

Payout Policy under Heterogeneous Beliefs: A Theory of Dividends versus Stock Repurchases, Price Impact, and Long-Run Stock Returns

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Abstract

We analyze a firm's choice between dividends and stock repurchases in a setting of heterogeneous beliefs and short sale constraints. We study a setting in which the insiders of a firm, owning a certain fraction of its equity and having a certain amount of cash to distribute to shareholders, choose between paying out cash dividends and buying back equity to shareholders, as well the scale of investment in their firm's new project. Outside equity holders in the firm have heterogeneous beliefs about the probability of success of the firm's project and therefore its long run prospects; they may also disagree with firm insiders about this probability. We show that, depending on the beliefs of firm insiders versus outsiders, the firm may distribute value by cash dividends alone; through a repurchase alone; or through a combination of a cash dividend and a stock repurchase. We also show that, in many situations, it is optimal for firm insiders to underinvest in the firm's positive net present value project and undertake a stock repurchase with the amount of cash saved by underinvesting. We then analyze the price impact of a cash dividend versus a share repurchase, where the price impact is defined as the abnormal return to the firm's equity upon the actual payment of a cash dividend or the implementation of a share repurchase respectively (rather than the announcement of these events). Finally, we analyze the long-run returns to a firm's equity following dividend payments and stock repurchases. Our model generates a number of testable predictions different from asymmetric information models of a firm's choice between dividends versus stock repurchases.

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1 Introduction

In recent years, the number of firms undertaking stock repurchases has increased dramatically, while the proportion of firms distributing value through cash dividends has declined (see, e.g., Fama and French (2001)). The popularity of share repurchases has not been mitigated even after the passage of the Jobs and Growth Tax Relief Act of 2003 in the U.S., which cut the dividend tax rate to 15%, thus substantially reducing the tax disadvantage of dividend payments to investors (see, e.g., Chetty and Saez (2006)). There have also been several articles in the popular and practitioner oriented press indicating that, in some situations, firms cut back on some positive net present value projects and use the cash saved for stock repurchases.¹ Other such articles indicate that firms that do not have cash on hand go to the extent of borrowing money to undertake stock repurchase programs.² Finally, a growing number of firms seem to be using a combination of dividends and stock repurchases to distribute value to shareholders.

The above evidence and anecdotes raise several interesting questions: What are the advantages and disadvantages of dividends and share repurchases from the point of view of maximizing shareholder wealth? Are there any situations where shareholders are better off if the firm underinvests in positive net present value projects and uses the cash to buy back equity? Is there an optimal combination of share repurchases and dividend payments that a firm should undertake? Is it ever optimal for a firm to fund a repurchase or a dividend payment by raising external financing? What are the longer term implications (price impact and long-run stock returns) of dividend payments and stock repurchase programs?

Given that, under perfect capital markets, stock repurchases and dividends are equivalent from the

¹See, e.g., the *New York Times* article, “As Layoffs Rise, Stock Buybacks Consume Cash,” November 21, 2011, describing how, while Pfizer cut back on its research budget and laid off 1,100 employees, it added \$5 billion to the \$ 4 billion it already earmarked for stock repurchases in 2011 and beyond.

²See, e.g., the *Wall Street Journal* article, “Intel Borrows \$6 Billion to Help Fund Stock Buyback,” December 4, 2012, mentioning that Intel borrowed \$6 billion in 2012 partly to fund a stock repurchase.

point of view of market value maximization, the usual theoretical justification given for stock repurchases rests on asymmetric information. In particular, a number of authors have advanced signaling models of dividends: see, e.g., Dann (1981), Vermaelen (1981), Ofer and Thakor (1987), and Constantinides and Grundy (1989). While the details vary across these models, the basic idea underlying signaling models is that firm insiders have private information about its future prospects, and buy back equity when they believe that its equity is undervalued, thus signaling their private information to outside shareholders. Thus, signaling models can explain the announcement effects of share repurchase programs, especially in the context of tender offers: see, e.g., Vermaelen (1981), Asquith and Mullins (1986), Comment and Jarrell (1991), D'Mello and Shroff (2000), or Louis and White (2007) for evidence. However, while share repurchases can serve as a credible signal in the context of tender offer repurchases, it is more difficult to believe that open market repurchases can serve as a credible signal, given that the repurchasing firm does not need to commit to repurchasing the entire amount of the share repurchase authorized by its board. It is worth noting here that open market repurchases constitute around 90% of the stock repurchases consummated in recent years: see, e.g., Comment and Jarrell (1991) or Grullon and Michaely (2004).³ Further, asymmetric information models with fully rational investors cannot explain the positive abnormal long-run stock returns that have been documented by the empirical literature (see, e.g., Ikenberry, Lakonishok, and Vermaelen (1995), and Peyer and Vermaelen (2009)) following stock repurchases.⁴ The above suggest the need for alternative theories that can better explain many aspects of a firm's choice between dividends and share repurchases, and the long-run impact of dividend payments and share repurchase programs on shareholder wealth.

The objective of this paper is to fill this gap in the literature by developing a new theory of a firm's

³See, however, Oded (2005), who develops a theoretical model demonstrating that, under suitable assumptions, open market share repurchase programs can signal firm insiders' private information even in the absence of a commitment by the firm to repurchase the entire amount of equity authorized to be repurchased by its board.

⁴Asymmetric information models will not be able to generate a significant price impact for a stock repurchase as well, since there is no new information-flow from firm insiders to outsiders when a stock repurchase (that has already been announced) is actually implemented.

choice between dividends and stock repurchases in a setting of heterogeneous beliefs and short sale constraints. We study a setting in which the insiders of a firm, owning a certain fraction of its equity and having a certain amount of cash to distribute to shareholders, choose between paying out cash dividends and buying back equity to shareholders, as well the scale of investment in their firm's new project. Outside equity holders in the firm have heterogeneous beliefs about the probability of success of the firm's project and therefore its long run prospects; they may also disagree with firm insiders about this probability.

Our theoretical analysis is in four parts. In the next section (section 2) we develop our basic model where we do not allow the firm to raise external financing to fund a stock repurchase. In this section the decision facing firm insiders is regarding how to allocate the cash available in the firm between investing in the firm's project (they may choose to undertake it up to the full investment level or to underinvest in it) and distributing it to shareholders; they also decide on the optimal combination of a cash dividend and a stock repurchase to distribute this value to shareholders. We show that, depending on the beliefs of firm insiders versus different groups of outsiders, the firm may distribute value by cash dividends alone; through a repurchase alone; or through a combination of a cash dividend and a stock repurchase. We also show that, in many situations, it is optimal for firm insiders to underinvest in the firm's positive net present value project and undertake a stock repurchase with the amount of cash saved by underinvesting.

In the following section (section 3), we analyze the price impact of cash dividends and stock repurchases. Here the price impact is defined as the abnormal return to the firm's equity upon the actual payment of a cash dividend or the implementation of a share repurchase respectively (not the abnormal stock return upon the announcement of these events). Here we show that, while the price impact of a dividend payment will be zero, the price impact of a stock repurchase will be positive; further, the

larger the number of shares the firm repurchases, the greater the price impact on average.

In the subsequent section (section 4), we study an extension of our basic model where we allow the firm to raise external financing (in the form of equity or debt) that may be used to fund either its new project or its dividend payment or stock repurchase. We show that, in some situations, it is optimal for a firm to issue equity (but never debt) to fund a dividend payment; we show that, in other situations, it is optimal for it to issue debt (but never equity) to fund a stock repurchase. As in our basic model, the choice between dividends and repurchases is driven by the relative optimism of firm insiders and different groups of outside shareholders about the firm's future prospects.

Finally (in section 5), we study an extension to our basic model where we analyze the long-run returns to a firm's equity following dividend payments and stock repurchases. Here we characterize the conditions under which the long run stock returns following dividend payments and repurchases will be positive and those under which it will be negative. We show that, if a firm does not underinvest in its project, the long-run stock returns following a stock repurchase will always be positive. Further, the long-run stock returns of a sample of firms that do not underinvest in their project and distribute value to shareholders through a stock repurchase will, on average, exceed that of a similar sample of firms that distributes value through cash dividends.

Our model generates a number of new testable predictions (detailed in section 6) regarding a firm's choice between dividends and stock repurchases. To the best of our knowledge, all the predictions of our model regarding the price impact of dividends and stock repurchases and the long-run stock returns following dividend payments and stock repurchases are new to the literature. While some of these predictions provide a theoretical rationale for observed empirical regularities, there is no evidence so far in the literature regarding some of our other model predictions: they can therefore serve to generate testable hypotheses for new empirical tests.

The paper in the existing literature closest in spirit to our paper is that of Huang and Thakor (2012). While developing testable hypotheses for their empirical analysis, they build on the existing theoretical literature on the consequences of disagreement between managers and outsiders (e.g., Boot, Gopalan, and Thakor (2006)) to argue that open market share repurchases may be a way for managers to buy back equity from outsiders who disagree with them, thus increasing firm value. The predictions here are that firms are more likely to repurchase equity when there is a greater disagreement regarding financial policies between firm managers and outsiders, and that this disagreement is reduced following a share repurchase. They find support for these predictions in their empirical analysis. Huang and Thakor (2012) assume that disagreement between managers and outside decrease firm value; in contrast, in our model any valuation effects of differences in beliefs between firm insiders and outsiders arise endogenously. However, the empirical finding of Huang and Thakor (2012) that stock repurchases are more likely when there is more disagreement between firm insiders and outside equity holders provide support for the theoretical predictions of our model as well. It is worth noting that, while Huang and Thakor (2012) provide a disagreement-based explanation for open market stock repurchases, they do not analyze a firm's choice between cash dividends and stock repurchases. Neither do they analyze the price impact or the long-run stock returns following cash dividends or stock repurchases.

Apart from the literature on payout policy discussed above, our paper is also related to the emerging literature in economics and finance on the effect of heterogeneity in investor beliefs on long-run stock returns and valuations and on trading among investors. Starting with Miller (1977), a number of authors have theoretically examined the stock price implications of heterogeneous beliefs and short sale constraints on stock valuations. Miller (1977) argues that when investors have heterogeneous beliefs about the future prospects of a firm, its stock price will reflect the valuation that optimists attach to it, because the pessimists will simply sit out the market (if they are constrained from short-selling). A

number of subsequent authors have developed theoretical models that derive some of the most interesting cross-sectional implications of Miller's logic. In an important paper, Morris (1996) shows that the greater the divergence in the valuations of the optimists and the pessimists, the higher the current price of a stock in equilibrium, and hence lower the subsequent returns (see also Allen and Morris (1998)). In another important paper, Duffie, Garleanu, and Pedersen (2002) show that, even when short-selling is allowed (but requires searching for security lenders and bargaining over the lending fee), the price of a security will be elevated and can be expected to decline subsequently in an environment of heterogeneous beliefs among investors if lendable securities are difficult to locate. Another important implication of heterogeneous beliefs among investors is that it can lead to a significant amount of speculative trading: see, e.g., Harris and Raviv (1993), who use differences in opinion among investors to explain empirical regularities about the relationship between stock price and volume.⁵

The corporate finance implications of heterogeneous beliefs have been relatively less widely studied. Allen and Gale (1999) examine how heterogeneous priors among investors affect the source of financing (banks versus equity) of new projects. Garmaise (2001) analyzes the optimal design of securities by a cash-constrained firm facing investors with diverse beliefs: however, his focus is on comparing optimal designs when investors have rational beliefs (in the sense of Kurz (1994)) versus rational expectations. Dittmar and Thakor (2007) study a firm's choice between issuing debt and equity when insiders and outsiders disagree about the firm's choice of project to invest in. Finally Bayar, Chemmanur, and Liu (2011), develop a model of equity carve-outs and negative stub values in a setting of heterogeneous beliefs.

The rest of the paper is organized as follows. In Section 2, we describe the structure of our basic model and analyze a firm's choice between cash dividends and stock repurchases. Section 3 analyzes

⁵Several other authors have also examined the asset pricing and trading implications of heterogeneous beliefs (see, e.g., Harrison and Kreps (1978), Varian (1985, 1989), Kandel and Pearson (1995), and Chen, Hong, and Stein (2002) for contributions to this literature, and Scheinkman and Xiong (2004) for a review.)

the price impact of cash dividends and stock repurchases on a firm's equity. Section 4 extends the analysis in our basic model to allow the firm to raise external financing in the form of equity or debt to fund its new project or its dividend payments or stock repurchases. Section 5 extends our basic model to analyze the long-run stock returns to the firm following a dividend payment or a stock repurchase. Section 6 describes the testable implications of our model, and Section 7 concludes. The proofs of all propositions are confined to the Appendix.

2 The Basic Model

There are three dates in the model: times 0, 1, and 2. At time 0, insiders of a firm own a fraction α of the firm's equity. The remaining fraction $(1 - \alpha)$ is held by a group of outside shareholders. The total number of shares in the firm is normalized to 1, so that α can be thought of as either the fraction of shares or the number of shares held by insiders. At time 0, the firm learns about its time-1 earnings E and chooses the scale of investment in its project. It then announces its time-1 cash dividend payment amount D_c and/or stock repurchase amount D_b . At time 1, the firm distributes the cash dividend and repurchases stock as announced at time 0, and simultaneously invests in its project. At time 2, the cash flows from the firm's project are realized and become common knowledge to all market participants. The sequence of events is shown in Figure 1.

The firm can invest in two different scales. If the firm invests the smaller amount I in the project, the cash flow from the firm's project will be either X^H or X^L , where $X^H > X^L \geq 0$. For simplicity, we normalize X^L to 0 and denote X^H by X throughout the paper. The firm can also choose to invest a larger amount of λI in the project, and in this case, the cash flow from the firm's project will be either $\lambda' X$ or 0, where λ and λ' are known constants with $\lambda \geq \lambda' > 1$. The condition $\lambda \geq \lambda'$ implies that the project has decreasing return to scale. From now onwards, we will refer to the case where the firm

At time 0, insiders of a firm own a fraction α of the firm's equity. The remaining $1-\alpha$ is held by a group of outside shareholders.

The total number of shares outstanding in the firm is normalized to 1.

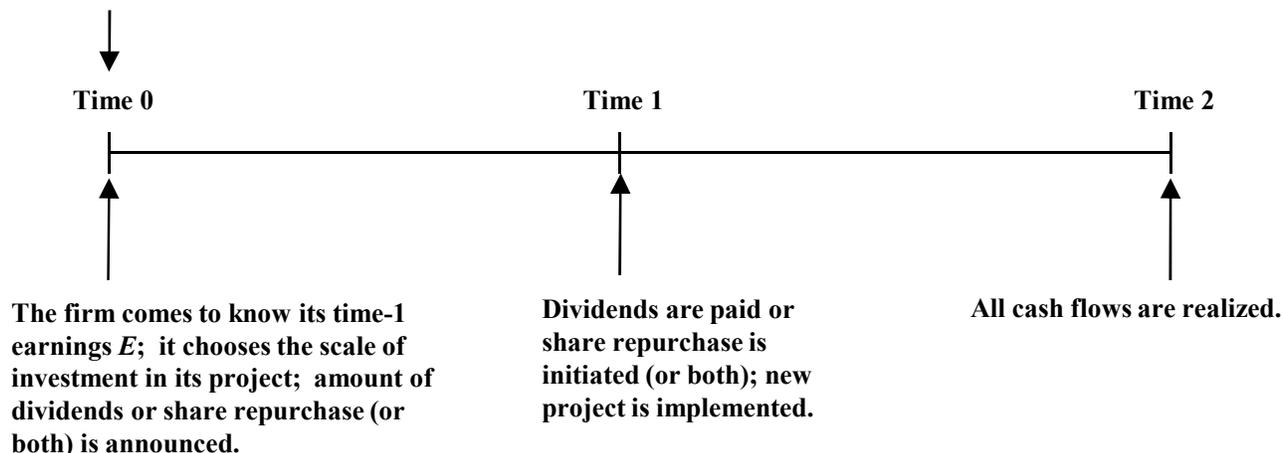


Figure 1: Sequence of Events

invests the larger amount as “investing up to the full investment level” and to the case where the firm invests only up to the smaller investment level as “underinvesting.”

The payout policy of the firm involves the following decisions. First, what amount of the time-1 earnings E should the firm distribute to its shareholders, and what amount should the firm reinvest? Second, how should the firm distribute value to its shareholders: cash dividend, stock repurchase, or a combination of the two?⁶

The capital market is characterized by heterogeneous beliefs and short-sale constraints. Specifically, the beliefs about the future (time 2) cash flows of the firm’s project are different between firm insiders and outsiders, and these beliefs are also different among outsiders. Firm insiders believe that with probability θ^f , the cash flow will be X , and with probability $(1 - \theta^f)$, the cash flow will be 0 if the

⁶Throughout the paper, we assume that the firm carries out its stock repurchase through an open market repurchase program, thus allowing it to pay different prices to the two groups of shareholders (optimists and pessimists). Note that, in practice, more than 90% of stock repurchases currently occur through an open market repurchase program: see, e.g., Grullon and Michaely (2004).

smaller amount I is invested. If the larger amount λI is invested, insiders believe that with probability θ^f the firm's time-2 cash flow will be $\lambda' X$, and with probability $(1 - \theta^f)$, the cash flow will be 0. We assume that regardless of whether the firm invests the larger amount λI or the smaller amount I in its project, the net present value of its project is positive conditional on insider beliefs about the probability of project success: i.e.,

$$\theta^f \lambda' X - \lambda I > 0, \quad \theta^f X - I > 0. \quad (1)$$

Further, given our assumption of decreasing returns to scale, it follows that the net present value per dollar of investment will be smaller if the firm undertakes its project at the larger scale (i.e., full investment level) compared to the case where it makes only a smaller investment in its project (i.e., it underinvests). However, the incremental investment from the underinvestment level up to the full investment level has still a positive net present value. In other words, we assume that

$$\theta^f (\lambda' - 1) X > (\lambda - 1) I, \quad (2)$$

which can be equivalently stated as $\theta^f > \frac{(\lambda-1)I}{(\lambda'-1)X}$.

The firm's current shareholders have heterogeneous beliefs about its future prospects. One group of outsiders, who hold a fraction δ of the firm's shares ($\delta < 1 - \alpha$), are relatively less optimistic about the success probability of the firm's project: they believe that this probability is $\underline{\theta}$. The second group of outsiders, who hold the fraction $(1 - \alpha - \delta)$ of the firm's equity are relatively more optimistic of the success probability of the firm's project: they believe that this probability is $\bar{\theta}$, where $0 < \underline{\theta} < \bar{\theta} < 1$ (Recall that firm insiders hold the remaining fraction α of the firm's equity). Finally, we assume that outside investors in the equity market who currently do not own shares in the firm also have a probability

assessment $\underline{\theta}$ about the success probability of the firm's project.^{7, 8}

In our basic model, we assume that the firm cannot issue any new equity to raise external financing to fund its investment in its new project. Further, we assume that the earnings E at time 1 are large enough for the firm to fund the project at the full investment level: i.e., $E \geq \lambda I$. We will relax these two assumptions in our extended model. Consistent with much of the literature on heterogeneous beliefs, we assume that all investors are subject to a short-sale constraint; i.e., short selling in the firm's equity is not allowed in this economy.

The objective of firm insiders is to choose the optimal payout policy in order to maximize the expected sum of the time-1 and time-2 payoffs to firm insiders, based on insiders' belief, θ^f , about the firm's future prospects. There is a risk-free asset in the economy, the net return on which is normalized to 0. All agents are assumed to be risk-neutral.

2.1 The Case where the Firm is Allowed to Distribute Value only through a Cash Dividend

We first analyze the case where the firm is allowed to pay only a cash dividend in order to distribute earnings to shareholders (i.e., no stock repurchase). Firm insiders choose the cash dividend amount D_c to maximize the sum of their time-1 and time-2 expected payoffs based on insiders' belief about the firm's future cash flow from the new project. The remainder of the earnings, amount $E - D_c$, will be invested in the firm's project. Thus, the firm insiders' objective is as follows:

$$\max_{D_c \in \{E-I, E-\lambda I\}} \alpha D_c + E_1 \left[\alpha C F_2^{equity} \middle| \theta^f \right] \quad (3)$$

⁷This assumption is appropriate, since one would expect that outsiders who are more optimistic about the firm's future prospects to be the first to buy shares in the firm (at any given price), so that investors who are not current shareholders would be those who are relatively less optimistic about the firm's future prospects.

⁸If we assume that all current outside shareholders have belief $\bar{\theta}$, then the firm's repurchase price is always the same no matter how many shares a firm repurchases.

where CF_2^{equity} , the cash flow realized from the firm's project at time 2, will be either X or 0.

The following proposition analyzes the firm's investment and dividend policy when the firm is allowed to make use of only dividends to distribute value to shareholders.

Proposition 1. (*The Firm is Allowed to Distribute Value only through a Cash Dividend*)
When the firm can distribute value to shareholders only through dividends, it invests an amount λI in its project (i.e., up to the full investment level) and distributes its excess cash ($E - \lambda I$) to shareholders as dividends.

The intuition behind the above proposition is that when the firm does not undertake any stock repurchase (i.e., it is allowed to distribute value only through dividends), the equity valuation of outside shareholders (and therefore their beliefs about the firm's future prospects) does not affect its investment or payout policy. In other words, in this setting, the investment and distribution policy of the firm is very similar to the benchmark case where there is no heterogeneity in beliefs either among outsiders or between firm insiders and outsiders: i.e., the firm invests to the fullest extent in any positive NPV project available to it, and distributes the remaining cash to outsiders in the form of dividends.

