

# Information Acquisition, Resource Allocation and Managerial Incentives

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## **Abstract**

A manager's incentives to acquire information about different investment alternatives and then to choose how to allocate resources among them are jointly influenced by his compensation contract and the level of resources allocated to him. We show how the level of resources can be used, over and above the manager's monetary compensation, as an instrument to induce additional information acquisition and may thus be set above or below the first-best level. Second, we show how the optimal compensation contract induces overconfidence, in the sense that the manager will choose a resource allocation that is more aggressive than the allocation that would maximize expected return. Finally, we show how the choice of the level of resources can be delegated to the manager without any loss in efficiency through appropriately tying managerial compensation to the level of resources requested (thus highlighting the risk of analyzing capital allocations in isolation from compensation), while induced overconfidence remains an unavoidable consequence of efficiently motivating information acquisition.

# 1 Introduction

Much research in finance and economics examines how firms can effectively use organizational systems and processes to deal with various frictions within the firm. A voluminous principal-agent literature following Holmstrom (1978) studies how firms can design compensation systems to address motivational problems, encourage information acquisition and improve biased decisions of agent managers. Another strand of literature following Harris, Kriebel and Raviv (1982), Antle and Eppen (1985) and Harris and Raviv (1996,1998) studies how firms can design capital budgeting processes to mitigate informational problems and minimize efficiency losses in resource allocation.

While both literatures separately shed light on two organizational design problems that significantly affect firm performance (compensation systems and capital allocation processes), little work simultaneously considers them. Our paper combines elements of both literatures to begin building a comprehensive picture of how the two problems may interact. First and foremost, our model has general normative value as it applies to any manager in charge of multiple projects or investments, e.g., CEOs, division managers, etc. From a positive perspective, our model generates equilibrium behavior by managers that is systematically biased from the firm's perspective but arises from standard preferences through the structure of the optimal compensation contract.

We consider a simple agency problem where a manager needs to first acquire information regarding the quality of different investment alternatives and then allocate the available resources among them. To manage the agency problem, the firm has two design parameters at its disposal. First, the firm determines the level of resources available to the manager for investment. Second, the firm decides on the compensation structure of the manager. In contrast to the existing literature on capital budgeting, we assume that the manager cares only about his expected compensation. There are thus no behavioral biases relative to the firm except to the extent that such behavior is induced in equilibrium by the optimal compensation structure of the manager.

If there were no additional constraints on the problem, the solution would be simple. The firm would simply sell the investment opportunities to the manager, who then becomes a full residual claimant and makes first-best choices. To avoid this trivial solution, we introduce two restrictions. First, and which is the key restriction, the manager is wealth-constrained and protected by limited liability, so that his compensation can never fall below zero. Second, only the overall investment performance of the manager is measurable. That is, the manager cannot be compensated on the individual performance of different investments – arbitrariness in setting internal transfer prices all too often makes individual assessment of different investments difficult.

The results that follow from the model are two-fold. First, complementing the literature on capital allocation, we show that in addition to playing a role in inducing truthful communication, distortions in the capital allocation also play a direct motivational role. Specifically, we show that when the manager's desired level of investment is decreasing in the quality of information, restricting the manager's access to capital below the first-best level induces the manager to make more careful decisions on how to allocate the funds and this increases his motivation to learn more about the investment alternatives. The converse, of course, is also true. When the desired level of investment is increasing in the quality of information, the manager acquires more information when provided access to a resource budget above the first-best level.

Second, we derive the optimal compensation contract and consider its implications for the behavior of the manager. With limited liability, we show that the optimal contract induces the manager to exhibit over-confidence, in the sense that given the quality of information held by the manager, he chooses a resource allocation that is more aggressive than the allocation that would maximize the expected return given the quality of his information. In other words, the manager invests too much in ex ante attractive projects and too little in ex ante unattractive projects. The intuition behind this result is straightforward. The value of information is clearly increasing in the aggressiveness of the equilibrium resource allocation because such aggressiveness makes mistakes more costly. Thus, by offering a compensation contract that induces more aggressive investment behavior, the firm is able to increase the value of information to the manager and thus achieve more information acquisition at the same expected monetary cost.

Finally, we consider, instead of centrally determining the level of resources available to the manager, delegating this choice to the manager. Here, we reach the natural conclusion that by linking managerial compensation appropriately to the level of resources requested, we can delegate this decision without any loss of efficiency. However, if we naively linked managerial compensation directly to the true cost of resources, then the manager will generically either over-invest or under-invest from the firm's perspective. The reason for this result is also simple. When choosing how much to invest, the manager wants to maximize his expected compensation, while the firm would like to maximize expected value. Because the optimal compensation contract biases the manager away from value maximization, his perceived return to investment will also be affected. Thus, empire-building behavior may also arise simply as the by-product of efficiently motivating information acquisition.

Our result on apparent overconfidence is related to works on risk-seeking behavior induced by convex compensation structures. The main modeling difference is that the manager is not gambling by turning a single risk dial but by making resource allocation decisions with greater upside potential and in favor of ex ante more attractive projects. In addition,

we show that the induced overconfidence result continues to hold at the optimal contract, which consists of at most two bonuses for exceeding given performance targets. It is thus not the convexity of the contract that induces the behavior, but the location of the bonuses. Both biases could be eliminated by particular bonus structures, but that is suboptimal from the perspective of effort incentives. The main message of our analysis is thus that capital allocation processes cannot be fully understood without at the same time considering compensation systems, because managerial compensation is one of the key determinants behind how managers will actually value and use the capital allocated.

## 2 Related literature (rewritten)

Our paper is related to the literature on capital budgeting that has followed the contributions of Harris, Kriebel and Raviv (1982), Antle and Eppen (1985) and Harris and Raviv (1996,1998), where some of the more recent contributions include Berkovitch and Israel (2004) and Marino and Matsusaka (2005).<sup>1</sup> The main difference is that while much of the literature has focused on the problem of information revelation given particular managerial preferences, we focus on the motivational effects of capital budgeting and derive explicitly the optimal compensation contract, which endogenously determines the extent to which the manager's preferences will differ from the firm's.

The key building blocks of the model are (i) the ex ante information acquisition step by the manager, (ii) the resource allocation decision following the information acquisition stage, and (iii) the joint role of the compensation contract and the initial resource allocation in motivating the manager. Since none of the individual components are fully new to the literature, it is instructive to relate the overall framework to the various papers that have already touched upon these topics.

First, there is a small but growing literature that has examined the dual agency problem where an agent first needs to acquire information regarding the value of a risky investment alternative and then choose whether to take the risky or risk-free alternative. Contributions examining this tradeoff include Lambert (1986), Holmstrom and Ricart I Cost (1986), Levitt and Snyder (1997) and Inderst and Klein (2007). The key similarity with this literature is that the agents are not inherently biased but may differ in their risk preferences or become biased due to the compensation contract that is offered to them. The key difference is that these papers deal with the binary choice between a risky and a risk-free alternative, whereas we consider a continuous allocation problem between two risky alternatives. To see

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<sup>1</sup>A generalized variant of the basic idea of Berkovitch and Israel (2004) can be found in Armstrong and Vickers (2010).

the key difference in these two formulations, note that if the choice between the two risky alternatives would be binary as in this literature, the agency problem would completely disappear. In other words, there is no agency conflict in our model for the agent to reveal which alternative is more attractive. Instead, the key interaction is between the value of information and the aggressiveness of the chosen allocation, which introduces the dual role for the level of resources and the compensation structure of the manager. Indeed, as models of binary choice, these papers are silent on the level of resources as a motivational tool, while we are able to consider the effects of both the level of resources allocated to the manager and the aggressiveness of the final resource allocation, as induced by the compensation contract of the manager. To illustrate this difference, consider Levitt and Snyder (1997), which is most similar to ours. They show that the manager will, in equilibrium, end up choosing the risky alternative more frequently than optimal, one interpretation for which is overinvestment (if the risk-free alternative is taken to be no investment), and the other is over-aggressiveness (choosing the riskier alternative too frequently). In contrast, in our setting, because the level of resources is endogenous and explicitly modeled, we show that the equilibrium may exhibit either underinvestment or overinvestment in levels, but has over-aggressiveness as a general feature of the solution, in terms of excessive skewing of the final resource allocation between the alternatives (as opposed to choosing a riskier alternative).<sup>2</sup>

Other papers related to motivating information acquisition are Szalay (2005), who shows how in a decision-making context it is optimal to rule out decisions that seem ex ante optimal because that will increase the cost of mistakes by preventing the agent from choosing compromise solutions, Lewis and Sappington (1997), who consider how to motivate an agent to learn his costs in a procurement setting, and Bernardo, Cai and Luo (2009), who consider how to use a menu of capital allocations and incentive contracts to motivate entrepreneurial effort and truthful communication of project qualities. The logic of Szalay (2005) is similar to ours, in that the reason why the optimal compensation contract in our setting induces over-aggressiveness in the resource allocation is that it increases the value of information, just like ruling out compromise solutions. Bernardo, Cai and Luo (2009) is probably the most similar to our setting in its focus, in that they also derive the result that low-quality projects should receive less than first-best level of funding while high-quality projects should receive more than the first-best level of funding, equivalent to our result for over-aggressiveness, but in a complex mechanism design setting involving interactions among entrepreneurial effort to improve investment alternatives, incentives to reveal the value of investment opportunities and ex post effort to implement the projects. We distill

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<sup>2</sup>See also Chen and Jiang (2004), who show that when a manager has empire-building preferences, then biasing the project acceptance rule against investment will increase his incentives to acquire information when the information generated is observable.

the problem to highlight the motivational benefits of over-aggressiveness for information acquisition and how to efficiently induce that with a single compensation contract, and further derive the additional effects of the *level* of resources available for the allocation problem.

Some papers considering the general effort consequences of capital allocations are Paik and Sen (1995), Bernardo, Cai and Luo (2001) and Han, Hirshleifer and Persons (2009). Paik and Sen (1995) consider how the capital allocation menu is distorted if there is a direct complementarity or substitutability between capital and effort and the agent is endowed with private information regarding the production technology. Bernardo, Cai and Luo (2001) consider a model where the principal hires an agent with empire-building preferences to both reveal the quality of an investment opportunity and then exert effort to implement it, with assumed complementarity between effort and capital allocation. The key difference, in addition to qualitative predictions, is that we do not assume any *ex ante* relationship between the value of effort and the level of resources. Indeed, the key observation is that depending on the particular setting, the value of effort may be either increasing or decreasing in the level of resources available to the agent, which is exactly what may make the level of resources made available to the agent to be either above or below the first-best level. Finally, Han, Hirshleifer and Persons (2009) also examine the interaction between incentives and capital allocation but they do not consider optimal incentives. They show that it can be optimal to restrict the managers' access to capital when they are participating in a promotion tournament, because the value of being first and obtaining a promotion can lead to excessive risk-seeking in the absence of capital restrictions.

Finally, our work is related to the large literature on internal capital markets in that one of the stages in our model is a resource allocation decision between two investment opportunities. The difference is that the literature on internal capital markets deals with the case where the opportunities belong to separate strategic agents and the analysis focuses on the conflicting interests between the two recipients of funding to compete for that funding. In our setting, a single agent is responsible for both and the conflict arises with that agent being still misaligned in equilibrium with corporate headquarters. Future extensions of our framework to account for the possibility of multiple agents appears promising since as with the capital budgeting literature, the literature on internal capital markets has paid only limited attention to the interaction between managerial compensation and the use of internal capital markets, instead assuming empire-building preferences. Two exceptions are Friebel and Raith (2010), who consider a resource allocation problem where two managers first attempt to generate good quality projects and then make claims regarding their need for resources and Bernardo, Cai and Luo (2004), which extends Bernardo, Cai and Luo (2001) to account for competing divisions.

### 3 Model

Suppose that a manager (say, a division head) has two projects or tasks and he needs to allocate resources between them. One of the projects is more attractive in that the project has greater expected marginal productivity than the other project for any given level of investment  $x$ . The manager, however, does not initially know enough to rank the two projects.

At a personal cost  $C(q)$ , the manager can acquire information to rank the two projects correctly with probability  $q$  ( $\geq 1/2$ ). Having formed his interim belief regarding which of the projects is more productive, the manager then allocates  $x_h$  to the more attractive project and  $x_l$  to the less attractive project (with  $x_h > x_l$ ), subject to a resource budget constraint  $x_h + x_l \leq I$ . As the manager's goal is to channel more resources to the more productive project, he is therefore "right" ( $H$ ) with probability  $q$  and "wrong" ( $L$ ) with probability  $(1 - q)$ . Given the investments  $x_h$  and  $x_l$ , the two projects then yield a total cash flow  $\pi \in [0, +\infty)$ , distributed according to  $f_H(\pi|x_h, x_l)$  when the allocation decision is right and  $f_L(\pi|x_h, x_l)$  when the allocation is wrong. The expected total gross cash flow given the investments is thus

$$\begin{aligned} E[\pi|x_h, x_l] &= q \int_0^{+\infty} \pi f_H(\pi|x_h, x_l) d\pi + (1 - q) \int_0^{+\infty} \pi f_L(\pi|x_h, x_l) d\pi \\ &= qE_H[\pi|x_h, x_l] + (1 - q)E_L[\pi|x_h, x_l], \end{aligned}$$

where the properties of  $f_H(\pi|x_h, x_l)$  and  $f_L(\pi|x_h, x_l)$  are discussed in more detail below.

The agency problem arises from the fact that the manager does not directly care about either the gross cash flow or the cost of resources. Instead, the manager cares only about his compensation. To motivate the manager to perform the dual task of first acquiring information  $q$  and then choosing the resource allocation  $(x_h, x_l)$ , the organization has two control instruments at its disposal. First, the organization can offer the manager a wage contract  $w(\pi)$ , which ties the manager's compensation on the realized gross cash flow. The only restrictions we place on the wage contract are (i) limited liability, so that  $w(\pi) \geq 0$  and (ii) monotonicity, so that  $w'(\pi) \geq 0$ .<sup>3</sup> Second, the organization determines ex ante the level of resources made available to the manager,  $I$ , where the cost of resources to the organization is  $r$ .<sup>4</sup> We can thus write the organizational design problem as

<sup>3</sup>The second restriction will arise naturally if it is possible for the manager to destroy some of the realized output.

