(\Sigma_r), \det(\Sigma_{r|s}))$. Given that the cost of the information acquisition is also a function of $(\det(\Sigma_r), \det(\Sigma_{r|s}))$, the investor is indifferent about how it allocates its total information acquired as long as $\sigma_{1|s} \leq \sigma_1$, $\sigma_{2|s} \leq \sigma_2$, and equation (12) are satisfied.

5.3.2 Solution with Short-Sale Constraints

In problem (10) if the investor faces short-sale constraints, then the investor also faces the constraint that $a_1 \geq 0$ and $a_2 \geq 0$. If the investor is constrained, then his optimal choice of the risky assets is

$$a_1^* = \max\left\{0, \frac{\mu_{1|s} - r_f}{\rho\sigma_{1|s}^2}\right\}, \; a_2^* = \max\left\{0, \frac{\mu_{2|s} - r_f}{\rho\sigma_{2|s}^2}\right\}.$$

Proposition 6: The optimal precision of the signal for the investor that faces short-sale constraints and acquires information about the return of the risky assets is the solution to the maximization problem specified in the proof of Proposition 6 in Appendix A.

We solve the optimization problems numerically and then discuss the solution. The optimal signal precision, along with the signal the investor receives in the second subperiod would determine the optimal portfolio choice for the investor. Note that, unlike in the case of no short-sale constraints, the solutions to the optimization problem with short-sale constraints are *not* indeterminate.

We present the numerical results with two risky assets in Table 6. In Panel A, we fix the expected return of the first risky asset at 7% and vary the expected return of the second risky asset. The standard deviations of both risky assets are fixed at 15%. In Panel B, we fix the standard deviation of the first risky asset at 15% and vary the standard deviation of the second risky asset. The expected returns of both risky assets are fixed at 7%. As discussed previously, for the optimization problem without short-sale constraints, we can obtain only results for overall information acquisition, not information acquisition for each individual asset. We denote I_U as total information acquisition without short-sale constraints. With short-sale constraints, we obtain total information acquisition I_C, as well as information acquisition at the individual asset level. We report results on information acquisition and investment decisions for the two risky assets with short-sale constraints in the table.

Starting with total information acquisition with no short-sale constraints, if the expected return of the second asset declines, total information acquisition increases. This result is the same as the one risky asset result with no short-sale constraints. Results for the standard deviations; however, are different. Because the standard deviation of the first asset is fixed, an increase in the standard deviation of the second risky asset does not lead to an increase in information acquisition. In fact, total information acquisition generally declines when the standard deviation is higher. Note that the expected returns of the risky assets are both fixed at 7%. This result suggests a "substitution" effect from forming a portfolio with two risky assets. When the standard deviation is high for one asset, the investor may choose to acquire less information by investing more in the other asset.[11]

With short-sale constraints, we can obtain the information acquisition and investment results at the individual asset level. Thus, we can assess the effects of short-sale constraints regarding total information acquisition by comparing results with and without short-sale constraints. We can further assess the effects on the individual assets by comparing the information acquisition and investment decisions between the two risky assets with short-sale constraints. As shown in Table 6, with short-sale constraints, total information acquisition (I_C) is lower across different levels of expected returns (Panel A) and across different levels of standard deviations (Panel B). The magnitude of the differences is comparable to the one risky asset case.

We next examine the effects of short-sale constraints on the two risky assets. From Panel A, when the two expected returns are the same, the information acquisition is the same for the two assets. When the expected return of the second asset is lower and thus is more likely to face short-sale constraints, the information acquisition is lower (because the standard deviations are fixed at

[11]The relation between standard deviation and decreasing information acquisition is not monotonic. For a robustness test, we examine cases where the expected returns are different and obtain similar results.

15%, we can compare the conditional standard deviation, $\sigma_{1|s}$, to assess the level of information acquisition). The relation is not monotonic, however. When the expected return of the second asset is higher and both assets are unlikely to face short-sale constraints, the total information acquisition is lower, and the information acquisition of the higher expected return asset is also lower. We can further compare the results in Panel A with Panel A of Table 2. The results show that with two risky assets, for the asset with a low expected return (say 0.5%), the level of information acquisition is even lower than in the one risky asset case. The reason is that the investor can invest in a portfolio of two risky assets, and is thus less dependent on the second risky asset if this asset is more likely to be subject to short-sale constraints. So he would acquire "relatively" more information in the first risky asset that is less likely to be affected by the short-sale constraints.

Similar results can be seen in Panel B. In this panel, both assets have expected returns of 7%, and the first asset has a standard deviation of 15%. When the second asset has a standard deviation of 19% and is slightly more likely to be subject to short-sale constraints than the first asset, the investor will stop acquiring any information on the second asset.

Overall, the results from the two risky assets model provide evidence on the robustness of the results we developed earlier in the one risky asset model. Furthermore, the results suggest that with multiple risky assets, the effects of short-sale constraints on information acquisition and investments at the individual asset level are substantially stronger than in the one risky asset model. Because of the substitution effects among the different risky assets, investors can easily abandon information acquisition efforts in assets that are likely to be subject to short-sale constraints by acquiring information on and investing in other risky assets that are less likely to be subject to short-sale constraints.

6 Model Predictions and the Empirical Evidence

This section taps into the existing empirical literature to connect the predictions of our model to the evidence, and offers further predictions on new empirical tests. In the discussions, we focus on the observed level and activities of short-selling, the cross-sectional and time series of short-selling activities, the information contents of short interest and short-selling activities, and the effects of short-sale constraints on asset prices.

6.1 Short-Selling Activities

One salient feature of short-selling activity is that the distribution of short interest across stocks is highly skewed. Asquith, Pathak, and Ritter (2005) find that most firms have less than 0.5% of their outstanding shares shorted, but a small number of firms have very large short positions (see also Dechow et al., 2001). Why is there so little aggregate short interest in the stock market and why is the short interest across stocks so highly dispersed? Although the cost of short-selling can be a significant impediment to shorting stocks, the direct cost does not have a drastic impact on shorting activities across stocks (see D'Avolio, 2002; Geczy, Musto, and Reed, 2002; and Jones and Lamont, 2002). In the same vein, Almazan et al. (2004) document that only 10% of mutual funds that were allowed to short-sell actually did so. In other words, most managers voluntarily decide not to sell stocks short.

Almazan et al. (2004) and He and Xiong (2013) argue that agency considerations in delegated portfolio management could potentially explain the contractual constraints on short-selling in the mutual fund industry. Our model offers an explanation for the voluntary restraints on short-selling by fund managers. Even without an explicit short-selling restriction, with a high cost for short-selling, investors who are likely to hold long positions will endogenize their information acquisition decision and tilt their portfolio to long-only portfolios. Similarly, with costly information acquisition, professional short-sellers target a small number of stocks to acquire information and consequently take short positions in an even smaller number of stocks. The activities of the long and short investors, combined, lead to low short interest in most stocks and highly concentrated short positions in a small number of stocks.

Our model further suggests that the observed short-selling activities should differ systematically across stocks. In particular, stocks with low expected returns and high variance would be likely to have higher short interest. Such predictions are consistent with the empirical results in Asquith and Meulbroek (1996) and Dechow et al. (2001). As such, our model provides a theoretical justification for controlling for the various stock characteristics when examining the short-selling activities and the levels of short interest. The model can also yield insight on the time series evidence on aggregate short-selling activity. Lamont and Stein (2004) find that aggregate short interest moves in a countercyclical fashion and is low when the market valuation is higher. Our results suggest that during market runup with high expected returns, short-sellers are less likely to acquire information. Consequently, short-selling activities are low when the contribution of short-selling to the financial market is more valuable.

6.2 Short-Sale Constraints and Trading Activities Across Markets

The results from our model offer potential explanations on the relation between short-sale constraints in the stock market and trading activities in the equity option market. The general view is that the options market provides another venue in which informed investors can trade, thereby potentially reducing the effect of short-sale constraints in the stock market. Because bearish option strategies and short-selling are close substitute, some investors with negative information may migrate from the equity market to the options market. However, the empirical evidence does not provide strong support for this substitution effect. Figlewski and Webb (1993), for example, find that optionable stocks exhibit a significantly higher level of short interest than stocks without options, and that individual stock's short interest increases after option listings. Studying the 2008 short-sale ban in the US, Battalio and Schultz (2011) and Grundy, Lim, and Verwijmeren (2012) find that the options market did not undo the short-sale restrictions in stock market, and short-sale restrictions in stock market in fact lead to reduced trading volume and deteriorating market quality in the equity option market.

Battalio and Schultz (2011) and Grundy, Lim, and Verwijmeren (2012) argue that short-sale restrictions adversely affect the option market because the restrictions affect the hedging strategies of investors and market makers. Our model provides an explanation for these empirical observations based on the information acquisition effects of short-sale constraints. The introduction of options reduces the constraints on short-selling, thereby increasing the incentive to acquire information. The increased information acquisition leads to increasing short positions in the stock market. Similarly, restricting short-selling in the stock market reduces incentive to acquire information, thus reduces information based trading in the options market.

6.3 Information Content of Short-Selling Activities

Various studies have examined the information content of short-selling activities, in particular the relation between short interest and subsequent stock returns. The empirical findings are mixed. Perhaps the most comprehensive and surprising results are "the good news in short interest" documented by Boehmer, Huszar, and Jordan (2010): stocks with high short interest experience weakly negative abnormal returns, whereas relatively heavily traded stocks with low short interest experience both statistically and economically significant positive abnormal returns. Consequently, the return difference between high and low short-interest stocks is largely driven by stocks with low (or no) short interest.

Boehmer, Huszar, and Jordan interpret their results as suggesting that the positive information associated with low short-interest is only slowly incorporated into prices. Our model provides a theoretical justification for the observation and interpretation. In contrast to traditional models of short-sale constraints, our model shows that because of the information acquisition effect, short-sale constraints moderately inflate the prices of the stocks that are subject to short-sale constraints but suppress the prices of those stocks that are less likely to be subject to the constraints. The latter effect can drive the "good news" in low short interest stocks as they are highly liquid stocks that are less likely to face the constraints.

Our model offers new insights on the information content of short interest through the combined information acquisition effects of long investors and short sellers. If the costs of short-selling vary across stocks, those with low costs encourage greater information acquisition and can lead to higher short interest as both negative and positive information is incorporated in stock prices. So, high short interest by itself may not predict abnormal returns. The combination of short interest and proxies of positive information yields a richer cross-sectional relation between short interest and stock returns.

6.4 The Effects of Short-Sale Constraints on Asset Prices

One central question on short-sale constraints involves their effects on asset prices. In the traditional models, investors acquire information but cannot use the negative information when facing short-sale constraints. As a result, for stocks with higher costs of short-selling, and when investors have divergent opinions or information, these stocks tend to be overvalued (see Miller, 1977). Our model provides a new direction for studying the asset pricing effects of short-sale constraints. Perhaps the most important insight from the results is that the first-order effect from short-sale constraints is on the acquisition of information, not the use of information. With short-sale constraints, investors acquire less information and there is less divergence in opinions. Consequently, asset prices reflect less information, both positive and negative.