2.2 The Case where the Firm is Allowed to Distribute Value only through a Stock Repurchase

We now assume that the firm is allowed to use stock repurchase alone to distribute value to shareholders.⁹

Further, for ease of exposition, we will specify two ranges of the earnings E available at time 1 to the firm: small or large. We assume that if the earnings available to the firm is small, then if the firm implements its project at the full investment level (i.e., invests an amount λI in its project), the amount of cash

⁹Throughout this section, we assume that the beliefs of pessimistic outside shareholders are below that of firm insiders: i.e., $\theta^f > \underline{\theta}$. If this were not the case, firm insiders would be better off not repurchasing any shares (since they would view outsiders' valuation of their firm's equity as being higher than their own valuation) and instead retain any cash in excess of investment requirements within the firm, to be distributed to shareholders at time 2. Note that, this assumption is required only in this section, where we allow the firm to distribute value to shareholders only in the form of a stock repurchase. As we will see in the next section, if the firm were allowed to use dividends as well as a stock repurchase to distribute value to shareholders, then insiders would distribute only cash in excess of its investment requirements to shareholders in the form of dividends when the above assumption is violated (i.e., if $\theta^f < \underline{\theta} < \bar{\theta}$).

it will have left over will be adequate to buy back only a fraction of equity less than the fraction δ held by pessimistic shareholders. On the other hand, we assume that if the firm underinvests (i.e., it invests only an amount I in its project), then the amount left over to distribute to shareholders is large enough to buy back all the shares held by pessimistic outside shareholders, but also (some shares) from optimistic outside shareholders.¹⁰ Thus, when the earnings available to the firm is small, the following parametric restriction holds:¹¹

$$\frac{E - \lambda I}{E + \underline{\theta}\lambda'X - \lambda I} < \delta < \frac{E - I}{E + \underline{\theta}X - I}. \quad (4)$$

If, however, the earnings available to the firm at time 1 is large, our assumption is that, even if the firm implements its project at its full investment level, the amount of cash left over will be adequate to buy back the entire equity δ held by pessimistic outside shareholders, but also some shares from optimistic outside shareholders. In other words, when the earnings available to the firm is large, the relevant parametric restriction is:

$$\delta < \frac{E - \lambda I}{E + \underline{\theta}\lambda'X - \lambda I} < \frac{E - I}{E + \underline{\theta}X - I}.^{12} \quad (5)$$

Finally, we also assume that, regardless of whether the amount of earnings E available inside the firm at time 1 is small or large, it is not large enough to buy back all the equity held by outsiders (i.e.,

¹⁰If we instead assume that if the firm implements the large project, the firm buys back more than δ shares of stock or that if the firm implements the small project, the firm buys back less than δ shares of stock, then the marginal repurchase price will be the same no matter which project the firm chooses and how many shares the firm repurchases. This amounts to the uninteresting case where the amount of repurchase does not affect the repurchase price. In reality, outsiders' beliefs are likely to be continuously distributed (instead of having only two discrete beliefs, which we assume for tractability). In such a setting, as firms repurchase more and more shares, they have to buy from investors with higher and higher beliefs about the firm's prospects and this will drive up the repurchase price. Thus, the assumption we make here captures the above realistic setting while maintaining tractability.

¹¹If the firm invests to the full extent in its project, the expected cash flow from the project is $\underline{\theta}\lambda'X$ and the earnings after investment is $E - \lambda I$. Further, the NPV of the new project is then $\underline{\theta}\lambda'X - \lambda I$. Thus, in this case, the repurchase price will be $E + \underline{\theta}\lambda'X - \lambda I$. If the firm underinvests, then the firm buys back more than δ shares. In this case, the repurchase price will be $E + \underline{\theta}X - I$ for the first δ shares and $E + \underline{\theta}X - I$ for the remaining shares.

¹²As one can see in equations (4) and (5), the number of shares the firm can potentially repurchase from pessimistic outside shareholders when the firm implements its project at the full investment level is always less the number of shares the firm can repurchase from pessimistic outside shareholders when the firm underinvests: i.e., $\frac{E - \lambda I}{E + \underline{\theta}\lambda'X - \lambda I} < \frac{E - I}{E + \underline{\theta}X - I}$. This is an implication of our assumption that the firm's new project has decreasing returns to scale.

optimistic and pessimistic shareholders combined).

Firm insiders choose the dollar amount of the stock repurchase D_b to maximize their time-2 expected payoff based on insiders' belief about the firm's future cash flow from the new project. The remainder of the earnings, amount $E - D_b$, will be invested in the firm's project. Thus, the firm insiders' objective is now given by:

$$\max_{D_b \in \{E-I, E-\lambda I\}} E_1 \left[\frac{\alpha}{1 - \alpha_b} CF_2^{equity} \mid \theta^f \right] \quad (6)$$

where α_b is the number of shares the firm repurchases, and CF_2^{equity} is the cash flow realized from the firm's project at time 2.

The following proposition characterizes the firm's investment and distribution policy when the firm is allowed to use only a stock repurchase to distribute value to shareholders.

Proposition 2. (The Firm is Allowed to Distribute Value only through a Stock Repurchase)
Let $\theta^f > \underline{\theta}$ and $\theta^f > (\delta \underline{\theta} + (1 - \delta) \bar{\theta}) \frac{E-I}{E-I+\delta(\bar{\theta}-\underline{\theta})X}$, so that the NPV of a stock repurchase based on insiders' belief is positive.

(i) If the earnings E available to the firm at time 1 is small, so that the parametric restriction (4) holds, then:

(a) If the incremental NPV obtained from increasing the scale of the firm's project from the underinvestment to the full investment level is relatively large (λ' is relatively high), and the belief $\bar{\theta}$ of optimistic outsiders is high enough so that $\bar{\theta} \geq \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\theta X}$, the firm will implement its project at the full investment level (λI), and repurchase $\frac{E-\lambda I}{E+\theta\lambda'X-\lambda I}$ shares from current outside shareholders with belief $\underline{\theta}$.

(b) If the incremental NPV obtained from increasing the scale of the firm's project from the underinvestment to the full investment level is relatively small (λ' is relatively low), and the belief $\bar{\theta}$ of optimistic outsiders is low enough so that $\underline{\theta} < \bar{\theta} < \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\theta X}$, the firm will underinvest (I) in its project and repurchase a larger number of shares, $\delta + \frac{E-I-\delta(E+\theta X-I)}{E+\theta X-I}$, from current outside shareholders. Of the shares repurchased, δ shares are bought back from shareholders with belief $\underline{\theta}$ and the remaining from those with belief $\bar{\theta}$.

(ii) If the earnings E available to the firm at time 1 is large, so that the parametric restriction (5) holds, then:

(a) If $\bar{\theta} \geq \frac{(\lambda-1)I}{(\lambda'-1)X}$, the firm will implement its project at the full investment level (λI), and repurchase $\delta + \frac{E-\lambda I-\delta(E+\theta\lambda'X-\lambda I)}{E+\theta\lambda'X-\lambda I}$ shares from current outside shareholders. Of the shares

repurchased, δ shares are bought back from shareholders with belief $\underline{\theta}$ and the remaining from those with belief $\bar{\theta}$.

(b) If $\bar{\theta} < \frac{(\lambda-1)I}{(\lambda'-1)X}$, the firm will underinvest (I) in its project and repurchase a larger number of shares, $\delta + \frac{E-I-\delta(E+\underline{\theta}X-I)}{E+\underline{\theta}X-I}$, from current outside shareholders. Of the shares repurchased, δ shares are bought back from shareholders with belief $\underline{\theta}$ and the remaining from those with belief $\bar{\theta}$.

The intuition behind the above proposition is as follows. The firm chooses the scale of investment and the number of shares to repurchase simultaneously. In making the above choice, firm insiders compare the incremental NPV obtained from increasing the scale of investment in its project from the underinvestment level to the full investment level to the NPV obtained from the financial transaction of repurchasing shares from outsiders. The NPV of repurchasing shares from outsiders depends on the price the firm has to pay each group of outsiders (the optimists and the pessimists) to repurchase their shares, which in turn, depends on their beliefs $\bar{\theta}$ and $\underline{\theta}$, respectively, about the firm's future prospects.

If the earnings E available to the firm is small as in part (i) of Proposition 2, the firm has to repurchase shares from pessimists alone when it implements its new project at the full investment level. On the other hand, the firm has to repurchase shares from both optimists and pessimists when it underinvests in its project. Therefore, if the optimists' belief ($\bar{\theta}$) is significantly higher than the belief $\underline{\theta}$ of pessimists, the NPV of repurchasing shares from both optimists and pessimists will be significantly smaller than that of repurchasing shares from pessimists alone (since the price paid to repurchase equity from pessimists will be much smaller). Firm insiders are better off implementing the firm's project at the full investment level, and repurchasing shares only from pessimistic outside shareholders if the following condition holds:

$$\alpha \left[\frac{E + \underline{\theta}\lambda'X - \lambda I}{\underline{\theta}\lambda'X} \right] \theta^f \lambda'X \geq \alpha \left[\frac{E + \bar{\theta}X - I}{\delta\underline{\theta}X + (1-\delta)\bar{\theta}X} \right] \theta^f X. \quad (7)$$

In this case, the incremental NPV obtained from increasing the scale of the firm's project from the

underinvestment to the full investment level will be greater than the NPV of repurchasing shares, if the firm were to repurchase equity from optimistic as well as pessimistic outside shareholders. After rearranging (7), we equivalently obtain a threshold on optimists' belief $\bar{\theta}$, which is equal to

$$\frac{\underline{\theta}(E - I - \delta(E + \underline{\theta}\lambda'X - \lambda I))}{(1 - \delta)(E + \underline{\theta}\lambda'X - \lambda I) - \underline{\theta}X}. \quad (8)$$

If optimistic shareholders' belief $\bar{\theta}$ is above this threshold, the firm makes its choice of investment and stock repurchase as characterized in Proposition 2(i)(a), i.e., it implements its new project at the full investment level and repurchases shares from pessimistic shareholders only.¹³

If, on the other hand, the optimists' belief $\bar{\theta}$ is lower than the threshold given in (8), so that it is closer to that of pessimists' belief $\underline{\theta}$, then the price paid to repurchase shares from optimistic outside shareholders will also be lower (and closer to the price paid to repurchase shares from the pessimists). This, in turn, implies that the NPV of repurchasing shares from both pessimists and optimists will be significantly larger in this case than in the scenario where the optimists' belief $\bar{\theta}$ is higher than the threshold given in (8). Consequently, the NPV from a stock repurchase from both optimists and pessimists will be greater than the incremental NPV obtained from increasing the scale of the firm's project from the underinvestment to the full investment level. Hence, the firm will underinvest in its project, and repurchase a larger amount of equity. In other words, the firm will buy back all the shares (δ) held by pessimistic outsiders and also some of the shares held by optimistic outsiders as outlined in Proposition 2(i)(b). Note that for the parameter condition given in part (i)(b) of Proposition 2 to be satisfied, it must be the case that the threshold for $\bar{\theta}$ defined in (8) must be strictly greater than the pessimists' belief $\underline{\theta}$, i.e. $\frac{\underline{\theta}(E - I - \delta(E + \underline{\theta}\lambda'X - \lambda I))}{(1 - \delta)(E + \underline{\theta}\lambda'X - \lambda I) - \underline{\theta}X} > \underline{\theta}$. This, in turn, implies that the following restriction

¹³Note that the threshold defined in (8) is increasing in δ , increasing in λ , increasing in I , decreasing in λ' , decreasing in X , and decreasing in E . In other words, the higher the NPV of increasing the scale of the new project, the lower is this threshold. Further, the greater the number of shares δ held by pessimistic shareholders relative to the earnings E available to the firm at time 1, the higher is this threshold.

must hold in part (i)(b) of Proposition 2:

$$\underline{\theta} < \frac{(\lambda - 1)I}{(\lambda' - 1)X}. \quad (9)$$

In other words, a necessary condition for the firm to underinvest and repurchase shares from both pessimistic and optimistic groups of shareholders is that the NPV of the new project based on pessimistic shareholders' belief $\underline{\theta}$ is negative. If the condition given in (9) is not satisfied, the firm will implement its project at the full investment level, and repurchase shares from pessimistic outside shareholders only.

If the earnings E available to the firm is large as in part (ii) of Proposition 2, the firm has to repurchase shares from both optimists and pessimists even when it implements its new project at the full investment level λI . However, in this case, we can still show that the total number of shares the firm has to repurchase from optimists will still be substantially less than the number of shares it has to repurchase from optimists in the case when it underinvests in its project:

$$\frac{E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)}{E + \bar{\theta}\lambda'X - \lambda I} < \frac{E - I - \delta(E + \underline{\theta}X - I)}{E + \bar{\theta}X - I}. \quad (10)$$

Thus, if the amount of earnings E is large, firm insiders are better off implementing the firm's project at the full investment level, and repurchasing a smaller number of shares from optimistic outside shareholders if the following condition holds:

$$\alpha \left[\frac{E + \bar{\theta}\lambda'X - \lambda I}{(\delta\underline{\theta} + (1 - \delta)\bar{\theta})\lambda'X} \right] \theta^f \lambda'X \geq \alpha \left[\frac{E + \bar{\theta}X - I}{(\delta\underline{\theta} + (1 - \delta)\bar{\theta})X} \right] \theta^f X. \quad (11)$$

After rearranging (11), we find that the firm will prefer to implement its project at the full investment level λI and repurchase a smaller number of shares if the belief of optimistic shareholders is sufficiently

high, i.e., if

$$\bar{\theta} \geq \frac{(\lambda - 1)I}{(\lambda' - 1)X}. \quad (12)$$

Otherwise, if $\bar{\theta} < \frac{(\lambda - 1)I}{(\lambda' - 1)X}$, the beliefs of both pessimistic and optimistic shareholders ($\underline{\theta}$ and $\bar{\theta}$) will be substantially less than the belief θ^f of firm insiders. In this case, firm insiders find that the NPV from a stock repurchase (when the firm underinvests) is large enough that it is greater than the incremental NPV obtained from increasing the scale of the firm's project from the underinvestment to the full investment level. Therefore, the firm chooses to underinvest in its project, and repurchases a larger amount of equity.

2.3 The Case where the Firm Distributes Value through a Combination of a Stock Repurchase and a Dividend Payment

We now assume that the firm is allowed to use a combination of a stock repurchase and a dividend payment to distribute value to shareholders. In this section, we want to determine the optimal structure of such a combination: i.e., the amount of cash dividend D_c paid out and the number of shares α_b repurchased by the firm. We condition on the amount of earnings E available to the firm at time 1 and the firm's investment policy, and determine the optimal combination in three separate cases: (i) the case where the firm implements its new project at the full investment level and E is small; (ii) the case where the firm underinvests in its new project; (iii) the case where the firm implements its new project at the full investment level and E is large.

Note that if firm insiders think that the firm's equity is undervalued by some existing shareholders (i.e., the NPV of repurchasing their shares is positive based on insiders' belief), they will first conduct the stock repurchase and then pay dividends to the remaining shareholders after the repurchase. Thus,

when the firm distributes value through a combination of a stock repurchase and a dividend payment, the sequence of events is as follows: 1) The firm repurchases shares first, and the dollar amount D_b of the repurchase is $E - \lambda I - D_c$, if the firm implements its new project at the full investment level or $E - I - D_c$, if the firm underinvests; 2) The firm makes a dividend payment D_c to the existing shareholders remaining after the stock repurchase.

Proposition 3. (The Case where the Firm Distributes Value through a Combination of a Stock Repurchase and a Dividend Payment)

- (i) *If the firm implements its project at the full investment level and the earnings E available to the firm at time 1 is small (so that the parametric restriction (4) holds), then it is not optimal for the firm to distribute value through a combination of a stock repurchase and a dividend payment.*
- (ii) *If the firm underinvests in its new project and $\underline{\theta} < \theta^f < \delta\underline{\theta} + (1 - \delta)\bar{\theta}$, then the firm will repurchase δ shares from pessimistic shareholders with belief $\underline{\theta}$ and pay a cash dividend of $E - I - \delta(E + \underline{\theta}X - I)$ to the remaining shareholders.*
- (iii) *If the firm implements its project at the full investment level and the earnings E available to the firm at time 1 is large (so that the parametric restriction (5) holds), then the firm will repurchase δ shares from pessimistic shareholders with belief $\underline{\theta}$ and pay a cash dividend of $E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)$ to the remaining shareholders if $\underline{\theta} < \theta^f < \delta\underline{\theta} + (1 - \delta)\bar{\theta}$.*

If the firm implements its new project at the full investment level and E is small as in part (i) of Proposition 3, the maximum number of shares the firm can repurchase is equal to $\frac{E - \lambda I}{E + \underline{\theta}\lambda'X - \lambda I}$, which is strictly less than the number of shares δ held by pessimistic shareholders. In this case, if firm insiders are more optimistic about the firm's future prospects than pessimistic shareholders, i.e., if $\theta^f > \underline{\theta}$, the NPV of repurchasing their shares will be positive, and the firm will find it optimal to distribute value through a stock repurchase only, thereby maximizing the number of shares that it repurchases from pessimistic shareholders. On the contrary, if $\theta^f \leq \underline{\theta}$, the NPV of repurchasing shares will be negative, and the firm will find it optimal to distribute value through a dividend payment only. Therefore, if the firm implements its project at the full investment level and E is small, the firm will never choose to distribute value through a combination of a stock repurchase and a dividend payment.

If the firm underinvests in its new project as in part (ii) of Proposition 3, the earnings E available to the firm at time 1 is large enough so that the maximum number of shares that firm can repurchase is greater than δ . However, if the firm repurchases more than δ shares, it means that, in addition to buying back all the shares held by pessimistic shareholders at the price $(E + \underline{\theta}X - I)$, the firm also has to buy back some shares from optimistic outside shareholders at the higher price $(E + \bar{\theta}X - I)$. In this case, if firm insiders are more optimistic about the firm's future prospects than pessimistic shareholders, i.e., if $\theta^f > \underline{\theta}$, the NPV of repurchasing their shares will be positive, and the firm will find it optimal to buy back at least all the shares (δ) held by pessimistic shareholders.¹⁴ To determine whether the firm will find it optimal to repurchase more than δ shares, or it will choose distribute some of the remaining excess earnings, $E - I - \delta(E + \underline{\theta}X - I)$, as a dividend payment (D_c), we set up the firm insiders' optimization problem as follows:

The firm buys back δ shares from pessimistic shareholders at the price $E + \underline{\theta}X - I$, and also some shares from optimistic shareholders at the price $E + \bar{\theta}X - I$ for a dollar amount of $E - I - D_c - \delta(E + \underline{\theta}X - I)$. Thus, the total number of shares repurchased by the firm is equal to

$$\alpha_b = \delta + \frac{E - I - D_c - \delta(E + \underline{\theta}X - I)}{E + \bar{\theta}X - I}. \quad (13)$$

After the repurchase, the firm pays a dollar amount of D_c to existing shareholders, who collectively hold

$$1 - \alpha_b = 1 - \delta - \frac{E - I - D_c - \delta(E + \underline{\theta}X - I)}{E + \bar{\theta}X - I} \quad (14)$$

shares. The dividend payment D_c can take a value in the closed interval $[0, E - I - \delta(E + \underline{\theta}X - I)]$.

¹⁴If $\theta^f \leq \underline{\theta}$, the NPV of repurchasing shares will be nonpositive, and the firm will find it optimal to distribute value through a dividend payment only.

Thus, the firm insiders' objective function is

$$\max_{D_c \in [0, E-I-\delta(E+\underline{\theta}X-I)]} \frac{\alpha}{\left(1 - \delta - \frac{E-I-D_c-\delta(E+\underline{\theta}X-I)}{E+\bar{\theta}X-I}\right)} [D_c + \theta^f X], \quad (15)$$

which is equivalent to

$$\max_{D_c \in [0, E-I-\delta(E+\underline{\theta}X-I)]} \alpha(E + \bar{\theta}X - I) \left[1 + \frac{(\theta^f - (\delta\underline{\theta} + (1-\delta)\bar{\theta}))X}{D_c + (\delta\underline{\theta} + (1-\delta)\bar{\theta})X} \right]. \quad (16)$$

The optimal solution for the firm is

$$D_c = \begin{cases} E - I - \delta(E + \underline{\theta}X - I) & \text{if } \theta^f \leq \delta\underline{\theta} + (1-\delta)\bar{\theta}, \\ 0 & \text{if } \theta^f > \delta\underline{\theta} + (1-\delta)\bar{\theta} \end{cases} \quad (17)$$

Thus, if $\theta^f > \delta\underline{\theta} + (1-\delta)\bar{\theta}$, the firm will find it optimal to distribute value through a stock repurchase alone.¹⁵ Otherwise, if $\underline{\theta} < \theta^f \leq \delta\underline{\theta} + (1-\delta)\bar{\theta}$, the firm will choose to distribute value through a combination of a stock repurchase and a dividend payment, by repurchasing exactly δ shares from pessimistic shareholders and making a dividend payment of $E - I - \delta(E + \underline{\theta}X - I)$ to the remaining shareholders. In this case, the expected payoff of firm insiders is given by:

$$\frac{\alpha}{1-\delta} [\theta^f X + E - I - \delta(E + \underline{\theta}X - I)]. \quad (18)$$

Finally, if we consider the case (part (iii) of Proposition 3), where the firm implements its new project at the full investment level and E is large, the optimal combination of a stock repurchase and a dividend payment is very similar to that of the above case where the firm underinvests. Conditional on the firm fully investing in its project and E being large, the optimal combination of a stock repurchase and a

¹⁵It will buy back δ shares from pessimistic shareholders and $\frac{E-I-\delta(E+\underline{\theta}X-I)}{E+\bar{\theta}X-I}$ shares from optimistic shareholders.

dividend payment is as follows: the firm will repurchase exactly δ shares from pessimistic shareholders, and pay out a cash dividend in the amount of $E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)$ to the remaining shareholders.

In this case, the expected payoff of firm insiders is given by:

$$\frac{\alpha}{1 - \delta} [\theta^f \lambda'X + E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)]. \quad (19)$$

2.4 The Choice Between Stock Repurchase and Cash Dividend

In this section, we analyze the firm's choice between a stock repurchase and a cash dividend. The firm has to choose between three possible ways of paying out excess cash to shareholders: (i) Stock repurchase alone; (ii) Paying out dividends alone; (iii) A combination of a stock repurchase and dividend payout.

While determining its payout policy, the firm also simultaneously chooses its investment policy: i.e., it decides whether to invest in its new project up to the full investment level, or to underinvest in it. Clearly, if the firm undertakes its project at the full investment level, it will pay out only a smaller amount compared to the case where its underinvests in its project.

Five possible payout and investment choices made by the firm in equilibrium can be summarized as follows: (i) the firm chooses to invest up to the full investment level in its project, and distributes the remaining cash available to it at time 1 as dividend alone to outside shareholders; (ii) the firm chooses to invest up to the full investment level in its project, and distributes the remaining cash available to it at time 1 in the form of a stock repurchase to outside shareholders; (iii) the firm chooses to underinvest in its project, and distributes the remaining cash available to it at time 1 in the form of a stock repurchase; (iv) the firm chooses to underinvest in its project, and distributes the remaining cash available to it at time 1 through a combination of a stock repurchase and a dividend payment; (v) the firm chooses to invest up to the full investment level in its project, and distributes the remaining cash available to it at time 1 through a combination of a stock repurchase and a dividend payment.

The three factors that drive firm insiders' equilibrium choices are the following. First, the incremental NPV of the firm's project implemented at its full investment level relative to the same project implemented at its underinvestment level. Second, whether the NPV of a stock repurchase is positive or negative, and if positive, the magnitude of this NPV. This, in turn, will depend upon the relative levels of the beliefs of firm insiders, θ^f , and that of the two groups of outside shareholders (optimists and pessimists), $\bar{\theta}$ and $\underline{\theta}$, respectively. Note that the latter two beliefs $\bar{\theta}$ and $\underline{\theta}$ will determine the price firm insiders need to pay to repurchase equity from optimistic and pessimistic outside shareholders, respectively. Third, the NPV of a dividend payment by the firm, which is zero regardless of the beliefs of either of the two groups of outside shareholders.