<sup>4</sup>In the extensions, we discuss how the choice of  $I$  could be delegated to the manager without any loss of

$$\begin{aligned}
& \max_{w(\pi), I} E[\pi - w(\pi)|x_h, x_l] - rI \\
& \text{s.t. } \{q, (x_h, x_l)\} \in \arg \max E[w(\pi)|x_h, x_l] - C(q) \\
& \quad x_h + x_l \leq I. \\
& \quad w(\pi), w'(\pi) \geq 0
\end{aligned}$$

That is, the organization will choose the compensation contract  $w(\pi)$  and the level of resources  $I$  to maximize its cash flow net of managerial compensation and the cost of resources, subject to the quality of information  $q$  and the resource allocation  $(x_h, x_l)$ , as chosen by the manager to maximize his expected surplus  $E[w(\pi)|x_h, x_l] - C(q)$ .

### 3.1 Assumptions regarding the gross cash flow

Because one of the key decisions made by the manager is how to allocate the resources  $I$  between the two tasks, an important element of the model is how this allocation influences the cash flow distributions  $f_H(\pi|x_h, x_l)$  and  $f_L(\pi|x_h, x_l)$ . To provide additional structure to the problem, we first assume that the expected returns are concave and satisfy the standard Inada conditions, so that

$$\frac{\partial E_j(\pi|x_h, x_l)}{\partial x_i} > 0 \text{ and } \frac{\partial E_j^2(\pi|x_h, x_l)}{\partial x_i^2} < 0, \text{ with } \lim_{x \rightarrow 0} \frac{\partial E_j(\pi|x_h, x_l)}{\partial x_i} = +\infty \text{ and } \frac{\partial E_j(\pi|x_h, x_l)}{\partial x_i} = 0.$$

Also, while we assume that only the total cash flow can be measured, it is composed of the cash flows of the two independent projects, so that  $\frac{\partial^2 E_j(\pi|x_h, x_l)}{\partial x_i \partial x_h} = 0$ . Finally, the assumption that one of the projects has a higher marginal productivity than the other project implies that

$$\frac{\partial E_H(\pi|x_h, x_l)}{\partial x_h} > \frac{\partial E_L(\pi|x_h, x_l)}{\partial x_h} \quad \forall x_h.$$

In other words, an additional dollar invested in the ex ante more attractive project always improves the expected return more when it is indeed allocated to the truly more productive project (while the opposite holds for  $x_l$ ).

Now, if we focused only on linear contracts, this would be (almost) enough for the analysis to go through. However, because we are dealing with general forms of compensation contracts, we need to put some additional structure on how the allocation choice influences the shape of the returns as well. The first assumption is that the investments improve returns in the sense of first-order stochastic dominance, so that  $\frac{\partial F_H(\pi|x_h, x_l)}{\partial x_i} < 0$  and  $\frac{\partial F_L(\pi|x_h, x_l)}{\partial x_i} < 0$ .

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efficiency through a contract  $w(\pi, I)$  that ties managerial compensation to the level of resources invested.

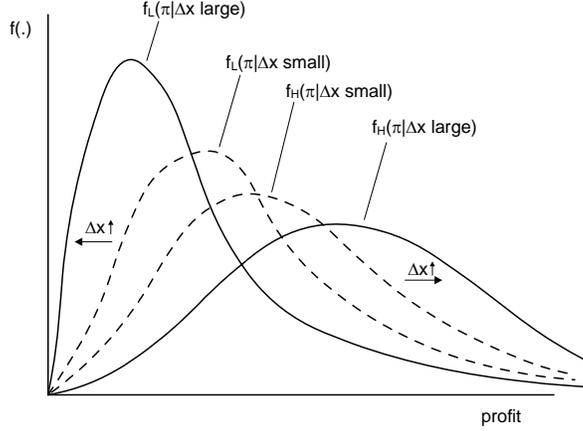


Figure 1: Illustration of distributional assumptions

In other words, the increase in the expected return does not come at the cost of increased risk. The second assumption we make is that

$$\frac{\partial}{\partial x_h} \left( \frac{1-F_H(\pi|x_h, x_l)}{1-F_L(\pi|x_h, x_l)} \right) > 0 \text{ and } \frac{\partial}{\partial x_l} \left( \frac{1-F_H(\pi|x_h, x_l)}{1-F_L(\pi|x_h, x_l)} \right) < 0 \quad \forall \pi, x_h, x_l.$$

This assumption is the distributional equivalent of the assumption that one of the projects has a higher marginal productivity and states that a dollar invested in the truly more productive project increases the likelihood of meeting any given cash flow threshold more than that same dollar invested in the less productive project, not only the expected cash flow.

Finally, note that given a particular investment level,  $I$ , the choice of  $(x_h, x_l)$  comes down to choosing  $\Delta x = x_h - x_l$  and so we can express the cash flow distributions also as  $F_i(\pi|\Delta x, I)$ . To illustrate the role of  $\Delta x$  in influencing the expected cash flow, note that if  $\Delta x = 0$ , then  $F_H(\pi|\Delta x, I) = F_L(\pi|\Delta x, I)$  as both tasks receive the same amount of resources and so the final distributions are equivalent. But as the manager increases  $\Delta x$ , then  $F_H(\pi|\Delta x, I) < F_L(\pi|\Delta x, I)$ . Further, given our assumptions above, it is clear that  $\frac{\partial F_L(\pi|\Delta x, I)}{\partial \Delta x} \leq 0$  as the allocation becomes increasingly "wrong," while  $\frac{\partial F_H(\pi|\Delta x, I)}{\partial \Delta x} \geq 0$  for  $\Delta x \leq \overline{\Delta x}$ , as the allocation becomes increasingly "right," until the investment allocation matches that which would be chosen if it was known for sure that the allocation is right:  $\overline{\Delta x}$  solves  $\frac{\partial E_H(\pi|\Delta x, I)}{\partial \Delta x} = 0$ . This relationship between the aggressiveness of the allocation and the return distributions is illustrated in figure 1.

A particularly transparent variant of the model is given by purely mean-shifting investments with additive shocks, where we can write the realized cash flow as

$$\pi = \pi_H(x_i) + \pi_L(x_j) + \varepsilon,$$

where  $H$  denotes the more productive task, and where  $\varepsilon \sim G(\varepsilon)$  with  $E(\varepsilon) = 0$  and  $\pi_j(x)$  denote the expected cash flows from the two projects, with  $\frac{\partial \pi_j(x)}{\partial x} > 0$ ,  $\frac{\partial^2 \pi_j(x)}{\partial x^2} < 0$  and  $\frac{\partial \pi_H(x)}{\partial x} > \frac{\partial \pi_L(x)}{\partial x} \forall x$ .<sup>5</sup> We will use this simplified framework to derive some more precise results. Finally, note that while we are making the assumption that only joint performance on the two tasks is measurable, it does not imply that there could not be individual accounting returns for both of the tasks. The assumption is simply that the manager has sufficient ability to transfer performance between the tasks so that the manager would game any differential compensation between the tasks.

**An example:** As an illustration of the logic behind the setup, suppose the manager in our question is the manager of a division that is responsible for two different products, the demand for which is uncertain and the resource allocation in question is the marketing budget for the division. The manager then needs to first acquire information regarding the relative attractiveness of the two products to the consumers, with the knowledge that one of them is going to be more popular, and then choose how to allocate the marketing budget between the two products. Note that even if the headquarters would earmark such budgets as  $(x_A, x_B)$  intended for the two products separately after consulting the manager and has accounting measures  $(\pi_A, \pi_B)$  regarding the revenue generated by the two products, the manager in charge of the division will have various actions available for both transferring the marketing resources and apparent performance between the two products.

## 4 Preliminary results

Before considering the full problem, we will illustrate the basic tradeoffs involved by solving (i) the first-best solution, where the firm can perform both the allocation and information acquisition tasks directly, and (ii) the mechanism design solution, where the firm is able to commit to both the compensation contract  $w(\pi)$  and the investment levels  $x_h, x_l$  but must rely on the manager for information. These results help to illustrate the basic tradeoffs of interest. The model itself is solved in the following section.

### 4.1 First-best

Consider first the case where the firm can perform both the information acquisition and the investment decisions. The headquarters thus solves

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<sup>5</sup>If we simply assume that the size of the shocks is bounded from below, we can also satisfy the non-negativity constraint on returns assumed.

$$\max_{x_h(q), x_l(q), q} \int_0^{\infty} \pi (q f_H(\pi|x_h, x_l) + (1-q) f_L(\pi|x_h, x_l)) d\pi - r(x_h + x_l) - C(q).$$

The solution is then given directly by the first-order conditions as summarized by the following proposition:

**Proposition 1 *First-best solution:***

(i) Investment levels solve  $q \frac{\partial E_H(\pi|x_h, x_l)}{\partial x_i} + (1-q) \frac{\partial E_L(\pi|x_h, x_l)}{\partial x_i} = r$ .

(ii) The quality of information solves  $E_H(\pi|x_h, x_l) - E_L(\pi|x_h, x_l) = C'(q)$ .

A simple investigation of these first-order conditions contains the basic relationships among the investment levels and the quality of information. First,  $x_h > x_l$  whenever  $q > \frac{1}{2}$  because the expected marginal productivity of the more productive task is higher at all levels of investment. Second,  $\frac{\partial x_h}{\partial q} > 0$  and  $\frac{\partial x_l}{\partial q} < 0$  because more precise information allows the organization to adopt a more aggressive investment policy. The reason is that the more confident the organization is regarding having identified the better task, the less it needs to worry about low returns to  $x_h$  if it accidentally allocates it to the low-productivity task and similarly the less it needs to worry about potentially high returns to  $x_l$ . The converse of this logic, which will play an important role below when considering the motivational effects of the resource allocation, is that the investment strategy influences the value of information. In particular, the aggressiveness of the allocation and the quality of information are strategic complements, because an increase in aggressiveness makes mistakes more costly.

While the investment policy becomes more aggressive with information, its impact on the level of overall investment is ambiguous, so that  $\frac{\partial(x_h(q)+x_l(q))}{\partial q} \gtrless 0$ . This is the second key element of the model, because it implies that the quality of information and the level of resources can be either strategic complements or substitutes. The intuition for this result is as follows. While the investment strategy is more aggressive, we don't know whether the increase in  $x_h$  is larger or smaller than the decrease in  $x_l$ . To return to our advertising example from above, suppose that if  $q = \frac{1}{2}$ , the firm decides to invest \$5M on advertising both products. Now, suppose that the prior increases to  $q > \frac{1}{2}$ . In response, the firm decides to invest \$7M on the more attractive product and \$2M on the less attractive product. The total expense is now \$9M and the additional information has reduced the total capital requirement because the funds can be targeted better. But, alternatively, the firm may increase advertising on the more attractive product to \$8M while decreasing the advertising on the other product to \$3M, in which case the additional information leads the

organization to invest more in advertising because it can be confident that the funds are invested appropriately.

## 4.2 Mechanism design

The second preliminary step is to consider the mechanism design problem where the firm is able to commit to given investment levels  $x_h$  and  $x_l$ , but the task of information acquisition must be undertaken by the manager who is then offered a contract  $w(\pi)$  to motivate that information acquisition. The design problem then becomes

$$\begin{aligned} & \max_{x_h, x_l, w(\pi)} \int_0^{\infty} (\pi - w(\pi)) (q f_H(\pi|x_h, x_l) + (1 - q) f_L(\pi|x_h, x_l)) - r(x_h + x_l) \\ \text{s.t.} \quad & q \in \arg \max \int_0^{\infty} w(\pi) (q f_H(\pi|x_h, x_l) + (1 - q) f_L(\pi|x_h, x_l)) - C(q) \\ & w(\pi), w'(\pi) \geq 0. \end{aligned}$$

The solution to this design problem is given by the following proposition.

### Proposition 2 *Optimal mechanism:*

(i) *The optimal wage contract consists of a single bonus  $B$  paid for performance exceeding  $\bar{\pi}$ , where  $\bar{\pi}$  solves  $\frac{1 - F_H(\bar{\pi})}{1 - F_L(\bar{\pi})} = \frac{f_H(\bar{\pi})}{f_L(\bar{\pi})} > 1$ .*

(ii) *The optimal investment levels solve*

$$q \frac{\partial E_H(\pi|x_h, x_l)}{\partial x_i} + (1 - q) \frac{\partial E_L(\pi|x_h, x_l)}{\partial x_i} = r + B \frac{\left( (1 - F_L(\bar{\pi})) \frac{\partial F_H(\bar{\pi})}{\partial x_i} - (1 - F_H(\bar{\pi})) \frac{\partial F_L(\bar{\pi})}{\partial x_i} \right)}{(F_L(\bar{\pi}) - F_H(\bar{\pi}))}.$$

(iii) *If the monotone likelihood ratio holds and  $\frac{f_H(\pi)}{f_L(\pi)} \rightarrow \infty$  as  $\pi \rightarrow \infty$ , then  $\bar{\pi}, B \rightarrow \infty$  and the bonus alone achieves the first-best*

(iv) *Otherwise,  $x_h^* > x_h^{FB}$  and  $x_l^* < x_l^{FB}$ , with  $(x_h^* + x_l^*) \gtrless (x_h^{FB} + x_l^{FB})$ .*

**Proof.** See Appendix A.1 ■

Dealing with the challenge of information acquisition alone is thus relatively straightforward. Following the simple logic of moral hazard problems, the manager should be paid

in the outcome states that are most informative about his effort. In the present case, this logic then leads to the solution familiar from hypothesis testing that the manager receives his bonus only after a threshold where the hazard rates are equal - it is after this point that the probability of the particular profit realization coming from the right resource allocation starts to exceed that of the wrong resource allocation, conditional on being at least  $\bar{\pi}$ . But recall that if the distributions satisfy the monotone likelihood ratio property, so that  $\frac{f_H(\pi)}{f_L(\pi)}$  is increasing in  $\pi$ , then  $\frac{1-F_H(\pi)}{1-F_L(\pi)} < \frac{f_H(\pi)}{f_L(\pi)}$  and the organization can always reduce the expected cost of compensation by increasing  $\bar{\pi}$  and  $B$  in a fashion that holds effort incentives constant. Then, as  $\frac{f_H(\pi)}{f_L(\pi)} \rightarrow \infty$ , the highest possible profit realization becomes infinitely informative and, as a result, the headquarters can still achieve the first-best solution with the compensation contract alone and the investment decisions are not distorted.