For the main results of the paper, we study the effects of short-sale constraints on information acquisition and investment decisions in a partial equilibrium setting. We provide a sketch of information acquisition in a general equilibrium setting in Appendix B and obtain numerical results that indicate that the main results from the partial equilibrium model should carry over to the general equilibrium model. Based on the information acquisition results, we conjecture that the effects of short-sale constraints on information acquisition, investment decisions, and thus the asset pricing effects are likely to be present in a general equilibrium setting.

Compared with traditional models of short-sale constraints, our model yields two distinctive results. First, based on our model, short-sale constraints may not necessarily lead to significant overvaluation in stocks that are likely to be subject to short-sale constraints and/or with higher costs of short-selling. With short-sale constraints, investors acquire less information, potentially both positive and negative. So asset prices are less informationally efficient, but the overvaluation effects are much smaller than suggested by the traditional models. These predictions are consistent with the empirical evidence obtained during the short-sale ban during the financial crisis of 2007-2009. Beber and Pagano (2013) and Boehmer, Jones, and Zhang (forthcoming) find that the outright ban on short-selling did not materially affect the valuation of the stocks that are subject to the ban, though it did affect liquidity, price discovery and overall market quality. These predictions are also in agreement with cross-country evidence on the relation between short-sale constraints and market efficiency (Bris, Goetzmann, and Zhu, 2007; and Saffi and Sigurdsson, 2011).

Second, our model shows that short-sale constraints affect stocks that are likely to be directly subject to the constraints as well as stocks that are less likely to be subject to the constraints. Notably, with reduced information acquisition, short-sale constraints can have a significant impact on the unconstrained stocks. Because of the different effects, short-sale constraints can lead to misvaluation — both overvaluation and undervaluation — across different stocks. This misvaluation effect can explain the "good news" in short interest documented in Boehmer, Huszar, and Jordan (2010) and further predicts that short-sale constraints can lead to negative as well as positive jumps in stock prices.

7 Conclusion

In this paper, we develop a model of information acquisition under short-sale constraints and examine the effect of short-sale constraints on information acquisition, portfolio choice, and asset prices. We find that short-sale constraints reduce equilibrium information acquisition, and affect investors' investment decisions both through restrictions on short-selling and through the effects on information acquisition. Our results show that the effects on information acquisition can be much greater than the direct effects of short-sale constraints on investment decisions. More important, with endogenous information acquisition, the effects of short-sale constraints vary across assets with different return and risk characteristics as well as with varying information acquisition costs and costs of short-selling.

We extend the model to cases of costly short-selling, the information acquisition decision of short-sellers, and multiple risky assets. The results from our model differ greatly from those of traditional models of short-sale constraints that do not incorporate information acquisition decisions, and can explain a wide range of empirical observations on the level and activities of short-selling, the cross-sectional and time series of evidence on short-selling activities, the information contents of short interest and short-selling activities, and the effects of short-sale constraints on asset prices.

Our results suggest that the extent of the effects of short-sale constraints may have been underestimated in the existing literature and in policy discussions on short-selling regulations. Restricting short-selling reduces information acquisition, and such a reduction can have profound effects on the functioning of financial markets. For example, Karpoff and Lou (2010) find a strong relation between short-selling and firm financial misconduct, suggesting that short sellers may be able to identify such misconduct through their information gathering and analysis, and their shorting activities (and possibly other efforts in relation to their shorting activities) can help speed up the discovery of such misconduct. With restrictions on short-selling, investors have little incentive to acquire such information; thus, restrictions on short-selling can prolong the life cycle of such misconduct.

Appendix A: Proofs

Lemma 1: The conditional (on signal s) expected utility of the investor that does not face a short-sale constraint is

$$E_2[u(wr_f + (r - r_f)a^*)] = -\exp\left(-\rho w r_f - \frac{1}{2}\frac{(\mu_{r|s} - r_f)^2}{\sigma^2_{r|s}}\right).$$

Proof: We know that if $x \sim N(\mu, \sigma^2)$, then

$$E\left[\exp(ax)\right] = \exp\left(a\mu + \frac{1}{2}a^2\sigma^2\right).$$

The conditional expected utility of the investor is

$$E_2[u(wr_f + (r - r_f)a^*)] \quad = \quad E_2\left[-\exp(-\rho(wr_f + (r - r_f)a^*))\right].$$

Given that $r|s \sim N(\mu_{r|s}, \sigma_{r|s})$, the expected utility of the investor conditional on signal s is

$$E_2[u(wr_f + (r - r_f)a^*)] \quad = \quad -\exp\left(-\rho w r_f - \frac{1}{2}\frac{(\mu_{r|s} - r_f)^2}{\sigma^2_{r|s}}\right).$$

$\square$

Proof of Proposition 1: The expected utility of the investor conditional on signal s can be written as

$$E_2[u(wr_f + (r - r_f)a^*)] \quad = \quad -\exp\left(-\rho w r_f - \frac{1}{2}\frac{(\mu_{r|s} - r_f)^2}{\sigma^2_{r|s}}\right).$$

We know that if $x \sim N(\mu, \sigma^2)$, then

$$E\left[\exp(ax^2)\right] = \frac{1}{\sqrt{1 - 2a\sigma^2}}\exp\left(\frac{a\mu^2}{1 - 2a\sigma^2}\right).$$

Given that $(\mu_{r|s} - r_f) \sim N\left(\mu_r - r_f, \frac{\sigma_r^4}{\sigma_r^2 + \sigma_\epsilon^2}\right)$, the (unconditional) expected utility of the investor can be written as

$$E_1[E_2[u(wr_f + (r - r_f)a^*)]] \quad = \quad -\frac{\exp(-\rho w r_f)}{\sqrt{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}}}\exp\left(\frac{-1}{2\sigma^2_{r|s}}\frac{(\mu_r - r_f)^2}{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}}\right).$$

Therefore, the information problem of the investor can be written as

$$\max -\frac{\exp(-\rho w r_f)}{\sqrt{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}}}\exp\left(\frac{-1}{2\sigma^2_{r|s}}\frac{(\mu_r - r_f)^2}{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}}\right) - \frac{\gamma}{2}\log_2\left(\frac{\sigma_r^2}{\sigma^2_{r|s}}\right). \tag{13}$$

Taking a derivative with respect to σ_ϵ and setting it equal to zero results in the following first-order condition:

$$\sigma_\epsilon^2 = \frac{\sigma_r^2}{\chi^2 - 1},$$

where χ is constant and can be written as $\chi = \frac{\log(2)}{\gamma}\exp\left(-\rho w r_f - \frac{(\mu_r - r_f)^2}{2\sigma_r^2}\right)$.

$\square$

Proof of Proposition 2: The conditional utility of the investor who is facing a short-sale constraint is

$$E_2[u(wr_f + (r - r_f)a^*)] = \begin{cases} -\exp(-\rho w r_f) & \text{if} \quad \mu_{r|s} \le r_f \\ -\exp\left(-\rho w r_f - \frac{1}{2}\frac{(\mu_{r|s} - r_f)^2}{\sigma^2_{r|s}}\right) & \text{if} \quad \mu_{r|s} \ge r_f. \end{cases}$$

We know that if $x \sim N(\mu \geq 0, \sigma^2)$, then

$$E\left[\exp(ax^2)\middle|\, x \geq 0\right] = \frac{e^{\frac{a\mu^2}{1-2a\sigma^2}}\left(1+\mathrm{Erf}\left(\frac{\mu}{\sigma\sqrt{2-4a\sigma^2}}\right)\right)}{\sqrt{1-2a\sigma^2}\,\mathrm{Erfc}\left(-\frac{\mu}{\sqrt{2}\sigma}\right)},$$

where Erf(.) and Erfc(.) are, respectively, the error function and the complementary error function and are defined as[12]

$$\begin{aligned}\mathrm{Erf(z)} &= \frac{2}{\sqrt{\pi}}\int_0^z \exp(-t^2)dt\\ \mathrm{Erfc(z)} &= \frac{2}{\sqrt{\pi}}\int_z^\infty \exp(-t^2)dt.\end{aligned}$$

Given that $(\mu_{r|s} - r_f) \sim N\left(\mu_r - r_f, \frac{\sigma_r^4}{\sigma_r^2+\sigma_\epsilon^2}\right)$, the (conditional) expected utility of the investor can be written as

$$\begin{aligned}E_1[E_2[u(wr_f + (r-r_f)a^*)]|(\mu_{r|s} - r_f) \geq 0] &= -\frac{\exp(-\rho w r_f)}{\sqrt{1+\frac{\sigma_r^2}{\sigma_\epsilon^2}}}\exp\left(\frac{-1}{2\sigma_{r|s}^2}\frac{(\mu_r - r_f)^2}{1+\frac{\sigma_r^2}{\sigma_\epsilon^2}}\right)\\ &\quad\times\frac{1+\mathrm{Erf}\left(\frac{\mu_r - r_f}{\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}\sqrt{2+2\frac{\sigma_r^2}{\sigma_\epsilon^2}}}\right)}{\mathrm{Erfc}\left(-\frac{\mu_r - r_f}{\sqrt{2}\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}}\right)}\\ E_1[E_2[u(wr_f + (r-r_f)a^*)]|(\mu_{r|s} - r_f) \leq 0] &= -\exp(-\rho w r_f).\end{aligned}$$

Therefore the investor's problem can be written as

$$\begin{aligned}\max \qquad & -\frac{1}{2}\mathrm{Erfc}\left(\frac{\mu_r - r_f}{\sqrt{2}\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}}\right)\exp(-\rho w r_f)\\ & -\frac{1}{2}\frac{\exp(-\rho w r_f)}{\sqrt{1+\frac{\sigma_r^2}{\sigma_\epsilon^2}}}\exp\left(\frac{-1}{2\sigma_{r|s}^2}\frac{(\mu_r - r_f)^2}{1+\frac{\sigma_r^2}{\sigma_\epsilon^2}}\right)\\ & \times\left(1+\mathrm{Erf}\left(\frac{\mu_r - r_f}{\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}\sqrt{2+2\frac{\sigma_r^2}{\sigma_\epsilon^2}}}\right)\right) - \frac{\gamma}{2}\log_2\left(\frac{\sigma_r^2}{\sigma_{r|s}^2}\right).\end{aligned}$$

Taking a derivative with respect to σ_ϵ and setting it equal to zero results in the following first-order condition:

$$\mathrm{Erf}\left(\frac{(\mu_r - r_f)\sigma_e}{\sqrt{2}\sigma_r^2}\right) - \frac{2\gamma}{\log(2)}\exp\left(\rho w r_f + \frac{(\mu - r_f)^2}{2\sigma_r^2}\right)\sqrt{1+\frac{\sigma_r^2}{\sigma_\epsilon^2}} + 1 = 0. \tag{14}$$

$$\tag{15}$$

□

Proof of Corollary 3: Let σ_ϵ^{-1} denote the precision of the signal for an investor that does not face short-sale constraints and let $\sigma_{\epsilon_c}^{-1}$ denote the precision of the signal for an investor who faces short-sale constraints. The optimal σ_ϵ^{-1} and $\sigma_{\epsilon_c}^{-1}$ are characterized by the following equations.

$$1 = \chi^{-1}\sqrt{1+\frac{\sigma_r^2}{\sigma_\epsilon^2}}$$

[12] $\mathrm{Erf}(z) = 2\Phi(z\sqrt{2}) - 1$, and $\mathrm{Erfc}(z) = 2\Phi(-z\sqrt{2})$ where $\Phi(z)$ is the standard cumulative distribution function.

$$\phi\left(\frac{(\mu_r - r_f)\sigma_{\epsilon_c}}{\sigma_r^2}\right) - \chi^{-1}\sqrt{1+\frac{\sigma_r^2}{\sigma_{\epsilon_c}^2}} = 0$$

where χ is a constant and can be written as $\chi = \frac{\log(2)}{\gamma} exp\left(-\rho w r_f - \frac{(\mu_r - r_f)^2}{2\sigma_r^2}\right)$.