We now characterize the conditions under which the firm uses a stock repurchase alone, a dividend payment alone, or a combination of a stock repurchase and a dividend payment to distribute value to shareholders. We first analyze the case where the earnings E available to the firm at time 1 is small.

Proposition 4. (The Choice Between Stock Repurchase and Cash Dividend when E is small) *Let the earnings E available to the firm at time 1 be small, so that the parametric restriction (4) holds. Then:*

(i) *If $\frac{(\lambda-1)I}{(\lambda'-1)X} \leq \underline{\theta} < \bar{\theta}$, the firm's optimal investment and payout policy is as follows:*

(a) *If $\theta^f \leq \underline{\theta}$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a dividend payment ($E - \lambda I$) alone.*

(b) *If $\underline{\theta} < \theta^f$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a stock repurchase alone, repurchasing $\frac{E-\lambda I}{E+\underline{\theta}\lambda'X-\lambda I}$ shares from pessimistic shareholders with belief $\underline{\theta}$.*

(ii) *If $\underline{\theta} < \frac{(\lambda-1)I}{(\lambda'-1)X} < \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\underline{\theta}\lambda'X-\lambda I)-\underline{\theta}X} \leq \bar{\theta}$, the firm's optimal investment and payout policy is as follows:*

(a) *If $\theta^f \leq \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\underline{\theta}\lambda'X-\lambda I)-\underline{\theta}X}$, the firm will choose to implement its project at the underinvestment level (\bar{I}), and will choose to distribute value through a combination of a dividend payment and a stock repurchase, repurchasing δ shares from pessimistic outside shareholders with belief $\underline{\theta}$ and paying a cash dividend of $E - I - \delta(E + \underline{\theta}X - I)$.*

(b) *If $\theta^f > \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\underline{\theta}\lambda'X-\lambda I)-\underline{\theta}X}$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a stock repurchase alone, repurchasing $\frac{E-\lambda I}{E+\underline{\theta}\lambda'X-\lambda I}$ shares from pessimistic shareholders with belief $\underline{\theta}$.*

(iii) If $\underline{\theta} < \frac{(\lambda-1)I}{(\lambda'-1)X} < \delta\underline{\theta} + (1-\delta)\bar{\theta} < \bar{\theta} < \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\underline{\theta}X}$, the firm's optimal investment and payout policy is as follows:

(a) If $\theta^f \leq \delta\underline{\theta} + (1-\delta)\bar{\theta}$, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a combination of a dividend payment and a stock repurchase, repurchasing δ shares from pessimistic outside shareholders with belief $\underline{\theta}$ and paying a cash dividend of $E - I - \delta(E + \underline{\theta}X - I)$.

(b) If $\theta^f > \delta\underline{\theta} + (1-\delta)\bar{\theta}$, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$ and the remaining $\left(\frac{E-I-\delta(E+\underline{\theta}X-I)}{E+\bar{\theta}X-I}\right)$ from optimistic shareholders with belief $\bar{\theta}$.

(iv) If $\underline{\theta} < \delta\underline{\theta} + (1-\delta)\bar{\theta} \leq \frac{(\lambda-1)I}{(\lambda'-1)X} < \bar{\theta} < \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\underline{\theta}X}$, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$ and the remaining $\left(\frac{E-I-\delta(E+\underline{\theta}X-I)}{E+\bar{\theta}X-I}\right)$ from optimistic shareholders with belief $\bar{\theta}$.

(v) If $\underline{\theta} < \bar{\theta} \leq \frac{(\lambda-1)I}{(\lambda'-1)X}$, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$ and the remaining $\left(\frac{E-I-\delta(E+\underline{\theta}X-I)}{E+\bar{\theta}X-I}\right)$ from optimistic shareholders with belief $\bar{\theta}$.

Part (i) of Proposition 4 characterizes a situation where both groups of outside shareholders are very optimistic about the prospects of the firm's new project so that even the belief of pessimistic shareholders, $\underline{\theta}$, is higher than the critical threshold $\frac{(\lambda-1)I}{(\lambda'-1)X}$. If this condition holds, we know from the discussion of Proposition 2(i) that the firm will choose to invest up to the full investment level in its project in case it distributes value to shareholders through a stock repurchase. Further, from Proposition 1, we also know that the firm will implement its project at the full investment level in case it distributes value through a dividend payment. In this case, firm insiders prefer a dividend payment to a stock repurchase if the following condition holds:

$$\alpha(E - \lambda I + \theta^f \lambda' X) \geq \alpha \left(\frac{E - \lambda I + \underline{\theta} \lambda' X}{\underline{\theta}} \right) \theta^f. \quad (20)$$

which is equivalent to $\theta^f \leq \underline{\theta}$. Intuitively, if firm insiders are more pessimistic about the new project

than both groups of outside shareholders (i.e., if $\theta^f \leq \underline{\theta} < \bar{\theta}$), they assess that the firm is overvalued by outside shareholders. In other words, the NPV of a stock repurchase will be negative based on insiders' belief. Therefore, the firm chooses to distribute the remaining cash available to it at time 1 as a dividend payment alone to outside shareholders. If, however, firm insiders are more optimistic than the pessimistic group of shareholders, repurchasing shares from these shareholders is a positive-NPV transaction for firm insiders, and the firm chooses to distribute value through a stock repurchase alone, repurchasing $\frac{E-\lambda I}{E+\underline{\theta}\lambda'X-\lambda I}$ shares from pessimistic shareholders only.

In part (ii) of Proposition 4, the belief of pessimistic outside shareholders, $\underline{\theta}$, is less than the $\frac{(\lambda-1)I}{(\lambda'-1)X}$, and the belief of optimistic outside shareholders, $\bar{\theta}$, is greater than the threshold given in (8).¹⁶ In this case, since insiders are more optimistic than pessimistic shareholders, i.e., $\theta^f > \underline{\theta}$, the NPV of repurchasing shares from pessimists is positive, and therefore, paying dividends alone, which is a zero-NPV transaction, is not an optimal choice for the firm. From part (i) of Proposition 2, we also know that since $\bar{\theta} \geq \frac{\theta(E-I-\delta(E+\underline{\theta}\lambda'X-\lambda I))}{(1-\delta)(E+\underline{\theta}\lambda'X-\lambda I)-\underline{\theta}X}$, the firm would prefer to implement its project at the full investment level and distribute value through a stock repurchase alone if paying some dividends was not an available option. However, this proposition shows that if the incremental NPV obtained by increasing the scale of the firm's project is relatively small (λ' and θ^f are relatively lower), the firm may prefer to underinvest in its project, and distribute value to shareholders through a combination of a stock repurchase and a dividend payment if the following condition holds:

$$\alpha \left(\frac{E + \underline{\theta}\lambda'X - \lambda I}{\underline{\theta}} \right) \theta^f \leq \frac{\alpha}{1-\delta} (\theta^f X + E - I - \delta(E + \underline{\theta}X - I)), \quad (21)$$

which is equivalent to the condition that $\theta^f \leq \frac{\theta(E-I-\delta(E+\underline{\theta}X-I))}{(1-\delta)(E+\underline{\theta}\lambda'X-\lambda I)-\underline{\theta}X}$. By conducting a stock repurchase program and paying dividends at the same time, the firm can first repurchase all the shares held by

¹⁶Since we assume that $\theta^f > \frac{(\lambda-1)I}{(\lambda'-1)X}$ throughout the paper, it follows that $\theta^f > \underline{\theta}$ in parts (ii) to (v) of Proposition 4.

pessimistic shareholders (δ) and then avoid repurchasing shares from optimistic shareholders at a higher price by paying dividends. Thus, if (21) holds, the incremental NPV from repurchasing all the shares (δ) held by pessimistic investors will exceed the incremental NPV of increasing the scale of the project.¹⁷ Therefore, the firm will underinvest in its project, and use the remaining cash available to it ($E - I$) at time 1 to distribute value through a combination of a stock repurchase and a dividend payment. If, however, $\theta^f > \frac{\theta(E-I-\delta(E+\theta X-I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\theta X}$, the incremental NPV obtained by increasing the scale of the project will be sufficiently large so that the firm will implement its project at the full investment level and distribute value through a stock repurchase alone.

Parts (iii) and (iv) of Proposition 4 characterize a situation, where the belief of pessimistic outside shareholders, $\underline{\theta}$, is less than the threshold $\frac{(\lambda-1)I}{(\lambda'-1)X}$ as in part (ii), but the belief of optimistic outside shareholders, $\bar{\theta}$, is less than the threshold given in (8). Since $\theta^f > \underline{\theta}$, a dividend payment alone is not optimal for the firm. From part (i) of Proposition 2, we also know that since $\bar{\theta} < \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\theta X}$, the firm would prefer to underinvest in its project, and distribute the remaining cash available to it through a stock repurchase alone if paying some dividends was not an available option. However, this proposition shows that, while the firm still chooses to underinvest in its project, it may prefer to distribute value to shareholders through a combination of a stock repurchase and a dividend payment if the following condition holds:

$$\alpha \left(\frac{E + \bar{\theta}X - I}{\delta\underline{\theta} + (1 - \delta)\bar{\theta}} \right) \theta^f \leq \frac{\alpha}{1 - \delta} (\theta^f X + E - I - \delta(E + \underline{\theta}X - I)), \quad (22)$$

which is equivalent to the condition that $\theta^f \leq \delta\underline{\theta} + (1 - \delta)\bar{\theta}$. Thus, if firm insiders' belief θ^f is between the pessimistic shareholders' belief $\underline{\theta}$ and the optimistic shareholders' belief $\bar{\theta}$, and it is not sufficiently high, the firm will find that the NPV of repurchasing shares from pessimistic shareholders will be greater than

¹⁷Note that when E is small, if the firm implements its project at the full investment level, the remaining cash $E - \lambda I$ is not enough to repurchase all the shares held by pessimistic outside shareholders.

the NPV of repurchasing shares from both pessimistic and optimistic shareholders. In this case, the firm will be better off by first repurchasing all the shares (δ) held by pessimistic shareholders and then distributing the remaining cash through a dividend payment. On the other hand, if $\theta^f > \delta\underline{\theta} + (1 - \delta)\bar{\theta}$, repurchasing shares from optimistic shareholders will also be a positive-NPV transaction. Therefore, in this case, the firm will find it optimal to distribute value through a stock repurchase alone while it underinvests in its new project as described in parts (iii)(b) and (iv) of Proposition 4.

Part (v) of Proposition 4 characterizes a situation where both groups of outside shareholders are very pessimistic about the prospects of the firm's new project so that $\underline{\theta} < \bar{\theta} \leq \frac{(\lambda-1)I}{(\lambda'-1)X}$. In this case, both groups of shareholders are more pessimistic about the future prospects of the firm than firm insiders, since $\underline{\theta} < \bar{\theta} < \theta^f$. Clearly, the NPV of repurchasing shares from both groups of shareholders will be much larger than the NPV of repurchasing shares from pessimistic shareholders only, and it will exceed the NPV of increasing the scale of the firm's new project from I to λI . Hence, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a stock repurchase alone in this situation.

We next analyze the case where the earnings E available to the firm at time 1 is large, so that the parameter condition in (5) holds. In this case, if the firm implements its new project at the full investment level, the remaining cash available to it at time 1, $E - \lambda I$, is more than enough to repurchase all the shares (δ) held by pessimistic shareholders.

Proposition 5. (*The Choice Between Stock Repurchase and Cash Dividend when E is large*)
Let the earnings E available to the firm at time 1 be large, so that the parametric restriction (5) holds. Then:

(i) *If $\frac{(\lambda-1)I}{(\lambda'-1)X} \leq \underline{\theta} < \bar{\theta}$, the firm's optimal investment and payout policy is as follows:*

- (a) *If $\theta^f \leq \underline{\theta}$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a dividend payment ($E - \lambda I$) alone.*
- (b) *If $\underline{\theta} < \theta^f < \delta\underline{\theta} + (1 - \delta)\bar{\theta}$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a combination of a dividend payment and*

a stock repurchase, repurchasing δ shares from pessimistic outside shareholders with belief $\underline{\theta}$ and paying a cash dividend of $E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)$.

(c) If $\delta\underline{\theta} + (1 - \delta)\bar{\theta} \leq \theta^f$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$ and the remaining $\left(\frac{E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)}{E + \underline{\theta}\lambda'X - \lambda I}\right)$ from optimistic shareholders with belief $\bar{\theta}$.

(ii) If $\underline{\theta} < \frac{(\lambda-1)I}{(\lambda'-1)X} < \delta\underline{\theta} + (1 - \delta)\bar{\theta} < \bar{\theta}$, the firm's optimal investment and payout policy is as follows:

(a) If $\theta^f < \delta\underline{\theta} + (1 - \delta)\bar{\theta}$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a combination of a dividend payment and a stock repurchase, repurchasing δ shares from pessimistic outside shareholders with belief $\underline{\theta}$ and paying a cash dividend of $E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)$.

(b) If $\delta\underline{\theta} + (1 - \delta)\bar{\theta} \leq \theta^f$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$ and the remaining $\left(\frac{E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)}{E + \underline{\theta}\lambda'X - \lambda I}\right)$ from optimistic shareholders with belief $\bar{\theta}$.

(iii) If $\underline{\theta} < \delta\underline{\theta} + (1 - \delta)\bar{\theta} \leq \frac{(\lambda-1)I}{(\lambda'-1)X} < \bar{\theta}$, the firm will choose to implement its project at the full investment level (λI), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$ and the remaining $\left(\frac{E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)}{E + \underline{\theta}\lambda'X - \lambda I}\right)$ from optimistic shareholders with belief $\bar{\theta}$.

(iv) If $\underline{\theta} < \bar{\theta} \leq \frac{(\lambda-1)I}{(\lambda'-1)X}$, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a stock repurchase alone. Of the shares repurchased, δ shares are bought back from pessimistic shareholders with belief $\underline{\theta}$ and the remaining $\left(\frac{E - I - \delta(E + \underline{\theta}X - I)}{E + \underline{\theta}X - I}\right)$ from optimistic shareholders with belief $\bar{\theta}$.

Part (i) of Proposition 5 characterizes a situation, where both groups of outside shareholders are relatively optimistic about the future prospects of the firm so that $\frac{(\lambda-1)I}{(\lambda'-1)X} \leq \underline{\theta} < \bar{\theta}$. If insiders are more pessimistic about the firm's future prospects than pessimistic outside shareholders, i.e., if $\theta^f \leq \underline{\theta}$ (as in part (i)(a) of Proposition 5), the firm will choose to distribute its excess earnings through a dividend payment alone, since the NPV of a stock repurchase will be negative if both groups of outside shareholders overvalue the firm's shares relative to firm insiders. However, if insiders are more optimistic about the firm's future prospects than pessimistic outside shareholders (as in parts (i)(b) and (i)(c) of Proposition 5), repurchasing shares from pessimistic shareholders will be optimal for the firm, since

the NPV of doing so will be positive based on insiders' belief. If $\underline{\theta} < \theta^f < \delta\underline{\theta} + (1 - \delta)\bar{\theta}$, the firm will prefer to distribute value to its shareholders through a combination of a stock repurchase and a dividend payment. It will repurchase all the shares (δ) held by pessimistic shareholders and distribute the remaining cash through a dividend payment, since repurchasing shares from optimistic shareholders will be too costly for the firm, when insiders' belief θ^f is closer to the belief $\underline{\theta}$ of pessimistic shareholders than it is to that of optimistic shareholders ($\bar{\theta}$). If insiders' belief θ^f is sufficiently high, however, so that it is greater than $\delta\underline{\theta} + (1 - \delta)\bar{\theta}$, the firm will prefer to distribute its excess earnings to its shareholders through a stock repurchase alone, since, in this case, the NPV of repurchasing shares from both pessimistic and optimistic shareholders will be greater than the NPV of repurchasing shares from pessimistic shareholders alone. The same intuition also applies in parts (ii) and (iii) of Proposition 5.¹⁸

Part (iv) of Proposition 5 characterizes a situation where both groups of outside shareholders are very pessimistic about the prospects of the firm's new project so that $\underline{\theta} < \bar{\theta} \leq \frac{(\lambda-1)I}{(\lambda'-1)X}$. In this case, the NPV of repurchasing shares from both groups of shareholders will be substantially large, since $\underline{\theta} < \bar{\theta} < \theta^f$. Thus, the firm will have an incentive to maximize the number of shares it repurchases by underinvesting in its project, since the NPV of repurchasing a larger number of shares from both pessimistic and optimistic shareholders will exceed the NPV of increasing the scale of the firm's new project from I to λI . Hence, the firm will choose to implement its project at the underinvestment level (I), and will choose to distribute value through a stock repurchase alone in this situation.

3 The Price Impact of Cash Dividends and Stock Repurchases

In this section, we investigate the price impact of cash dividend payments or stock repurchases on the current stock price of the firm on the day the payout becomes effective ("the execution day" from

¹⁸Since we assume that $\theta^f > \frac{(\lambda-1)I}{(\lambda'-1)X}$ throughout the paper, it follows that $\theta^f > \underline{\theta}$ in parts (ii) to (iv) of Proposition 5.

now on). The price impact is measured as the abnormal return to the firm's equity from the price prevailing before a cash dividend payment or a stock repurchase (not the announcement date) to the price prevailing after the corporate payout.¹⁹ Since the market is already aware that a payout has been announced at time 0, one would expect a price impact of zero in the absence of heterogeneity in investor beliefs at time 1 when the payout policy is executed.

We measure the price impact by the total return on the execution day, which is the sum of dividend yield and capital gains yield. In the case of a stock repurchase, the dividend yield is zero and the capital gains yield is potentially nonzero. In contrast, in the case of a cash dividend payment, both the dividend yield and capital gains yield are potentially nonzero. Therefore, we must take both the dividend yield and capital gains yield into account if we are to compare the total returns on the execution day for a dividend payment versus a stock repurchase.

Proposition 6. (*The Price Impact of Cash Dividends and Stock Repurchases*)

- (i) *If the firm pays a cash dividend, then the price impact will be zero.*
- (ii) *If the earnings E available to the firm at time 1 is small, so that the parametric restriction (4) holds, then:*
 - (a) *The price impact of a stock repurchase will be weakly positive.*
 - (b) *The price impact of a stock repurchase will be weakly greater when the firm repurchases a larger number of shares compared to the situation where it repurchases a smaller number of shares.*
- (iii) *If the earnings E available to the firm at time 1 is large, so that the parametric restriction (5) holds, the price impact of a stock repurchase will be strictly positive.*

Before the cash dividend or stock repurchase comes into effect, the firm's stock price at time 0 is either $(E + \underline{\theta}X - I)$ or $(E + \underline{\theta}\lambda'X - \lambda I)$, depending on whether the firm is known to implement its project at the underinvestment or the full investment level. Note that the pricing of equity at time 0 is

¹⁹Note that, empirically, the price impact is quite different from the announcement effect measured on the day of the announcement of the corporate payout (time 0 in our model), while the price impact is the abnormal return measured on the day the cash dividend or stock repurchase actually comes into effect (time 1 in our model).

driven by the belief (and therefore, the valuation) of the marginal investor in the firm's equity, which is $\underline{\theta}$.²⁰

If the firm makes a cash dividend payment, the share price at time 1 will decrease by the amount of the cash dividend. Without loss of generality, if the firm chooses to implement its project at the full investment level and makes a dividend payment of $(E - \lambda I)$, the firm's stock price will be $(E + \underline{\theta}\lambda'X - \lambda I)$ at time 0 before the dividend payment and $\underline{\theta}\lambda'X$ at time 1 after the payment. However, the total stock return at time 1 will be zero, because the capital gains yield will be $-\left(\frac{E - \lambda I}{\underline{\theta}\lambda'X + E - \lambda I}\right)$, and the dividend yield will be $\left(\frac{E - \lambda I}{\underline{\theta}\lambda'X + E - \lambda I}\right)$, so that the total return, which is the sum of the two, will be zero.

If the earnings E available to the firm at time 1 is small and the firm chooses to undertake a stock repurchase, the price impact of the stock repurchase will depend on the number of shares the firm repurchases. If the firm implements its project at the full investment level, the firm's share price will be $(E + \underline{\theta}\lambda'X - \lambda I)$ before the stock repurchase (since the total firm value is $(E + \underline{\theta}\lambda'X - \lambda I)$ and the firm has one share outstanding). After the stock repurchase, the total firm value will be $\underline{\theta}\lambda'X$, but the total number of shares outstanding will be reduced to $\left(1 - \frac{E - \lambda I}{E + \underline{\theta}\lambda'X - \lambda I}\right)$, so that the share price will be

$$\frac{\underline{\theta}\lambda'X}{1 - \frac{E - \lambda I}{E + \underline{\theta}\lambda'X - \lambda I}} = E + \underline{\theta}\lambda'X - \lambda I, \quad (23)$$

yielding a total return of zero on the firm's equity (i.e., the price impact is zero). Notice that, in this case, the number of shares repurchased by the firm is less than δ , and the firm repurchases shares from pessimistic outside shareholders only. Thus, the price of the firm's equity does not change after the stock repurchase, since the marginal investor in the firm's equity after the repurchase is the same as that before the repurchase, namely, a pessimistic investor with belief $\underline{\theta}$.

²⁰In any equity market, the price at which a trade takes place will be the lowest price at which sellers are willing to sell and the highest price at which buyers are willing to buy. In our setting, this will clearly be determined by the belief $\underline{\theta}$, since this is the belief of current pessimistic equity holders about the firm's future prospects, which also equals the belief of non-shareholders.

On the other hand, if the firm undertakes a stock repurchase while it underinvests in its new project, it will have to repurchase a larger number of shares (from both pessimistic and optimistic outside shareholders) compared to the above case where it undertakes a stock repurchase while implementing its project at the full investment level. Before the stock repurchase, the firm's share price will be $(E + \underline{\theta}X - I)$. After the repurchase, the total firm value will be $\bar{\theta}X$, but the total number of shares outstanding will be reduced to $\left(1 - \delta - \frac{E - I - \delta(E + \underline{\theta}X - I)}{E + \bar{\theta}X - I}\right)$, so that the share price will be

$$\frac{\bar{\theta}X}{1 - \delta - \frac{E - I - \delta(E + \underline{\theta}X - I)}{E + \bar{\theta}X - I}} = \frac{\bar{\theta}}{\delta\underline{\theta} + (1 - \delta)\bar{\theta}}(E + \bar{\theta}X - I). \quad (24)$$

Thus, the total return on the firm's equity will be

$$\frac{\frac{\bar{\theta}}{\delta\underline{\theta} + (1 - \delta)\bar{\theta}}(E + \bar{\theta}X - I) - (E + \underline{\theta}X - I)}{E + \underline{\theta}X - I} > 0, \quad (25)$$

yielding a positive price impact. Here, the share price after the stock repurchase is higher than that prevailing before the stock repurchase, since the marginal investor in the firm's equity after the stock repurchase is an optimistic investor with belief $\bar{\theta}$, while the marginal investor prior to the repurchase was a pessimistic investor with belief $\underline{\theta}$.^{21,22}

The intuition behind part (ii)(b) of Proposition 6 is that, when a firm repurchases a larger number of shares, it is more likely that the marginal investor in the firm's equity changes to a more optimistic

²¹Note that when a firm undertakes a pure stock repurchase and underinvests in its new project, the price impact is given by (25) regardless of whether the earnings E is small or large.