Suppose, however, that this is not the case, so that an interior solution to  $\frac{1-F_H(\bar{\pi})}{1-F_L(\bar{\pi})} = \frac{f_H(\bar{\pi})}{f_L(\bar{\pi})}$  actually exists. Then, the mechanism can provide additional incentives by distorting the capital allocation to support further information acquisition. The logic of these distortions follows directly from above. By increasing the aggressiveness of the investment strategy, the headquarters increases the value of information by making the meeting of any given performance standard more dependent on having the right information. In other words, the means of increasing the quality of information acquired by the manager is to commit to a pattern of behavior that is as if the manager had better information than he really does because that makes mistakes more costly to the manager. As a result,  $x_H^* > x_H^{FB}$  and  $x_L^* < x_L^{FB}$ .

Finally, the same logic also leads to a distortion in the overall level of investment, but unfortunately, in the general case, there is no easy solution as to the direction of the distortion as it now depends on how the *manager* values the level of resources. However, in the case of additive investments ( $\pi = \pi_H(x_i) + \pi_L(x_j) + \varepsilon$ ), we can further show that  $x_h^* + x_l^* < x_h^{FB} + x_l^{FB}$  iff  $\frac{\partial(x_h^* + x_l^*)}{\partial q} < 0$ . Intuitively, if better information leads the firm to be able to economize on the total level of investment, then restricting the overall level of resources available to the manager will induce him to acquire more information.

## 5 Analysis

Having considered the cases where the firm is able to do both tasks and the case where only information acquisition is undertaken by the manager, we can now consider the case where the manager is responsible for both the capital allocation  $(x_h, x_l)$  and information acquisition  $(q)$ . The design problem for the headquarters becomes now

$$\begin{aligned}
& \max_{I, w(\pi)} \int_0^{\infty} (\pi - w(\pi)) (q f_H(\pi|x_h, x_l) + (1-q) f_L(\pi|x_h, x_l)) d\pi - r(x_h + x_l) \\
\text{s.t.} \quad & q^M \in \arg \max_q \int_0^{\infty} w(\pi) (q f_H(\pi|x_h, x_l) + (1-q) f_L(\pi|x_h, x_l)) d\pi - C(q) \\
& x_i^M \in \arg \max_{x_i} \int_0^{\infty} w(\pi) (q f_H(\pi|x_h, x_l) + (1-q) f_L(\pi|x_h, x_l)) d\pi, \quad i \in \{h, l\} \\
& x_h^M + x_l^M \leq I \\
& w(\pi), w'(\pi) \geq 0.
\end{aligned}$$

In other words, the problem is thus identical to the one considered above, with the exception that instead of being able to commit to individual resource allocations, the headquarters can now only provide a total resource allocation  $I$ , which is then invested by the manager in an incentive-compatible fashion. To examine this problem, we will consider it in three steps. First, we will highlight the general motivational effect of the resource allocation, which is operates for all  $w(\pi)$ . Second, we will consider the structure of the optimal compensation contract  $w(\pi)$  and its implications for both managerial behavior and the optimal level of resources. Third, we will consider how the same solution can be implemented through allowing the manager to choose  $I$  but linking his compensation on the resource allocation requested. For the analysis, we assume that the MLRP holds so that  $\frac{f_H(\pi|x_h, x_l)}{f_L(\pi|x_h, x_l)}$  is increasing in  $\pi$  and  $\frac{f_H(\pi|x_h, x_l)}{f_L(\pi|x_h, x_l)} \rightarrow \infty$  as  $\pi \rightarrow \infty$ .

## 5.1 Motivational effects of the level of resources available

To examine the motivational effects of the level of resources allocated to the manager, we can write the manager's problem as

$$\max_{x_h, x_l, q} \int_0^{\infty} w(\pi) (q f_H(\pi|x_h, x_l) + (1-q) f_L(\pi|x_h, x_l)) d\pi - \lambda(x_h + x_l - I) - C(q),$$

where  $\lambda$  is the Lagrange multiplier for the constraint  $x_h + x_l \leq I$ . The analysis of the maximization problem establishes the following proposition:

**Proposition 3** *The motivational effect of resource allocation:*

*The effort level of the manager is decreasing in the level of resources if and only if the*

shadow value of capital ( $\lambda$ ) is decreasing the in the quality of information:  $\frac{\partial q}{\partial I} < 0$  iff  $\frac{\partial \lambda}{\partial q} < 0$ .

**Proof.** See Appendix A.2 ■

As expressed, the motivational effect of the resource allocation is thus very simple: the effort of the manager is increasing in the level of resources if effort and resources are complements and vice versa. Intuitively, if resources and effort are substitutes, then by artificially holding some resources away from the manager, we can increase the value of effort and thus induce the manager to work harder. The novel element of this observation is how this mechanism arises from how the investment choices  $(x_h, x_l)$  respond to both the quality of information,  $q$ , and the total resources available,  $I$ , and whereby the strategic manipulation of  $I$  influences the value of information through the investment choices. Further, the effect can go in either direction, depending on both the structure of compensation  $w(\pi)$  and the structure of the returns  $\frac{\partial F_j(\pi|x_h, x_l)}{\partial x_i}$ .<sup>6</sup>

This basic observation thus qualifies the common approach of viewing capital and information as complements, whereby the more resources the organization has available, the more valuable information is to the organization because returns to information scale with the level of investment. What the present paper highlights is that resources and information can also be substitutes: by having more precise information, the organization can actually save on the level of overall resources because it is able to target the funds available more precisely. And when this is the case, then restricting the manager's access to capital functions as a motivational tool over and above any direct compensation to the manager.

## 5.2 Optimal compensation contracts

Having isolated the motivational effects of the level of resources, the next step is to consider the structure of the optimal compensation contract. The structure of the optimal compensation contract is of interest for two reasons. First, it turns out to be relatively simple in the present setting and, second, it will have clear implications for the behavior of the manager. These properties of the first-best compensation contract are summarized by the following proposition:

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<sup>6</sup>We are dealing with the Lagrange multiplier because the level of resources allocated to the manager is given in the present setting. Section 5.3 discusses the case where the manager is allowed to choose  $I$ , where the equivalent statement is then that  $\frac{\partial q^M}{\partial r} > 0$  iff  $\frac{\partial(x_h^M + x_l^M)}{\partial q} < 0$  or the total investment by the manager is decreasing in the precision of information.

**Proposition 4 Optimal compensation contracts:**

(i) The optimal compensation contract consists of at most two bonuses  $\underline{B}$  and  $\overline{B}$ , paid for exceeding performance thresholds  $\underline{\pi}$  and  $\overline{\pi}$ , respectively

(ii) A single bonus  $B$  paid for exceeding performance  $\tilde{\pi}$  may be optimal. In the case of additive shocks, a single bonus will be optimal if the following condition holds:

$$\frac{[\Delta g_H(\pi_2, \pi)g_L(\pi) - \Delta g_L(\pi_2, \pi)g_H(\pi)]}{[\Delta G_L(\pi_2, \pi)(1 - G_H(\pi)) - \Delta G_H(\pi_2, \pi)(1 - G_L(\pi))]} \geq \frac{[\Delta g_H(\pi, \pi_1)g_L(\pi) - \Delta g_L(\pi, \pi_1)g_H(\pi)]}{[\Delta G_L(\pi, \pi_1)(1 - G_H(\pi)) - \Delta G_H(\pi, \pi_1)(1 - G_L(\pi))]}.$$

(iii) The optimal compensation contract induces the manager to exhibit over-confidence, in the sense that he will invest more aggressively than the first-best allocation, conditional on the information available to him:  $\frac{x_h^M(q)}{x_h^M(q) + x_l^M(q)} > \frac{x_h^{FB}(q)}{x_h^{FB}(q) + x_l^{FB}(q)}$ .

**Proof.** See Appendix A.3 ■

In other words, the optimal compensation contract consists at most of two bonuses. While this might appear a little surprising given the level of freedom we have given to the design of  $w(\pi)$ , the intuition behind the result is quite simple. First, recall from above that if effort provision was the only problem faced by the firm, then the optimal contract would consist of a single bonus. Second, if we were only worried about the allocation decision, then there are a number of different contracts that would be able to achieve the right allocation. For example, any linear sharing rule  $w(\pi) = \alpha\pi + \beta$  would achieve the desired allocation. But there exists also a single bonus contract that is able to achieve it, where  $\hat{\pi}$  is chosen so that the relative marginal contribution of  $x_h$  and  $x_l$  to meeting the performance threshold would exactly match their relative marginal contribution to expected cash flow. And since each of the agency problems (effort and allocation choice) could be solved with a single bonus, it comes as no surprise that the joint problem can be solved with a contract that consists of two bonuses.

Of more interest is the result that a single bonus can still dominate two bonuses. While we have not been able to derive a necessary and sufficient condition for this result to hold in the general case, the proposition gives such a condition in the case of additive shocks ( $\pi = \pi_H(x_i) + \pi_L(x_j) + \varepsilon$ ). The general intuition relates simply to whether the distortion in the equilibrium decisions under two bonuses at  $\underline{\pi}$  and  $\overline{\pi}$  needed to induce given equilibrium effort exceeds that under a single bonus at  $\pi \in [\underline{\pi}, \overline{\pi}]$ . If over the relevant range, the distortion in the equilibrium allocation is convex ( $\frac{\partial^2 \Delta x}{\partial \pi^2} > 0$ ), then it will generally be optimal to use a single bonus for the simple reason that under the two bonuses, the decision distortion will be a linear combination of the distortion at the two bonus thresholds.

The second result relates to the investment behavior induced by the optimal contract, which states that the investment behavior of the manager will be more aggressive than the first-best investment behavior. The intuition for this result is clear from the mechanism design solution discussed earlier. Because the value of information is increasing in the aggressiveness of the resource allocation, inducing more aggressive behavior by the manager following the acquisition of information through the compensation contract will induce the manager to acquire more information and thus allow the firm to economize on the monetary cost of information acquisition. In addition, such aggressiveness will make the realized cash flow more informative of whether the manager made the right choice, thus further reducing the expected monetary compensation that needs to be paid to the manager. To summarize, while there exists a compensation contract that is able to induce  $(x_h^{FB}(q), x_l^{FB}(q))$ , it is optimal for the firm to distort the contract to induce more aggressive investment behavior as it allows it to reduce the expected monetary compensation both because the manager will be induced to acquire more information and the realized cash flow becomes more informative of the correctness of the allocation.

### 5.2.1 The interaction between the compensation contract and the level of resources

From the discussion above, it is clear that both the level of resources and the compensation contract are valuable tools for motivating the manager to acquire more information, where one of the avenues is through influencing the aggressiveness of the resource allocation. As a result, the two will naturally also interact. Suppose the current compensation contract is optimal. Then, if  $\frac{\partial q}{\partial I} < 0$ , the principal can induce further information acquisition and thus economize on the compensation contract by restricting the manager's access to resources below the first-best level. But suppose that the current compensation contract is already inducing suboptimally aggressive investment behavior for reasons outside the model. Then, even if  $\frac{\partial q}{\partial I} < 0$ , it may be optimal to actually *increase* the manager's access to resources. The reason is that now the manager is already excessively aggressive and giving additional resources will dampen the aggressiveness of the allocation.

Finally, there is a simple distortion caused by the positive level of managerial compensation, whereby the simple fact that the manager will receive positive pay will reduce the value of investment to the principal. Thus, the principal will mechanically want to invest less than the first-best level due to the lower perceived expected return. These interactions are highlighted in more detail in the example below. However, before analyzing the example, let us consider the feasibility of delegating the resource level choice to the manager as well.

### 5.2.2 Delegating the choice of $I$ to the manager

So far, we have assumed that the manager is simply allocated a given level of resources,  $I$ . However, it is straightforward to delegate this choice to the manager without any loss in efficiency. The logic is as follows. While we clearly cannot charge the manager for the amount of resources he will use because of limited liability, what we can do is to condition the compensation contract of the manager on the total level of investment. In particular, we can set the bonus thresholds to be  $\pi_i = \tilde{\pi}_i + \tilde{r}I$ , where  $\tilde{r}$  will then be the sensitivity of the bonus thresholds to the level of resources used by the manager. The manager will then choose  $(x_h, x_l)$  that will solve

$$\max_{x_h, x_l} \sum_i B_i (q(1 - F_H(\pi_i|x_h, x_l)) + (1 - q)(1 - F_L(\pi_i|x_h, x_l))),$$

with the resulting first-order conditions of

$$\begin{aligned} - \sum_i B_i \left( q \frac{\partial F_H(\pi_i|x_h, x_l)}{\partial x_h} + (1 - q) \frac{\partial F_L(\pi_i|x_h, x_l)}{\partial x_h} \right) &= \tilde{r} \sum_i B_i (q f_H(\pi_i|x_h, x_l) + (1 - q) f_L(\pi_i|x_h, x_l)) \\ - \sum_i B_i \left( q \frac{\partial F_H(\pi_i|x_h, x_l)}{\partial x_l} + (1 - q) \frac{\partial F_L(\pi_i|x_h, x_l)}{\partial x_l} \right) &= \tilde{r} \sum_i B_i (q f_H(\pi_i|x_h, x_l) + (1 - q) f_L(\pi_i|x_h, x_l)). \end{aligned}$$

The left-hand side of the equations is equivalent to the marginal return to additional investment while the right-hand side is now the marginal cost of additional investment, equivalent to the shadow cost of capital,  $\lambda$ , in the case of fixed allocation  $I$ . But because the marginal cost of additional investment is the same for  $x_h$  and  $x_l$ , we can use  $\tilde{r}$  to induce any desired level of investment without influencing the efficiency of the original contract.