Therefore

$$\phi\left(\frac{(\mu_r - r_f)\sigma_{\epsilon_c}}{\sigma_r^2}\right) - 1 - \chi^{-1}\sqrt{1+\frac{\sigma_r^2}{\sigma_{\epsilon_c}^2}} + \chi^{-1}\sqrt{1+\frac{\sigma_r^2}{\sigma_{\epsilon}^2}} = 0$$

Given that $\chi \geq 0$ and $\phi(.) \leq 1$ we get that $\sigma_\epsilon \leq \sigma_{\epsilon_c}$ which implies that an investor who faces short-sale constraints acquires less or equal information than an investor who does not face such constraints.

□

Proof of Proposition 3: The conditional utility of the investor who faces costly short-selling is

$$E_2[u(wr_f + (r-r_f)a^*)] = \begin{cases} -\exp\left(-\rho w r_f - \frac{1}{2}\frac{(\mu_{r|s}-r_f)^2}{\sigma_{r|s}^2}\right) & \text{if} \quad \mu_{r|s} - r_f \geq 0 \\ -\exp(-\rho w r_f) & \text{if} \quad \mu_{r|s} - r_f \leq 0 \ \& \ \mu_{r|s} - r_f + \delta \geq 0 \\ -\exp\left(-\rho w r_f - \frac{1}{2}\frac{(\mu_{r|s}-r_f+\delta)^2}{\sigma_{r|s}^2}\right) & \text{if} \quad \mu_{r|s} - r_f + \delta \leq 0. \end{cases}$$

Given that $(\mu_{r|s} - r_f) \sim N\left(\mu_r - r_f, \frac{\sigma_r^4}{\sigma_r^2+\sigma_\epsilon^2}\right)$, the (conditional) expected utility of the investor can be written as

$$\begin{aligned}
E_1[E_2[u(wr_f + (r-r_f)a^*)]|(\mu_{r|s} - r_f) \geq 0] &= -\frac{\exp(-\rho w r_f)}{\sqrt{1+\frac{\sigma_r^2}{\sigma_\epsilon^2}}}\exp\left(\frac{-1}{2\sigma_{r|s}^2}\frac{(\mu_r - r_f)^2}{1+\frac{\sigma_r^2}{\sigma_\epsilon^2}}\right) \\
&\times \frac{1+\operatorname{Erf}\left(\frac{\mu_r - r_f}{\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}\sqrt{2+2\frac{\sigma_r^2}{\sigma_\epsilon^2}}}\right)}{\operatorname{Erfc}\left(-\frac{\mu_r - r_f}{\sqrt{2}\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}}\right)} \\
E_1[E_2[u(wr_f + (r-r_f)a^*)]|\mu_{r|s} - r_f \leq 0 \ \& \ \mu_{r|s} - r_f + \delta \geq 0] &= -\exp(-\rho w r_f) \\
E_1[E_2[u(wr_f + (r-r_f+\delta)a^*)]|(\mu_{r|s} - r_f + \delta) \leq 0] &= -\frac{\exp(-\rho w r_f)}{\sqrt{1+\frac{\sigma_r^2}{\sigma_\epsilon^2}}}\exp\left(\frac{-1}{2\sigma_{r|s}^2}\frac{(\mu_r - r_f+\delta)^2}{1+\frac{\sigma_r^2}{\sigma_\epsilon^2}}\right) \\
&\times \frac{\operatorname{Erfc}\left(\frac{\mu_r - r_f+\delta}{\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}\sqrt{2+2\frac{\sigma_r^2}{\sigma_\epsilon^2}}}\right)}{\operatorname{Erfc}\left(\frac{\mu_r - r_f+\delta}{\sqrt{2}\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}}\right)}.
\end{aligned}$$

Therefore, the information acquisition problem that the investor solves can be written as

$$\begin{aligned}
\max \quad & -\frac{1}{2}\frac{\exp(-\rho w r_f)}{\sqrt{1+\frac{\sigma_r^2}{\sigma_\epsilon^2}}}\exp\left(-\frac{1}{2}\frac{(\mu_r - r_f)^2}{\sigma_r^2}\right) \times \left(1+\operatorname{Erf}\left(\frac{(\mu_r - r_f)\sigma_\epsilon}{\sqrt{2}\sigma_r^2}\right)\right) \\
& -\frac{1}{2}\frac{\exp(-\rho w r_f)}{\sqrt{1+\frac{\sigma_r^2}{\sigma_\epsilon^2}}}\exp\left(-\frac{1}{2}\frac{(\mu_r - r_f+\delta)^2}{\sigma_r^2}\right) \times \operatorname{Erfc}\left(\frac{(\mu_r - r_f+\delta)\sigma_\epsilon}{\sqrt{2}\sigma_r^2}\right) \\
& -\frac{1}{2}\exp(-\rho w r_f)\left(\operatorname{Erfc}\left(\frac{\mu_r - r_f}{\sqrt{2}\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}}\right) - \operatorname{Erfc}\left(\frac{\mu_r - r_f+\delta}{\sqrt{2}\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}}\right)\right) - \frac{\gamma}{2}\log_2\left(\frac{\sigma_r^2}{\sigma_{r|s}^2}\right).
\end{aligned} \tag{16}$$

Taking a derivative with respect to σ_ϵ and setting it equal to zero results in

$$\mathrm{Erf}\left(\frac{(\mu_r - r_f)\sigma_\epsilon}{\sqrt{2}\sigma_r^2}\right) + \exp\left(-\frac{\delta(2\mu_r - 2r_f + \delta)}{2\sigma_r^2}\right)\mathrm{Erfc}\left(\frac{(\mu_r - r_f + \delta)\sigma_\epsilon}{\sqrt{2}\sigma_r^2}\right) + 1$$
$$= \frac{2\gamma}{\log(2)}\exp\left(\rho w r_f + \frac{(\mu_r - r_f)^2}{2\sigma_r^2}\right)\sqrt{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}}.$$

□

Proof of Proposition 4: The conditional utility of the investor who is facing a short-only constraint is

$$E_2[u(wr_f + (r - r_f)a^*)] = \begin{cases} -\exp\left(-\rho w r_f - \frac{1}{2}\frac{(\mu_{r|s} - r_f)^2}{\sigma_{r|s}^2}\right) & \text{if} \quad \mu_{r|s} \le r_f \\ -\exp(-\rho w r_f) & \text{if} \quad \mu_{r|s} \ge r_f. \end{cases}$$

We know that if $x \sim N(\mu \ge 0, \sigma^2)$, then

$$E\left[\exp(ax^2)\middle| x \le 0\right] = \frac{e^{\frac{a\mu^2}{1-2a\sigma^2}}\left(\mathrm{Erfc}\left(\frac{\mu}{\sigma\sqrt{2-4a\sigma^2}}\right)\right)}{\sqrt{1-2a\sigma^2}\,\mathrm{Erfc}\left(\frac{\mu}{\sqrt{2}\sigma}\right)}$$

Given that $(\mu_{r|s} - r_f) \sim N\left(\mu_r - r_f, \frac{\sigma_r^4}{\sigma_r^2 + \sigma_\epsilon^2}\right)$, the (conditional) expected utility of the investor can be written as

$$\begin{aligned}
E_1[E_2[u(wr_f + (r - r_f)a^*)]|(\mu_{r|s} - r_f) \le 0] &= -\frac{\exp(-\rho w r_f)}{\sqrt{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}}}\exp\left(\frac{-1}{2\sigma_{r|s}^2}\frac{(\mu_r - r_f)^2}{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}}\right) \\
&\quad \times \frac{\mathrm{Erfc}\left(\frac{\mu_r - r_f}{\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}\sqrt{2 + 2\frac{\sigma_r^2}{\sigma_\epsilon^2}}}\right)}{\mathrm{Erfc}\left(\frac{\mu_r - r_f}{\sqrt{2}\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}}\right)} \\
E_1[E_2[u(wr_f + (r - r_f)a^*)]|(\mu_{r|s} - r_f) \ge 0] &= -\exp(-\rho w r_f).
\end{aligned}$$

Therefore, the information acquisition problem that the investor solves can be written as

$$\begin{aligned}
\max \qquad & -\frac{1}{2}\frac{\exp(-\rho w r_f)}{\sqrt{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}}}\exp\left(\frac{-1}{2\sigma_{r|s}^2}\frac{(\mu_r - r_f)^2}{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}}\right) \\
& \times \mathrm{Erfc}\left(\frac{\mu_r - r_f}{\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}\sqrt{2 + 2\frac{\sigma_r^2}{\sigma_\epsilon^2}}}\right) - \left(1 - \frac{1}{2}\mathrm{Erfc}\left(\frac{\mu_r - r_f}{\sqrt{2}\frac{\sigma_r}{\sigma_\epsilon}\sigma_{r|s}}\right)\right)\exp(-\rho w r_f) \\
& -\frac{\gamma}{2}\log_2\left(\frac{\sigma_r^2}{\sigma_{r|s}^2}\right).
\end{aligned} \tag{17}$$

Taking a derivative with respect to σ_ϵ and setting it equal to zero results in

$$\mathrm{Erfc}\left(\frac{(\mu_r - r_f)\sigma_e}{\sqrt{2}\sigma_r^2}\right) - \frac{2\gamma}{\log(2)}\exp\left(\rho w r_f + \frac{(\mu - r_f)^2}{2\sigma_r^2}\right)\sqrt{1 + \frac{\sigma_r^2}{\sigma_\epsilon^2}} = 0 \tag{18}$$