²²When E is small, if the firm chooses to distribute value through a combination of a stock repurchase and a dividend payment, and underinvest in its new project, it first repurchases δ shares from pessimistic shareholders, and then pays a cash dividend of $(E - I - \delta(E + \underline{\theta}X - I))$ to the remaining shareholders. The share price at time 0 will be $(E + \underline{\theta}X - I)$. After the repurchase, the total number of shares outstanding will be $(1 - \delta)$. If the number of shares repurchased by the firm is slightly (ϵ) less than δ , the marginal investor in the firm's equity will still be a pessimistic investor with belief $\underline{\theta}$ after the payout at time 1. Therefore, the share price at time 1 after the stock repurchase and the dividend payment is $\frac{\underline{\theta}X}{1 - \delta}$, which implies a capital gain of $\left(\frac{\underline{\theta}X}{1 - \delta} - (E + \underline{\theta}X - I)\right)$, while the dividend per share is $\left(\frac{E - I - \delta(E + \underline{\theta}X - I)}{1 - \delta}\right)$. The total return on the firm's equity, and therefore, the price impact is zero, since the dividend payment and the capital gain cancel each other out.

investor after the repurchase compared to that before the repurchase.²³ Thus, if we compare the two situations characterized above, the price impact when the firm invests up to the full investment level and repurchases a smaller number of shares is zero, while it is positive when the firm underinvests and repurchases a larger number of shares.

Part (iii) of Proposition 6 shows that if the earnings E is large and the firm chooses to undertake a stock repurchase alone to distribute value to shareholders, the price impact of the stock repurchase will be strictly positive, since the firm will go up the ladder of shareholder beliefs regardless of the optimal level of investment in its new project. Unlike in the above case where E is small, if the firm chooses to implement its project at the full investment level when E is large, the firm will repurchase shares from both pessimistic and optimistic outside shareholders. In this case, we show that the total return on the firm's equity will be

$$\frac{\frac{\bar{\theta}}{\delta\bar{\theta}+(1-\delta)\underline{\theta}}(E + \bar{\theta}\lambda'X - \lambda I) - (E + \underline{\theta}\lambda'X - \lambda I)}{E + \underline{\theta}\lambda'X - \lambda I} > 0, \quad (26)$$

also yielding a positive price impact in the case where the firm chooses a stock repurchase alone to distribute value while implementing its project at the full investment level. Thus, when E is large, a stock repurchase has a positive price impact regardless of whether the firm underinvests in its project or implements it at the full investment level, since after the repurchase, the belief of the marginal investor in the firm's equity increases from $\underline{\theta}$ to $\bar{\theta}$.

²³In other words, when the firm repurchases a larger number of shares, the firm will be more likely to go up the ladder of shareholder beliefs, and therefore, the price impact will be greater. The price impact of a stock repurchase is not continuously increasing in our model due to our assumption of two discrete groups of investors, one with more pessimistic belief than the other. If instead we assume that there are atomistic investors and their beliefs are continuously distributed, then any number of shares repurchased will change marginal investor beliefs and therefore, the price of the firm's stock. In such a setting, the price impact will be strictly increasing in the number of shares repurchased.

4 Analysis of the Case Where the Firm May Raise External Financing

In this section, we allow the firm to issue debt or equity to raise external financing from outside investors. Our objective here is to analyze whether the a firm has an incentive to raise external financing by issuing debt or equity while distributing value to shareholders, and if so, to study the method the firm would adopt to implement this payout (cash dividend or stock repurchase). In this section, we assume that the low cash flow X^L (λ') from the firm's new project when the investment level is I (λI) is positive: i.e., $X^H > X^L > 0$. We make this assumption in order to model the notion that debt is a less belief-sensitive security than equity.²⁴

4.1 Analysis of the Case Where the Firm May Issue Debt

In this subsection, we show that when the earnings E is small, and the firm undertakes a stock repurchase while implementing its new project at the full investment level, debt issuance can be the optimal course of action for firm insiders. In particular, when outside investors and some of the firm's current outside shareholders are more pessimistic about the firm's future prospects than firm insiders, i.e., $\theta^f > \underline{\theta}$, issuing debt (the less belief-sensitive security) and using the proceeds from the debt issue to buy back more of the firm's (undervalued) equity (the more belief-sensitive security) can benefit firm insiders.²⁵ The debt issued by the firm can be either risk-free or risky.

We will now characterize the conditions under which the firm issues new debt to raise external financing while distributing value to shareholders through a stock repurchase.

Proposition 7. *(The Case Where the Firm May Simultaneously Issue Debt and Distribute*

²⁴Note that risk-free debt is not belief-sensitive at all; i.e., it is belief-neutral.

²⁵In all other scenarios, where the firm has enough cash to buy out all current shareholders with low belief $\underline{\theta}$ anyway, the firm's incentive to raise external financing is much weaker. Therefore, our analysis here focuses on the case where E is small and the firm implements its new project at the full investment level.

Value) Let the earnings E available to the firm at time 1 be small, so that the parametric restriction (4) holds. If $\theta^f > \underline{\theta}$, then:

- (i) If $\theta^f < \delta\underline{\theta} + (1 - \delta)\bar{\theta}$ and $\delta\lambda'(\underline{\theta}X^H + (1 - \underline{\theta})X^L) - (1 - \delta)(E - \lambda I) \leq \lambda'X^L$, the firm will issue new risk-free debt worth $P_D = \delta\lambda'(\underline{\theta}X^H + (1 - \underline{\theta})X^L) - (1 - \delta)(E - \lambda I)$, repurchase all the shares (δ) held by its current outside shareholders with belief $\underline{\theta}$, and implement its project at the full investment level (λI) simultaneously, if the following condition holds:

$$\frac{E + \lambda'(\theta^f X^H + (1 - \theta^f)X^L) - \lambda I - \delta(E + \lambda'(\underline{\theta}X^H + (1 - \underline{\theta})X^L) - \lambda I)}{(1 - \delta) \frac{(\theta^f X^H + (1 - \theta^f)X^L)(E + \bar{\theta}X^H + (1 - \bar{\theta})X^L - I)}{\delta(\underline{\theta}X^H + (1 - \underline{\theta})X^L) + (1 - \delta)(\bar{\theta}X^H + (1 - \bar{\theta})X^L)}} > 1. \quad (27)$$

- (ii) If $\theta^f < \delta\underline{\theta} + (1 - \delta)\bar{\theta}$ and $\delta\lambda'(\underline{\theta}X^H + (1 - \underline{\theta})X^L) - (1 - \delta)(E - \lambda I) > \lambda'X^L$, the firm will issue new risky debt worth $P_D = \delta\lambda'(\underline{\theta}X^H + (1 - \underline{\theta})X^L) - (1 - \delta)(E - \lambda I)$, repurchase all the shares (δ) held by its current outside shareholders with belief $\underline{\theta}$, and implement its project at the full investment level (λI) simultaneously, if the following condition holds:

$$\frac{\theta^f}{\underline{\theta}} (E + \lambda'(\underline{\theta}X^H + (1 - \underline{\theta})X^L) - \lambda I) > \frac{(\theta^f X^H + (1 - \theta^f)X^L)(E + \bar{\theta}X^H + (1 - \bar{\theta})X^L - I)}{\delta(\underline{\theta}X^H + (1 - \underline{\theta})X^L) + (1 - \delta)(\bar{\theta}X^H + (1 - \bar{\theta})X^L)}. \quad (28)$$

- (iii) The firm will never choose to issue equity and simultaneously repurchase shares from its current outside shareholders. Further, the firm will never choose to issue debt and simultaneously pay dividends to its current outside shareholders.

From our earlier discussion, we know that if the earnings E available to the firm at time 1 is small and firm insiders are more optimistic than the pessimistic group of outside shareholders, the firm's earnings left over after implementing its project at the full investment level, $E - \lambda I$, are not large enough to buy back all the undervalued equity held by the pessimistic outside shareholders, even though the NPV of repurchasing these shares is positive. Therefore, we showed that if the incremental NPV from increasing the scale of the project is not too large, the firm may even choose to underinvest in its new project in order to buy back all the shares held by pessimistic outside shareholders, and thereby maximize the NPV of repurchasing undervalued shares. This proposition shows that if the incremental NPV from increasing the scale of the project is not too small, the firm's optimal choice will be to raise some external financing by issuing some debt (which is less belief-sensitive) in order to finance its desired repurchases of equity held by pessimistic outside shareholders while implementing its new project at

the full investment level.

Further, this proposition shows that there is no point for firm insiders to issue new equity to outside investors with belief $\underline{\theta}$, and then buy back stock from current outside shareholders with belief $\underline{\theta}$, since the valuation effects of these two transactions will cancel each other out. Finally, if $\theta^f > \underline{\theta}$, the firm will never choose to issue debt and simultaneously pay dividends to its current outside shareholders, since in this case, the NPV of repurchasing shares is positive while the NPV of paying out dividends is zero.

4.2 Analysis of the Case Where the Firm May Issue Equity

We assume that the total wealth of outsiders who are interested in investing in the firm's equity is W . Recall also our assumption that outside investors who currently do not hold equity in the firm believe that the success probability of the firm's project is $\underline{\theta}$. We now characterize the conditions under which the firm simultaneously issues equity while distributing value to shareholders.

Proposition 8. *(The Case Where the Firm May Simultaneously Issue Equity and Distribute Value) If $\theta^f < \underline{\theta}$, then:*

- (i) *The firm will issue new equity and make a dividend payment to its current outside shareholders simultaneously. In particular, the firm will implement its project at the full investment level (λI), issue $\frac{W}{E + \lambda'(\underline{\theta}X^H + (1-\underline{\theta})X^L) - \lambda I}$ new shares, and pay a cash dividend of $E - \lambda I + W$ to its current shareholders.*
- (ii) *The firm will never choose to issue equity and simultaneously repurchase shares from current outside shareholders. Further, the firm will never choose to issue debt and simultaneously pay dividends to its current outside shareholders.*

In Propositions 4 and 5, we showed that if $\theta^f < \underline{\theta}$, the firm will implement its project at the full investment level (λI), and distribute value to its shareholders through a dividend payment ($E - \lambda I$) alone at time 1.²⁶ When firm insiders are more pessimistic than both groups of outside shareholders about the future prospects of the firm, the NPV of repurchasing shares is negative, and the firm prefers a dividend

²⁶Recall also that the firm chooses to implement its new project at the full investment level, since the incremental NPV of increasing the project's scale is positive.

payment alone, which is a zero-NPV transaction, to distribute its excess earnings to shareholders. The intuition underlying Proposition 8 is that if $\theta^f < \underline{\theta}$, firm insiders will also have an incentive to issue new equity simultaneously in order to take advantage of the optimism of outside investors by selling new shares to them at a price higher than the insiders' own valuation of the firm, thereby increasing the firm's dividend payout to its shareholders.²⁷ They have to do this at time 1, before the cash flows are realized to take advantage of the temporary heterogeneity in beliefs between insiders and outsiders.

Consider the case when the firm implements its project at the full investment level, issues β_b new shares to outsiders, and pays an amount D_c as cash dividend. Since all outsiders who currently do not hold equity in the firm have belief $\underline{\theta}$, the price per share in the equity issue will be $(E + \lambda'(\underline{\theta}X^H + (1 - \underline{\theta})X^L) - \lambda I)$. Before the stock issue, the firm has one share of stock outstanding. After stock issue, the firm will have $(1 + \beta_b)$ shares outstanding. After investing an amount λI in its new project, the firm pays out a cash dividend of

$$D_c = E - \lambda I + \beta_b (E + \lambda'(\underline{\theta}X^H + (1 - \underline{\theta})X^L) - \lambda I) \quad (29)$$

to its shareholders. The objective function of firm insiders is given by:

$$\max_{\beta_b} EU = \frac{\alpha}{1 + \beta_b} [E - \lambda I + \beta_b (E + \lambda'(\underline{\theta}X^H + (1 - \underline{\theta})X^L) - \lambda I) + \lambda'(\theta^f X^H + (1 - \theta^f)X^L)] \quad (30)$$

where $\beta_b \in \left[0, \frac{W}{E + \lambda'(\underline{\theta}X^H + (1 - \underline{\theta})X^L) - \lambda I}\right]$.²⁸ The partial derivative of this objective function with respect to

²⁷Since outside investors' belief is $\underline{\theta}$, the NPV of issuing new equity at time 1 will be positive based on insiders' belief if and only if $\theta^f < \underline{\theta}$.

²⁸ $\frac{W}{E + \lambda'(\underline{\theta}X^H + (1 - \underline{\theta})X^L) - \lambda I}$ is the maximum number of shares the firm can issue to outside investors, if it implements the project at the full investment level.

the choice variable β_b is given by:

$$\frac{\partial EU}{\partial \beta_b} = \frac{\alpha(\underline{\theta} - \theta^f) \lambda' (X^H - X^L)}{(1 + \beta_b)^2}. \quad (31)$$

Note that if $\theta^f < \underline{\theta}$, this partial derivative is positive, and the optimal choice for the firm is to issue as many new shares as possible by choosing $\beta_b = \frac{W}{E + \lambda'(\underline{\theta}X^H + (1-\underline{\theta})X^L) - \lambda I}$. On the other hand, if $\theta^f \geq \underline{\theta}$, $\frac{\partial EU}{\partial \beta_b} \leq 0$, and the optimal choice for the firm is to issue no new equity by setting $\beta_b = 0$.²⁹

Earlier, we showed in Propositions 4 and 5 that the firm will have an incentive to repurchase equity from current outside shareholders only if insiders are more optimistic than pessimistic shareholders, i.e., if $\theta^f > \underline{\theta}$. Given that firm insiders have an incentive to issue equity only if they are more pessimistic about the future prospects of the firm than the marginal investor in the firm's equity with belief $\underline{\theta}$, i.e. if $\theta^f < \underline{\theta}$, it follows that insiders never find it optimal to issue new equity and repurchase shares from current shareholders at the same time. In other words, in our setting with heterogeneous beliefs between insiders and outsiders, when the NPV of issuing new equity based on insiders' belief is positive, the NPV of repurchasing shares from current shareholders is negative, and vice versa. On the other hand, the NPV of a dividend payment is always zero, and if firm insiders are more pessimistic than outside investors about the firm's future prospects (so that the NPV of issuing equity is positive), it will be optimal for the firm to issue equity and distribute value through a dividend payment simultaneously.

²⁹Note that in this case, i.e., when $\theta^f > \underline{\theta}$, if we don't restrict β_b to be nonnegative and E is sufficiently large so that (5) holds, the optimal value of β_b will be equal to $-\delta$, and the firm will optimally repurchase the δ shares held by pessimistic shareholders, and distribute the remaining earnings as a dividend payment. The optimality of a combination of a share repurchase and a dividend payment (when E is large) in this case was already shown in Proposition 5.

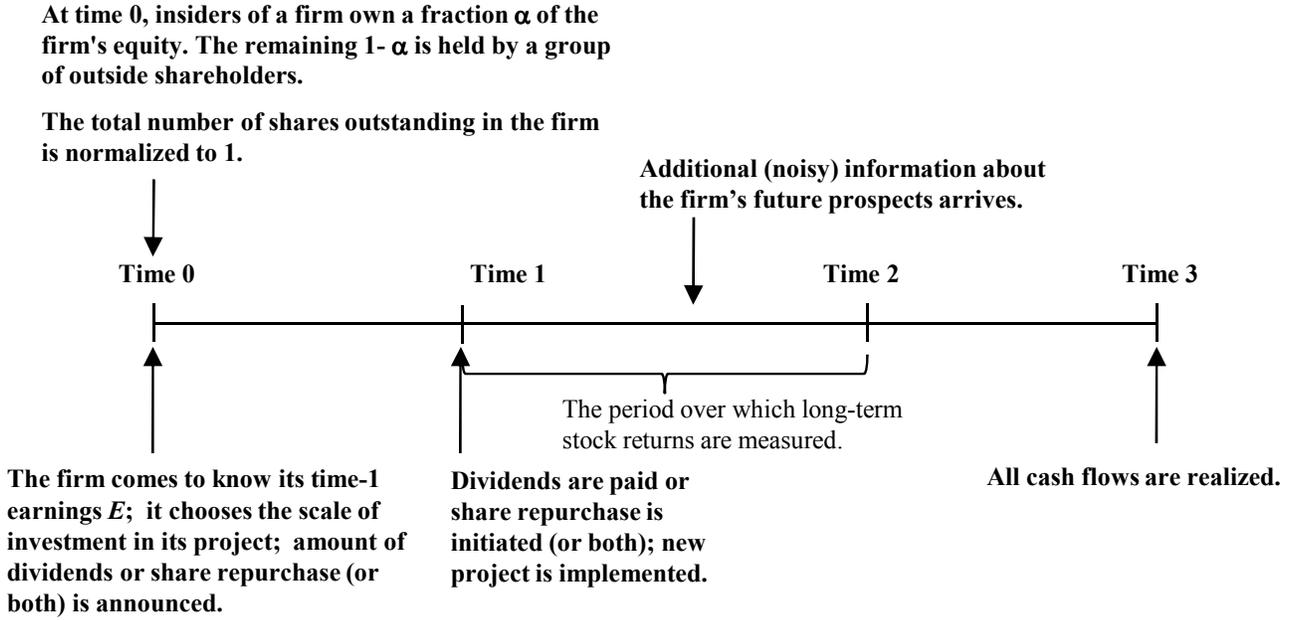


Figure 2: Sequence of Events in the Extended Model

5 Long-Run Stock Returns following Dividend Payments and Stock Repurchases

In this section, we will analyze the long-run stock returns of firms following cash dividend payments and stock repurchases using an extended model. In extending our basic model, we will make the additional assumption that, in the long run, some additional noisy public information arrives about the firm's future performance. We denote the final date in our model as time 3, and introduce another date (time 2) between the corporate payout date (time 1) and the final cash flow realization date (time 3); noisy new information about the firm's future operating performance becomes available to outsiders at time 2. The sequence of events in this extended model is given in Figure 2.

The noisy new information arriving at time 2 is hard information, and may be collected from the firm's annual reports, earnings announcements, and other public sources. Thus, due to the arrival of this new information, we assume that investor beliefs about the firm's cash flows become less heterogeneous

in the long run. The motivation for this assumption is that the beliefs of each agent is a function of their prior beliefs and the additional information they receive from various sources. Thus, at time 0, the beliefs of each of the three groups of agents in our model (firm insiders, optimistic outside shareholders, and pessimistic outside shareholders) is driven by their (heterogeneous) prior beliefs. On the other hand, at time 2, the beliefs of these three groups of agents is determined not only by their prior beliefs, but also by the common (hard) information that arrives between the payout date and time 2. Since the proportion of information common to the above three groups of agents increases at time 2, their beliefs become less heterogenous at this date. Therefore, we assume that in the long run (i.e., at time 2), the three different beliefs θ^f , $\bar{\theta}$, and $\underline{\theta}$ will converge to each other. Let us define the beliefs of the insiders, optimistic outsiders, and pessimistic outsiders at time 2 as ν^f , $\bar{\nu}$, and $\underline{\nu}$, respectively. The beliefs converge as follows in the long run (i.e., at time 2):

$$\nu^f = \theta^f + \frac{d}{3}(\bar{\theta} - \theta^f) + \frac{d}{3}(\underline{\theta} - \theta^f), \quad (32)$$

$$\bar{\nu} = \bar{\theta} + \frac{d}{3}(\theta^f - \bar{\theta}) + \frac{d}{3}(\underline{\theta} - \bar{\theta}), \quad (33)$$

$$\underline{\nu} = \underline{\theta} + \frac{d}{3}(\theta^f - \underline{\theta}) + \frac{d}{3}(\bar{\theta} - \underline{\theta}), \quad (34)$$

where $d \in [0, 1]$ measures the degree of convergence. When $d = 0$, there is no convergence, and we have $\nu^f = \theta^f$, $\bar{\nu} = \bar{\theta}$, and $\underline{\nu} = \underline{\theta}$. On the other extreme, when $d = 1$, there is full convergence, and we have $\nu^f = \bar{\nu} = \underline{\nu} = \frac{1}{3}(\theta^f + \bar{\theta} + \underline{\theta})$.

We first characterize the long-run stock return in a situation where the firm uses only a dividend payment to distribute value to shareholders.

Proposition 9. (Long-run stock returns following cash dividends) *If the firm makes a dividend payment and undertakes its new project at the full investment level, the long-run stock return will be $\frac{d}{3} \frac{(\theta^f - \underline{\theta}) + (\bar{\theta} - \underline{\theta})}{\underline{\theta}}$. The long-run stock return will be positive if $\underline{\theta} - (\bar{\theta} - \underline{\theta}) < \theta^f \leq \underline{\theta}$, and it will be negative if $\theta^f < \underline{\theta} - (\bar{\theta} - \underline{\theta})$.*

If the firm distributes value through a dividend payment alone while implementing its new project at the full investment level, the firm's stock price will be $P_1 = \underline{\theta}\lambda'X$ at time 1 after the dividend payment, and the number of shares outstanding will still be 1. At time 2, after the marginal equity investor updates her belief, the stock price will be

$$P_2 = \left[\underline{\theta} + \frac{d}{3}(\theta^f - \underline{\theta}) + \frac{d}{3}(\bar{\theta} - \underline{\theta}) \right] \lambda'X. \quad (35)$$

The long-run stock return, $\frac{P_2 - P_1}{P_1}$, will be given by $\frac{d}{3} \frac{(\theta^f - \underline{\theta}) + (\bar{\theta} - \underline{\theta})}{\underline{\theta}}$. We know that a firm will choose to distribute value through a dividend payment alone (while implementing its new project at the full investment level) only if $\theta^f < \underline{\theta}$. Further, we know that $\bar{\theta} - \underline{\theta} > 0$ by assumption. Thus, the long-run stock return will be positive if $\underline{\theta} + (\bar{\theta} - \underline{\theta}) < \theta^f \leq \underline{\theta}$. However, if firm insiders are much more pessimistic than the firm's current shareholders so that $\theta^f < \underline{\theta} - (\bar{\theta} - \underline{\theta})$, then the long-run stock return will be negative.