This cost of resources,  $\tilde{r}$ , resembles an internal rate of return in the sense that it measures how the manager will evaluate the use of resources when making his investment decisions. The key observation here is simply that because the manager naturally cares only about his compensation, there is no clear link between  $r$ , the true cost of resources, and  $\tilde{r}$ , how the manager will value the resources. In the case of additive shocks, which provides sufficient structure to the model to allow us to derive additional results, we obtain the following proposition:

**Proposition 5** *In the case of additive shocks, the manager will exhibit empire-building preferences, in the sense that he will want to invest more than the first-best level of resources if given access to resources at cost  $r$ , if and only if his desired level of investment is increasing in the quality of information:  $I^M > I^{FB}$  if and only if  $\frac{\partial I^M}{\partial q} > 0$ .*

**Proof.** See Appendix A.4 ■

The intuition behind this result is as follows. From above, we know that the optimal contract will induce the manager to behave as if he is right more frequently than he truly is. But this bias will then naturally influence also the value that he places on having access to additional resources. If his desire for capital is decreasing in the quality of his information, then he will naturally also desire less resources at any given cost, while if his desire for capital is increasing in the quality of information, then he will desire more capital at any given cost. In the case of additive shocks, unbiased decision-making will further lead the manager to desire exactly the desired level of capital if given access at cost  $r$ , which establishes the direction of the distortion.<sup>7</sup>

As a final observation, it is worth noting that the proposition relates the investment behavior of the manager only to the first-best level of investment, absent any agency problems. Above, we have seen how the principal may want to distort the level of resources away from the first-best level to induce better information acquisition incentives, in which case  $\tilde{r}$  needs to be further adjusted to account for these effects.

## 6 An example

To illustrate the results discussed above, we will consider a numerical solution to the problem in a particular setting. We will consider a model where the returns to each task are exponentially distributed with shape parameter  $\lambda_i(x_j)$ , where  $\lambda_H(x) = \frac{1}{K\theta(1-e^{-\alpha x})}$  while  $\lambda_L(x) = \frac{1}{K(1-e^{-\alpha x})}$ . Here,  $x$  is the amount invested,  $\alpha$  is the marginal return to investment and  $\theta > 1$  measures how much better the better task is, thus measuring the value of information. The cost of information is given by  $C(q) = \mu((2q-1)^\gamma + \ln(1-(2q-1)^\gamma))$ , so that we have a well-behaved cost function with the marginal cost of information zero at  $q = \frac{1}{2}$  and  $\lim_{q \rightarrow 1} C'(q) = \infty$ . The parameter  $\gamma \geq 1$  controls the convexity of the cost function. The reason why  $\lim_{q \rightarrow 1/2} C'(q) = 0$  alone is not enough for positive levels of information acquisition is that the returns to information are also convex: the more precise the information, the more the investment responds to that information and thus the higher the marginal value. Alternatively, very little information is practically worthless because the equilibrium resource allocation will barely respond to that information.

This simple parameterization is attractive because it allows us to derive in closed-form the optimal investment decisions and confirm that the optimal total level of investment is

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<sup>7</sup>While the direction of the effect is clear as a function of  $\frac{\partial I^M}{\partial q}$ , in other cases unbiased allocation incentives may not lead to unbiased level incentives because generally  $\frac{\partial F_H(\pi|x_h, x_l)}{\partial x_i}$  need not be proportional to  $\frac{\partial E_H(\pi|x_h, x_l)}{\partial x_i}$ . In these cases, there needs to be a baseline distortion to  $\tilde{r}$  even if the contract induces unbiased allocation decisions.

indeed decreasing in the value of information. A quick calculation reveals that the optimal investment levels are

$$x_h = \frac{1}{\alpha} \ln \left( \frac{\alpha K(q\theta + (1-q))}{r} \right) \quad \text{and} \quad x_l = \frac{1}{\alpha} \ln \left( \frac{\alpha K(q + (1-q)\theta)}{r} \right),$$

so that the impact of information on total investment is  $\frac{\partial(x_h+x_l)}{\partial q} = \frac{r(\theta-1)^2(1-2q)}{\alpha(q\theta+(1-q))(q+(1-q)\theta)} < 0$  since  $q > \frac{1}{2}$ .<sup>8</sup> In this case, both the decision distortion and the likelihood ratio are convex in  $\pi$ , suggesting that a single bonus is optimal in equilibrium. As a result, in solving the model we will focus on the location of a single bonus.

We will provide three illustrations of the results. First, to highlight the competing forces in the determination of the level of resources, we will let the threshold  $\pi$  to be exogenous and consider how the rest of the optimal contract depends on that threshold, given the induced distortions. Second, we will consider the endogenous determination of the threshold as a function of the cost of information, which determines the tension between acquiring information and then using that information appropriately in the resource allocation decision. Finally, we will illustrate how to implement the desired resource level by delegating the resource level decision to the manager but tying his compensation appropriately to the level of resources used.

## 6.1 Investment distortions caused by the bonus threshold

The first illustration is regarding the impact of the threshold used on the equilibrium distortions. We solve the model above, with the exception that we take the performance threshold,  $\bar{\pi}$ , to be exogenous and consider how that influences the optimal resource allocation to the manager, and contrast that with (i) the first-best level of resources and (ii) how the firm would prefer to invest ex post, conditional on the quality of information acquired by the manager. We are thus considering how, after the fact, the organization would like to respond to the information generated, both from ex ante and ex post perspectives. However, it is worth noting that this is not an equilibrium comparison, since if we truly changed the resource allocation decision, that would in turn influence managerial incentives to acquire information and thus the quality of information on the basis of which the resource allocation decision is made. We are assuming that manager is fundamentally in control of that allocation and use these benchmark alternatives simply to highlight the biases in the manager's behavior and the role of the resource allocation in managing these biases.

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<sup>8</sup>While the focus here is on the role of restricted capital allocation to motivate effort, as discussed earlier, that result depends on the particular structure of the payoff returns. A threshold case is provided by returns  $\theta \ln(1 + \alpha x)$  and  $\ln(1 + \alpha x)$ , in which case the total investment level is independent of the quality of information.

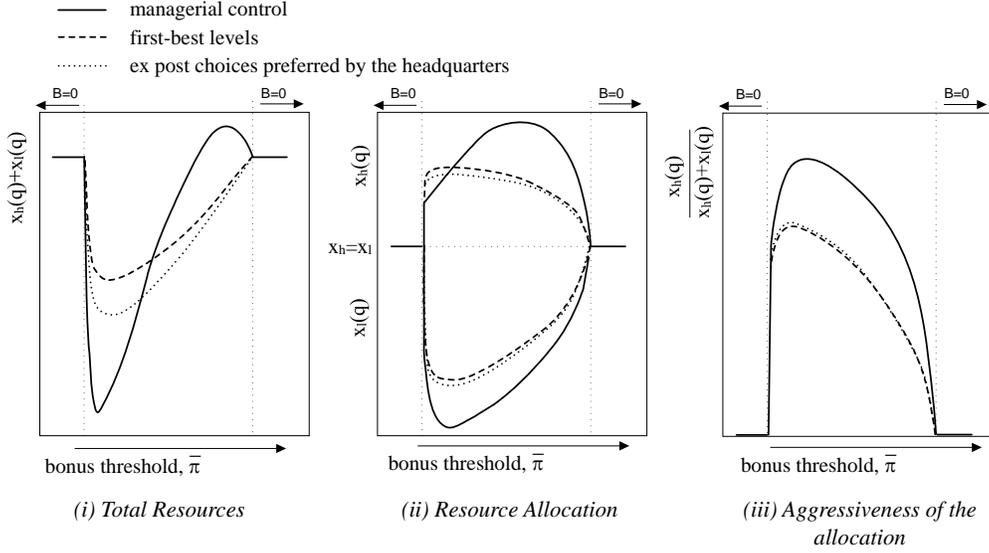


Figure 2: An illustration of the optimal distortion in the resource allocation given the performance threshold

This solution is illustrated in figure 2. Panel (i) plots the total resource levels as a function of the performance threshold. First, recall that the optimal total level of investment was decreasing in the quality of information. As a result, the better the information, the lower the first-best level of investment. Second, whenever the threshold is too high or too low, the optimal bonus is zero and thus all the outcomes are the same, with an equal allocation to both tasks. The reason is that when the threshold is too low, motivating information acquisition is too inefficient and, as a result, the optimal bonus is zero. Conversely, when the threshold is too high, then the distortion in the managerial decision-making caused by that threshold is so large that the optimal bonus is again zero. Third, as suggested above, the ex post desired level of investment by the headquarters is below that of the first-best level because they have to share the returns with the manager.

Finally, and what is the key feature of the first panel, the level of resources allocated to the manager can be either below or above both the first-best level and the ex post level that would be invested by the headquarters. The reason for this result follows from the interaction between the compensation contract and the level of resources. When the bonus threshold is low, then the resource allocation induced by the compensation contract is not too aggressive, and the level of resources is further restricted to utilize its motivational benefit. But as the bonus threshold increases, the manager becomes increasingly aggressive in his resource allocation decision, which the firm then begins to counter by actually increasing the level of resources allocated to the manager even above the first-best level.

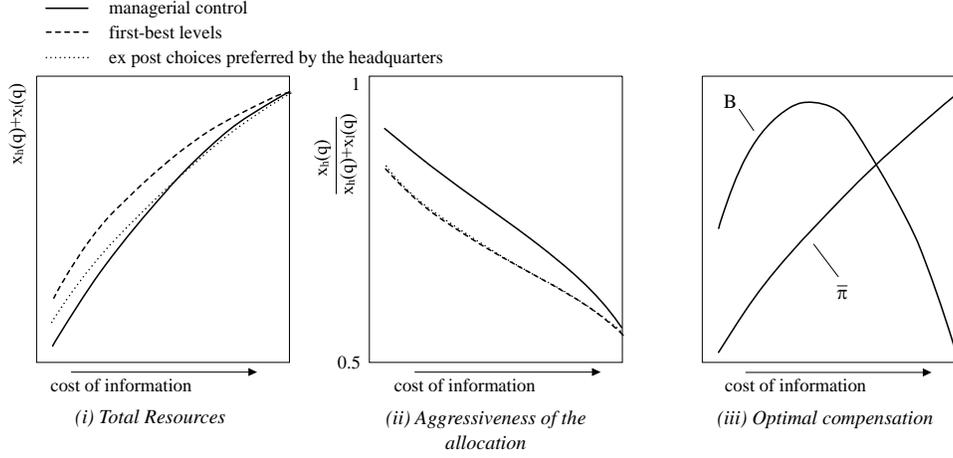


Figure 3: Illustration of the equilibrium resource allocation and the optimal compensation contract as a function of the cost of information

The other two panels then illustrate the bias itself, with panel (ii) giving the decomposition of how the resources of panel (i) are actually allocated between the two tasks. We can see that due to the resource constraint, for low enough bonus thresholds, the manager under-invests in both tasks, while the common theme is the ex ante more attractive task receiving too many and the less attractive task too few resources. Panel (iii) gives an alternative perspective of the same decomposition by considering the ratio of  $x_h(q)$  on the total level of resources. Again, the induced over-confidence result is immediate, and as we increase the threshold, it becomes increasingly worse in relative terms, while being attenuated by the two forces of (i) increased level of total resources allocated to induce less aggressive investment behavior and (ii) smaller optimal bonus to induce less information acquisition and thus less aggressive investment behavior.

## 6.2 Equilibrium compensation structure and investment levels

The above subsection maintained that the performance threshold  $\bar{\pi}$  was exogenously given, to highlight the basic tensions involved in the determination of the optimal resource allocation. Having discussed these forces, we can now endogenize  $\bar{\pi}$  and consider the full equilibrium. An illustration of the overall solution is given in figure 3, which plots the equilibrium total resource allocation and the aggressiveness of investment and benchmarks them against the first-best and headquarters' ex post solutions, as above, together with the optimal compensation contract  $(\bar{\pi}, B)$ , as a function of the cost of information.

Recall from the main analysis that the basic tension in the present model is between inducing the manager to acquire information and then inducing him to use that information appropriately when choosing how to allocate the available resources between the two tasks. Then, understanding the role of the cost of information is straightforward. When information is cheap, the firm can induce a lot of information acquisition even with limited incentives. As a result, both  $B$  and  $\bar{\pi}$  will be low (panel (iii)). Further, the firm provides additional incentives for information acquisition by restricting the level of resources available to the manager below both the first-best level and the level that the headquarters would invest if having access to information of quality  $q$ . The fact that information acquisition still needs to be motivated leads, however, to the manager being overly aggressive or over-confident from the perspective of the organization in equilibrium (panel (ii)).

As information becomes costlier, motivating its acquisition efficiently becomes more of a concern to the firm. As a result, both  $B$  and  $\bar{\pi}$  are increase, although, in equilibrium, the quality of information naturally decreases. As the quality of information decreases, the overall level of resources allocated to the manager increases. However, it continues to remain below the first-best level and the relative distortion  $\left(\frac{x_h^M(q)/I^M}{x_h^{FB}(q)/I^{FB}}\right)$  in the allocation decision actually increases (panel (ii)) to economize on the expected monetary compensation. Eventually, the distortion in the manager's allocation decision caused by increasing  $\bar{\pi}$  becomes so severe that the headquarters start to actually reduce the optimal bonus to limit the quality of information and thus the aggressiveness of the resource allocation. Then, as the quality of information starts to approach  $\frac{1}{2}$ , the relative distortion starts to vanish and the resources allocated to the manager converges to the first-best level. Finally, note that while the level of resources allocated to the manager remains below the first-best level as a tool for motivating information acquisition, it does exceed the level of resources that would be invested by the firm ex post. These two levels are not, however, directly comparable because they solve different problems.