□

Lemma 2: The conditional (on signal s) expected utility of the investor that does not face a short-sale constraint and can invest in two risky assets is

$$E_2[u(wr_f + (r_1 - r_f)a_1^* + (r_2 - r_f)a_2^*)] = -\exp\left(-\rho w r_f - \frac{1}{2}\frac{(\mu_{1|s_1} - r_f)^2}{\sigma_{1|s_1}^2} - \frac{1}{2}\frac{(\mu_{2|s_2} - r_f)^2}{\sigma_{2|s_2}^2}\right).$$

Proof of Proposition 5: The expected utility of the investor conditional on signal s can be written as

$$E_2[u(wr_f + (r_1 - r_f)a_1^* + (r_2 - r_f)a_2^*)] = -\exp\left(-\rho w r_f - \frac{1}{2}\frac{(\mu_{1|s_1} - r_f)^2}{\sigma^2_{1|s_1}} - \frac{1}{2}\frac{(\mu_{2|s_2} - r_f)^2}{\sigma^2_{2|s_2}}\right).$$

We know that if $x \sim N(\mu, \sigma^2)$, then

$$E\left[\exp(ax^2)\right] = \frac{1}{\sqrt{1 - 2a\sigma^2}}\exp\left(\frac{a\mu^2}{1 - 2a\sigma^2}\right).$$

Given that $(\mu_{r_i|s_i} - r_f) \sim N\left(\mu_{r_i} - r_f, \frac{\sigma^4_{r_i}}{\sigma^2_{r_i} + \sigma^2_{\epsilon_i}}\right)$, for $i = 1, 2$, the (unconditional) expected utility of the investor can be written as

$$E_1[E_2[u(wr_f + (r_1 - r_f)a_1^*) + (r_2 - r_f)a_2^*)]] =$$
$$-\frac{\exp(-\rho w r_f)}{\sqrt{1 + \frac{\sigma_1^2}{\sigma^2_{\epsilon_1}}}\sqrt{1 + \frac{\sigma_2^2}{\sigma^2_{\epsilon_2}}}}\exp\left(\frac{-1}{2\sigma^2_{1|s_1}}\frac{(\mu_1 - r_f)^2}{1 + \frac{\sigma_1^2}{\sigma^2_{\epsilon_1}}}\right)\exp\left(\frac{-1}{2\sigma^2_{2|s_2}}\frac{(\mu_2 - r_f)^2}{1 + \frac{\sigma_2^2}{\sigma^2_{\epsilon_2}}}\right).$$

Therefore, the information problem of the investor can be written as

$$\begin{aligned} \max \qquad & -\frac{\exp(-\rho w r_f)}{\sqrt{1 + \frac{\sigma^2_{r_1}}{\sigma^2_{\epsilon_1}}}\sqrt{1 + \frac{\sigma^2_{r_2}}{\sigma^2_{\epsilon_2}}}}\exp\left(\frac{-1}{2\sigma^2_{1|s_1}}\frac{(\mu_1 - r_f)^2}{1 + \frac{\sigma_1^2}{\sigma^2_{\epsilon_1}}}\right)\exp\left(\frac{-1}{2\sigma^2_{2|s_2}}\frac{(\mu_2 - r_f)^2}{1 + \frac{\sigma_2^2}{\sigma^2_{\epsilon_2}}}\right) \\ & -\frac{\gamma}{2}\log_2\left(\frac{\sigma_1^2\sigma_2^2}{\sigma^2_{1|s_1}\sigma^2_{2|s_2}}\right). \end{aligned} \tag{19}$$

Taking derivatives with respect to σ_{ϵ_1} and σ_{ϵ_2} and setting them equal to zero results in

$$\gamma = \frac{e^{-\rho w r_f - \frac{(\mu_1 - r_f)^2}{2\sigma_1^2} - \frac{(\mu_2 - r_f)^2}{2\sigma_2^2}}\log(2)}{\sqrt{1 + \frac{\sigma_1^2}{\sigma^2_{\epsilon_1}}}\sqrt{1 + \frac{\sigma_2^2}{\sigma^2_{\epsilon_2}}}}$$
$$\gamma = \frac{e^{-\rho w r_f - \frac{(\mu_1 - r_f)^2}{2\sigma_1^2} - \frac{(\mu_2 - r_f)^2}{2\sigma_2^2}}\log(2)}{\sqrt{1 + \frac{\sigma_1^2}{\sigma^2_{\epsilon_1}}}\sqrt{1 + \frac{\sigma_2^2}{\sigma^2_{\epsilon_2}}}},$$

which implies that

$$\frac{\sigma_1\sigma_2}{\sigma_{1|s_1}\sigma_{2|s_2}} = \frac{\log(2)}{\gamma}\exp\left(-\rho w r_f - \frac{(\mu_1 - r_f)^2}{2\sigma_1^2} - \frac{(\mu_2 - r_f)^2}{2\sigma_2^2}\right). \tag{20}$$

□

Proof of Proposition 6: The conditional utility of the investor who is facing short-sale constraints is

$$\begin{aligned} & E_2[u(wr_f + (r_1 - r_f)a_1^* + (r_2 - r_f)a_2^*)] \\ = & \begin{cases} -\exp(-\rho w r_f) & \text{if} \quad \mu_{r_1|s_1} \leq r_f \;\&\; \mu_{r_2|s_2} \leq r_f \\ -\exp\left(-\rho w r_f - \frac{1}{2}\frac{(\mu_{2|s_2} - r_f)^2}{\sigma^2_{2|s_2}}\right) & \text{if} \quad \mu_{1|s_1} \leq r_f \;\&\; \mu_{2|s_2} \geq r_f \\ -\exp\left(-\rho w r_f - \frac{1}{2}\frac{(\mu_{1|s_1} - r_f)^2}{\sigma^2_{1|s_1}}\right) & \text{if} \quad \mu_{1|s_1} \geq r_f \;\&\; \mu_{2|s_2} \leq r_f \\ -\exp\left(-\rho w r_f - \frac{1}{2}\frac{(\mu_{1|s_1} - r_f)^2}{\sigma^2_{1|s_1}} - \frac{1}{2}\frac{(\mu_{2|s_2} - r_f)^2}{\sigma^2_{2|s_2}}\right) & \text{if} \quad \mu_{1|s_1} \geq r_f \;\&\; \mu_{2|s_2} \geq r_f. \end{cases} \end{aligned}$$

Therefore, the expected utility of the investor can be written as

$$
\begin{aligned}
&E_1[E_2[u(wr_f + (r_1 - r_f)a_1^* + (r_2 - r_f)a_2^*)]] = \\
&\quad -\frac{1}{4}\mathrm{Erfc}\left(\frac{\mu_1 - r_f}{\sqrt{2}\frac{\sigma_1}{\sigma_{\epsilon_1}}\sigma_{1|s_1}}\right)\mathrm{Erfc}\left(\frac{\mu_2 - r_f}{\sqrt{2}\frac{\sigma_2}{\sigma_{\epsilon_2}}\sigma_{2|s_2}}\right)\exp(-\rho w r_f) \\
&\quad -\frac{1}{4}\mathrm{Erfc}\left(\frac{\mu_1 - r_f}{\sqrt{2}\frac{\sigma_1}{\sigma_{\epsilon_1}}\sigma_{1|s_1}}\right)\frac{\exp(-\rho w r_f)}{\sqrt{1+\frac{\sigma_2^2}{\sigma_{\epsilon_2}^2}}}\exp\left(\frac{-1}{2\sigma_{2|s_2}^2}\frac{(\mu_2 - r_f)^2}{1+\frac{\sigma_2^2}{\sigma_{\epsilon_2}^2}}\right)\times\left(1+\mathrm{Erf}\left(\frac{\mu_2 - r_f}{\frac{\sigma_2}{\sigma_{\epsilon_2}}\sigma_{2|s_2}\sqrt{2+2\frac{\sigma_2^2}{\sigma_{\epsilon_2}^2}}}\right)\right) \\
&\quad -\frac{1}{4}\mathrm{Erfc}\left(\frac{\mu_2 - r_f}{\sqrt{2}\frac{\sigma_2}{\sigma_{\epsilon_2}}\sigma_{2|s_2}}\right)\frac{\exp(-\rho w r_f)}{\sqrt{1+\frac{\sigma_1^2}{\sigma_{\epsilon_1}^2}}}\exp\left(\frac{-1}{2\sigma_{1|s_2}^2}\frac{(\mu_1 - r_f)^2}{1+\frac{\sigma_1^2}{\sigma_{\epsilon_1}^2}}\right)\times\left(1+\mathrm{Erf}\left(\frac{\mu_1 - r_f}{\frac{\sigma_1}{\sigma_{\epsilon_1}}\sigma_{1|s_1}\sqrt{2+2\frac{\sigma_1^2}{\sigma_{\epsilon_1}^2}}}\right)\right) \\
&\quad -\frac{1}{4}\frac{\exp(-\rho w r_f)}{\sqrt{1+\frac{\sigma_1^2}{\sigma_{\epsilon_1}^2}}}\exp\left(\frac{-1}{2\sigma_{1|s_1}^2}\frac{(\mu_1 - r_f)^2}{1+\frac{\sigma_1^2}{\sigma_{\epsilon_1}^2}}\right)\times\left(1+\mathrm{Erf}\left(\frac{\mu_1 - r_f}{\frac{\sigma_{r_1}}{\sigma_{\epsilon_1}}\sigma_{1|s_1}\sqrt{2+2\frac{\sigma_1^2}{\sigma_{\epsilon_1}^2}}}\right)\right) \\
&\qquad \times\frac{1}{\sqrt{1+\frac{\sigma_2^2}{\sigma_{\epsilon_2}^2}}}\exp\left(\frac{-1}{2\sigma_{2|s_2}^2}\frac{(\mu_2 - r_f)^2}{1+\frac{\sigma_2^2}{\sigma_{\epsilon_2}^2}}\right)\times\left(1+\mathrm{Erf}\left(\frac{\mu_2 - r_f}{\frac{\sigma_2}{\sigma_{\epsilon_2}}\sigma_{2|s_1}\sqrt{2+2\frac{\sigma_2^2}{\sigma_{\epsilon_2}^2}}}\right)\right).
\end{aligned}
$$