We now characterize the long-run stock return in a situation where the firm uses only a stock repurchase to distribute value to shareholders.

Proposition 10. (Long-run stock returns following stock repurchases)

- (i) If E is small, and the firm implements a stock repurchase and undertakes its project at the full investment level, then its long-run stock return will be $\frac{d}{3} \frac{(\theta^f - \underline{\theta}) + (\bar{\theta} - \underline{\theta})}{\underline{\theta}}$. In this case, the long-run stock return will be positive, since the firm will choose to distribute value through a stock repurchase only if $\theta^f > \underline{\theta}$.
- (ii) If the firm implements a stock repurchase and underinvests in its project, then its long-run stock return will be $\frac{d}{3} \frac{(\theta^f - \bar{\theta}) + (\underline{\theta} - \bar{\theta})}{\bar{\theta}}$. The long-run stock return will be positive if $\theta^f > \bar{\theta} + (\bar{\theta} - \underline{\theta})$, and it will be negative if $\underline{\theta} < \theta^f < \bar{\theta} + (\bar{\theta} - \underline{\theta})$.
- (iii) If E is large, and the firm implements a stock repurchase and undertakes its project at the full investment level, then its long-run stock return will be $\frac{d}{3} \frac{(\theta^f - \bar{\theta}) + (\underline{\theta} - \bar{\theta})}{\bar{\theta}}$. The long-run stock return will be positive if $\theta^f > \bar{\theta} + (\bar{\theta} - \underline{\theta})$, and it will be negative if $\underline{\theta} < \theta^f < \bar{\theta} + (\bar{\theta} - \underline{\theta})$.

When E is small, if the firm chooses to implement its project at the full investment level and distribute value through a stock repurchase alone, after the repurchase, the total firm value is $\underline{\theta}\lambda'X$,

but the number of shares outstanding is reduced to $\left(1 - \frac{E-\lambda I}{E+\underline{\theta}\lambda'X-\lambda I}\right)$, so that the price per share is

$$P_1 = \frac{\underline{\theta}\lambda'X}{\left(1 - \frac{E-\lambda I}{E+\underline{\theta}\lambda'X-\lambda I}\right)}, \quad (36)$$

at time 1. At time 2, after the marginal equity investor updates her belief, the price per share will change to

$$P_2 = \frac{\left[\underline{\theta} + \frac{d}{3}(\theta^f - \underline{\theta}) + \frac{d}{3}(\bar{\theta} - \underline{\theta})\right]\lambda'X}{\left(1 - \frac{E-\lambda I}{E+\underline{\theta}\lambda'X-\lambda I}\right)}. \quad (37)$$

Therefore, the long-run stock return, $\frac{P_2-P_1}{P_1}$, is equal to $\frac{d}{3} \frac{(\theta^f - \underline{\theta}) + (\bar{\theta} - \underline{\theta})}{\underline{\theta}}$. We know that a firm chooses to implement its project at the full investment level and distribute value through a stock repurchase alone only if $\theta^f > \underline{\theta}$. Further, we know that $\bar{\theta} - \underline{\theta} > 0$ by assumption. Thus, the long-run stock return will always be positive in this case.

If the firm chooses to distribute value through a stock repurchase alone while underinvesting in its new project, the firm's stock price per share will be $E + \underline{\theta}X - I$ before the stock repurchase (because the firm value is $E + \underline{\theta}X - I$ and the firm has one share outstanding). After the repurchase, the total firm value will be $\bar{\theta}X$, but the number of shares outstanding will be reduced to $\left(1 - \delta - \frac{E-I-\delta(E+\underline{\theta}X-I)}{E+\underline{\theta}X-I}\right)$, so the price per share will be

$$P_1 = \frac{\bar{\theta}X}{\left(1 - \delta - \frac{E-I-\delta(E+\underline{\theta}X-I)}{E+\underline{\theta}X-I}\right)} \quad (38)$$

at time 1. At time 2, after the marginal equity investor updates her belief, the stock price will change to

$$P_2 = \frac{\left[\bar{\theta} + \frac{d}{3}(\theta^f - \bar{\theta}) + \frac{d}{3}(\underline{\theta} - \bar{\theta})\right]X}{\left(1 - \delta - \frac{E-I-\delta(E+\underline{\theta}X-I)}{E+\underline{\theta}X-I}\right)}. \quad (39)$$

Therefore, in this case, the long-run stock return, $\frac{P_2-P_1}{P_1}$, is equal to $\frac{d}{3} \frac{(\theta^f - \bar{\theta}) + (\underline{\theta} - \bar{\theta})}{\bar{\theta}}$. Note that if both groups of outsiders (with beliefs $\underline{\theta}$ and $\bar{\theta}$, respectively) are much more pessimistic about the firm's

future prospects than firm insiders with belief θ^f , so that $\theta^f > \bar{\theta} + (\bar{\theta} - \underline{\theta})$, then the long-run stock return following the stock repurchase will be positive in this case.

When E is large, if the firm chooses to distribute value through a stock repurchase alone while implementing its project at the full investment level, the marginal investor in the firm's equity after the repurchase at time 1 will also have the belief $\bar{\theta}$ as in part (ii) of Proposition 10. Therefore, in this case, the long-run stock return following the stock repurchase will be positive if insiders' belief θ^f is sufficiently high so that $\theta^f > \bar{\theta} + (\bar{\theta} - \underline{\theta})$. Otherwise, it will be negative.

We now characterize the long-run stock return in a situation where the firm uses a combination of a dividend payment and a stock repurchase to distribute value to shareholders.

Proposition 11. (*Long-run stock returns following a combination of a stock repurchase and a cash dividend*) *If the firm chooses a combination of a stock repurchase and a dividend payment to distribute value to shareholders, the long-run stock return will be $\frac{d}{3} \frac{(\theta^f - \underline{\theta}) + (\bar{\theta} - \underline{\theta})}{\bar{\theta}}$. In this case, the long-run stock return will be positive, since the firm will choose to distribute value through a stock repurchase only if $\theta^f > \underline{\theta}$.*

If the firm chooses to distribute value through a combination of a stock repurchase and a dividend payment, the firm first repurchases δ shares from pessimistic shareholders and then makes a dividend payment of $(E - I - \delta(E + \underline{\theta}X - I))$ to the remaining shareholders.³⁰ The price per share at time 0 is $E + \underline{\theta}X - I$. Recall that after the repurchase, the total number of shares outstanding is $1 - \delta$. Therefore, the price per share at time 1 after the stock repurchase and the dividend payment is $P_1 = \frac{\underline{\theta}X}{1 - \delta}$.³¹ At time 2, the price will be $P_2 = \frac{[\underline{\theta} + \frac{d}{3}(\theta^f - \underline{\theta}) + \frac{d}{3}(\bar{\theta} - \underline{\theta})]X}{1 - \delta}$. So the long-run stock return is positive if $\theta^f > \bar{\theta}$. Note that for the firm to optimally choose a combination of a stock repurchase and a dividend payment to distribute value to its shareholders, it has to be the case that $\theta^f > \underline{\theta}$. Therefore, the long-run stock return will always be positive in this case.

³⁰Here, we assume that the firm underinvests in its project. But the logic and the result regarding the firm's long-run stock return is the same when E is large and the firm implements its project at the full investment level while distributing value to its shareholders through a combination of a stock repurchase and a dividend payment.

³¹We assume that the number of shares repurchased by the firm is slightly (ϵ) less than δ so that the marginal investor in the firm's equity has still the belief $\underline{\theta}$ after the payout at time 1.

5.1 Comparison of Long-Run Stock Returns Following Dividends versus Stock Repurchases

We now compare the long-run stock returns of firms using dividend payments versus those using stock repurchases to distribute value.

Proposition 12. *(Comparison of the long-run stock return following a cash dividend versus that following a stock repurchase) Let the earnings E available at time 1 be small, firm insiders' belief θ^f be fixed, and $k = \bar{\theta} - \underline{\theta}$ be a constant for all firms. Consider two samples of firms: (i) a sample of firms, which distribute value through dividend payments alone and implement their projects at the full investment level; (ii) a sample of firms, which distribute value through stock repurchases alone and implement their projects at the full investment level. Then, the average long-run stock return of the latter sample of firms will be greater than the average long-run stock return of the former sample of firms.*

Earlier we showed that if E is small, conditional on implementing its new project at the full investment level, a firm's long-run stock return following a pure cash dividend payment or a pure stock repurchase is given by:

$$\frac{d}{3} \frac{(\theta^f - \underline{\theta}) + (\bar{\theta} - \underline{\theta})}{\underline{\theta}}. \quad (40)$$

Assuming that $k = \bar{\theta} - \underline{\theta}$, this long-run stock return is equal to $\frac{d}{3} \frac{(\theta^f - \underline{\theta} + k)}{\underline{\theta}}$.³²

Proposition 9 showed that, for a firm which distributes value through a dividend payment alone (while implementing its new project at the full investment level), this long-run stock return is positive if $\underline{\theta} - (\bar{\theta} - \underline{\theta}) < \theta^f \leq \underline{\theta}$, and it is negative if $\theta^f < \underline{\theta} - (\bar{\theta} - \underline{\theta})$. We also know that a firm will choose to distribute value through a dividend payment only if $\theta^f \leq \underline{\theta}$ (see Proposition 4(i)(a)). If we assume that for the cross section firms paying dividends only and implementing their projects at the full investment level, optimistic shareholders' belief $\bar{\theta}$ is uniformly distributed over the interval $[\theta^f + k, 1)$, it follows that for the same cross section of firms, pessimistic shareholders' belief $\underline{\theta}$ is uniformly distributed over the interval $[\theta^f, 1 - k)$. While the long-run stock return will be positive for some fraction of these firms

³²Note that the long-run stock return given in (40) is decreasing in $\underline{\theta}$ (regardless of whether $k = \bar{\theta} - \underline{\theta}$ is fixed or not).

($\theta^f < \underline{\theta} \leq \theta^f + k$), it will be negative for the remaining fraction of them ($\theta^f + k < \underline{\theta} < 1 - k$). Thus, the *average* long-run stock return \bar{r}_{div} of firms paying dividends only and implementing their projects at the full investment level may be positive or negative, and it is given by:³³

$$\bar{r}_{div} = \int_{\theta^f}^{(1-k)} \frac{\frac{d}{3}(\theta^f - \theta + k)}{\theta} \frac{1}{(1-k-\theta^f)} d\theta. \quad (41)$$

In contrast, a firm will optimally choose to distribute value through a stock repurchase alone if firm insiders are more optimistic than the pessimistic group of outside shareholders, i.e., if $\theta^f > \underline{\theta}$. Further, we know from Proposition 10(i) that, in this case (when E is small), the marginal equity investor's belief will be $\underline{\theta}$ after the stock repurchase. Thus, the long-run valuation of the firm at time 2 will be determined by the updated time-2 belief of the marginal investor in the firm's equity at time 1, who has the belief $\underline{\theta}$ at time 1. Note that for a firm distributing value through a stock repurchase alone, the pessimistic shareholders' belief $\underline{\theta}$ is always lower than both the optimistic shareholders' belief $\bar{\theta}$ and the firm insiders' belief θ^f . When the three beliefs converge in the long run, the ex-post value of $\underline{\theta}$ will be above the ex-ante value of $\underline{\theta}$. Therefore, the long-run stock return of a firm that distributes value through a stock repurchase alone and implements its new project at the full investment level will always be positive as described in part (i) of Proposition 10. When E is small, for a firm to repurchase shares only and fully invest in its new project, we know from Proposition 4(i)(b) that $\frac{(\lambda-1)I}{(\lambda'-1)X} \leq \underline{\theta} < \theta^f$.³⁴

Thus, the *average* long-run stock return \bar{r}_{rep} of firms repurchasing shares only and implementing their

³³Note that $\theta^f - \theta < 0$ in the expression for \bar{r}_{div} given in (41).

³⁴Note that strictly speaking, for some parameter specifications, a small fraction of the share-repurchasing (and fully-investing) firms may actually satisfy the condition given in Proposition 4(ii)(b) rather than the condition given in Proposition 4(i)(b). For these firms, it holds that $\frac{(\lambda-1)I}{(\lambda'-1)X} - k < \underline{\theta} < \frac{(\lambda-1)I}{(\lambda'-1)X}$. Since the long-run stock return given in (40) is decreasing in $\underline{\theta}$, the average long-run stock return of these firms is greater than the average long-run stock return of firms satisfying the condition that $\frac{(\lambda-1)I}{(\lambda'-1)X} \leq \underline{\theta} < \theta^f$. Therefore, it suffices to show that the average long-run stock return of share-repurchasing firms, for which $\frac{(\lambda-1)I}{(\lambda'-1)X} \leq \underline{\theta} < \theta^f$, is greater than the average long-run stock return of dividend-paying firms satisfying the condition given in Proposition 4(i)(a).

projects at the full investment level will always be positive, and it is given by:

$$\bar{r}_{rep} = \int_{\frac{(\lambda-1)I}{(\lambda'-1)X}}^{\theta^f} \frac{\frac{d}{3}(\theta^f - \theta + k)}{\theta} \frac{1}{\left(\theta^f - \frac{(\lambda-1)I}{(\lambda'-1)X}\right)} d\theta. \quad (42)$$

It is straightforward to verify that $\bar{r}_{rep} > \bar{r}_{div}$. Thus, the average long-run stock return of firms that distribute value through a stock repurchase alone and implement their new projects at the full investment level will be greater than the average long-run stock return of firms that distribute value through a dividend payment only and implement their new projects at the full investment level.

6 Testable Implications

In this section, we describe some of the testable implications of our model. In developing testable implications based on our model, one has to interpret dividend payment in our model carefully. This is because our model of a firm's choice of payout policy is a static one, and it is well known that, in practice, many dynamic considerations enter into a firm's choice of dividend policy. For example, as documented by Lintner (1956) based on a survey of how managers choose dividend policy in practice, firm managers are usually reluctant to cut dividends (see, e.g., Lambrecht and Myers (2012) for a formal model of the dynamics of a firm's dividend policy). This means that, in many cases, dynamic considerations may prevent firms from reducing dividends and committing all available cash to make a stock repurchase, even though the purely static considerations based on heterogeneous beliefs that we model here may motivate them to do so. However, these problems arising from the interaction between dynamic and static considerations that drive a firm's choice of dividend policy can be almost fully addressed if we think of a "dividend payment" in our model as being, in practice, a dividend increase above its existing dividend level (or a dividend initiation, if the firm is not currently paying any dividends). Under this interpretation, if, for example, if our model predicts that a firm will choose not to pay any dividends in

equilibrium (i.e., it will choose to pay out all its cash through a repurchase), this translates, in practice, to the firm choosing not to raise its dividend above its existing level (so that the firm will maintain its current dividend level, and pay out all available remaining cash to its shareholders through a stock repurchase). In reading our testable implications below, it is useful to keep in mind this “dynamic” interpretation of the static results in our model.

(i) Relationship between insider and outsider beliefs and payout policy: Our model predicts that, if the level of optimism (average belief) among outside shareholders and the dispersion in beliefs among outsiders is such that a significant proportion of outside shareholders are less optimistic than firm insiders, then the firm is more likely to distribute value through a stock repurchase rather than paying a cash dividend. Conversely, if the bulk of outside shareholders are at the same or higher optimism level about the firm’s future prospects compared to firm insiders, the firm is more likely to distribute value through a cash dividend. This is because the net present value of a stock repurchase to firm insiders will be greater (more positive) when outsiders are more pessimistic while a dividend payment is a zero net present value transaction regardless of outsider beliefs. The above prediction can be tested using proxies for investor optimism developed by Baker and Wurgler (2006), and the two standard proxies for heterogeneity in investor beliefs used in the literature, namely, the dispersion in analyst earnings forecasts and abnormal share turnover.

(ii) Relationship between dividends and stock repurchases and investment policy: Our model predicts that firms implementing stock repurchases are more likely to cut back on positive net present value projects compared to those initiating or increasing dividends. This is because, while a stock repurchases can be a positive net present value transaction for firm insiders (if outside investors in the firm are less optimistic), paying dividends is at most a zero net present value transaction in our setting regardless of the relative optimism of firm insiders and outside shareholders about the firm’s future prospects. This

means that, when a firm is cash-constrained, firm insiders may compare the NPV of the real investment opportunities available to it to the NPV of undertaking a stock repurchase, and may prefer to implement a stock repurchase at the expense of undertaking the real investment opportunity if the latter NPV is greater.

(iii) Effects of macro-events on the propensity of a stock repurchase: Our model predicts that stock repurchases will increase after market crashes. A market crash is likely to make outside investors more pessimistic about the future prospects of firms (while insiders' beliefs are likely to be relatively less affected) resulting in an increase in the difference in beliefs of the two groups of agents (insiders versus outsiders). This, in turn, will lead to an increase in the number of firms repurchasing equity in periods where the difference in optimism between firm insiders and outsiders is greater.

(iv) Choice between stock repurchases and cash dividend payments under external financing: Our model predicts that firms are more likely to undertake a repurchase programs with borrowed cash than initiate (or increase) dividend payments with borrowed cash. The idea here is that, when outsiders are pessimistic compared to firm insiders, a stock repurchase can be a positive NPV transaction, whereas a dividend payment always has zero NPV. Given that there are transaction costs involved, this means that firms won't consistently pay dividends using borrowed money, whereas it may be optimal for them to undertake a stock repurchase under these conditions. On the other hand, our model implies that, when outside shareholders are relatively more optimistic about the firm's future prospects compared to firm insiders, the firm will simultaneously issue equity and pay out dividends since this allows them to take advantage of the optimism of outside investors. We also show that it never makes sense for a firm to initiate stock repurchases by issuing equity (something we never see firms doing in practice).

(v) Relationship between outside investor beliefs, cash holdings, and payout policy: Our model predicts that when the bulk of outside shareholders in the firm are more pessimistic about the firm's

prospects compared to firm insiders, then the firm will distribute value through either a stock repurchase alone, or a combination of a stock repurchase and a dividend increase (depending upon the amount of cash available to distribute to shareholders). If the amount of cash available to distribute is relatively small, the firm will undertake only a repurchase, since, in this case, a repurchase will be a positive NPV transaction. If, however, the amount of cash available to distribute is large, the firm will distribute only a fraction of this using a stock repurchase, since repurchasing a larger number of shares would involve buying back overvalued equity (from shareholders who are more optimistic than firm insiders). The firm will therefore use the remaining cash to make a dividend increase (recall that dividend payments are zero NPV transactions regardless of outsider beliefs).

(vi) Price impact of dividend payments and stock repurchases: Our model predicts that the price impact of a dividend payment (measured as the sum of any dividend yield and capital gain on the stock) will be zero while the price impact of a stock repurchase will be positive. Further, our model predicts that, the larger the number of shares repurchased by the firm, the larger the price impact. The intuition behind the latter prediction is that, as the firm repurchases more and more shares, it goes up the ladder of outside shareholder beliefs (repurchasing shares from outsiders with higher and higher valuations) so that the marginal investor after the completion of the repurchase program will have a higher valuation for the firm's equity if the number of shares repurchased is greater. Since, in practice, open market repurchases are completed over a period of months rather than days, the above price impact needs to be measured empirically over such a period. this prediction can serve to distinguish our model from the predictions of asymmetric information models for stock repurchases: if the positive effects of stock repurchases on the repurchasing firm's share price is due to the information release associated with the announcement of a repurchase, one would expect the effect of this release to be incorporated into the firm's share price in a matter of days, not months.

(vii) Long-run stock returns following dividend payments and stock repurchases: Our model makes three predictions regarding the above. First, the long-run stock return following dividend increases and stock repurchases may be positive or negative. However, the long-run stock return following a stock repurchase undertaken when a significant proportion of outsiders are pessimistic about the firm's prospects will be positive; the more pessimistic these outsiders, the larger the long-run stock return following the stock repurchase. Second, holding the real investment level constant, the long-run stock return following a stock repurchase will be greater than that following a dividend initiation or increase. Third, the long-run stock return following a dividend initiation (or increase) and stock repurchase combination (i.e., where both methods of payout are undertaken roughly around the same time) will be positive on average.

7 Conclusion

We analyzed a firm's choice between dividends and stock repurchases in a setting of heterogeneous beliefs and short sale constraints. We studied a setting in which the insiders of a firm, owning a certain fraction of its equity and having a certain amount of cash to distribute to shareholders, choose between paying out cash dividends and buying back equity to shareholders, as well the scale of investment in their firm's new project. Outside equity holders in the firm have heterogeneous beliefs about the probability of success of the firm's project and therefore its long run prospects; they may also disagree with firm insiders about this probability. We showed that, depending on the beliefs of firm insiders versus outsiders, the firm may distribute value by cash dividends alone; through a repurchase alone; or through a combination of a cash dividend and a stock repurchase. We also showed that, in many situations, it is optimal for firm insiders to underinvest in the firm's positive net present value project and undertake a stock repurchase with the amount of cash saved by underinvesting. We then analyzed

the price impact of a cash dividend versus a share repurchase, where the price impact is defined as the abnormal return to the firm's equity upon the actual payment of a cash dividend or the implementation of a share repurchase respectively (rather than the announcement of these events). Finally, we analyzed the long-run returns to a firm's equity following dividend payments and stock repurchases. Our model generates a number of testable predictions different from asymmetric information models of a firm's choice between dividends versus stock repurchases.

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Appendix: Proofs of Propositions

Proof of Proposition 1: If the investment level is I (underinvestment), the firm insiders' expected payoff is

$$\alpha D_c + E_1[\alpha C F_2^{equity} | \theta^f] = \alpha[E - I] + \alpha\theta^f X, \quad (\text{A.1})$$

If the investment level is λI (full investment), the firm insiders' expected payoff is

$$\alpha D_c + E_1[\alpha C F_2^{equity} | \theta^f] = \alpha[E - \lambda I] + \alpha\theta^f \lambda' X. \quad (\text{A.2})$$

Comparing the expected payoffs in the two cases, we find that the firm will implement its project at the full investment level λI if and only if

$$\theta^f X - I < \theta^f \lambda' X - \lambda I. \quad (\text{A.3})$$

From (2), it follows that the firm will invest an amount λI in its project and distribute its excess cash, $E - \lambda I$ to shareholders as dividends. Q.E.D.