### 6.3 Implied internal price of resources

The discussion so far has illustrated the over-confidence induced by the optimal compensation contract, as highlighted by the excessively aggressive resource allocation decision by the manager from the perspective of the firm. We can now consider delegating the choice of resource level to the manager. From above, we know that we can do this by tying the manager's compensation to the amount of resources requested, with the threshold given by  $\tilde{\pi} = \underline{\pi} + \tilde{r}I$ , where  $\tilde{r}$  is then the imputed price charged to the manager. This modification allows us to write the first-order condition for the allocations as

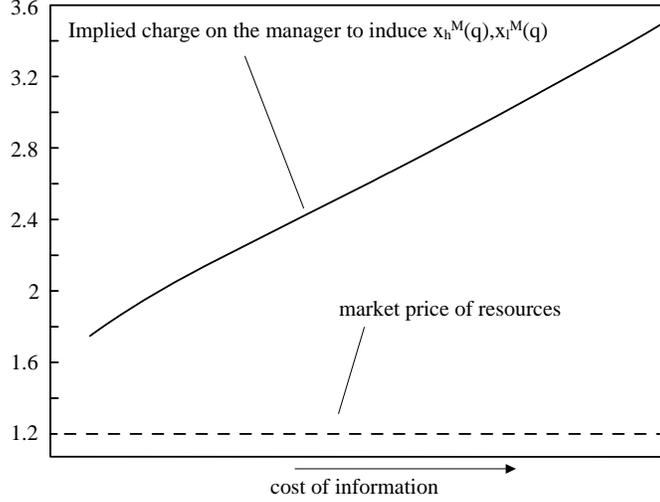


Figure 4: The implied price of resources charged to the manager to induce the desired resource allocation  $x_h^M(q), x_l^M(q)$ .

$$x_i : - \frac{\left[ \left( q \frac{\partial F_H(\tilde{\pi}|x_h, x_l)}{\partial x_i} + (1-q) \frac{\partial F_L(\tilde{\pi}|x_h, x_l)}{\partial x_i} \right) \right]}{[q f_H(\tilde{\pi}|x_h, x_l) + (1-q) f_L(\tilde{\pi}|x_h, x_l)]} = \tilde{r}.$$

Since from above we already know the target  $(x_h, x_l)$  and optimal  $\tilde{\pi}$ , we can simply plug in the values to solve for the imputed  $\tilde{r}$  that will induce these choices, which we plot in figure 4. This solution naturally blends two effects, where the first is the manager's bias in valuing the resources as induced by the compensation contract, and second, the distortion in the desired level of resources itself. In this particular example, we thus have that the manager exhibits empire-building preferences which need to be curtailed by an imputed charge that is significantly above the true cost of capital and increasing in the aggressiveness of the resource allocation. As discussed earlier, this result is not general, but simply illustrates how such preferences *can* arise through the structure of managerial compensation alone.

## 7 Extensions

There are three natural extensions to the framework, where the complete analysis are left for future research but the implications of which we will discuss here based on simple intuition. First, we have assumed that the manager is fundamentally in charge of the resource allocation  $(x_h, x_l)$ . Alternatively, the resource allocation decision itself may be a strategic variable that may be allocated either to the headquarters or the manager. Second, we have

assumed that only the total performance of the manager is measurable, as opposed to measuring the value generated separately by the two resource allocations. Third, by assuming that the returns to the two tasks are perfectly inversely correlated, we have eliminated any variation in the optimal *level* of resources. In reality, there is also clearly variability in the overall productivity of the available tasks, which the manager may also acquire information about.

## 7.1 Resource allocation decision as a strategic variable

The analysis assumed that the resource allocation decision is inalienable from the manager responsible for the division. It may, however, be possible that the decision can be used as a strategic design variable, where it is either retained by the headquarters or delegated to the division manager. If the information acquisition problem discussed in this paper would be the only organizational challenge, then the headquarters would generally retain the decision right. The reason is that by retaining the decision right, the headquarters can customize the incentive contract to only motivate information acquisition and then simply respond to the information generated in a profit-maximizing way.

If, however, there are other constraints on the design of incentives, then strategic delegation of the resource allocation decision may become optimal. The reason is that, as discussed above, any over-responsiveness to the information acquired by the manager through the existing compensation contract implies under-responsiveness by the headquarters. As a result, under the *same* compensation contract, the manager will be more motivated to acquire information under delegation because he will, in equilibrium, respond more to that information, thus increasing its value. The headquarters is then facing a tradeoff familiar from Aghion and Tirole (1997): by delegating authority to the manager, the headquarters suffers a loss of control in the sense that the manager will choose resource allocations that are too aggressive from the perspective of the headquarters, but at the same time, and indeed, because of it, the manager will be motivated to acquire more precise information on which to base his investment decisions. When the motivational advantage is sufficiently large relative to the biased decisions, delegation will be optimal.

## 7.2 Separate performance measures

The second question is whether the manager could be induced to make better choices if the performance in the two tasks would be separable. In this case, the simple logic is that clearly the optimal use of two performance measures cannot be worse than the optimal use

of a single performance measure, but the baseline tension between efficient effort incentives and efficient resource allocation decisions persists, so that the first-best still cannot be attained and the optimal contract will exhibit over-responsiveness from the perspective of the headquarters. The simple reason is that as long as the manager is in charge of both tasks, then the cost-efficient way of inducing information acquisition involves inducing excessive responsiveness to the information generated. The only thing afforded by having two separate performance measures is that the excessive responsiveness may be induced in a more cost-efficient way than with a single performance measure.

### 7.3 Variable overall productivity

The final extension would be to eliminate the simple binomial and perfectly negatively correlated productivity structure and instead allow for the productivity parameters  $\theta_i, \theta_j$  to be drawn from some joint distribution  $F(\boldsymbol{\theta})$ , and where the manager then acquires some information  $\Omega$  regarding the realization of those productivities. The key added level of realism provided by such a variant is that now there will be uncertainty over both (i) how aggressive the investment allocation should be, as determined by  $E(|\theta_i - \theta_j| \Omega)$ , and what will the overall need for resources be, as determined by  $E(\theta_i | \Omega)$  and  $E(\theta_j | \Omega)$ .

From above, it is clear that it will continue to be the case that as long as the manager is protected by limited liability, it will remain optimal to induce some over-responsiveness because the value of information will continue to be increasing in the aggressiveness of allocation decision and thus allow us to economize on the monetary compensation costs. But what is unclear is whether the variability in the actual *level* of desired investment can also be utilized through the combination of the compensation contract and the allocation schedule  $I(\theta_i, \theta_j)$  in a similar fashion, and whether the optimal compensation contract will now induce more systematic empire-building preferences much like it induced systematic over-responsiveness over the allocation decision in the present analysis (and its corresponding implications for the schedule  $r(I)$  which will achieve the desired investment levels under delegated resource level choices). These questions are left for future research.

## 8 Conclusion

We analyze a simple model of resource allocation where a manager first needs to acquire information regarding the relative productivity of different investment alternatives and then allocate the resources available to him among the alternatives. The firm has two design parameters at its disposal: the structure of the compensation contract offered to the manager

and the level of resources allocated to the manager. The main results from the analysis are two-fold. First, with respect to the level of resources, we identify an important motivational role whereby restricting the manager's access to resources below the first-best level could either sharpen or lessen the manager's incentive to acquire information. If more precise information decreases the manager's desired level of investment, then by restricting the manager's access to resources below the first-best level, the firm can induce the manager to acquire more information. Conversely, if more precise information increases the manager's desired level of investment, then by increasing the manager's access to resources above the first-best level, the firm can induce the manager to acquire more information.

Second, with respect to the optimal structure of compensation, we show how the optimal compensation contract induces the manager to be overconfident, in the sense that he allocates resources in a fashion that is more aggressive than what would maximize the expected cash flow. The intuition for this result is that by inducing more aggressive investment behavior by the manager, the firm could increase the value of information to him and thus economize on the expected monetary cost inducing any given level of information acquisition.

Third, we make the simple observation that we can delegate the choice of how much to invest overall simply by tying the compensation of the manager appropriately to the level of resources invested. However, if we linked the compensation only to the true cost of resources, the investment choices of the manager will generally be biased. The reason for this result is simply that because the manager invests to maximize his expected compensation, and the optimal compensation contract biases the manager away from maximizing expected value, the value that the manager places on the resources invested will generally be different from their true value. In particular, the manager may exhibit empire-building preferences, in the sense that he would prefer to invest more resources than the first-best level if given access to resources at their true cost. To counter this, the imputed charge on capital that affects the compensation of the manager must then be above the true cost of resources.

Of these two biases, it is the empire-building preferences that have received the most attention in the literature, with most of the capital budgeting literature exogenously assuming such preferences and then proceeding with the analysis to understand how the allocation should be distorted to induce truth-telling. Our analysis can be thus viewed as a model that is able to generate such preferences endogenously, providing some justification for such an assumption, but at the same time the analysis illustrates a simple solution to the problem: given the preference for excess capital, firms should simply link managerial compensation to the level of investment in a fashion that eliminates the effects of the bias. That way, truth-telling can be achieved for free and no agency problem would remain in the overall level of investment. The analysis thus suggests that it is dangerous to analyse compensation

and capital budgeting processes in isolation, because one of the main drivers how managers will behave and evaluate different investment alternatives is how they are compensated in equilibrium. Once one allows managerial compensation to be endogenous, fixing the conflict in the desired level of capital is simple. The more challenging problem identified here and which has received less attention in the literature is how to fix the way managers invest the resources allocated to them, with the present setting identifying induced overconfidence as an unavoidable side consequence of motivating information acquisition.

As a final observation, our analysis shows that the internal rate of return imposed on managers may need to be above the true cost of resources because of induced empire-building preferences. Our analysis thus provides a simple justification for the empirical regularity that many firms use internal rates of return significantly above the market rate, but at the same time highlights that having a higher IRR does not necessarily mean suboptimally low levels of investment if part of the purpose of a higher IRR is to counter excessive desire for capital as illustrated by our example.

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## A Proofs and derivations

### A.1 Proof of proposition 2

Recall that the design problem is given by

$$\begin{aligned} \max_{x_h, x_l, w(\pi)} & \int_0^{\infty} (\pi - w(\pi)) (qf_H(\pi|x_h, x_l) + qf_L(\pi|x_h, x_l)) - r(x_h + x_l) \\ \text{s.t.} \quad & q \in \arg \max \int_0^{\infty} w(\pi) (qf_H(\pi|x_h, x_l) + qf_L(\pi|x_h, x_l)) - C(q) \\ & w(\pi), w'(\pi) \geq 0 \end{aligned}$$

Let us begin with the optimal  $w(\pi)$ , which we can approximate through a sequence of bonuses paid for exceeding performance thresholds  $\pi_i$ , so that

$$E(w(\pi)) = \sum_i B_i [q(1 - F_H(\pi_i)) + (1 - q)(1 - F_L(\pi_i))].$$

The manager maximizes  $E(w(\pi)) - C(q)$ , which then gives the first-order condition

$$\sum B_i [F_L(\pi_i) - F_H(\pi_i)] = C'(q),$$

while the cost of inducing that effort to the firm is given by

$$E(w(\pi)) = \sum qB_i [(F_L(\pi_i) - F_H(\pi_i))] + \sum B_i (1 - F_L(\pi_i)).$$

Now, we can minimize this cost of effort for any given effort level, as determined by holding  $\sum B_i [F_L(\pi_i) - F_H(\pi_i)]$  constant. Thus, we solve  $\min \sum B_i (1 - F_L(\pi_i))$

$$\text{s.t.} \quad \sum B_i [F_L(\pi_i) - F_H(\pi_i)] = K.$$

Now, consider changing  $\pi_i$  and  $B_i$  so that the effort incentives are unchanged. This implies that  $\frac{\partial B_i}{\partial \pi_i} = -B_i \frac{[f_L(\pi_i) - f_H(\pi_i)]}{[F_L(\pi_i) - F_H(\pi_i)]}$ , while the change in cost is given by  $\frac{\partial B_i}{\partial \pi_i} (1 - F_L(\pi_i)) - B_i f_L(\pi_i)$ . The location of the bonus is thus optimal if

$$\frac{\partial B_i}{\partial \pi_i} (1 - F_L(\pi_i)) = B_i f_L(\pi_i),$$

and substituting the condition for constant effort we obtain

$$- [f_L(\pi_i) - f_H(\pi_i)] (1 - F_L(\pi_i)) = [F_L(\pi_i) - F_H(\pi_i)] f_L(\pi_i),$$

which then simplifies to  $\frac{f_H(\pi_i)}{f_L(\pi_i)} = \frac{(1-F_H(\pi_i))}{(1-F_L(\pi_i))}$ . And since this choice doesn't depend on the rest of the compensation structure, there will thus be only one bonus paid after performance exceeding this level. The optimal level of the bonus is then simply

$$\begin{aligned} \frac{\partial}{\partial B} : [E_H(\pi|x_h, x_l) - E_L(\pi|x_h, x_l)] \frac{\partial q}{\partial B} \\ - B [(F_L(\bar{\pi}) - F_H(\bar{\pi}))] \frac{\partial q}{\partial B} - [q(F_L(\bar{\pi}) - F_H(\bar{\pi})) + (1 - F_L(\bar{\pi}))] = 0, \end{aligned}$$

while from the manager's FOC we know that  $\frac{\partial q}{\partial B} = \frac{(F_L(\bar{\pi}) - F_H(\bar{\pi}))}{C''(q)}$ .

Next, considering the optimal choice of  $x_h$  and  $x_l$ , we can write the first-order conditions as

$$\frac{\partial}{\partial x_i} : q \frac{\partial E_H(\pi|x_i)}{\partial x_i} + (1 - q) \frac{\partial E_L(\pi|x_i)}{\partial x_i} - r - \frac{\partial E(w(\pi))}{\partial x_i} + [E_H(\pi|x_h, x_l) - E_L(\pi|x_h, x_l)] \frac{\partial q}{\partial x_i} = 0,$$

which is equivalent to the case under the first-best solution, with the exception that the investment level will now influence also both the expected level of compensation,  $E(w(\pi))$ , and the quality of information,  $q$ . From the manager's first-order condition we obtain

$$B [F_L(\bar{\pi}) - F_H(\bar{\pi})] = C'(q),$$

which implies that  $\frac{\partial q}{\partial x_i} = \frac{B \left( \frac{\partial F_L(\bar{\pi})}{\partial x_i} - \frac{\partial F_H(\bar{\pi})}{\partial x_i} \right)}{C''(q)}$ .