The information acquisition problem of the agent can be written as

$$
\begin{aligned}
\max \quad & -\frac{1}{4}\mathrm{Erfc}\left(\frac{\mu_1 - r_f}{\sqrt{2}\frac{\sigma_1}{\sigma_{\epsilon_1}}\sigma_{1|s_1}}\right)\mathrm{Erfc}\left(\frac{\mu_2 - r_f}{\sqrt{2}\frac{\sigma_2}{\sigma_{\epsilon_2}}\sigma_{2|s_2}}\right)\exp(-\rho w r_f) \\
& -\frac{1}{4}\mathrm{Erfc}\left(\frac{\mu_1 - r_f}{\sqrt{2}\frac{\sigma_1}{\sigma_{\epsilon_1}}\sigma_{1|s_1}}\right)\times\left(1+\mathrm{Erf}\left(\frac{(\mu_2 - r_f)\sigma_{\epsilon_2}}{\sqrt{2}\sigma_2^2}\right)\right)\frac{\exp(-\rho w r_f)}{\sqrt{1+\frac{\sigma_2^2}{\sigma_{\epsilon_2}^2}}}\exp\left(-\frac{1}{2}\frac{(\mu_2 - r_f)^2}{\sigma_2^2}\right) \\
& -\frac{1}{4}\left(1+\mathrm{Erf}\left(\frac{(\mu_1 - r_f)\sigma_{\epsilon_1}}{\sqrt{2}\sigma_1^2}\right)\right)\mathrm{Erfc}\left(\frac{\mu_2 - r_f}{\sqrt{2}\frac{\sigma_2}{\sigma_{\epsilon_2}}\sigma_{2|s_2}}\right)\frac{\exp(-\rho w r_f)}{\sqrt{1+\frac{\sigma_1^2}{\sigma_{\epsilon_1}^2}}}\exp\left(-\frac{1}{2\sigma_1^2}\frac{(\mu_1 - r_f)^2}{\sigma_1^2}\right) \\
& -\frac{1}{4}\left(1+\mathrm{Erf}\left(\frac{(\mu_1 - r_f)\sigma_{\epsilon_1}}{\sqrt{2}\sigma_1^2}\right)\right)\left(1+\mathrm{Erf}\left(\frac{(\mu_2 - r_f)\sigma_{\epsilon_2}}{\sqrt{2}\sigma_2^2}\right)\right) \\
& \quad \times\frac{\exp(-\rho w r_f)}{\sqrt{1+\frac{\sigma_1^2}{\sigma_{\epsilon_1}^2}}\sqrt{1+\frac{\sigma_2^2}{\sigma_{\epsilon_2}^2}}}\exp\left(-\frac{1}{2}\frac{(\mu_1 - r_f)^2}{\sigma_1^2}-\frac{1}{2}\frac{(\mu_2 - r_f)^2}{\sigma_2^2}\right)-\frac{\gamma}{2}\log_2\left(\frac{\sigma_1^2\sigma_2^2}{\sigma_{1|s_1}^2\sigma_{2|s_2}^2}\right).
\end{aligned}
\tag{21}
$$

□

Appendix B: A General Equilibrium Model

In this appendix we sketch a general equilibrium model with endogenous information acquisition. We closely follow the model presented in Section 3 and only present the modifications to that model.

There is a unit measure of ex ante identical investors who are indexed by $j \in [0, 1]$. Two assets are traded in the market: a risk-free asset with a fixed rate of r_f and a risky asset whose price at time $t = 2$ is p and has a payoff of f. All investor have the prior belief that $f \sim N(\mu_r, \sigma_r^2)$, where μ_r and σ_r are the (unconditional) mean and standard deviation of the payoff of the risky asset. The supply of risky asset is $\bar{x} + x$, where $\bar{x}$ is a constant and $x \sim N(0, \sigma_x^2)$. Each investor receives its own signal, denoted by s_j, about the payoff of the risky asset. The signals that the investors receive are independent. The supply of the risky asset is a random variable which implies that investors cannot perfectly infer the signal of other investors.

It is straightforward to show that the price of the risky asset in the second subperiod is

$$p = \frac{1}{r_f}\left(\bar{\mu}_f - \rho\bar{\sigma}_f^2(\bar{x} + x)\right)$$

where $\bar{\mu}_f = \int_0^1 \mu_j dj$, and $\bar{\sigma}_f^2 = \left(\int_0^1 \sigma_j^{-2} dj\right)^{-1}$, are respectively the posterior mean and variance of the payoff of the risky asset for the 'average' investor and $\mu_j = \mathrm{E}[f|\mu_r, s_j, p]$ and $\sigma_j^2 = \mathrm{V}[f|\sigma_r, s_j, p]$ are the posteriors for investor j. In addition, the the expected excess return is

$$E_1[f - r_f p] = \rho\bar{x}\bar{\sigma}_f^2$$

which implies that the expected excess return is increasing in the risk-aversion of the investors and is decreasing as the investors acquire more information.

Let a_j^* denote the optimal holding of the risky asset for investor j, then

$$a_j^* = \frac{\mu_j - pr_f}{\rho\sigma_j^2}$$

Given that the investors are ex ante identical, the information problem of all of them are the same. The information problem of the representative investor can be written as

$$\begin{aligned}
\max_{\sigma_\epsilon^{-1}\geq 0} \quad & -\frac{\exp(-\rho w r_f)}{\sqrt{1 + \frac{\rho^2\sigma_r^2\sigma_x^2\left(\sigma_r^2+\sigma_\epsilon^2\right)\left(1+\rho^2\sigma_x^2\left(\sigma_r^2+\sigma_\epsilon^2\right)\right)}{2\rho^2\sigma_r^4\sigma_x^2+\rho^2\sigma_x^2\sigma_\epsilon^4+\sigma_r^2\left(1+3\rho^2\sigma_x^2\sigma_\epsilon^2\right)}}} \\
& \exp\left(-\frac{\bar{x}^2\rho^4\sigma_r^2\sigma_x^2\left(\sigma_r^2+\sigma_\epsilon^2\right)^2}{2\left(\rho^4\sigma_r^6\sigma_x^4 + \rho^2\sigma_x^2\sigma_\epsilon^4 + \rho^2\sigma_r^4\sigma_x^2\left(3 + 2\rho^2\sigma_x^2\sigma_\epsilon^2\right) + \sigma_r^2\left(1 + 4\rho^2\sigma_x^2\sigma_\epsilon^2 + \rho^4\sigma_x^4\sigma_\epsilon^4\right)\right)}\right) \\
& -\frac{\gamma}{2}\log_2\left(1 + \frac{\sigma_r^2}{\sigma_r^2+\sigma_\epsilon^2} + \frac{\sigma_r^2}{\rho^2\sigma_x^2\left(\sigma_r^2+\sigma_\epsilon^2\right)^2}\right).
\end{aligned}$$

We solve this problem numerically and find that similar to the partial equilibrium model, in general, there is a monotone relationship between the amount of information acquired and the standard deviation of the payoff of the risky asset. In addition, there is an inverse relationship between the information acquired and the variable σ_x.

We conjecture that if the investors are facing short-sale constraints then 1) they acquire less information, and 2) the higher the volatility of the payoff of the risky-asset, the more prominent the effects of short-sale constraints on information acquisition. Two forces interact with each other to produce such effects. Short-sale constraints may reduce the expected utility of an investor due to the restriction on asset holdings. This reduction in expected utility will lead to a reduction in the amount of information acquired if the price of the risky asset is exogenous. However, if the price of the risky asset is endogenous, the price may be less informative as it may not reflect the demand of the investors that receive negative signals but cannot short the asset due to short-sale constraints. This reduction in the informativeness of the price may lead investors to acquire more information about the payoff of the asset. We expect that the reduction in the information acquired due to holding constraints dominates the increase in the information acquired due to the less informative price. This is because the posterior variance of the payoff of the risky asset can be written as

$$\mathrm{V}[f|\sigma_r, s, p] = \left((\sigma_r^2)^{-1} + (\sigma_s^2)^{-1} + \left(\rho^2\sigma_x^2\sigma_s^4\right)^{-1}\right)^{-1}$$

The second term on the right-hand-side of this equation represents the contribution to the posterior variance due to the information in a signal and the third term represents the contribution due to the information in the price. This equation shows that for reasonable parameters ($\rho\sigma_x \gg \sigma_s^{-1}$) the informativeness of signal that is mainly determined by asset holding constraints has first order effect while the informativeness of the price has a second order effect on total information acquired. Therefore, we expect that short-sale constraints reduce information acquisition. In addition, the higher the volatility of the payoff of the risky asset, the higher the probability of a binding short-sale constraint, and therefore the more prominent the effects of short-sale constraints on information acquisition.

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Table 1: Information Acquisition and Investment Decisions *without* Short-Sale Constraints

The table presents the numerical results for information acquisition and investment decisions without short-sale constraints. Panels A, B, and C present the solutions for three levels of information acquisition cost (γ). The first six columns present comparative statics with respect to the (unconditional) expected return of the risky asset, μ_r. The second six columns present comparative statics with respect to the (unconditional) standard deviation of the return of the risky asset, σ_r. The numerical results include information acquisition measured by the entropy I; the post-information acquisition, conditional standard deviation of the risky asset, $\sigma_{r|s}$; the expected investment and the standard deviation of the investment in the risky asset, $E_1[a]$ and $\sigma_1[a]$; and the level of investment in the risky asset without information acquisition, $\bar{a}$.

Panel A: Information Acquisition Cost, $\gamma = 0.1$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	1.350	0.059	0.144	3.983	0.022	0.05	0.629	0.032	4.784	3.650	2.000
1.03	1.347	0.059	0.288	3.968	0.044	0.07	0.982	0.035	3.983	4.810	1.020
1.05	1.322	0.060	0.833	3.817	0.133	0.11	1.201	0.048	2.185	4.329	0.413
1.07	1.270	0.062	1.293	3.530	0.222	0.15	1.270	0.062	1.293	3.530	0.222
1.09	1.193	0.066	1.627	3.136	0.311	0.19	1.301	0.077	0.840	2.918	0.139
1.11	1.091	0.070	1.815	2.670	0.400	0.23	1.316	0.092	0.586	2.470	0.095
1.13	0.963	0.077	1.857	2.173	0.489	0.27	1.326	0.108	0.431	2.134	0.069

Panel B: Information Acquisition Cost, $\gamma = 0.05$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	2.350	0.029	0.577	16.983	0.022	0.05	1.629	0.016	19.136	18.109	2.000
1.03	2.347	0.029	1.151	16.926	0.044	0.07	1.982	0.018	15.934	21.581	1.020
1.05	2.322	0.030	3.332	16.323	0.133	0.11	2.201	0.024	8.741	18.771	0.413
1.07	2.270	0.031	5.172	15.179	0.222	0.15	2.270	0.031	5.172	15.179	0.222
1.09	2.193	0.033	6.508	13.609	0.311	0.19	2.301	0.039	3.361	12.507	0.139
1.11	2.091	0.035	7.258	11.759	0.400	0.23	2.316	0.046	2.345	10.567	0.095
1.13	1.963	0.038	7.426	9.788	0.489	0.27	2.326	0.054	1.724	9.121	0.069

Panel C: Information Acquisition Cost, $\gamma = 0.15$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	0.765	0.088	0.064	1.556	0.022	0.05	0.044	0.048	2.126	0.518	2.000
1.03	0.762	0.088	0.128	1.549	0.044	0.07	0.397	0.053	1.770	1.613	1.020
1.05	0.737	0.090	0.370	1.481	0.133	0.11	0.616	0.072	0.971	1.620	0.413
1.07	0.685	0.093	0.575	1.350	0.222	0.15	0.685	0.093	0.575	1.350	0.222
1.09	0.608	0.098	0.723	1.170	0.311	0.19	0.716	0.116	0.373	1.126	0.139
1.11	0.506	0.106	0.806	0.954	0.400	0.23	0.731	0.139	0.261	0.957	0.095
1.13	0.378	0.115	0.825	0.718	0.489	0.27	0.741	0.162	0.192	0.829	0.069

Table 2: Information Acquisition and Investment Decisions *with* Short-Sale Constraints

The table presents the numerical results for information acquisition and investment decisions with short-sale constraints. Panels A, B, and C present the solutions for three levels of information acquisition cost (γ). The first six columns present comparative statics with respect to the (unconditional) expected return of the risky asset, μ_r. The second six columns present comparative statics with respect to the (unconditional) standard deviation of the return of the risky asset, σ_r. The numerical results include information acquisition measured by the entropy I; the post-information acquisition, conditional standard deviation of the risky asset, $\sigma_{r|s}$; the expected investment and the standard deviation of the investment in the risky asset, $E_1[a]$ and $\sigma_1[a]$; and the level of investment in the risky asset without information acquisition, $\bar{a}$.