Proof of Proposition 2: Let the earnings E available to the firm at time 1 be small, so that the parametric restriction (4) holds. If the firm decides to implement its project at the full investment level by investing an amount λI in the project and repurchase a dollar amount $E - \lambda I$ of shares from outsiders, the number of shares the firm buys back is

$$\alpha_b = \frac{E - \lambda I}{\theta \lambda' X + E - \lambda I} \quad (\text{A.4})$$

In this case, the firm insiders' expected payoff is given by:

$$\frac{\alpha}{1 - \frac{E - \lambda I}{\theta \lambda' X - \lambda I + E}} \theta^f \lambda' X = \frac{E + \theta \lambda' X - \lambda I}{\theta \lambda' X} \alpha \theta^f \lambda' X = \frac{E + \theta \lambda' X - \lambda I}{\theta} \alpha \theta^f. \quad (\text{A.5})$$

If the firm decides to underinvest by investing an amount I in its project and repurchase $E - I$ from outsiders, the number of shares the firm buys back is

$$\alpha_b = \delta + \frac{E - I - \delta(\theta X + E - I)}{\bar{\theta} X + E - I}. \quad (\text{A.6})$$

In this case, the firm buys the first δ shares at a price of $\theta X + E - I$ from outside shareholders with belief θ and the remaining shares at a price of $\bar{\theta} X + E - I$ from outside shareholders with belief $\bar{\theta}$. The firm insiders' expected payoff is given by:

$$\frac{\alpha}{1 - \left(\delta + \frac{E - I - \delta(\theta X + E - I)}{\bar{\theta} X + E - I} \right)} \theta^f X = \frac{E + \bar{\theta} X - I}{\delta \theta X + (1 - \delta) \bar{\theta} X} \alpha \theta^f X = \frac{E + \bar{\theta} X - I}{\delta \theta + (1 - \delta) \bar{\theta}} \alpha \theta^f. \quad (\text{A.7})$$

From the comparison of insiders' expected payoffs, it follows that firm insiders will choose to implement the firm's project at the full investment level if the following condition holds:

$$\frac{E + \theta \lambda' X - \lambda I}{\theta} > \frac{E + \bar{\theta} X - I}{\delta \theta + (1 - \delta) \bar{\theta}}. \quad (\text{A.8})$$

Otherwise, it is optimal for the firm to underinvest and repurchase a larger number of shares.

When is condition (A.8) more likely to be satisfied? First, the LHS of (A.8) increases with λ' (the NPV of the project at the full investment level). Second, the RHS of (A.8) decreases with $\bar{\theta}$, since

$$\frac{\partial \left[\frac{E + \bar{\theta} X - I}{\delta \theta + (1 - \delta) \bar{\theta}} \right]}{\partial \bar{\theta}} = \frac{\delta(E + \theta X - I) - (E - I)}{[\delta \theta + (1 - \delta) \bar{\theta}]^2} < 0. \quad (\text{A.9})$$

This inequality follows from $\delta < \frac{E-I}{E+\underline{\theta}X-I}$ in (4). This means that when $\bar{\theta}$ is high, condition (A.8) is more likely to be satisfied so that it will be optimal for the firm to implement its project at the full investment level. After rearranging (A.8), we obtain the following equivalent restriction on $\bar{\theta}$:

$$\bar{\theta} \geq \frac{\theta(E-I-\delta(E+\underline{\theta}\lambda'X-\lambda I))}{(1-\delta)(E+\underline{\theta}\lambda'X-\lambda I)-\underline{\theta}X}, \quad (\text{A.10})$$

where $\frac{\theta(E-I-\delta(E+\underline{\theta}\lambda'X-\lambda I))}{(1-\delta)(E+\underline{\theta}\lambda'X-\lambda I)-\underline{\theta}X} > \underline{\theta}$ if and only if $\underline{\theta} < \frac{(\lambda-1)I}{(\lambda'-1)X}$.

Let the earnings E available to the firm at time 1 be large, so that the parametric restriction (5) holds. If the firm decides to implement its project at the full investment level by investing an amount λI in the project and repurchase a dollar amount $E - \lambda I$ of shares from outsiders, the number of shares the firm buys back is

$$\alpha_b = \delta + \frac{E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)}{E + \bar{\theta}\lambda'X - \lambda I}. \quad (\text{A.11})$$

In this case, the firm buys the first δ shares at a price of $(E + \underline{\theta}\lambda'X - \lambda I)$ from outside shareholders with belief $\underline{\theta}$ and the remaining shares at a price of $(E + \bar{\theta}\lambda'X - \lambda I)$ from outside shareholders with belief $\bar{\theta}$. The firm insiders' expected payoff is given by:

$$\frac{\alpha}{1 - \left(\delta + \frac{E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)}{E + \bar{\theta}\lambda'X - \lambda I} \right)} \theta^f \lambda'X = \frac{E + \bar{\theta}\lambda'X - \lambda I}{\delta \underline{\theta}\lambda'X + (1 - \delta)\bar{\theta}\lambda'X} \alpha \theta^f \lambda'X = \frac{E + \bar{\theta}\lambda'X - \lambda I}{\delta \underline{\theta} + (1 - \delta)\bar{\theta}} \alpha \theta^f. \quad (\text{A.12})$$

Comparing this expected payoff (A.12) to that of underinvestment in (A.7), the firm insiders will prefer to implement the project at the full investment level if the following condition holds:

$$\bar{\theta}\lambda'X - \lambda I > \bar{\theta}X - I. \quad (\text{A.13})$$

which is equivalent to the following parameter restriction: $\bar{\theta} > \frac{(\lambda-1)I}{(\lambda'-1)X}$. Q.E.D.

Proof of Proposition 3: If E is small and the firm implements its new project at the full investment level, the firm's share price at time 1 prior to the value distribution is $E + \underline{\theta}\lambda'X - \lambda I$ and the total number of shares outstanding is 1. Therefore, the maximum number of shares the firm can repurchase is equal to $\frac{E - \lambda I}{E + \underline{\theta}\lambda'X - \lambda I}$, which is strictly less than δ (when E is small).

If the firm buys back a dollar amount of $D_b = E - \lambda I - D_c$ from its outside shareholders at the price $E + \underline{\theta}\lambda'X - \lambda I$ per share, the number of shares repurchased is given by:

$$\alpha_b = \frac{E - \lambda I - D_c}{E + \underline{\theta}\lambda'X - \lambda I}. \quad (\text{A.14})$$

After the repurchase, the firm will make a dividend payment of D_c to the existing shareholders who collectively hold $1 - \alpha_b = 1 - \frac{E - \lambda I - D_c}{E + \underline{\theta}\lambda'X - \lambda I}$ shares outstanding. So, the firm insiders' objective is given by

$$\max_{D_c \in [0, E - \lambda I]} \frac{\alpha}{1 - \frac{E - \lambda I - D_c}{E + \underline{\theta}\lambda'X - \lambda I}} [D_c + \theta^f \lambda'X], \quad (\text{A.15})$$

which is equivalent to

$$\max_{D_c \in [0, E - \lambda I]} \alpha(E + \underline{\theta}\lambda'X - \lambda I) \left[1 + \frac{(\theta^f - \underline{\theta})\lambda'X}{D_c + \underline{\theta}\lambda'X} \right]. \quad (\text{A.16})$$

The optimal solution for the firm is

$$D_c = \begin{cases} E - \lambda I & \text{if } \theta^f \leq \underline{\theta}, \\ 0 & \text{if } \theta^f > \underline{\theta}. \end{cases}$$

Thus, if $\theta^f \leq \underline{\theta}$, the firm finds it optimal to distribute value through a dividend payment only. Otherwise, if $\theta^f > \underline{\theta}$, the firm finds it optimal to distribute value through a stock repurchase only, and buys back as many shares as

possible. Therefore, if the firm implements its project at the full investment level and E is small, the firm will never choose to distribute value through a combination of a stock repurchase and a dividend payment.

Consider the case where the firm underinvests in its new project. In this case, E is always large enough so that the firm is able to repurchase more than δ shares. However, if the firm repurchases more than δ shares, it means that it also has to buy back shares from optimistic outside shareholders at a higher price. First, consider the case where we restrict the number of shares that the firm can repurchase to be less than or equal to δ , i.e., $0 \leq \alpha_b \leq \delta$. Then, the amount of cash dividend D_c that the firm can choose to distribute will be in the closed interval $[E - I - \delta(E + \underline{\theta}X - I), E - I]$.

Before the stock repurchase and the dividend payment, the firm value is $E + \underline{\theta}X - I$ and the total number of shares outstanding is 1. The firm buys back shares at a dollar amount of $D_b = E - I - D_c$ from outsiders, at the price of $E + \underline{\theta}X - I$ per share, so that the number of shares repurchased is equal to

$$\alpha_b = \frac{E - I - D_c}{E + \underline{\theta}X - I}.$$

After the share repurchase, the firm makes a dividend payment of D_c to existing shareholders (who collectively hold $1 - \alpha_b = 1 - \frac{E - I - D_c}{E + \underline{\theta}X - I}$ shares). So the firm insiders' objective function is

$$\max_{D_c \in [E - I - \delta(E + \underline{\theta}X - I), E - I]} \frac{\alpha}{\left(1 - \frac{E - I - D_c}{E + \underline{\theta}X - I}\right)} [D_c + \theta^f X],$$

which is equivalent to

$$\max_{D_c \in [E - I - \delta(E + \underline{\theta}X - I), E - I]} \alpha(E + \underline{\theta}X - I) \left[1 + \frac{(\theta^f - \underline{\theta})X}{D_c + \underline{\theta}X}\right].$$

The optimal solution for the firm is

$$D_c = \begin{cases} E - I & \text{if } \theta^f \leq \underline{\theta} \\ E - I - \delta(E + \underline{\theta}X - I) & \text{if } \theta^f > \underline{\theta} \end{cases}$$

Thus, if $\theta^f \leq \underline{\theta}$, the firm finds it optimal to distribute value through a dividend payment only. Otherwise, if $\theta^f > \underline{\theta}$ the firm finds it optimal to distribute value through a combination of a stock repurchase and a dividend payment, by repurchasing δ shares from pessimistic shareholders and distribute the remaining earnings as a cash dividend payment.

If we allow the firm to repurchase more than δ shares (but not less) when it underinvests in its project, we can describe the firm insiders' optimization problem as follows. The firm buys back δ shares from pessimistic shareholders at price $E + \underline{\theta}X - I$, and also some shares from optimistic shareholders at price $E + \bar{\theta}X - I$ for a dollar amount of $E - I - D_c - \delta(E + \underline{\theta}X - I)$. Thus, the total number of shares repurchased by the firm is equal to

$$\alpha_b = \delta + \frac{E - I - D_c - \delta(E + \underline{\theta}X - I)}{E + \bar{\theta}X - I}.$$

After the repurchase, the firm pays a dollar amount of D_c to existing shareholders (who collectively hold $1 - \alpha_b = 1 - \delta - \frac{E - I - D_c - \delta(E + \underline{\theta}X - I)}{E + \bar{\theta}X - I}$ shares). The dividend payment D_c can take a value in the closed interval $[0, E - I - \delta(E + \underline{\theta}X - I)]$. Thus, the firm insiders' objective function is

$$\max_{D_c \in [0, E - I - \delta(E + \underline{\theta}X - I)]} \frac{\alpha}{\left(1 - \delta - \frac{E - I - D_c - \delta(E + \underline{\theta}X - I)}{E + \bar{\theta}X - I}\right)} [D_c + \theta^f X],$$

which is equivalent to

$$\max_{D_c \in [0, E - I - \delta(E + \underline{\theta}X - I)]} \alpha(E + \bar{\theta}X - I) \left[1 + \frac{(\theta^f - (\delta\underline{\theta} + (1 - \delta)\bar{\theta}))X}{D_c + (\delta\underline{\theta} + (1 - \delta)\bar{\theta})X}\right].$$

The optimal solution for the firm is

$$D_c = \begin{cases} E - I - \delta(E + \underline{\theta}X - I) & \text{if } \theta^f \leq \delta\underline{\theta} + (1 - \delta)\bar{\theta}, \\ 0 & \text{if } \theta^f > \delta\underline{\theta} + (1 - \delta)\bar{\theta} \end{cases}$$

Thus, if $\theta^f > \delta\underline{\theta} + (1 - \delta)\bar{\theta}$, the firm finds it optimal to distribute value through a stock repurchase alone. Otherwise, if $\theta^f \leq \delta\underline{\theta} + (1 - \delta)\bar{\theta}$, the firm will choose to distribute value through a combination of a stock repurchase and a dividend payment, by repurchasing δ shares from pessimistic shareholders and distributing the remaining value as a cash dividend payment.

Thus, conditional on the firm underinvesting in its project, its optimal combination of a stock repurchase and a dividend payment is as follows: the firm will repurchase exactly δ shares from pessimistic shareholders, and pay out a cash dividend in the amount of $E - I - \delta(E + \underline{\theta}X - I)$. In this case, the expected payoff of firm insiders is given by:

$$\frac{\alpha}{1 - \delta} [\theta^f X + E - I - \delta(E + \underline{\theta}X - I)]. \quad (\text{A.17})$$

Finally, if we consider the case where the firm implements its new project at the full investment and E is large, the proof is very similar to that of the above case where the firm underinvests. Conditional on the firm fully investing in its project and E being large, the optimal combination of a stock repurchase and a dividend payment is as follows: the firm will repurchase exactly δ shares from pessimistic shareholders, and pay out a cash dividend in the amount of $E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)$. In this case, the expected payoff of firm insiders is given by:

$$\frac{\alpha}{1 - \delta} [\theta^f \lambda'X + E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)]. \quad (\text{A.18})$$

Proof of Proposition 4: When E is small, the firm insiders have the following menu of choices: 1) Implement the new project at the full investment level and make a dividend payment only; 2) Underinvest in the new project and make a dividend payment only; 3) Implement the new project at the full investment level and repurchase shares only; 4) Underinvest in the new project and repurchase shares only; 5) Underinvest in the new project and undertake a combination of a stock repurchase and a dividend payment.

Since $\frac{(\lambda-1)I}{(\lambda'-1)X} < \theta^f$, we know from Proposition 1 that choice 1 strictly dominates choice 2. Further, from part (i) of Proposition 2, we know that when E is small, choice 4 is preferred to choice 3 if and only if the following constraint on $\underline{\theta}$ holds:

$$\underline{\theta} < \bar{\theta} < \frac{\theta(E - I - \delta(E + \underline{\theta}\lambda'X - \lambda I))}{(1 - \delta)(E + \underline{\theta}\lambda'X - \lambda I) - \underline{\theta}X}, \quad (\text{A.19})$$

For this constraint to be feasible, i.e., $\frac{\theta(E - I - \delta(E + \underline{\theta}\lambda'X - \lambda I))}{(1 - \delta)(E + \underline{\theta}\lambda'X - \lambda I) - \underline{\theta}X} > \underline{\theta}$, the following condition must hold:

$$\underline{\theta} < \frac{(\lambda - 1)I}{(\lambda' - 1)X}. \quad (\text{A.20})$$

If (A.20) is not satisfied, choice 3 will strictly dominate choice 4. Further, it is straightforward to show that if E is small so that (4) holds, and $\underline{\theta} < \frac{(\lambda-1)I}{(\lambda'-1)X}$, then the following relationship will hold:

$$\frac{(\lambda - 1)I}{(\lambda' - 1)X} < \frac{\theta(E - I - \delta(E + \underline{\theta}\lambda'X - \lambda I))}{(1 - \delta)(E + \underline{\theta}\lambda'X - \lambda I) - \underline{\theta}X}. \quad (\text{A.21})$$

Let us first consider the firm's optimal choices conditional on the firm implementing its new project at the full investment level. The firm will prefer choice 3 to choice 1 if and only if insiders' expected payoff from choice 3 is greater than insiders' expected payoff from choice 1:

$$\alpha\theta^f \frac{E + \underline{\theta}\lambda'X - \lambda I}{\underline{\theta}} > \alpha(E + \theta^f \lambda'X - \lambda I), \quad (\text{A.22})$$

which is equivalent to the following condition:

$$\theta^f > \underline{\theta}, \quad (\text{A.23})$$

Let us next consider the firm's optimal choices conditional on the firm underinvesting in its new project. The firm will prefer choice 4 to choice 2 if and only if insiders' expected payoff from choice 4 is greater than insiders' expected payoff from choice 2:

$$\alpha\theta^f \frac{E + \bar{\theta}X - I}{\delta\underline{\theta} + (1 - \delta)\bar{\theta}} > \alpha(E + \theta^f X - \lambda I), \quad (\text{A.24})$$

which is equivalent to the following condition:

$$\theta^f > (\delta\underline{\theta} + (1 - \delta)\bar{\theta}) \frac{E - I}{E - I + \delta(\bar{\theta} - \underline{\theta})X}, \quad (\text{A.25})$$

where $(\delta\underline{\theta} + (1 - \delta)\bar{\theta}) \frac{E - I}{E - I + \delta(\bar{\theta} - \underline{\theta})X} > \underline{\theta}$, since $\delta < \frac{E - I}{E + \bar{\theta}X - I}$ by assumption. The firm will prefer choice 4 to choice 5 if and only if insiders' expected payoff from choice 4 is greater than insiders' expected payoff from choice 5:

$$\alpha\theta^f \frac{E + \bar{\theta}X - I}{\delta\underline{\theta} + (1 - \delta)\bar{\theta}} > \frac{\alpha}{1 - \delta} (\theta^f X + E - I - \delta(E + \underline{\theta}X - I)), \quad (\text{A.26})$$

which is equivalent to the following condition:

$$\theta^f > \delta\underline{\theta} + (1 - \delta)\bar{\theta}. \quad (\text{A.27})$$

The firm will prefer choice 5 to choice 2 if and only if insiders' expected payoff from choice 5 is greater than insiders' expected payoff from choice 2:

$$\frac{\alpha}{1 - \delta} (\theta^f X + E - I - \delta(E + \underline{\theta}X - I)) > \alpha(E + \theta^f X - I), \quad (\text{A.28})$$

which is equivalent to the following condition:

$$\theta^f > \underline{\theta}. \quad (\text{A.29})$$

Let us also compare choice 5 and choice 3 with investment levels I and λI , respectively. The firm will prefer choice 5 to choice 3 if and only if insiders' expected payoff from choice 5 is greater than insiders' expected payoff from choice 3:

$$\frac{\alpha}{1 - \delta} (\theta^f X + E - \lambda I - \delta(E + \underline{\theta}X - I)) > \alpha\theta^f \frac{E + \underline{\theta}\lambda'X - \lambda I}{\underline{\theta}}, \quad (\text{A.30})$$

which is equivalent to the following condition:

$$\theta^f < \frac{\underline{\theta}(E - I - \delta(E + \underline{\theta}X - I))}{(1 - \delta)(E + \underline{\theta}\lambda'X - \lambda I) - \underline{\theta}X}, \quad (\text{A.31})$$

where $\frac{\underline{\theta}(E - I - \delta(E + \underline{\theta}X - I))}{(1 - \delta)(E + \underline{\theta}\lambda'X - \lambda I) - \underline{\theta}X} > \underline{\theta}$ if and only if $\underline{\theta} < \frac{(\lambda - 1)I}{(\lambda' - 1)X}$. Further, $\frac{\underline{\theta}(E - I - \delta(E + \underline{\theta}X - I))}{(1 - \delta)(E + \underline{\theta}\lambda'X - \lambda I) - \underline{\theta}X} < \frac{\underline{\theta}(E - I - \delta(E + \underline{\theta}\lambda'X - \lambda I))}{(1 - \delta)(E + \underline{\theta}\lambda'X - \lambda I) - \underline{\theta}X}$ if and only if $\underline{\theta} < \frac{(\lambda - 1)I}{(\lambda' - 1)X}$.

In part (i) of Proposition 4, we consider the case where $\frac{(\lambda - 1)I}{(\lambda' - 1)X} \leq \underline{\theta} < \bar{\theta}$. Since $\underline{\theta} \geq \frac{(\lambda - 1)I}{(\lambda' - 1)X}$, it follows from our earlier discussion that in this choice 3 strictly dominates choice 4.

If $\theta^f \leq \underline{\theta}$, it follows from (A.23) that choice 1 is preferred to choice 3. Choice 1 is preferred to choice 4 by transitivity (choice 1 > choice 3 > choice 4). Similarly, from inequality (A.29), it follows that choice 1 is preferred to choice 5 by transitivity (choice 1 > choice 2 > choice 5). Thus, choice 1 is optimal in this case.

If $\underline{\theta} < \theta^f$, it follows from (A.23) that choice 3 is preferred to choice 1. Since $\frac{(\lambda - 1)I}{(\lambda' - 1)X} \leq \underline{\theta}$ in this case, it follows from (A.31) that $\frac{\underline{\theta}(E - I - \delta(E + \underline{\theta}X - I))}{(1 - \delta)(E + \underline{\theta}\lambda'X - \lambda I) - \underline{\theta}X} \leq \underline{\theta}$, and therefore, choice 3 is preferred to choice 5. Choice 3 is preferred to choice 2 by transitivity (choice 3 > choice 1 > choice 2). Thus, choice 3 is optimal in this case.

In part (ii) of Proposition 4, we consider the case where $\underline{\theta} < \frac{(\lambda-1)I}{(\lambda'-1)X} < \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\underline{\theta}X} \leq \bar{\theta}$. Note that in this case, $\underline{\theta} < \theta^f$ due to our global assumption that $\frac{(\lambda-1)I}{(\lambda'-1)X} < \theta^f$. Thus, it follows from (A.33) that choice 3 is preferred to choice 1, and choice 3 is preferred to choice 2 by transitivity (choice 3 > choice 1 > choice 2). Further, since $\frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\underline{\theta}X} \leq \bar{\theta}$, it follows that choice 3 is preferred to choice 4.

If $\theta^f > \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\underline{\theta}X}$, it follows from (A.31) that choice 3 is preferred to choice 5. Thus, choice 3 is optimal in this case.

If $\theta^f \leq \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\underline{\theta}X}$, it follows from (A.31) that choice 5 is preferred to choice 3. Since $\underline{\theta} < \theta^f$, it follows from (A.29) that choice 5 is preferred to choice 2. Choice 5 is preferred to choice 1 and choice 4 by transitivity (choice 5 > choice 3 > choice 1, and choice 5 > choice 3 > choice 4). Thus, choice 5 is optimal in this case.

In part (iii) of Proposition 4, we consider the case where $\underline{\theta} < \frac{(\lambda-1)I}{(\lambda'-1)X} < \delta\underline{\theta} + (1-\delta)\bar{\theta} < \bar{\theta} < \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\underline{\theta}X}$. Note that in this case again, $\underline{\theta} < \theta^f$ due to our global assumption that $\frac{(\lambda-1)I}{(\lambda'-1)X} < \theta^f$. Thus, it follows from (A.33) that choice 3 is preferred to choice 1, and choice 3 is preferred to choice 2 by transitivity (choice 3 > choice 1 > choice 2). Further, since $\bar{\theta} < \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\underline{\theta}X}$, it follows that choice 4 is preferred to choice 3.