Further,  $\frac{\partial E(w(\pi))}{\partial x_i} = B \left[ q \left( \frac{\partial F_L(\bar{\pi})}{\partial x_i} - \frac{\partial F_H(\bar{\pi})}{\partial x_i} \right) - \frac{\partial F_L(\bar{\pi})}{\partial x_i} \right] + B (F_L(\bar{\pi}) - F_H(\bar{\pi})) \frac{\partial q}{\partial x_i}$ , so that we can write the above as

$$\begin{aligned} \frac{\partial}{\partial x_i} : q \frac{\partial E_H(\pi|x_i)}{\partial x_i} + (1 - q) \frac{\partial E_L(\pi|x_i)}{\partial x_i} = r + B \left[ q \left( \frac{\partial F_L(\bar{\pi})}{\partial x_i} - \frac{\partial F_H(\bar{\pi})}{\partial x_i} \right) - \frac{\partial F_L(\bar{\pi})}{\partial x_i} \right] \\ + [B (F_L(\bar{\pi}) - F_H(\bar{\pi})) - [E_H(\pi|x_h, x_l) - E_L(\pi|x_h, x_l)]] \frac{\partial q}{\partial x_i}. \end{aligned}$$

But then since  $\frac{\partial q}{\partial B} = \frac{(F_L(\bar{\pi}) - F_H(\bar{\pi}))}{C''(q)}$ ,  $\frac{\partial q}{\partial x_i} = \frac{B \left( \frac{\partial F_L(\bar{\pi})}{\partial x_i} - \frac{\partial F_H(\bar{\pi})}{\partial x_i} \right)}{(F_L(\bar{\pi}) - F_H(\bar{\pi}))} \frac{\partial q}{\partial B}$  and further rearranging the first-order condition for  $B$  gives us

$$\frac{\partial q}{\partial B} = \frac{[q(F_L(\bar{\pi}) - F_H(\bar{\pi})) + (1 - F_L(\bar{\pi}))]}{[[E_H(\pi|x_h, x_l) - E_L(\pi|x_h, x_l)] - B[(F_L(\bar{\pi}) - F_H(\bar{\pi}))]]},$$

we obtain

$$\frac{\partial E_H(\pi|x_i)}{\partial x_i} + (1 - q) \frac{\partial E_L(\pi|x_i)}{\partial x_i} =$$

$$r + B \left[ q \left( \frac{\partial F_L(\bar{\pi})}{\partial x_i} - \frac{\partial F_H(\bar{\pi})}{\partial x_i} \right) - \frac{\partial F_L(\bar{\pi})}{\partial x_i} \right] - \frac{B \left( \frac{\partial F_L(\bar{\pi})}{\partial x_i} - \frac{\partial F_H(\bar{\pi})}{\partial x_i} \right)}{(F_L(\bar{\pi}) - F_H(\bar{\pi}))} [q(F_L(\bar{\pi}) - F_H(\bar{\pi})) + (1 - F_L(\bar{\pi}))],$$

which we can then finally rearrange to

$$q \frac{\partial E_H(\pi|x_i)}{\partial x_i} + (1 - q) \frac{\partial E_L(\pi|x_i)}{\partial x_i} = r + B \frac{[(1 - F_L(\bar{\pi})) \frac{\partial F_H(\bar{\pi})}{\partial x_i} - (1 - F_H(\bar{\pi})) \frac{\partial F_L(\bar{\pi})}{\partial x_i}]}{(F_L(\bar{\pi}) - F_H(\bar{\pi}))}.$$

From where we can conclude then that  $x_h^* > x_{FB}^*$  and  $x_l^* < x_{FB}^*$  as long as  $\frac{d}{dx_h} \left( \frac{1 - F_H(\pi)}{1 - F_L(\pi)} \right) > 0$  and  $\frac{d}{dx_h} \left( \frac{1 - F_H(\pi)}{1 - F_L(\pi)} \right) < 0$  as assumed. Note that the distortion is increasing in  $B$ , which measures the importance of additional savings, with  $x_h^* \rightarrow x_h^{FB}$  and  $x_l^* \rightarrow x_l^{FB}$  as  $B \rightarrow 0$ . Thus, establishing the sign of  $\frac{\partial(x_h^* + x_l^*)}{\partial B}$  establishes the sign of the distortion in the total investment level. In the additive case, we have

$$\begin{aligned} \frac{\partial F_H(\pi)}{\partial x_H} &= -g_H(\pi) \frac{\partial \pi_1}{\partial x_h} & \frac{\partial F_L(\pi)}{\partial x_H} &= -g_L(\pi) \frac{\partial \pi_2}{\partial x_h} \\ \frac{\partial F_H(\pi)}{\partial x_L} &= -g_H(\pi) \frac{\partial \pi_1}{\partial x_l} & \frac{\partial F_L(\pi)}{\partial x_L} &= -g_L(\pi) \frac{\partial \pi_2}{\partial x_l}, \end{aligned}$$

So let us write the first-order conditions as

$$\begin{aligned} D_H : q \frac{\partial \pi_H}{\partial x_h} + (1 - q) \frac{\partial \pi_L}{\partial x_h} - r - B \frac{[(1 - G_H(\bar{\pi}))g_L(\bar{\pi}) \frac{\partial \pi_L}{\partial x_h} - (1 - G_L(\bar{\pi}))g_H(\bar{\pi}) \frac{\partial \pi_H}{\partial x_h}]}{(G_L(\bar{\pi}) - F_H(\bar{\pi}))} &= 0 \\ D_L : q \frac{\partial \pi_H}{\partial x_l} + (1 - q) \frac{\partial \pi_L}{\partial x_l} - r - B \frac{[(1 - G_H(\bar{\pi}))g_L(\bar{\pi}) \frac{\partial \pi_L}{\partial x_l} - (1 - G_L(\bar{\pi}))g_H(\bar{\pi}) \frac{\partial \pi_H}{\partial x_l}]}{(G_L(\bar{\pi}) - F_H(\bar{\pi}))} &= 0, \end{aligned}$$

from where we have that

$$\frac{\partial x_h}{\partial B} = \frac{\frac{[(1 - G_H(\bar{\pi}))g_L(\bar{\pi}) \frac{\partial \pi_L}{\partial x_h} - (1 - G_L(\bar{\pi}))g_H(\bar{\pi}) \frac{\partial \pi_H}{\partial x_h}]}{(G_L(\bar{\pi}) - F_H(\bar{\pi}))}}{\frac{\partial D_H}{\partial x_h}}, \quad \frac{\partial x_l}{\partial B} = \frac{\frac{[(1 - G_H(\bar{\pi}))g_L(\bar{\pi}) \frac{\partial \pi_L}{\partial x_l} - (1 - G_L(\bar{\pi}))g_H(\bar{\pi}) \frac{\partial \pi_H}{\partial x_l}]}{(G_L(\bar{\pi}) - F_H(\bar{\pi}))}}{\frac{\partial D_L}{\partial x_l}},$$

$$\text{while } \frac{\partial x_h}{\partial q} = -\frac{\left( \frac{\partial \pi_H}{\partial x_h} - \frac{\partial \pi_L}{\partial x_h} \right)}{\frac{\partial D_H}{\partial x_h}} \quad \text{and} \quad \frac{\partial x_l}{\partial q} = -\frac{\left( \frac{\partial \pi_H}{\partial x_l} - \frac{\partial \pi_L}{\partial x_l} \right)}{\frac{\partial D_L}{\partial x_l}},$$

so that  $\frac{\partial x_h}{\partial B} + \frac{\partial x_l}{\partial B}$

$$= -\frac{\frac{[(1 - G_H(\bar{\pi}))g_L(\bar{\pi}) \frac{\partial \pi_L}{\partial x_h} - (1 - G_L(\bar{\pi}))g_H(\bar{\pi}) \frac{\partial \pi_H}{\partial x_h}]}{(G_L(\bar{\pi}) - F_H(\bar{\pi}))}}{\left( \frac{\partial \pi_H}{\partial x_h} - \frac{\partial \pi_L}{\partial x_h} \right)} \frac{\partial x_h}{\partial q} - \frac{\frac{[(1 - G_H(\bar{\pi}))g_L(\bar{\pi}) \frac{\partial \pi_L}{\partial x_l} - (1 - G_L(\bar{\pi}))g_H(\bar{\pi}) \frac{\partial \pi_H}{\partial x_l}]}{(G_L(\bar{\pi}) - F_H(\bar{\pi}))}}{\left( \frac{\partial \pi_H}{\partial x_l} - \frac{\partial \pi_L}{\partial x_l} \right)} \frac{\partial x_l}{\partial q}.$$

Thus,  $\frac{\partial x_h}{\partial B} + \frac{\partial x_l}{\partial B} < 0$  iff

$$\frac{1}{(G_L(\bar{\pi}) - F_H(\bar{\pi}))} \left[ \begin{aligned} & \frac{[(1 - G_H(\bar{\pi}))g_L(\bar{\pi}) \frac{\partial \pi_L}{\partial x_h} - (1 - G_L(\bar{\pi}))g_H(\bar{\pi}) \frac{\partial \pi_H}{\partial x_h}]}{\left( \frac{\partial \pi_H}{\partial x_h} - \frac{\partial \pi_L}{\partial x_h} \right)} \frac{\partial x_h}{\partial q} \\ & + \frac{[(1 - G_H(\bar{\pi}))g_L(\bar{\pi}) \frac{\partial \pi_L}{\partial x_l} - (1 - G_L(\bar{\pi}))g_H(\bar{\pi}) \frac{\partial \pi_H}{\partial x_l}]}{\left( \frac{\partial \pi_H}{\partial x_l} - \frac{\partial \pi_L}{\partial x_l} \right)} \frac{\partial x_l}{\partial q} \end{aligned} \right] > 0,$$

but recall that at the optimal bonus,  $\frac{(1-G_H(\bar{\pi}))g_L(\bar{\pi})}{(1-G_L(\bar{\pi}))g_H(\bar{\pi})} = 1$ , so we can simplify the above further to

$$\frac{(1-G_L(\bar{\pi}))g_H(\bar{\pi})}{(G_L(\bar{\pi})-F_H(\bar{\pi}))} \left[ \frac{\partial x_h}{\partial q} + \frac{\partial x_l}{\partial q} \right] < 0$$

## A.2 Proof of proposition 3

Recall the agent's maximization problem

$$\begin{aligned} \max_{x_h, x_l, q} \int_0^{\infty} w(\pi) (q f_H(\pi|x_h, x_l) + (1-q) f_L(\pi|x_h, x_l)) d\pi - C(q) \\ \text{s.t.} \quad x_h + x_l \leq I. \end{aligned}$$

The first-order conditions with respect to effort and investment choices are

$$\begin{aligned} q : \int_0^{\infty} w(\pi) (f_H(\pi|x_h, x_l) - f_L(\pi|x_h, x_l)) d\pi = C'(q) \\ x_i : \int_0^{\infty} w(\pi) \left( q \frac{\partial f_H(\pi|x_h, x_l)}{\partial x_i} + (1-q) \frac{\partial f_L(\pi|x_h, x_l)}{\partial x_i} \right) d\pi = \lambda. \end{aligned}$$

Then, we know that

$$\frac{\partial q}{\partial \lambda} > 0 \text{ iff } \int_0^{\infty} w(\pi) \left[ \left( \frac{\partial f_H(\pi|x_h, x_l)}{\partial x_H} - \frac{f_L(\pi|x_h, x_l)}{\partial x_H} \right) \frac{\partial x_h}{\partial \lambda} + \left( \frac{\partial f_H(\pi|x_h, x_l)}{\partial x_L} - \frac{f_L(\pi|x_h, x_l)}{\partial x_L} \right) \frac{\partial x_l}{\partial \lambda} \right] d\pi > 0$$

But from the agent's FOC for the investment choices we have that

$$\begin{aligned} \frac{\partial x_h}{\partial \lambda} &= \frac{1}{\frac{d}{dx_H} \int_0^{\infty} w(\pi) \left( q \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_H} + (1-q) \frac{\partial f_L(\pi|x_H, x_L)}{\partial x_H} \right)} \\ \frac{\partial x_l}{\partial \lambda} &= \frac{1}{\frac{d}{dx_L} \int_0^{\infty} w(\pi) \left( q \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_L} + (1-q) \frac{\partial f_L(\pi|x_H, x_L)}{\partial x_L} \right)}. \end{aligned}$$

Also, we can infer the change in the desired investment level by holding the shadow value of investment constant and obtaining

$$\frac{\partial x_H}{\partial q} = - \frac{\int_0^{\infty} w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_H} - \frac{\partial f_L(\pi|x_H, x_L)}{\partial x_H} \right) d\pi - \frac{\partial \lambda}{\partial q}}{\frac{d}{dx_H} \int_0^{\infty} w(\pi) \left( q \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_H} + (1-q) \frac{\partial f_L(\pi|x_H, x_L)}{\partial x_H} \right)}$$

$$\frac{\partial x_L}{\partial q} = - \frac{\int_0^\infty w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_L} - \frac{\partial f_L(\pi|x_H, x_L)}{\partial x_L} \right) d\pi - \frac{\partial \lambda}{\partial q}}{\frac{d}{dx_L} \int_0^\infty w(\pi) \left( q \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_L} + (1-q) \frac{\partial f_L(\pi|x_H, x_L)}{\partial x_L} \right) d\pi},$$

so that we have the sign of  $\frac{\partial q}{\partial \lambda}$  as

$$\left[ \frac{\int_0^\infty w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_H} - \frac{f_L(\pi|x_H, x_L)}{\partial x_H} \right) d\pi}{\int_0^\infty w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_H} - \frac{\partial f_L(\pi|x_H, x_L)}{\partial x_H} \right) d\pi - \frac{\partial \lambda}{\partial q}} \frac{\partial x_H}{\partial q} + \frac{\int_0^\infty w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_L} - \frac{f_L(\pi|x_H, x_L)}{\partial x_L} \right) d\pi}{\int_0^\infty w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_L} - \frac{\partial f_L(\pi|x_H, x_L)}{\partial x_L} \right) d\pi - \frac{\partial \lambda}{\partial q}} \frac{\partial x_L}{\partial q} \right].$$

Now, with fixed allocation,  $\frac{\partial x_L}{\partial q} + \frac{\partial x_H}{\partial q} = 0$ , so the expression is negative if

$$\frac{\int_0^\infty w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_H} - \frac{f_L(\pi|x_H, x_L)}{\partial x_H} \right) d\pi}{\int_0^\infty w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_H} - \frac{\partial f_L(\pi|x_H, x_L)}{\partial x_H} \right) d\pi - \frac{\partial \lambda}{\partial q}} < \frac{\int_0^\infty w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_L} - \frac{f_L(\pi|x_H, x_L)}{\partial x_L} \right) d\pi}{\int_0^\infty w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_L} - \frac{\partial f_L(\pi|x_H, x_L)}{\partial x_L} \right) d\pi - \frac{\partial \lambda}{\partial q}}$$

$$\frac{\frac{\partial \lambda}{\partial q}}{\int_0^\infty w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_H} - \frac{f_L(\pi|x_H, x_L)}{\partial x_H} \right) d\pi} < \frac{\frac{\partial \lambda}{\partial q}}{\int_0^\infty w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_L} - \frac{f_L(\pi|x_H, x_L)}{\partial x_L} \right) d\pi}.$$

And since we have that  $\int_0^\infty w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_L} - \frac{f_L(\pi|x_H, x_L)}{\partial x_L} \right) d\pi < 0$  and

$\int_0^\infty w(\pi) \left( \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_H} - \frac{f_L(\pi|x_H, x_L)}{\partial x_H} \right) d\pi > 0$ , the only way to satisfy this expression is if  $\frac{\partial \lambda}{\partial q} < 0$ , i.e. information relaxes the constraint.