Panel A: Information Acquisition Cost, $\gamma = 0.1$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	0.394	0.114	0.317	0.449	0.022	0.05	0.425	0.037	3.675	2.266	2.000
1.03	0.430	0.111	0.365	0.501	0.044	0.07	0.559	0.048	2.414	1.960	1.020
1.05	0.527	0.104	0.551	0.674	0.133	0.11	0.594	0.073	1.200	1.200	0.413
1.07	0.577	0.101	0.730	0.802	0.222	0.15	0.577	0.101	0.730	0.802	0.222
1.09	0.594	0.099	0.894	0.886	0.311	0.19	0.555	0.129	0.503	0.584	0.139
1.11	0.585	0.100	1.033	0.922	0.400	0.23	0.536	0.159	0.375	0.452	0.095
1.13	0.553	0.102	1.139	0.910	0.489	0.27	0.519	0.188	0.295	0.365	0.069

Panel B: Information Acquisition Cost, $\gamma = 0.05$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	1.366	0.058	1.703	2.433	0.022	0.05	1.113	0.023	9.889	7.382	2.000
1.03	1.379	0.058	1.815	2.532	0.044	0.07	1.303	0.028	7.197	6.506	1.020
1.05	1.412	0.056	2.258	2.870	0.133	0.11	1.400	0.042	4.053	4.344	0.413
1.07	1.417	0.056	2.663	3.094	0.222	0.15	1.417	0.056	2.663	3.094	0.222
1.09	1.397	0.057	2.999	3.189	0.311	0.19	1.418	0.071	1.939	2.362	0.139
1.11	1.355	0.059	3.240	3.157	0.400	0.23	1.414	0.086	1.509	1.895	0.095
1.13	1.293	0.061	3.370	3.009	0.489	0.27	1.410	0.102	1.228	1.577	0.069

Panel C: Information Acquisition Cost, $\gamma = 0.15$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	0.014	0.149	0.050	0.062	0.022	0.05	0.044	0.048	2.126	0.518	2.000
1.03	0.042	0.146	0.093	0.114	0.044	0.07	0.220	0.060	1.421	0.924	1.020
1.05	0.146	0.136	0.236	0.256	0.133	0.11	0.235	0.093	0.644	0.557	0.413
1.07	0.208	0.130	0.364	0.351	0.222	0.15	0.208	0.130	0.364	0.351	0.222
1.09	0.237	0.127	0.483	0.412	0.311	0.19	0.180	0.168	0.235	0.241	0.139
1.11	0.239	0.127	0.589	0.439	0.400	0.23	0.156	0.206	0.165	0.176	0.095
1.13	0.215	0.129	0.674	0.427	0.489	0.27	0.136	0.246	0.123	0.135	0.069

Table 3: Information Acquisition and Investment Decisions in Traditional Models of Short-Sale Constraints

The table presents the numerical results for information acquisition and investment decisions in traditional models of short-sale constraints. In such models, investors acquire information *as if* they do not face short-sale constraints but face short-sale constraints in their investment decisions. Panels A, B, and C present the solutions for three levels of information acquisition cost (γ). The first six columns present comparative statics with respect to the (unconditional) expected return of the risky asset, μ_r. The second six columns present comparative statics with respect to the (unconditional) standard deviation of the return of the risky asset, σ_r. The numerical results include information acquisition measured by the entropy I; the post-information acquisition, conditional standard deviation of the risky asset, $\sigma_{r|s}$; the expected investment and the standard deviation of the investment in the risky asset, $E_1[a]$ and $\sigma_1[a]$; and the level of investment in the risky asset without information acquisition, $\bar{a}$.

Panel A: Information Acquisition Cost, $\gamma = 0.1$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	1.350	0.059	1.662	2.374	0.022	0.05	0.629	0.032	4.946	3.354	2.000
1.03	1.347	0.059	1.731	2.415	0.044	0.07	0.982	0.035	4.533	3.991	1.020
1.05	1.322	0.060	1.975	2.507	0.133	0.11	1.201	0.048	3.035	3.227	0.413
1.07	1.270	0.062	2.148	2.485	0.222	0.15	1.270	0.062	2.148	2.485	0.222
1.09	1.193	0.066	2.229	2.350	0.311	0.19	1.301	0.077	1.632	1.983	0.139
1.11	1.091	0.070	2.209	2.118	0.400	0.23	1.316	0.092	1.306	1.638	0.095
1.13	0.963	0.077	2.094	1.816	0.489	0.27	1.326	0.108	1.084	1.390	0.069

Panel B: Information Acquisition Cost, $\gamma = 0.05$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	2.350	0.029	7.068	10.112	0.022	0.05	1.629	0.016	20.489	15.892	2.000
1.03	2.347	0.029	7.343	10.273	0.044	0.07	1.982	0.018	18.822	17.441	1.020
1.05	2.322	0.030	8.313	10.647	0.133	0.11	2.201	0.024	12.657	13.776	0.413
1.07	2.270	0.031	8.990	10.564	0.222	0.15	2.270	0.031	8.990	10.564	0.222
1.09	2.193	0.033	9.292	10.038	0.311	0.19	2.301	0.039	6.849	8.420	0.139
1.11	2.091	0.035	9.187	9.134	0.400	0.23	2.316	0.046	5.491	6.954	0.095
1.13	1.963	0.038	8.691	7.959	0.489	0.27	2.326	0.054	4.565	5.904	0.069

Panel C: Information Acquisition Cost, $\gamma = 0.15$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	0.765	0.088	0.653	0.930	0.022	0.05	0.044	0.048	2.126	0.518	2.000
1.03	0.762	0.088	0.684	0.948	0.044	0.07	0.397	0.053	1.882	1.428	1.020
1.05	0.737	0.090	0.794	0.988	0.133	0.11	0.616	0.072	1.245	1.250	0.413
1.07	0.685	0.093	0.874	0.975	0.222	0.15	0.685	0.093	0.874	0.975	0.222
1.09	0.608	0.098	0.915	0.909	0.311	0.19	0.716	0.116	0.660	0.780	0.139
1.11	0.506	0.106	0.912	0.795	0.400	0.23	0.731	0.139	0.526	0.645	0.095
1.13	0.378	0.115	0.870	0.642	0.489	0.27	0.741	0.162	0.435	0.548	0.069

Table 4: Information Acquisition and Investment Decisions with Costly Short-Selling

The table presents the numerical results for information acquisition and investment decisions with costly selling constraints. Panels A, B, and C present the solutions of the model with information acquisition cost, $\gamma = 0.1$, and with three levels of costs of short-selling. Panels D, E, and F present the solutions of the model with information acquisition cost, $\gamma = 0.15$, and with three levels of costs of short-selling. In all panels, the first six columns present comparative statics with respect to the (unconditional) expected return of the risky asset, μ_r. The second six columns present comparative statics with respect to the (unconditional) standard deviation of the return of the risky asset, σ_r. The numerical results include information acquisition measured by the entropy I; the post-information acquisition, conditional standard deviation of the risky asset, $\sigma_{r|s}$; the expected investment and the standard deviation of the investment in the risky asset, $E_1[a]$ and $\sigma_1[a]$; and the level of investment in the risky asset without information acquisition, $\bar{a}$.

Panel A: Short-Selling Cost, $\delta = 0.02$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	1.306	0.061	0.384	3.516	0.022	0.05	0.471	0.036	3.908	2.531	2.000
1.03	1.301	0.061	0.508	3.488	0.044	0.07	0.799	0.040	3.275	3.271	1.020
1.05	1.263	0.062	0.966	3.301	0.133	0.11	1.094	0.052	2.086	3.419	0.413
1.07	1.202	0.065	1.330	3.009	0.222	0.15	1.202	0.065	1.330	3.009	0.222
1.09	1.116	0.069	1.575	2.640	0.311	0.19	1.252	0.080	0.900	2.592	0.139
1.11	1.006	0.075	1.693	2.228	0.400	0.23	1.279	0.095	0.643	2.249	0.095
1.13	0.875	0.082	1.694	1.804	0.489	0.27	1.296	0.110	0.480	1.975	0.069

Panel B: Short-Selling Cost, $\delta = 0.01$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	1.330	0.060	0.273	3.754	0.022	0.05	0.527	0.035	4.206	2.891	2.000
1.03	1.326	0.060	0.407	3.732	0.044	0.07	0.888	0.038	3.619	3.943	1.020
1.05	1.294	0.061	0.909	3.559	0.133	0.11	1.150	0.050	2.152	3.859	0.413
1.07	1.237	0.064	1.320	3.266	0.222	0.15	1.237	0.064	1.320	3.266	0.222
1.09	1.156	0.067	1.607	2.882	0.311	0.19	1.277	0.078	0.875	2.754	0.139
1.11	1.049	0.072	1.757	2.441	0.400	0.23	1.298	0.094	0.617	2.359	0.095
1.13	0.918	0.079	1.775	1.979	0.489	0.27	1.311	0.109	0.457	2.055	0.069

Panel C: Short-Selling Cost, $\delta = 0.05$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	1.212	0.065	0.611	2.783	0.022	0.050	0.428	0.037	3.690	2.282	2.000
1.030	1.203	0.065	0.704	2.747	0.044	0.070	0.625	0.045	2.639	2.258	1.020
1.050	1.152	0.068	1.033	2.554	0.133	0.110	0.917	0.058	1.787	2.352	0.413
1.070	1.079	0.071	1.276	2.301	0.222	0.150	1.079	0.071	1.276	2.301	0.222
1.090	0.987	0.076	1.425	2.008	0.311	0.190	1.164	0.085	0.925	2.121	0.139
1.110	0.877	0.082	1.486	1.701	0.400	0.230	1.212	0.099	0.690	1.922	0.095
1.130	0.754	0.089	1.475	1.400	0.489	0.270	1.243	0.114	0.530	1.738	0.069