If $\theta^f \leq \delta\underline{\theta} + (1-\delta)\bar{\theta}$, it follows from (A.27) that choice 5 is preferred to choice 4. Then, choice 5 is preferred to choice 3 and choice 1 by transitivity (choice 5 > choice 4 > choice 3, and choice 5 > choice 3 > choice 1). From (A.29) and $\theta^f > \underline{\theta}$, it follows that choice 5 is preferred to choice 2 as well. Thus, choice 5 is optimal in this case.

If $\theta^f > \delta\underline{\theta} + (1-\delta)\bar{\theta}$, it follows from (A.27) that choice 4 is preferred to choice 5. Then, choice 4 is preferred to choice 1 by transitivity (choice 4 > choice 3 > choice 1). From (A.25), it follows that choice 4 is preferred to choice 2 since $\theta^f > \delta\underline{\theta} + (1-\delta)\bar{\theta}$. Thus, choice 4 is optimal in this case.

In part (iv) of Proposition 4, we consider the case where $\underline{\theta} < \delta\underline{\theta} + (1-\delta)\bar{\theta} \leq \frac{(\lambda-1)I}{(\lambda'-1)X} < \bar{\theta} < \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\underline{\theta}X}$. Note that in this case, $\theta^f > \delta\underline{\theta} + (1-\delta)\bar{\theta}$ due to our global assumption that $\frac{(\lambda-1)I}{(\lambda'-1)X} < \theta^f$. In this case, the optimality of choice 4 follows from the above proof for part (iii) of Proposition 4.

Finally, in part (v) of Proposition 4, we consider the case where $\underline{\theta} < \bar{\theta} \leq \frac{(\lambda-1)I}{(\lambda'-1)X}$. Note that in this case, $\theta^f > \bar{\theta}$ due to our global assumption that $\frac{(\lambda-1)I}{(\lambda'-1)X} < \theta^f$. Since $\frac{(\lambda-1)I}{(\lambda'-1)X} < \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\underline{\theta}X}$ in this case as well, it follows that $\bar{\theta} < \frac{\theta(E-I-\delta(E+\theta\lambda'X-\lambda I))}{(1-\delta)(E+\theta\lambda'X-\lambda I)-\underline{\theta}X}$. Therefore, choice 4 is preferred to choice 3. Since $\theta^f > \delta\underline{\theta} + (1-\delta)\bar{\theta}$, the overall optimality of choice 4 in this case follows the above proofs for part (iii) and (iv) of Proposition 4. Q.E.D.

Proof of Proposition 5: When E is large, the firm insiders have the following menu of choices: 1) Implement the new project at the full investment level and make a dividend payment only; 2) Underinvest in the new project and make a dividend payment only; 3) Implement the new project at the full investment level and repurchase shares only; 4) Underinvest in the new project and repurchase shares only; 5) Underinvest in the new project and undertake a combination of a stock repurchase and a dividend payment; 6) Implement the new project at the full investment level and undertake a combination of a stock repurchase and a dividend payment.

Since $\frac{(\lambda-1)I}{(\lambda'-1)X} < \theta^f$, we know from Proposition 1 that choice 1 strictly dominates choice 2. Further, from part (ii) of Proposition 2, we know that choice 3 is preferred to choice 4 if $\frac{(\lambda-1)I}{(\lambda'-1)X} \leq \bar{\theta}$, and choice 4 is preferred to choice 3 if $\bar{\theta} < \frac{(\lambda-1)I}{(\lambda'-1)X}$.

Let us first consider the firm's optimal choices conditional on the firm implementing its new project at the full investment level. The firm will prefer choice 3 to choice 1 if and only if insiders' expected payoff from choice 3 is greater than insiders' expected payoff from choice 1:

$$\alpha\theta^f \frac{E + \bar{\theta}\lambda'X - \lambda I}{\delta\underline{\theta} + (1-\delta)\bar{\theta}} > \alpha(E + \theta^f\lambda'X - \lambda I), \quad (\text{A.32})$$

which is equivalent to the following condition:

$$\theta^f > (\delta\underline{\theta} + (1-\delta)\bar{\theta}) \frac{E - \lambda I}{E - \lambda I + \delta(\bar{\theta} - \underline{\theta})\lambda'X}, \quad (\text{A.33})$$

where $(\delta\underline{\theta} + (1 - \delta)\bar{\theta}) \frac{E - \lambda I}{E - \lambda I + \delta(\bar{\theta} - \underline{\theta})\lambda'X} > \underline{\theta}$, since $\delta < \frac{E - \lambda I}{E + \underline{\theta}\lambda'X - \lambda I}$ if E is large (see inequality (5)). The firm will prefer choice 3 to choice 6 if and only if insiders' expected payoff from choice 3 is greater than insiders' expected payoff from choice 6:

$$\alpha\theta^f \frac{E + \bar{\theta}\lambda'X - \lambda I}{\delta\underline{\theta} + (1 - \delta)\bar{\theta}} > \frac{\alpha}{1 - \delta} (\theta^f \lambda'X + E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)), \quad (\text{A.34})$$

which is equivalent to the following condition:

$$\theta^f > \delta\underline{\theta} + (1 - \delta)\bar{\theta}. \quad (\text{A.35})$$

The firm will prefer choice 6 to choice 1 if and only if insiders' expected payoff from choice 6 is greater than insiders' expected payoff from choice 1:

$$\frac{\alpha}{1 - \delta} (\theta^f \lambda'X + E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)) > \alpha(E + \theta^f \lambda'X - \lambda I), \quad (\text{A.36})$$

which is equivalent to the following condition:

$$\theta^f > \underline{\theta}. \quad (\text{A.37})$$

Let us next consider the firm's optimal choices conditional on the firm underinvesting in its new project. The firm will prefer choice 4 to choice 2 if and only if insiders' expected payoff from choice 4 is greater than insiders' expected payoff from choice 2:

$$\alpha\theta^f \frac{E + \bar{\theta}X - I}{\delta\underline{\theta} + (1 - \delta)\bar{\theta}} > \alpha(E + \theta^f X - \lambda I), \quad (\text{A.38})$$

which is equivalent to the following condition:

$$\theta^f > (\delta\underline{\theta} + (1 - \delta)\bar{\theta}) \frac{E - I}{E - I + \delta(\bar{\theta} - \underline{\theta})X}, \quad (\text{A.39})$$

where $(\delta\underline{\theta} + (1 - \delta)\bar{\theta}) \frac{E - I}{E - I + \delta(\bar{\theta} - \underline{\theta})X} > \underline{\theta}$, since $\delta < \frac{E - I}{E + \underline{\theta}X - I}$ by assumption. The firm will prefer choice 4 to choice 5 if and only if insiders' expected payoff from choice 4 is greater than insiders' expected payoff from choice 5:

$$\alpha\theta^f \frac{E + \bar{\theta}X - I}{\delta\underline{\theta} + (1 - \delta)\bar{\theta}} > \frac{\alpha}{1 - \delta} (\theta^f X + E - I - \delta(E + \underline{\theta}X - I)), \quad (\text{A.40})$$

which is equivalent to the following condition:

$$\theta^f > \delta\underline{\theta} + (1 - \delta)\bar{\theta}. \quad (\text{A.41})$$

The firm will prefer choice 5 to choice 2 if and only if insiders' expected payoff from choice 5 is greater than insiders' expected payoff from choice 2:

$$\frac{\alpha}{1 - \delta} (\theta^f X + E - I - \delta(E + \underline{\theta}X - I)) > \alpha(E + \theta^f X - I), \quad (\text{A.42})$$

which is equivalent to the following condition:

$$\theta^f > \underline{\theta}. \quad (\text{A.43})$$

Finally, we compare choice 5 and choice 6 with investment levels I and λI , respectively. The firm will prefer choice 6 to choice 5 if and only if insiders' expected payoff from choice 6 is greater than insiders' expected payoff from choice 5:

$$\frac{\alpha}{1 - \delta} (\theta^f \lambda'X + E - \lambda I - \delta(E + \underline{\theta}\lambda'X - \lambda I)) > \frac{\alpha}{1 - \delta} (\theta^f X + E - I - \delta(E + \underline{\theta}X - I)) \quad (\text{A.44})$$

which is equivalent to the following condition:

$$\theta^f > \delta\underline{\theta} + (1 - \delta)\frac{(\lambda - 1)I}{(\lambda' - 1)X}. \quad (\text{A.45})$$

In part (i) of Proposition 5, we consider the case where $\frac{(\lambda - 1)I}{(\lambda' - 1)X} \leq \underline{\theta} < \bar{\theta}$. Since $\frac{(\lambda - 1)I}{(\lambda' - 1)X} < \bar{\theta}$, it follows from part (ii) of Proposition 2 that firm insiders prefer choice 3 to choice 4.

If $\theta^f \leq \underline{\theta}$, it follows from inequalities (A.33) and (A.37) that choice 1 is preferred to both choice 3 and choice 6. Choice 1 is preferred to choice 4 by transitivity (choice 1 > choice 3 > choice 4). Similarly, from inequality (A.43), it follows that choice 1 is preferred to choice 5 by transitivity (choice 1 > choice 2 > choice 5). Thus, choice 1 is optimal if $\theta^f \leq \underline{\theta}$.

If $\underline{\theta} < \theta^f < \delta\underline{\theta} + (1 - \delta)\bar{\theta}$, it follows from inequalities (A.35) and (A.37) that choice 6 is preferred to both choice 3 and choice 1. Since $\frac{(\lambda - 1)I}{(\lambda' - 1)X} \leq \underline{\theta} < \theta^f$ in this case, it follows from (A.45) that choice 6 is preferred to choice 5. From (A.43), choice 6 is preferred to choice 2 by transitivity (choice 6 > choice 5 > choice 2). Thus, choice 6 is optimal in this case.

If $\delta\underline{\theta} + (1 - \delta)\bar{\theta} \leq \theta^f$, it follows from inequalities (A.33) and (A.35) that choice 3 is preferred to both choice 1 and choice 6. From (A.39) and (A.41), it follows that choice 3 is preferred to both choice 2 and choice 5 by transitivity (choice 3 > choice 4 > choice 2, and choice 3 > choice 4 > choice 5). Thus, choice 3 is optimal in this case.

In part (ii) of Proposition 5, we consider the case where $\underline{\theta} < \frac{(\lambda - 1)I}{(\lambda' - 1)X} < \delta\underline{\theta} + (1 - \delta)\bar{\theta} < \bar{\theta}$. Since $\frac{(\lambda - 1)I}{(\lambda' - 1)X} < \bar{\theta}$, it again follows from part (ii) of Proposition 2 that firm insiders prefer choice 3 to choice 4. Note that in this case, $\underline{\theta} < \theta^f$ due to our global assumption that $\frac{(\lambda - 1)I}{(\lambda' - 1)X} < \theta^f$. The optimality of choice 6 when $\theta^f < \delta\underline{\theta} + (1 - \delta)\bar{\theta}$, and the optimality of choice 3 when $\delta\underline{\theta} + (1 - \delta)\bar{\theta} \leq \theta^f$ follow from the discussion above.

In part (iii) of Proposition 5, we consider the case where $\underline{\theta} < \delta\underline{\theta} + (1 - \delta)\bar{\theta} \leq \frac{(\lambda - 1)I}{(\lambda' - 1)X} < \bar{\theta}$. Since $\frac{(\lambda - 1)I}{(\lambda' - 1)X} < \bar{\theta}$, it again follows from part (ii) of Proposition 2 that firm insiders prefer choice 3 to choice 4. Further, in this case, it holds that $\delta\underline{\theta} + (1 - \delta)\bar{\theta} < \theta^f$. The optimality of choice 3 when $\delta\underline{\theta} + (1 - \delta)\bar{\theta} < \theta^f$ follows from the discussion above.

Finally, in part (iv) of Proposition 5, we consider the case where $\underline{\theta} < \bar{\theta} < \frac{(\lambda - 1)I}{(\lambda' - 1)X}$. Since $\bar{\theta} < \frac{(\lambda - 1)I}{(\lambda' - 1)X}$, it follows from part (ii) of Proposition 2 that firm insiders prefer choice 4 to choice 3. Further, in this case, $\theta^f > \bar{\theta}$ due to our global assumption that $\frac{(\lambda - 1)I}{(\lambda' - 1)X} < \theta^f$. From inequalities (A.39) and (A.41), it follows that choice 4 is preferred to both choice 2 and choice 5. From (A.35) and (A.33), it follows that choice 4 is preferred to both choice 6 and choice 1 by transitivity (choice 4 > choice 3 > choice 6, and choice 4 > choice 3 > choice 1). Thus, choice 4 is optimal in this case. Q.E.D.

Proof of Proposition 6: If the firm makes a cash dividend payment, the share price at time 1 will decrease by the amount of the cash dividend. Without loss of generality, if the firm chooses to implement its project at the full investment level and makes a dividend payment of $(E - \lambda I)$, the firm's stock price will be $(E + \underline{\theta}\lambda'X - \lambda I)$ at time 0 before the dividend payment and $\underline{\theta}\lambda'X$ at time 1 after the payment. However, the total stock return at time 1 will be zero, because the capital gains yield will be

$$\frac{P_1 - P_0}{P_0} = -\left(\frac{E - \lambda I}{E + \underline{\theta}\lambda'X - \lambda I}\right), \quad (\text{A.46})$$

and the dividend yield will be

$$\frac{Div_1}{P_0} = \frac{E - \lambda I}{\underline{\theta}\lambda'X + E - \lambda I}, \quad (\text{A.47})$$

so that the total return, which is the sum of the two, will be zero. Hence, the price impact (measured as the sum of dividend yield and capital gains yield) of a cash dividend payment is 0.

If E is small and the firm chooses to undertake a stock repurchase, its price impact depends on the number of shares the firm repurchases. If the firm implements its project at the full investment level, the firm's share price will be $P_0 = (E + \underline{\theta}\lambda'X - \lambda I)$ before the stock repurchase (at time 0). After the stock repurchase, the total firm value will be $\underline{\theta}\lambda'X$, but the total number of shares outstanding will be reduced from 1 to $\left(1 - \frac{E - \lambda I}{E + \underline{\theta}\lambda'X - \lambda I}\right)$,

so that the share price at time 1 will be

$$P_1 = \frac{\theta \lambda' X}{1 - \frac{E - \lambda I}{E + \theta \lambda' X - \lambda I}} = E + \theta \lambda' X - \lambda I, \quad (\text{A.48})$$

yielding a total return of zero on the firm's equity (i.e., the price impact $\frac{P_1 - P_0 + Div_1}{P_0}$ is zero).

On the other hand, if the firm undertakes a stock repurchase while it underinvests in its new project, it will have to repurchase a larger number of shares (from both pessimistic and optimistic outside shareholders) compared to the above case where it undertakes a stock repurchase while implementing its project at the full investment level. Before the stock repurchase, the firm's share price will be $P_0 = (E + \theta X - I)$. After the repurchase, the total firm value will be $\bar{\theta} X$, but the total number of shares outstanding will be reduced to $\left(1 - \delta - \frac{E - I - \delta(E + \theta X - I)}{E + \theta X - I}\right)$, so that the share price at time 1 will be

$$P_1 = \frac{\bar{\theta} X}{1 - \delta - \frac{E - I - \delta(E + \theta X - I)}{E + \theta X - I}} = \frac{\bar{\theta}}{\delta \theta + (1 - \delta) \theta} (E + \bar{\theta} X - I). \quad (\text{A.49})$$

Thus, the total return on the firm's equity will be

$$\frac{P_1 - P_0}{P_0} = \frac{\frac{\bar{\theta}}{\delta \theta + (1 - \delta) \theta} (E + \bar{\theta} X - I) - (E + \theta X - I)}{E + \theta X - I} > 0, \quad (\text{A.50})$$

yielding a positive price impact. Thus, if E is small, the price impact of a stock repurchase is weakly positive, and it increases with the number of shares the firm buys back.

If E is large and the firm chooses to undertake a stock repurchase alone to distribute value to shareholders, the price impact of the stock repurchase will be strictly positive, since the firm will go up the ladder of shareholder beliefs regardless of the optimal level of investment in its new project. Unlike in the above case where E is small, if the firm chooses to implement its project at the full investment level when E is large, the firm will repurchase shares from both pessimistic and optimistic outside shareholders. Before the stock repurchase, the firm's share price will be $P_0 = (E + \theta \lambda' X - \lambda I)$. After the repurchase, the total firm value will be $\bar{\theta} \lambda' X$, but the total number of shares outstanding will be reduced to $\left(1 - \delta - \frac{E - \lambda I - \delta(E + \theta \lambda' X - \lambda I)}{E + \theta \lambda' X - \lambda I}\right)$, so that the share price at time 1 will be

$$P_1 = \frac{\bar{\theta} \lambda' X}{1 - \delta - \frac{E - \lambda I - \delta(E + \theta \lambda' X - \lambda I)}{E + \theta \lambda' X - \lambda I}} = \frac{\bar{\theta}}{\delta \theta + (1 - \delta) \theta} (E + \bar{\theta} \lambda' X - \lambda I). \quad (\text{A.51})$$

In this case, the total return on the firm's equity will be

$$\frac{P_1 - P_0}{P_0} = \frac{\frac{\bar{\theta}}{\delta \theta + (1 - \delta) \theta} (E + \bar{\theta} \lambda' X - \lambda I) - (E + \theta \lambda' X - \lambda I)}{E + \theta \lambda' X - \lambda I} > 0, \quad (\text{A.52})$$

also yielding a positive price impact in the case where the firm chooses a stock repurchase alone to distribute value while implementing its project at the full investment level. Thus, when E is large, a stock repurchase has a positive price impact regardless of whether the firm underinvests in its project or implements it at the full investment level, since after the repurchase, the belief of the marginal investor in the firm's equity increases from $\underline{\theta}$ to $\bar{\theta}$. Q.E.D.

Proof of Proposition 7: First let's consider the case of risk-free debt. If the firm implements its project at the full investment, then the amount of money needs to be raised is equal to

$$P_D = D_b + \lambda I - E, \quad (\text{A.53})$$

because the sources of funds are from debt issuance P_D and earnings E , and the uses of funds are stock repurchase D_b and investment λI . Suppose the sequence of events are as follows: raise funding through debt, then repurchase, then invest.

Since it's risk-free debt, the payoff to debtholders will be P_D in either state, where $P_D \leq \lambda'X^L$ because we normalized the risk-free rate to 0. When the firm buys back stocks from current shareholders with belief $\underline{\theta}$, they will pay a price of P_1 for each share, and the number of shares they will buy back will be δ (they will buy out all shareholders who have lower belief than insiders), so we will have

$$D_b = \delta \times P_1. \quad (\text{A.54})$$

Investors know that the firm value at time 2 will be either $\lambda'X^H - P_D$ or $\lambda'X^L - P_D$, with $1 - \delta$ shares outstanding, so current shareholders with belief $\underline{\theta}$ will value each share at a price of

$$P_1 = \frac{\underline{\theta}(\lambda'X^H - P_D) + (1 - \underline{\theta})(\lambda'X^L - P_D)}{1 - \delta} \quad (\text{A.55})$$

Solving the above three equations for P_D , D_b , and P_1 , we obtain:

$$\begin{aligned} P_D &= \delta \lambda' (\underline{\theta}X^H + (1 - \underline{\theta})X^L) - (1 - \delta)(E - \lambda I) \\ D_b &= \delta (E - \lambda I + \lambda' (\underline{\theta}X^H + (1 - \underline{\theta})X^L)) \\ P_1 &= E - \lambda I + \lambda' (\underline{\theta}X^H + (1 - \underline{\theta})X^L) \end{aligned}$$

Conditional on E being small, the firm now has essentially three choices: (i) implement the project at the full investment level, and use the left over earnings to buy back shares without external financing; (ii) implement the project at the full investment level and raise debt as above to buy back all δ shares held by pessimistic shareholders; (iii) underinvest in the firm's project, and buy back δ shares from pessimistic investors and some more shares from optimistic outside shareholders with belief $\bar{\theta}$ without external financing.

The expected payoff from implementing the new project at the full investment level and repurchasing all δ shares held by pessimistic shareholders by using debt financing is given by:

$$EU = \frac{\alpha}{1 - \delta} [\theta^f (\lambda'X^H - P_D) + (1 - \theta^f) (\lambda'X^L - P_D)] \quad (\text{A.56})$$

$$= \alpha (E - \lambda I) + \frac{\alpha}{1 - \delta} ((\theta^f \lambda'X^H + (1 - \theta^f) \lambda'X^L) - \delta (\underline{\theta} \lambda'X^H + (1 - \underline{\theta}) \lambda'X^L)) \quad (\text{A.57})$$

In contrast, the expected payoff from implementing the project and using the leftover earnings to buy back shares without external financing is given by:

$$EU = \frac{\alpha (\theta^f X^H + (1 - \theta^f) X^L)}{\underline{\theta} X^H + (1 - \underline{\theta}) X^L} (E - \lambda I + \underline{\theta} \lambda' X^H + (1 - \underline{\theta}) \lambda' X^L). \quad (\text{A.58})$$

It is straightforward to show that the expected payoff from implementing the new project at the full investment level and repurchasing all δ shares held by pessimistic shareholders by using debt financing is always greater than the expected payoff from implementing the project and using the leftover earnings to buy back shares without external financing.

The expected payoff from underinvesting in the new project and buying back δ shares from pessimistic shareholders and some more shares from optimistic shareholders with belief $\bar{\theta}$ without external financing, is given by:

$$EU = \frac{\alpha (\theta^f X^H + (1 - \theta^f) X^L) (E - I + \bar{\theta} X^H + (1 - \bar{\theta}) X^L)}{\delta (\underline{\theta} X^H + (1 - \underline{\theta}) X^L) + (1 - \delta) (\bar{\theta} X^H + (1 - \bar{\theta}) X^L)}. \quad (\text{A.59})$$

Thus, the choice with debt financing will be optimal if the following condition holds:

$$\alpha (E - \lambda I) + \frac{\alpha}{1 - \delta} ((\theta^f \lambda' X^H + (1 - \theta^f) \lambda' X^L) - \delta (\underline{\theta} \lambda' X^H + (1 - \underline{\theta}) \lambda' X^L)) \quad (\text{A.60})$$

$$> \frac{\alpha (\theta^f X^H + (1 - \theta^f) X^L) (E - I + \bar{\theta} X^H + (1 - \bar{\theta}) X^L)}{\delta (\underline{\theta} X^H + (1 - \underline{\theta}) X^L) + (1 - \delta) (\bar{\theta} X^H + (1 - \bar{\theta}) X^L)} \quad (\text{A.61})$$

Note that the firm can issue risk-free debt to finance the stock repurchase only if $P_D \leq \lambda'X^L$, i.e., if

$$(\lambda I - E)(1 - \delta) + \delta(\underline{\theta}\lambda'X^H + (1 - \underline{\theta})\lambda'X^L) \leq \lambda'X^L. \quad (\text{A.62})$$

The condition holds when δ is small and $\lambda'X^L$ is relatively large.