### A.3 Proof of proposition 4

Let us first show that a contract consisting of at most two bonuses will be optimal. First, approximate any non-decreasing compensation contract by a sequence of bonuses and then minimize the cost of inducing given equilibrium effort and decisions. Given the sequence of bonuses, we can write the cost as

$$q \sum B_i (F_L(\pi_i|x_h, x_l) - F_H(\pi_i|x_h, x_l)) + \sum B_i (1 - F_L(\pi_i|x_h, x_l))$$

subject to

$$q : \sum B_i (F_L(\pi|x_h, x_l) - F_H(\pi|x_h, x_l)) = C'(q)$$

$$x_i : \sum B_i \left( q \int_{\pi_i}^\infty \frac{\partial f_H(\pi|x_H, x_L)}{\partial x_i} + (1-q) \int_{\pi_i}^\infty \frac{\partial f_L(\pi|x_H, x_L)}{\partial x_i} \right) = \sum B_i \left( \frac{\partial E \Pr(\pi > \pi_i)}{\partial x_i} \right) = \lambda$$

Since the goal is to leave the equilibrium decisions unchanged, we can write this condition as

$$\sum B_i \left( \frac{\partial \Pr(\pi > \pi_i)}{\partial x_h} \right) = \sum B_i \left( \frac{\partial \Pr(\pi > \pi_i)}{\partial x_l} \right).$$

Now, take three bonuses  $(B_i, B_j, B_k)$ , and consider increasing one of them while changing the other two in a fashion that leaves both the equilibrium decisions and the equilibrium effort level unchanged. If at least one such change increases costs, that implies that three or more bonuses is suboptimal because I can use two bonuses to achieve the same decisions and effort at a lower cost.

From the effort condition we get that the change needs to satisfy

$$B_i (F_L(\pi_i) - F_H(\pi_i)) + \frac{\partial B_j}{\partial B_i} (F_L(\pi_j) - F_H(\pi_j)) + \frac{\partial B_k}{\partial B_i} (F_L(\pi_k) - F_H(\pi_k)) = 0,$$

while from the decision-making constraint we have that the change needs to satisfy

$$\left( \frac{\partial \Pr(\pi > \pi_i)}{\partial x_h} - \frac{\partial \Pr(\pi > \pi_i)}{\partial x_l} \right) + \frac{\partial B_j}{\partial B_i} \left( \frac{\partial \Pr(\pi > \pi_j)}{\partial x_h} - \frac{\partial \Pr(\pi > \pi_j)}{\partial x_l} \right) + \frac{\partial B_k}{\partial B_i} \left( \frac{\partial \Pr(\pi > \pi_k)}{\partial x_h} - \frac{\partial \Pr(\pi > \pi_k)}{\partial x_l} \right) = 0.$$

To simplify notation going forward, let  $\varphi_i(x) = \left( \frac{\partial \Pr(\pi > \pi_i)}{\partial x_H} - \frac{\partial \Pr(\pi > \pi_i)}{\partial x_L} \right)$ , so that the above becomes simply  $\varphi_i(x) + \frac{\partial B_j}{\partial B_i} \varphi_j(x) + \frac{\partial B_k}{\partial B_i} \varphi_k(x) = 0$ . From the effort constraint we have then that

$$\begin{aligned} \frac{\partial B_k}{\partial B_i} &= - \frac{\left( (F_L(\pi_i) - F_H(\pi_i)) + \frac{\partial B_j}{\partial B_i} (F_L(\pi_j) - F_H(\pi_j)) \right)}{(F_L(\pi_k) - F_H(\pi_k))} \\ \frac{\partial B_j}{\partial B_i} &= - \frac{\left( (F_L(\pi_i) - F_H(\pi_i)) + \frac{\partial B_k}{\partial B_i} (F_L(\pi_k) - F_H(\pi_k)) \right)}{(F_L(\pi_j) - F_H(\pi_j))}, \end{aligned}$$

and then substituting in the equilibrium decision constraint and rearranging we get the required rates of change as

$$\begin{aligned} \frac{(F_L(\pi_j) - F_H(\pi_j))\varphi_i(x) - (F_L(\pi_i) - F_H(\pi_i))\varphi_j(x)}{(F_L(\pi_k) - F_H(\pi_k))\varphi_j(x) - (F_L(\pi_j) - F_H(\pi_j))\varphi_k(x)} &= \frac{(\Gamma_{ji} - \Gamma_{ij})}{(\Gamma_{kj} - \Gamma_{jk})} = \frac{\partial B_k}{\partial B_i} \\ \frac{(F_L(\pi_k) - F_H(\pi_k))\varphi_i(x) - (F_L(\pi_i) - F_H(\pi_i))\varphi_k(x)}{[(F_L(\pi_j) - F_H(\pi_j))\varphi_k(x) - \varphi_j(x)(F_L(\pi_k) - F_H(\pi_k))]} &= \frac{(\Gamma_{ki} - \Gamma_{ik})}{(\Gamma_{jk} - \Gamma_{kj})} = \frac{\partial B_j}{\partial B_i}. \end{aligned}$$

The results for the other derivatives (changing  $B_j$  or  $B_k$ ) follow symmetrically. Thus, we have three changes, with the impact on cost as

$$\begin{aligned} \frac{\partial C}{\partial B_i} &: (1 - F_L(\pi_i)) + \frac{(\Gamma_{ki} - \Gamma_{ik})}{(\Gamma_{jk} - \Gamma_{kj})} (1 - F_L(\pi_j)) + \frac{(\Gamma_{ji} - \Gamma_{ij})}{(\Gamma_{kj} - \Gamma_{jk})} (1 - F_L(\pi_k)) \\ \frac{\partial C}{\partial B_k} &: (1 - F_L(\pi_k)) + \frac{(\Gamma_{ik} - \Gamma_{ki})}{(\Gamma_{ji} - \Gamma_{ij})} (1 - F_L(\pi_j)) + \frac{(\Gamma_{jk} - \Gamma_{kj})}{(\Gamma_{ij} - \Gamma_{ji})} (1 - F_L(\pi_i)) \end{aligned}$$

$$\frac{\partial C}{\partial B_j} : (1 - F_L(\pi_j)) + \frac{(\Gamma_{kj} - \Gamma_{jk})}{(\Gamma_{ik} - \Gamma_{ki})} (1 - F_L(\pi_i)) + \frac{(\Gamma_{ij} - \Gamma_{ji})}{(\Gamma_{ki} - \Gamma_{ik})} (1 - F_L(\pi_k)).$$

We have thus three linear equations. Then, for costs to decrease in  $B_i$  we would need that

$$(1 - F_L(\pi_i)) \leq \frac{1}{(\Gamma_{jk} - \Gamma_{kj})} [(\Gamma_{ji} - \Gamma_{ij})(1 - F_L(\pi_k)) - (\Gamma_{ki} - \Gamma_{ik})(1 - F_L(\pi_j))],$$

while for the costs to decrease in  $B_k$  we would need that

$$(1 - F_L(\pi_k)) \leq \frac{1}{(\Gamma_{ji} - \Gamma_{ij})} [(\Gamma_{jk} - \Gamma_{kj})(1 - F_L(\pi_i)) - (\Gamma_{ik} - \Gamma_{ki})(1 - F_L(\pi_j))].$$

Now, suppose first that  $(\Gamma_{jk} - \Gamma_{kj}), (\Gamma_{ji} - \Gamma_{ij}) > 0$ . Then,

$$(1 - F_L(\pi_i)) \leq \frac{1}{(\Gamma_{jk} - \Gamma_{kj})} [(\Gamma_{ji} - \Gamma_{ij})(1 - F_L(\pi_k)) - (\Gamma_{ki} - \Gamma_{ik})(1 - F_L(\pi_j))]$$

implies that  $\frac{(\Gamma_{jk} - \Gamma_{kj})(1 - F_L(\pi_i)) - (\Gamma_{ik} - \Gamma_{ki})(1 - F_L(\pi_j))}{(\Gamma_{ji} - \Gamma_{ij})} \leq 1 - F_L(\pi_k)$ ,

which implies that if cost is decreasing in  $B_i$ , it will be increasing in  $B_k$  and thus we should set  $B_k = 0$ . A symmetric arguments shows the same result if  $(\Gamma_{jk} - \Gamma_{kj}), (\Gamma_{ji} - \Gamma_{ij}) < 0$ . For opposite signs, say  $(\Gamma_{jk} - \Gamma_{kj}) > 0$  but  $(\Gamma_{ji} - \Gamma_{ij}) < 0$  we have that the first two conditions for increasing the bonus to be optimal become

$$\begin{aligned} (1 - F_L(\pi_i)) + \frac{(\Gamma_{ki} - \Gamma_{ik})}{(\Gamma_{jk} - \Gamma_{kj})} (1 - F_L(\pi_j)) + \frac{(\Gamma_{ji} - \Gamma_{ij})}{(\Gamma_{kj} - \Gamma_{jk})} (1 - F_L(\pi_k)) &< 0 \\ (1 - F_L(\pi_k)) + \frac{(\Gamma_{ik} - \Gamma_{ki})}{(\Gamma_{ji} - \Gamma_{ij})} (1 - F_L(\pi_j)) + \frac{(\Gamma_{jk} - \Gamma_{kj})}{(\Gamma_{ij} - \Gamma_{ji})} (1 - F_L(\pi_i)) &< 0, \end{aligned}$$

which we can rewrite as

$$\begin{aligned} (1 - F_L(\pi_i)) &< \frac{(\Gamma_{ji} - \Gamma_{ij})}{(\Gamma_{jk} - \Gamma_{kj})} (1 - F_L(\pi_k)) - \frac{(\Gamma_{ki} - \Gamma_{ik})}{(\Gamma_{jk} - \Gamma_{kj})} (1 - F_L(\pi_j)) \\ \frac{[(\Gamma_{ji} - \Gamma_{ij})(1 - F_L(\pi_k)) - (\Gamma_{ki} - \Gamma_{ik})(1 - F_L(\pi_j))]}{(\Gamma_{jk} - \Gamma_{kj})} &> (1 - F_L(\pi_i)). \end{aligned}$$

So that both of the first two conditions are satisfied but then the third condition is

$$\begin{aligned} (1 - F_L(\pi_j)) + \frac{(\Gamma_{kj} - \Gamma_{jk})}{(\Gamma_{ik} - \Gamma_{ki})} (1 - F_L(\pi_i)) + \frac{(\Gamma_{ij} - \Gamma_{ji})}{(\Gamma_{ki} - \Gamma_{ik})} (1 - F_L(\pi_k)) &> 0 \\ \frac{(\Gamma_{jk} - \Gamma_{kj})}{(\Gamma_{ki} - \Gamma_{ik})} (1 - F_L(\pi_i)) &> \frac{(\Gamma_{ji} - \Gamma_{ij})}{(\Gamma_{ki} - \Gamma_{ik})} (1 - F_L(\pi_k)) - (1 - F_L(\pi_j)). \end{aligned}$$

But for the first two conditions to be satisfied it needs to be that  $(\Gamma_{ji} - \Gamma_{ij})(1 - F_L(\pi_k)) - (\Gamma_{ki} - \Gamma_{ik})(1 - F_L(\pi_j)) > 0$  since  $(\Gamma_{jk} - \Gamma_{kj}) > 0$  which implies that since  $(\Gamma_{ji} - \Gamma_{ij}) < 0$ ,  $(\Gamma_{ki} - \Gamma_{ik}) < 0$ , and so the condition becomes

$$(1 - F_L(\pi_i)) < \frac{(\Gamma_{ji} - \Gamma_{ij})(1 - F_L(\pi_k)) - (\Gamma_{ki} - \Gamma_{ik})(1 - F_L(\pi_j))}{(\Gamma_{jk} - \Gamma_{kj})}.$$

In other words, if increasing  $B_i$  and  $B_k$  is profitable, so is decreasing  $B_j$  and so optimal  $B_j = 0$ . Finally, suppose that I am indifferent for the bonus level of the first bonus, so that  $\frac{\partial C}{\partial B_i} = 0$ . But then I could simply set  $B_i = 0$  and achieve the same outcome only with two bonuses. As a result, the most complicated optimal compensation contract will consist at most of two bonuses.