Table 4: Continued –

Panel D: Short-Selling Cost, $\delta = 0.02$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	0.694	0.093	0.163	1.282	0.022	0.05	0.044	0.048	2.126	0.518	2.000
1.03	0.689	0.093	0.215	1.270	0.044	0.07	0.263	0.058	1.505	1.070	1.020
1.05	0.653	0.095	0.410	1.191	0.133	0.11	0.484	0.079	0.879	1.151	0.413
1.07	0.593	0.099	0.566	1.068	0.222	0.15	0.593	0.099	0.566	1.068	0.222
1.09	0.512	0.105	0.674	0.913	0.311	0.19	0.648	0.121	0.386	0.945	0.139
1.11	0.413	0.113	0.733	0.743	0.400	0.23	0.678	0.144	0.278	0.833	0.095
1.13	0.301	0.122	0.754	0.571	0.489	0.27	0.697	0.167	0.208	0.739	0.069

Panel E: Short-Selling Cost, $\delta = 0.01$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	0.732	0.090	0.119	1.421	0.022	0.05	0.044	0.048	2.126	0.518	2.000
1.03	0.729	0.091	0.177	1.412	0.044	0.07	0.313	0.056	1.604	1.258	1.020
1.05	0.697	0.093	0.395	1.335	0.133	0.11	0.552	0.075	0.932	1.373	0.413
1.07	0.641	0.096	0.575	1.206	0.222	0.15	0.641	0.096	0.575	1.206	0.222
1.09	0.561	0.102	0.701	1.037	0.311	0.19	0.683	0.118	0.382	1.035	0.139
1.11	0.458	0.109	0.770	0.841	0.400	0.23	0.706	0.141	0.271	0.895	0.095
1.13	0.336	0.119	0.786	0.635	0.489	0.27	0.720	0.164	0.201	0.784	0.069

Panel F: Short-Selling Cost, $\delta = 0.05$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	0.534	0.104	0.229	0.842	0.022	0.050	0.044	0.048	2.126	0.518	2.000
1.030	0.527	0.104	0.265	0.831	0.044	0.070	0.223	0.060	1.428	0.935	1.020
1.050	0.486	0.107	0.393	0.772	0.133	0.110	0.314	0.088	0.723	0.715	0.413
1.070	0.432	0.111	0.498	0.700	0.222	0.150	0.432	0.111	0.498	0.700	0.222
1.090	0.372	0.116	0.581	0.624	0.311	0.190	0.524	0.132	0.370	0.688	0.139
1.110	0.309	0.121	0.648	0.549	0.400	0.230	0.582	0.154	0.282	0.651	0.095
1.130	0.243	0.127	0.700	0.471	0.489	0.270	0.620	0.176	0.220	0.606	0.069

Table 5: Information Acquisition and Investment Decisions of Short-Only Investors

The table presents the numerical results for information acquisition and investment decisions of short-only investors (i.e., investors with long constraints). Panels A, B, and C present the solutions for three levels of information acquisition cost (γ). The first six columns present comparative statics with respect to the (unconditional) expected return of the risky asset, μ_r. The second six columns present comparative statics with respect to the (unconditional) standard deviation of the return of the risky asset, σ_r. The numerical results include information acquisition measured by the entropy I; the post-information acquisition, conditional standard deviation of the risky asset, $\sigma_{r|s}$; the expected investment and the standard deviation of the investment in the risky asset, $E_1[a]$ and $\sigma_1[a]$; and the level of investment in the risky asset without information acquisition, $\bar{a}$.

Panel A: Information Acquisition Cost, $\gamma = 0.1$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	0.295	0.122	-0.215	0.328	0.000	0.05	0.000	0.050	0.000	0.000	0.000
1.03	0.000	0.150	0.000	0.000	0.000	0.07	0.000	0.070	0.000	0.000	0.000
1.05	0.000	0.150	0.000	0.000	0.000	0.11	0.000	0.110	0.000	0.000	0.000
1.07	0.000	0.150	0.000	0.000	0.000	0.15	0.000	0.150	0.000	0.000	0.000
1.09	0.000	0.150	0.000	0.000	0.000	0.19	0.000	0.190	0.000	0.000	0.000
1.11	0.000	0.150	0.000	0.000	0.000	0.23	0.000	0.230	0.000	0.000	0.000
1.13	0.000	0.150	0.000	0.000	0.000	0.27	0.000	0.270	0.000	0.000	0.000

Panel B: Information Acquisition Cost, $\gamma = 0.05$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	1.333	0.060	-1.480	2.219	0.000	0.05	0.000	0.050	0.000	0.000	0.000
1.03	1.313	0.060	-1.370	2.106	0.000	0.07	0.000	0.070	0.000	0.000	0.000
1.05	1.207	0.065	-0.953	1.624	0.000	0.11	0.000	0.110	0.000	0.000	0.000
1.07	1.041	0.073	-0.584	1.116	0.000	0.15	1.041	0.073	-0.584	1.116	0.000
1.09	0.000	0.150	0.000	0.000	0.000	0.19	1.138	0.086	-0.609	1.094	0.000
1.11	0.000	0.150	0.000	0.000	0.000	0.23	1.189	0.101	-0.588	1.017	0.000
1.13	0.000	0.150	0.000	0.000	0.000	0.27	1.221	0.116	-0.554	0.933	0.000

Panel C: Information Acquisition Cost, $\gamma = 0.15$

μ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$	σ_r	I	$\sigma_{r\|s}$	$E_1[a]$	$\sigma_1[a]$	$\bar{a}$
1.025	0.000	0.150	0.000	0.000	0.000	0.05	0.000	0.050	0.000	0.000	0.000
1.03	0.000	0.150	0.000	0.000	0.000	0.07	0.000	0.070	0.000	0.000	0.000
1.05	0.000	0.150	0.000	0.000	0.000	0.11	0.000	0.110	0.000	0.000	0.000
1.07	0.000	0.150	0.000	0.000	0.000	0.15	0.000	0.150	0.000	0.000	0.000
1.09	0.000	0.150	0.000	0.000	0.000	0.19	0.000	0.190	0.000	0.000	0.000
1.11	0.000	0.150	0.000	0.000	0.000	0.23	0.000	0.230	0.000	0.000	0.000
1.13	0.000	0.150	0.000	0.000	0.000	0.27	0.000	0.270	0.000	0.000	0.000

Table 6: Information Acquisition and Investment Decisions with Two Risky Assets

The table presents the numerical results for information acquisition and investment decisions with two risky assets. Panel A presents comparative statics with respect to the (unconditional) expected return of the second risky asset, μ_2, with the expected return of the first risky asset, μ_1, and the standard deviations of both risky asset returns, σ_1 and σ_2 fixed. Panel B presents comparative statics with respect to the (unconditional) standard deviation of the second risky asset return, σ_2, with the standard deviation of the first risky asset, σ_1, and the expected returns of both risky asset returns, μ_1 and μ_2 fixed. In both panels, the information acquisition costs are set to be $\gamma = 0.1$. The numerical results include the total information acquired measured by the entropy I_U (without short-sale constraints) and I_C (with short-sale constraints). The other results are obtained for the two risky assets with short-sale constraints: the conditional standard deviations of the two risky assets, $\sigma_{i|s}, (i = 1, 2)$; the expected value and the standard deviations of investment in the two risky assets, $E_1[a_i]$ and $\sigma_1[a_i]$, $(i = 1, 2)$; and the level of investment in the two risky assets without information acquisition (or the unconditional value), $\bar{a}_i, (i = 1, 2)$.

Panel A: Expected Returns and Asset Allocation with Two Risky Assets

μ_1	μ_2	I_U	I_C	$\sigma_{1\|s}$	$\sigma_{2\|s}$	$E_1[a_1]$	$E_1[a_2]$	$\sigma_1[a_1]$	$\sigma_1[a_2]$	$\bar{a}_1$	$\bar{a}_2$
1.070	1.025	1.270	0.725	0.112	0.121	0.555	0.256	0.591	0.360	0.222	0.022
1.070	1.030	1.267	0.726	0.113	0.121	0.544	0.281	0.578	0.382	0.222	0.044
1.070	1.050	1.241	0.715	0.116	0.118	0.508	0.383	0.534	0.455	0.222	0.133
1.070	1.070	1.190	0.686	0.118	0.118	0.480	0.480	0.499	0.499	0.222	0.222
1.070	1.090	1.113	0.637	0.120	0.120	0.458	0.561	0.471	0.507	0.222	0.311
1.070	1.110	1.011	0.555	0.121	0.126	0.447	0.598	0.458	0.450	0.222	0.400
1.070	1.130	0.882	0.318	0.121	0.149	0.449	0.495	0.460	0.076	0.222	0.489

Panel B: Standard Deviations and Asset Allocation with Two Risky Assets

σ_1	σ_2	I_U	I_C	$\sigma_{1\|s}$	$\sigma_{2\|s}$	$E_1[a_1]$	$E_1[a_2]$	$\sigma_1[a_1]$	$\sigma_1[a_2]$	$\bar{a}_1$	$\bar{a}_2$
0.150	0.050	1.341	0.150	0.141	0.048	0.274	2.177	0.222	0.621	0.222	2.000
0.150	0.070	1.452	0.292	0.136	0.063	0.311	1.277	0.279	0.736	0.222	1.020
0.150	0.110	1.345	0.541	0.128	0.089	0.385	0.735	0.379	0.668	0.222	0.413
0.150	0.150	1.190	0.686	0.118	0.118	0.480	0.480	0.499	0.499	0.222	0.222
0.150	0.190	1.050	0.540	0.103	0.190	0.685	0.139	0.749	0.000	0.222	0.139
0.150	0.230	0.928	0.552	0.102	0.230	0.699	0.095	0.765	0.000	0.222	0.095
0.150	0.270	0.822	0.558	0.102	0.270	0.707	0.069	0.775	0.000	0.222	0.069

Figure 1: Sequence of Events in the Model

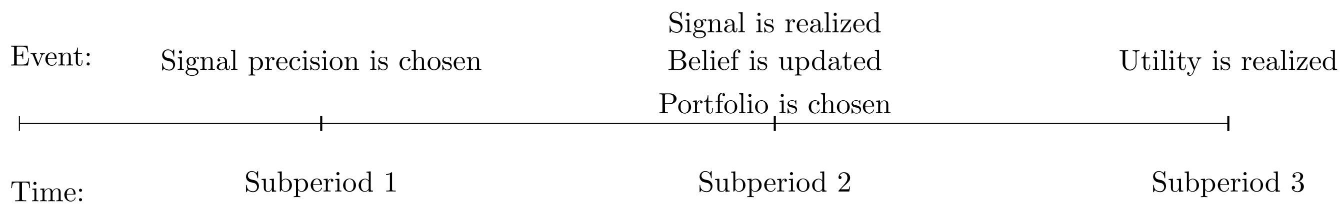

Figure 2: Information Acquisition without Short-Sale Constraints

The figure plots optimal information acquisition *without* short-sale constraints corresponding to different levels of expected returns and standard deviations of the risky asset. Information acquisition is measured by the entropy I and the information acquisition cost $\gamma = 0.1$.