Now consider the case of risky debt. If the firm implements its new project at the full investment, then the amount of money that needs to be raised is given by:

$$P_D = D_b + \lambda I - E \quad (\text{A.63})$$

because the sources of funds are from debt issuance P_D and earnings E , and the uses of funds are stock repurchase D_b and investment λI . Note that P_D is the market value of debt, which is also the dollar amount the firm raises from outsiders. Suppose the sequence of events are as follows: raise funding through debt, then repurchase, then invest.

Let the face value of debt be F . So in the good state, the payoff to debtholders will be F , and in the bad state, the payoff will be $\lambda'X^L$. Since the belief of outside investors is $\underline{\theta}$, the market value of debt will be given by:

$$P_D = \underline{\theta}F + (1 - \underline{\theta})\lambda'X^L \quad (\text{A.64})$$

When the firms buys back stocks from current shareholders with belief $\underline{\theta}$, they will pay a price of P_1 for each share, and the number of shares they will buy back will be δ (they will buy out all shareholders who have lower belief than insiders), so we will have

$$D_b = \delta \times P_1. \quad (\text{A.65})$$

At the same time, equity holders know that the total equity value at time 2 will be either $\lambda'X^H - F$ or 0, with $(1 - \delta)$ shares outstanding, so current shareholders with belief $\underline{\theta}$ will value each share at a price of

$$P_1 = \frac{\underline{\theta}(\lambda'X^H - F)}{1 - \delta}. \quad (\text{A.66})$$

Solving the above four equations for P_D , D_b , P_1 , and F , we obtain:

$$\begin{aligned} P_D &= (\lambda I - E)(1 - \delta) + \delta[\underline{\theta}\lambda'X^H + (1 - \underline{\theta})\lambda'X^L], \\ D_b &= \delta[E - \lambda I + \underline{\theta}\lambda'X^H + (1 - \underline{\theta})\lambda'X^L], \\ P_1 &= E - \lambda I + \underline{\theta}\lambda'X^H + (1 - \underline{\theta})\lambda'X^L, \\ F &= \frac{\delta\underline{\theta}\lambda'X^H + (1 - \delta)[\lambda I - E - (1 - \underline{\theta})\lambda'X^L]}{\underline{\theta}}. \end{aligned}$$

The expected payoff from implementing the new project at the full investment level and repurchasing all δ shares held by pessimistic shareholders by using debt financing is given by:

$$EU = \frac{\alpha}{1 - \delta}\theta^f(\lambda'X^H - F) \quad (\text{A.67})$$

$$= \frac{\alpha}{1 - \delta}\theta^f\left(\lambda'X^H - \frac{\delta\underline{\theta}\lambda'X^H + (1 - \delta)[\lambda I - E - (1 - \underline{\theta})\lambda'X^L]}{\underline{\theta}}\right) \quad (\text{A.68})$$

$$= \frac{\alpha\theta^f}{\underline{\theta}}(E - \lambda I + \underline{\theta}\lambda'X^H + (1 - \underline{\theta})\lambda'X^L). \quad (\text{A.69})$$

In contrast, the expected payoff from implementing the project and using the leftover earnings to buy back shares without external financing is given by:

$$EU = \frac{\alpha(\theta^f X^H + (1 - \theta^f)X^L)}{\underline{\theta}X^H + (1 - \underline{\theta})X^L}(E - \lambda I + \underline{\theta}\lambda'X^H + (1 - \underline{\theta})\lambda'X^L). \quad (\text{A.70})$$

It is straightforward to show that the expected payoff from implementing the new project at the full investment level and repurchasing all δ shares held by pessimistic shareholders by using debt financing is always greater than the expected payoff from implementing the project and using the leftover earnings to buy back shares without external financing.

The expected payoff from underinvesting in the new project and buying back δ shares from pessimistic shareholders and some more shares from optimistic shareholders with belief $\bar{\theta}$ without external financing, is given by:

$$EU = \frac{\alpha(\theta^f X^H + (1 - \theta^f)X^L)(E - I + \bar{\theta}X^H + (1 - \bar{\theta})X^L)}{\delta(\underline{\theta}X^H + (1 - \underline{\theta})X^L) + (1 - \delta)(\bar{\theta}X^H + (1 - \bar{\theta})X^L)}. \quad (\text{A.71})$$

Thus, the choice with debt financing will be optimal if the following condition holds:

$$\frac{\theta^f}{\underline{\theta}}(E + \lambda'(\underline{\theta}X^H + (1 - \underline{\theta})X^L) - \lambda I) > \frac{(\theta^f X^H + (1 - \theta^f)X^L)(E + \bar{\theta}X^H + (1 - \bar{\theta})X^L - I)}{\delta(\underline{\theta}X^H + (1 - \underline{\theta})X^L) + (1 - \delta)(\bar{\theta}X^H + (1 - \bar{\theta})X^L)}. \quad (\text{A.72})$$

Proof of Proposition 8: First, consider the case where the firm implements its project at the full investment level, issues β_b new shares to outsiders, and pays an amount D_c as cash dividend. Since all outsiders who currently do not hold equity in the firm have belief $\underline{\theta}$, the share price in the equity issue will be $(E + \underline{\theta}\lambda'X - \lambda I)$. Before the stock issue, the firm has one share of stock outstanding. After the stock issue, the firm will have $(1 + \beta_b)$ shares outstanding. After investing an amount λI in its new project, the firm pays out a cash dividend of

$$D_c = E - \lambda I + \beta_b(E + \underline{\theta}\lambda'X - \lambda I) \quad (\text{A.73})$$

to its shareholders. The objective function of firm insiders is given by:

$$\max_{\beta_b \in \left[0, \frac{W}{E + \underline{\theta}\lambda'X - \lambda I}\right]} EU = \frac{\alpha}{1 + \beta_b} [E - \lambda I + \beta_b(E + \underline{\theta}\lambda'X - \lambda I) + \theta^f \lambda'X] \quad (\text{A.74})$$

where $\frac{W}{E + \underline{\theta}\lambda'X - \lambda I}$ is the maximum number of shares the firm can issue to outside investors, if it implements the project at the full investment level. The partial derivative of this objective function with respect to the choice variable β_b is given by:

$$\frac{\partial EU}{\partial \beta_b} = \frac{\alpha(\underline{\theta} - \theta^f)\lambda'X}{(1 + \beta_b)^2}, \quad (\text{A.75})$$

Note that if $\theta^f < \underline{\theta}$, this partial derivative is positive, and the optimal choice for the firm is to issue as many new shares as possible by choosing $\beta_b = \frac{W}{E + \underline{\theta}\lambda'X - \lambda I}$. On the other hand, if $\theta^f \geq \underline{\theta}$, $\frac{\partial EU}{\partial \beta_b} \leq 0$, and the optimal choice for the firm is to issue no new equity by setting $\beta_b = 0$.

Second, consider the case where the firm underinvests in its new project, issues β_b new shares to outsiders, and pays an amount D_c as cash dividend. The share price in the equity issue will be $(E + \underline{\theta}X - I)$. After the stock issue, the firm will have $(1 + \beta_b)$ shares outstanding. After investing an amount I in its new project, the firm pays out a cash dividend of

$$D_c = E - I + \beta_b(E + \underline{\theta}X - I) \quad (\text{A.76})$$

to its shareholders. The objective function of firm insiders is given by:

$$\max_{\beta_b \in \left[0, \frac{W}{E + \underline{\theta}X - I}\right]} EU = \frac{\alpha}{1 + \beta_b} [E - I + \beta_b(E + \underline{\theta}X - I) + \theta^f X] \quad (\text{A.77})$$

The partial derivative of this objective function with respect to the choice variable β_b is given by:

$$\frac{\partial EU}{\partial \beta_b} = \frac{\alpha(\underline{\theta} - \theta^f)X}{(1 + \beta_b)^2}, \quad (\text{A.78})$$

If $\theta^f < \underline{\theta}$, this partial derivative is positive, and the optimal choice for the firm is to issue as many new shares as

possible by choosing $\beta_b = \frac{W}{E+\underline{\theta}X-I}$. On the other hand, if $\theta^f \geq \underline{\theta}$, $\frac{\partial EU}{\partial \beta_b} \leq 0$, and the optimal choice for the firm is to issue no new equity by setting $\beta_b = 0$.

Now the firm has to choose the level of investment in its new project when $\theta^f < \underline{\theta}$. If the firm chooses to fully invest in its project, the expected payoff of firm insiders is given by:

$$EU = \frac{\alpha}{1 + \frac{W}{E+\underline{\theta}\lambda'X-\lambda I}} \left(E - \lambda I + \frac{W}{E + \underline{\theta}\lambda'X - \lambda I} (E + \underline{\theta}\lambda'X - \lambda I) + \theta^f \lambda'X \right) \quad (\text{A.79})$$

$$= \left(\frac{E + \theta^f \lambda'X - \lambda I + W}{E + \underline{\theta}\lambda'X - \lambda I + W} \right) \alpha (E + \underline{\theta}\lambda'X - \lambda I). \quad (\text{A.80})$$

If the firm chooses to underinvest in its new project, the expected payoff of firm insiders is given by:

$$EU = \frac{\alpha}{1 + \frac{W}{E+\underline{\theta}X-I}} \left(E - I + \frac{W}{E + \underline{\theta}X - I} (E + \underline{\theta}X - I) + \theta^f X \right) \quad (\text{A.81})$$

$$= \frac{E + \theta^f X - I + W}{E + \underline{\theta}X - I + W} \alpha (E + \underline{\theta}X - I). \quad (\text{A.82})$$

Since $\frac{(\lambda-1)I}{(\lambda'-1)X} < \theta^f$ by assumption and $\theta^f < \underline{\theta}$ in this case, it follows that

$$\left(\frac{E + \theta^f \lambda'X - \lambda I + W}{E + \underline{\theta}\lambda'X - \lambda I + W} \right) \alpha (E + \underline{\theta}\lambda'X - \lambda I) > \frac{E + \theta^f X - I + W}{E + \underline{\theta}X - I + W} \alpha (E + \underline{\theta}X - I). \quad (\text{A.83})$$

Therefore, it is optimal for the firm to implement its new project at the full investment level.

Earlier, we showed in Propositions 4 and 5 that the firm will have an incentive to repurchase equity from current outside shareholders only if $\theta^f > \underline{\theta}$. Given that firm insiders have an incentive to issue equity only if $\theta^f < \underline{\theta}$, it follows that insiders never find it optimal to issue new equity and repurchase shares from current shareholders at the same time. In other words, when the NPV of issuing new equity based on insiders' belief is positive, the NPV of repurchasing shares from current shareholders is negative, and vice versa. On the other hand, the NPV of a dividend payment is always zero, and if firm insiders are more pessimistic than outside investors about the firm's future prospects (so that the NPV of issuing equity is positive), it will be optimal for the firm to issue equity and distribute value through a dividend payment simultaneously. Q.E.D.

Proof of Proposition 9: In this case, the firm's stock price will be $P_1 = \underline{\theta}\lambda'X$ at time 1 after the dividend payment, and the number of shares outstanding will still be 1. At time 2, after the marginal equity investor updates her belief as $\underline{\nu} = \underline{\theta} + \frac{d}{3}(\theta^f - \underline{\theta}) + \frac{d}{3}(\bar{\theta} - \underline{\theta})$, the stock price will be

$$P_2 = \left[\underline{\theta} + \frac{d}{3}(\theta^f - \underline{\theta}) + \frac{d}{3}(\bar{\theta} - \underline{\theta}) \right] \lambda'X. \quad (\text{A.84})$$

The long-run stock return, $\frac{P_2 - P_1}{P_1}$, will then be given by $\frac{d}{3} \frac{(\theta^f - \underline{\theta}) + (\bar{\theta} - \underline{\theta})}{\underline{\theta}}$. A firm will choose to distribute value through a dividend payment alone (while implementing its new project at the full investment level) only if $\theta^f < \underline{\theta}$. Further, $\bar{\theta} - \underline{\theta} > 0$ by assumption. Thus, the long-run stock return will be positive if $\underline{\theta} + (\bar{\theta} - \underline{\theta}) < \theta^f \leq \underline{\theta}$. Otherwise, if $\theta^f < \underline{\theta} - (\bar{\theta} - \underline{\theta})$, the long-run stock return will be negative. Q.E.D.

Proof of Proposition 10: When E is small, if the firm chooses to implement its project at the full investment level and distribute value through a stock repurchase alone, after the repurchase, the total firm value is $\underline{\theta}\lambda'X$, but the number of shares outstanding is reduced to $\left(1 - \frac{E - \lambda I}{E + \underline{\theta}\lambda'X - \lambda I} \right)$, so that the price per share is

$$P_1 = \frac{\underline{\theta}\lambda'X}{\left(1 - \frac{E - \lambda I}{E + \underline{\theta}\lambda'X - \lambda I} \right)}, \quad (\text{A.85})$$

at time 1. At time 2, after the marginal equity investor updates her belief as $\underline{\nu} = \underline{\theta} + \frac{d}{3}(\theta^f - \underline{\theta}) + \frac{d}{3}(\bar{\theta} - \underline{\theta})$, the price

per share will change to

$$P_2 = \frac{\left[\underline{\theta} + \frac{d}{3}(\theta^f - \underline{\theta}) + \frac{d}{3}(\bar{\theta} - \underline{\theta})\right] \lambda' X}{\left(1 - \frac{E - \lambda I}{E + \theta \lambda' X - \lambda I}\right)}. \quad (\text{A.86})$$

Therefore, the long-run stock return, $\frac{P_2 - P_1}{P_1}$, is equal to $\frac{d}{3} \frac{(\theta^f - \underline{\theta}) + (\bar{\theta} - \underline{\theta})}{\underline{\theta}}$. A firm conducts a stock repurchase only if $\theta^f > \underline{\theta}$. Further, $\bar{\theta} - \underline{\theta} > 0$ by assumption. Thus, the long-run stock return will always be positive in this case.

If the firm chooses to distribute value through a stock repurchase alone while underinvesting in its new project, the firm's stock price per share will be $E + \underline{\theta}X - I$ before the stock repurchase. After the repurchase, the belief of the marginal investor in the firm's equity will change from $\underline{\theta}$ to $\bar{\theta}$. Therefore, the total firm value will be $\bar{\theta}X$, but the number of shares outstanding will be reduced to $\left(1 - \delta - \frac{E - I - \delta(E + \underline{\theta}X - I)}{E + \bar{\theta}X - I}\right)$, so the share price will be

$$P_1 = \frac{\bar{\theta}X}{\left(1 - \delta - \frac{E - I - \delta(E + \underline{\theta}X - I)}{E + \bar{\theta}X - I}\right)} \quad (\text{A.87})$$

at time 1. At time 2, after the marginal equity investor updates her belief as $\bar{\nu} = \bar{\theta} + \frac{d}{3}(\theta^f - \bar{\theta}) + \frac{d}{3}(\underline{\theta} - \bar{\theta})$, the stock price will change to

$$P_2 = \frac{\left[\bar{\theta} + \frac{d}{3}(\theta^f - \bar{\theta}) + \frac{d}{3}(\underline{\theta} - \bar{\theta})\right] X}{\left(1 - \delta - \frac{E - I - \delta(E + \underline{\theta}X - I)}{E + \bar{\theta}X - I}\right)}. \quad (\text{A.88})$$

Therefore, in this case, the long-run stock return, $\frac{P_2 - P_1}{P_1}$, is equal to $\frac{d}{3} \frac{(\theta^f - \bar{\theta}) + (\underline{\theta} - \bar{\theta})}{\bar{\theta}}$. If $\theta^f > \bar{\theta} + (\bar{\theta} - \underline{\theta})$ in this case, then the long-run stock return following the stock repurchase will be positive. Otherwise, if $\underline{\theta} < \theta^f < \bar{\theta} + (\bar{\theta} - \underline{\theta})$, it will be negative.

When E is large, if the firm chooses to distribute value through a stock repurchase alone while implementing its project at the full investment level, the marginal investor in the firm's equity after the repurchase at time 1 will also have the belief $\bar{\theta}$ as in part (ii) of Proposition 10. Thus, the share price at time 1 (after the repurchase) will be

$$P_1 = \frac{\bar{\theta} \lambda' X}{\left(1 - \delta - \frac{E - \lambda I - \delta(E + \underline{\theta} \lambda' X - \lambda I)}{E + \bar{\theta} \lambda' X - \lambda I}\right)}. \quad (\text{A.89})$$

At time 2, after the marginal equity investor updates her belief as $\bar{\nu} = \bar{\theta} + \frac{d}{3}(\theta^f - \bar{\theta}) + \frac{d}{3}(\underline{\theta} - \bar{\theta})$, the stock price will change to

$$P_2 = \frac{\left[\bar{\theta} + \frac{d}{3}(\theta^f - \bar{\theta}) + \frac{d}{3}(\underline{\theta} - \bar{\theta})\right] \lambda' X}{\left(1 - \delta - \frac{E - \lambda I - \delta(E + \underline{\theta} \lambda' X - \lambda I)}{E + \bar{\theta} \lambda' X - \lambda I}\right)}. \quad (\text{A.90})$$

Therefore, in this case, the long-run stock return, $\frac{P_2 - P_1}{P_1}$, is equal to $\frac{d}{3} \frac{(\theta^f - \bar{\theta}) + (\underline{\theta} - \bar{\theta})}{\bar{\theta}}$ as well. It will be positive if $\theta^f > \bar{\theta} + (\bar{\theta} - \underline{\theta})$. Otherwise, if $\underline{\theta} < \theta^f < \bar{\theta} + (\bar{\theta} - \underline{\theta})$, it will be negative. Q.E.D.

Proof of Proposition 11: Without loss of generality, consider the case where the firm underinvests in its new project while it distributes value through a combination of a stock repurchase and a dividend payment. The firm first repurchases δ shares from pessimistic shareholders and then makes a dividend payment of $(E - I - \delta(E + \underline{\theta}X - I))$ to the remaining shareholders. The share price at time 0 is $E + \underline{\theta}X - I$. Recall that after the repurchase, the total number of shares outstanding is $1 - \delta$. If the number of shares repurchased by the firm is slightly (ϵ) less than δ , then the marginal investor in the firm's equity will still have the belief $\underline{\theta}$ after the payout at time 1. Therefore, the share price at time 1 after the stock repurchase and the dividend payment is

$$P_1 = \frac{\underline{\theta}X}{1 - \delta}. \quad (\text{A.91})$$

At time 2, after the marginal investor in the firm's equity updates her belief as $\underline{\nu} = \underline{\theta} + \frac{d}{3}(\theta^f - \underline{\theta}) + \frac{d}{3}(\bar{\theta} - \underline{\theta})$, the

share price will be

$$P_2 = \frac{\left[\underline{\theta} + \frac{d}{3}(\theta^f - \underline{\theta}) + \frac{d}{3}(\bar{\theta} - \underline{\theta}) \right] X}{1 - \delta}. \quad (\text{A.92})$$

Therefore, the long-run stock return, $\frac{P_2 - P_1}{P_1}$, is equal to $\frac{d}{3} \frac{(\theta^f - \underline{\theta}) + (\bar{\theta} - \underline{\theta})}{\underline{\theta}}$. Since $(\bar{\theta} - \underline{\theta}) > 0$ by assumption, the long-run stock return is positive if $\theta^f > \bar{\theta}$. Note that for a firm to optimally choose a combination of a stock repurchase and a dividend payment to distribute value to its shareholders, it has to be the case that $\theta^f > \underline{\theta}$. Therefore, the long-run stock return will always be positive in this case. Q.E.D.

Proof of Proposition 12: We assume that E is small so that the condition in (4) holds, θ^f is fixed, and $k = \bar{\theta} - \underline{\theta}$ is a constant, and $\underline{\theta}$ is uniformly distributed on the interval $(0, 1 - k)$. From Proposition 4(i)(a), it follows that for the sample of pure dividend-paying (and fully-investing) firms, the following parameter condition holds: $\theta^f \leq \underline{\theta} < 1 - k$. Then, the *average* long-run stock return \bar{r}_{div} of firms paying dividends only and implementing their projects at the full investment level is given by:

$$\bar{r}_{div} = \int_{\theta^f}^{(1-k)} \frac{\frac{d}{3}(\theta^f - \theta + k)}{\theta} \frac{1}{(1 - k - \theta^f)} d\theta. \quad (\text{A.93})$$

When E is small, we know from Proposition 4(i)(b) that firms, for which $\frac{(\lambda-1)I}{(\lambda'-1)X} \leq \underline{\theta} < \theta^f$, will repurchase shares only and fully invest in their new projects. Thus, the *average* long-run stock return \bar{r}_{rep} of these firms is given by:

$$\bar{r}_{rep} = \int_{\frac{(\lambda-1)I}{(\lambda'-1)X}}^{\theta^f} \frac{\frac{d}{3}(\theta^f - \theta + k)}{\theta} \frac{1}{\left(\theta^f - \frac{(\lambda-1)I}{(\lambda'-1)X}\right)} d\theta. \quad (\text{A.94})$$

After some steps of calculus and algebra,

$$\bar{r}_{div} = \frac{d(\theta^f + k)}{3} \frac{\ln(1 - k) - \ln(\theta^f)}{(1 - k) - \theta^f} - \frac{d}{3}, \quad (\text{A.95})$$

and

$$\bar{r}_{rep} = \frac{d(\theta^f + k)}{3} \frac{\ln(\theta^f) - \ln\left(\frac{(\lambda-1)I}{(\lambda'-1)X}\right)}{\theta^f - \frac{(\lambda-1)I}{(\lambda'-1)X}} - \frac{d}{3}. \quad (\text{A.96})$$

The result that $\bar{r}_{rep} > \bar{r}_{div}$ follows from the concavity of natural logarithm function $\ln(\cdot)$.

For some parameter specifications, a small fraction of the share-repurchasing (and fully-investing) firms will actually satisfy the condition given in Proposition 4(ii)(b) rather than the condition given in Proposition 4(i)(b). For these firms, it holds that $\frac{(\lambda-1)I}{(\lambda'-1)X} - k < \underline{\theta} < \frac{(\lambda-1)I}{(\lambda'-1)X}$. Since the long-run stock return given in (40) is decreasing in $\underline{\theta}$, the average long-run stock return of these firms is greater than the average long-run stock return \bar{r}_{rep} (given in (A.96)) of firms satisfying the condition that $\frac{(\lambda-1)I}{(\lambda'-1)X} \leq \underline{\theta} < \theta^f$. Therefore, for our purposes, it suffices to show that the average long-run stock return \bar{r}_{rep} of share-repurchasing firms (given in (A.96)), for which $\frac{(\lambda-1)I}{(\lambda'-1)X} \leq \underline{\theta} < \theta^f$, is greater than the average long-run stock return \bar{r}_{div} of dividend-paying firms (given in (A.95)) satisfying the condition given in Proposition 4(i)(a). Q.E.D.