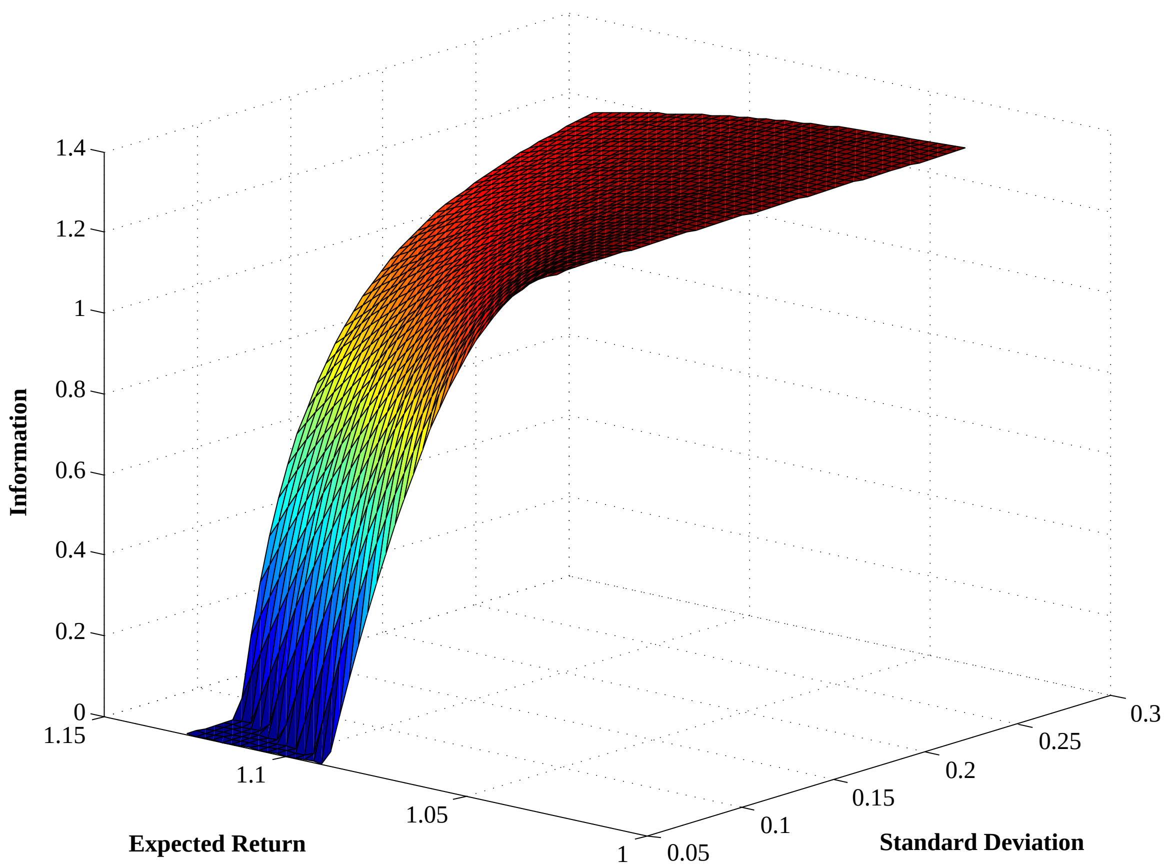

Figure 3: Information Acquisition with Short-Sale Constraints

The figure plots optimal information acquisition *with* short-sale constraints corresponding to different levels of expected returns and standard deviations of the risky asset. Information acquisition is measured by the entropy I and the information acquisition cost $\gamma = 0.1$.

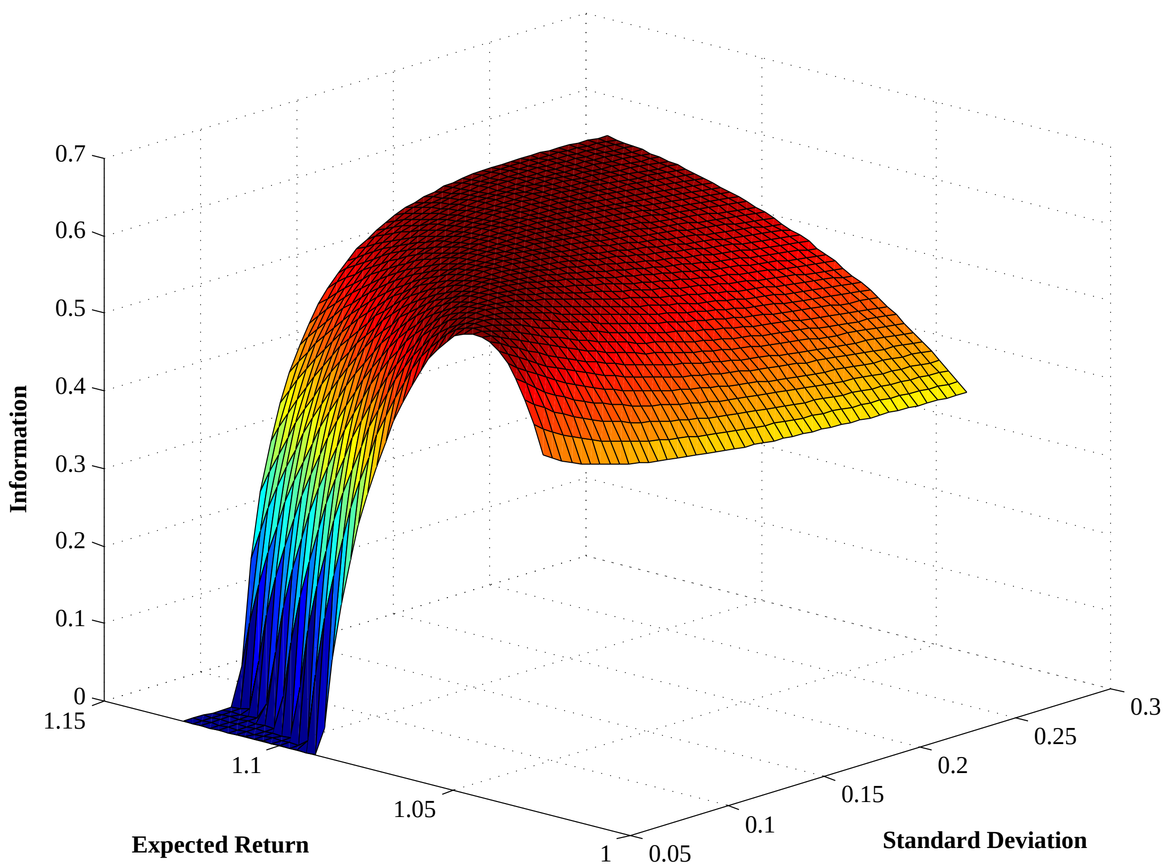

Figure 4: Differences in Information Acquisition with and without Short-Sale Constraints

The figure plots the differences in optimal information acquisition *with* and *without* short-sale constraints. The differences of information acquisition, measured by the entropy I, correspond to different levels of expected returns and standard deviations of the risky asset.

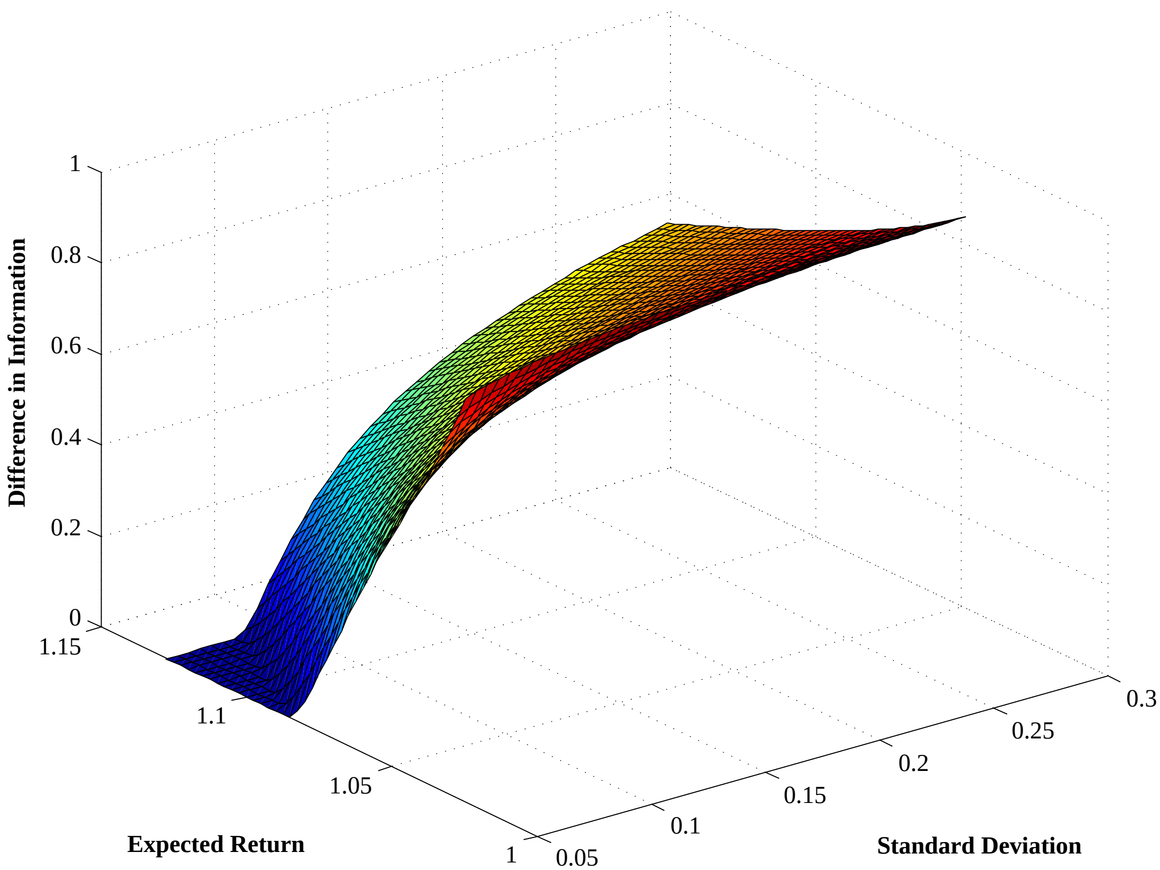

Figure 5: Ratios of Expected Investments under Short-Sale Constraints

The figure plots the ratios of expected investments in the risky asset under short-sale constraints. The ratio is the level of expected investment obtained from the traditional models relative to the level of expected investment from the endogenous information acquisition model.