Now, while we can show that we need at most two bonuses in the optimal compensation contract, even one bonus can sometimes be optimal. We have not been able to obtain a strict condition for this result to hold in the general case, but in the case of additive shocks, we can derive the exact condition for when this will be the case. The simple logic is as follows. Take a single bonus contract that will induce particular equilibrium effort and decisions. Then, take a two-bonus contract that implements exactly the same outcome and compare their expected costs. For a single-bonus contract, we have

$$\begin{aligned} \text{cost: } & B(q(G_L(\pi) - G_H(\pi)) + (1 - G_L(\pi))) \\ \text{effort: } & B((G_L(\pi) - G_H(\pi))) = C'(e) \\ \text{decisions: } & \frac{\partial E \Pr(\pi > \pi)}{\partial x_H} - \frac{\partial E \Pr(\pi > \pi)}{\partial x_L} = 0, \end{aligned}$$

while for a two-bonus contract, we have

$$\begin{aligned} \text{cost: } & q[B_1(G_L(\pi_1) - G_H(\pi_1)) + B_2(G_L(\pi_2) - G_H(\pi_2))] + B_1(1 - G_L(\pi_1)) + B_2(1 - G_L(\pi_2)) \\ \text{effort: } & B_1((G_L(\pi_1) - G_H(\pi_1))) + B_2((G_L(\pi_2) - G_H(\pi_2))) = C'(e) \\ \text{decisions: } & B_1\left(\frac{\partial E \Pr(\pi > \pi_1)}{\partial x_H} - \frac{\partial E \Pr(\pi > \pi_1)}{\partial x_L}\right) + B_2\left(\frac{\partial E \Pr(\pi > \pi_2)}{\partial x_H} - \frac{\partial E \Pr(\pi > \pi_2)}{\partial x_L}\right) = 0. \end{aligned}$$

Now, recall that that  $\frac{\partial E \Pr(\pi > \pi_j)}{\partial x_h} = qg_H(\pi_j) \frac{\partial \pi_H}{\partial x_h} + (1 - q)g_L(\pi_j) \frac{\partial \pi_L}{\partial x_h}$ , so we are achieving the same equilibrium investment levels if  $\frac{\partial \pi_i}{\partial x_j}$  are equal across the two bonus structures (since the marginal returns are unique). In the case of two bonuses, we have that, in equilibrium,

$$\begin{aligned} & q[B_1(g_H(\pi_1)) + B_2(g_H(\pi_2))]\left(\frac{\partial \pi_H}{\partial x_H} - \frac{\partial \pi_L}{\partial x_L}\right) \\ & - (1 - q)[B_1g_L(\pi_1) + B_2g_L(\pi_2)]\left(\frac{\partial \pi_H}{\partial x_L} - \frac{\partial \pi_L}{\partial x_H}\right) = 0, \end{aligned}$$

while in the case of a single bonus, we have

$$B\left(q\left(g_H(\pi)\left(\frac{\partial \pi_H}{\partial x_H} - \frac{\partial \pi_L}{\partial x_L}\right)\right) - (1 - q)g_L(\pi)\left(\frac{\partial \pi_H}{\partial x_L} - \frac{\partial \pi_L}{\partial x_H}\right)\right) = 0$$

$$\left(\frac{\partial \pi_H}{\partial x_H} - \frac{\partial \pi_L}{\partial x_L}\right) = \frac{(1-q)g_L(\pi)}{qg_H(\pi)} \left(\frac{\partial \pi_H}{\partial x_L} - \frac{\partial \pi_L}{\partial x_H}\right).$$

Substituting back in the two-bonus constraint, we obtain the first relationship between the two bonuses:

$$q [B_1 (g_H(\pi_1)) + B_2 (g_H(\pi_2))] \frac{(1-q)g_L(\pi)}{qg_H(\pi)} \left(\frac{\partial \pi_H}{\partial x_L} - \frac{\partial \pi_L}{\partial x_H}\right) = (1-q) [B_1 g_L(\pi_1) + B_2 g_L(\pi_2)] \left(\frac{\partial \pi_H}{\partial x_L} - \frac{\partial \pi_L}{\partial x_H}\right)$$

$$B_1 = B_2 \frac{[g_H(\pi)g_L(\pi_2) - g_H(\pi_2)g_L(\pi)]}{[g_H(\pi_1)g_L(\pi) - g_H(\pi)g_L(\pi_1)]}.$$

And finally from the effort constraint we have that

$$B((G_L(\pi) - G_H(\pi))) = B_1((G_L(\pi_1) - G_H(\pi_1))) + B_2((G_L(\pi_2) - G_H(\pi_2))),$$

which then allows us to solve the relationship between  $B$ ,  $B_1$  and  $B_2$ :

$$B_2 = B \left[ \frac{[g_H(\pi_1)g_L(\pi) - g_H(\pi)g_L(\pi_1)](G_L(\pi) - G_H(\pi))}{[g_H(\pi)g_L(\pi_2) - g_H(\pi_2)g_L(\pi)](G_L(\pi_1) - G_H(\pi_1)) + [g_H(\pi_1)g_L(\pi) - g_H(\pi)g_L(\pi_1)](G_L(\pi_2) - G_H(\pi_2))} \right]$$

$$B_1 = B \left[ \frac{[g_H(\pi)g_L(\pi_2) - g_H(\pi_2)g_L(\pi)](G_L(\pi) - G_H(\pi))}{[g_H(\pi)g_L(\pi_2) - g_H(\pi_2)g_L(\pi)](G_L(\pi_1) - G_H(\pi_1)) + [g_H(\pi_1)g_L(\pi) - g_H(\pi)g_L(\pi_1)](G_L(\pi_2) - G_H(\pi_2))} \right].$$

Having solved the bonuses, we can then compare the expected costs. The difference is simply (since the effort levels are equal across the contracts)

$$B_1(1 - G_L(\pi_1)) + B_2(1 - G_L(\pi_2)) - B(1 - G_L(\pi)),$$

substituting the bonuses in and rearranging gives us the condition for two (distinct) bonuses to yield a worse outcome as

$$\begin{aligned} & [[g_H(\pi_2)g_L(\pi) - g_H(\pi)g_L(\pi_2)](G_L(\pi) - G_H(\pi))] (1 - G_L(\pi_1)) \\ & + [[g_H(\pi)g_L(\pi_1) - g_H(\pi_1)g_L(\pi)](G_L(\pi) - G_H(\pi))] (1 - G_L(\pi_2)) \\ & \geq (1 - G_L(\pi)) [g_H(\pi_2)g_L(\pi) - g_H(\pi)g_L(\pi_2)] (G_L(\pi_1) - G_H(\pi_1)) \\ & + (1 - G_L(\pi)) [g_H(\pi)g_L(\pi_1) - g_H(\pi_1)g_L(\pi)] (G_L(\pi_2) - G_H(\pi_2)), \end{aligned}$$

which we can then rearrange to yield

$$\frac{[g_H(\pi_2)g_L(\pi) - g_L(\pi_2)g_H(\pi)]}{[g_H(\pi)g_L(\pi_1) - g_L(\pi)g_H(\pi_1)]} \geq \frac{[\Delta G_L(\pi_2, \pi)(1 - G_H(\pi)) - \Delta G_H(\pi_2, \pi)(1 - G_L(\pi))]}{[\Delta G_L(\pi, \pi_1)(1 - G_H(\pi_1)) - \Delta G_H(\pi, \pi_1)(1 - G_L(\pi_1))]}.$$

and since

$$\begin{aligned} & \Delta G_L(\pi, \pi_1) (1 - G_H(\pi_1)) - \Delta G_H(\pi, \pi_1) (1 - G_L(\pi_1)) \\ & = \Delta G_L(\pi, \pi_1) (1 - G_H(\pi)) - \Delta G_H(\pi, \pi_1) (1 - G_L(\pi)), \end{aligned}$$

we get

$$\frac{[\Delta g_H(\pi_2, \pi) g_L(\pi) - \Delta g_L(\pi_2, \pi) g_H(\pi)]}{[\Delta G_L(\pi_2, \pi)(1 - G_H(\pi)) - \Delta G_H(\pi_2, \pi)(1 - G_L(\pi))]} \geq \frac{[\Delta g_H(\pi, \pi_1) g_L(\pi) - \Delta g_L(\pi, \pi_1) g_H(\pi)]}{[\Delta G_L(\pi, \pi_1)(1 - G_H(\pi)) - \Delta G_H(\pi, \pi_1)(1 - G_L(\pi))]}.$$

In the general case, we are unable to derive as clean results because the impact of investment on the probability of winning is less precisely defined. However, the result on the equilibrium distortion in the manager's behavior is straightforward. First, note that we can write the principal's optimization problem as (noting that given  $I$ , the manager's problem boils down to choosing  $\Delta x$ )

$$\begin{aligned} & \max_{\underline{B}, \overline{B}, \underline{\pi}, \overline{\pi}} q E_H(\pi | \Delta x) + (1 - q) E_L(\pi | \Delta x) + \underline{B} (q F_H(\underline{\pi} | \Delta x) + [1 - q] F_L(\underline{\pi} | \Delta x) - 1) \\ & + \overline{B} (q F_H(\overline{\pi} | \Delta x) + (1 - q) F_L(\overline{\pi} | \Delta x) - 1) \end{aligned}$$

$$\begin{aligned} \text{s.t.} \quad & \underline{B} (F_L(\underline{\pi} | \Delta x) - F_H(\underline{\pi} | \Delta x)) + \overline{B} (F_L(\overline{\pi} | \Delta x) - F_H(\overline{\pi} | \Delta x)) = C'(q) \\ & - \left[ \underline{B} \left( q \left( \frac{\partial F_H(\underline{\pi} | \Delta x)}{\partial \Delta x} \right) + [1 - q] \left( \frac{\partial F_L(\underline{\pi} | \Delta x)}{\partial \Delta x} \right) \right) + \overline{B} \left( q \left( \frac{\partial F_H(\overline{\pi} | \Delta x)}{\partial \Delta x} \right) + [1 - q] \left( \frac{\partial F_L(\overline{\pi} | \Delta x)}{\partial \Delta x} \right) \right) \right] = \\ & 0, \end{aligned}$$

where the two constraints are the manager's information acquisition and investment constraints. Now, consider the choice of  $(\underline{\pi}, \overline{\pi})$  where we are adjusting the strength of bonuses  $\underline{B}$  and  $\overline{B}$  in a fashion that leaves both the expected compensation and the level of information acquisition directly unaffected (indeed, given MLRP, we could induce more information acquisition by increasing  $\overline{\pi}$  and  $\overline{B}$  while leaving costs unchanged). Then, consider the optimal choice of  $\Delta x$  (which the principal can manipulate through the combination of  $\underline{\pi}, \overline{\pi}$ ), which gives a first-order condition

$$\begin{aligned} & \left( q \frac{\partial E_H(\pi | \Delta x)}{\partial \Delta x} + (1 - q) \frac{\partial E_L(\pi | \Delta x)}{\partial \Delta x} \right) \\ & + \left( \underline{B} \left( q \frac{\partial F_H(\underline{\pi} | \Delta x)}{\partial \Delta x} + [1 - q] \frac{\partial F_L(\underline{\pi} | \Delta x)}{\partial \Delta x} \right) + \overline{B} \left( q \frac{\partial F_H(\overline{\pi} | \Delta x)}{\partial \Delta x} + (1 - q) \frac{\partial F_L(\overline{\pi} | \Delta x)}{\partial \Delta x} \right) \right) + \\ & (E_H(\pi | \Delta x) - E_L(\pi | \Delta x) - [\underline{B} (F_L(\underline{\pi} | \Delta x) - F_H(\underline{\pi} | \Delta x)) + \overline{B} (F_L(\overline{\pi} | \Delta x) - F_H(\overline{\pi} | \Delta x))]) \frac{\partial q}{\partial \Delta x} = \\ & 0 \end{aligned}$$

but from the agent's FOC we know that

$$\begin{aligned} & \left( \underline{B} \left( q \frac{\partial F_H(\underline{\pi} | \Delta x)}{\partial \Delta x} + [1 - q] \frac{\partial F_L(\underline{\pi} | \Delta x)}{\partial \Delta x} \right) + \overline{B} \left( q \frac{\partial F_H(\overline{\pi} | \Delta x)}{\partial \Delta x} + (1 - q) \frac{\partial F_L(\overline{\pi} | \Delta x)}{\partial \Delta x} \right) \right) = 0 \\ & [\underline{B} (F_L(\underline{\pi} | \Delta x) - F_H(\underline{\pi} | \Delta x)) + \overline{B} (F_L(\overline{\pi} | \Delta x) - F_H(\overline{\pi} | \Delta x))] = C'(q) \end{aligned}$$

so the above simplifies to

$$\left( q \frac{\partial E_H(\pi|\Delta x)}{\partial \Delta x} + (1-q) \frac{\partial E_L(\pi|\Delta x)}{\partial \Delta x} \right) + (E_H(\pi|\Delta x) - E_L(\pi|\Delta x) - C'(q)) \frac{\partial q}{\partial \Delta x} = 0$$

while from the agent's effort FOC we have

$$\frac{\partial q}{\partial \Delta x} = \frac{\left[ \underline{B} \left( \frac{\partial F_L(\underline{\pi}|\Delta x)}{\partial \Delta x} - \frac{\partial F_H(\underline{\pi}|\Delta x)}{\partial \Delta x} \right) + \overline{B} \left( \frac{\partial F_L(\overline{\pi}|\Delta x)}{\partial \Delta x} - \frac{\partial F_H(\overline{\pi}|\Delta x)}{\partial \Delta x} \right) \right]}{C''(q)},$$

and since  $\frac{\partial F_L(\underline{\pi}|\Delta x)}{\partial \Delta x} > \frac{\partial F_H(\underline{\pi}|\Delta x)}{\partial \Delta x}$ ,  $\frac{\partial q}{\partial \Delta x} > 0$  and since  $(E_H(\pi|\Delta x) - E_L(\pi|\Delta x) - C'(q)) > 0$ , it must be that, at the optimum,

$$\left( q \frac{\partial E_H(\pi|\Delta x)}{\partial \Delta x} + (1-q) \frac{\partial E_L(\pi|\Delta x)}{\partial \Delta x} \right) < 0,$$

which implies that  $(\Delta x)^M > (\Delta x)^{FB}$ , paralleling the mechanism design solution. The only exception is that this additional aggressiveness is now induced through the compensation contract instead, while if the MLRP held in the mechanism design case, we could rely only on the compensation contract and achieve  $(\Delta x)^{FB}$ .

#### A.4 Proof of proposition 5

For the level of investment, we need to endogenize the investment level, which we do by assuming that the bonus threshold is now  $\tilde{\pi}(I) = \underline{\pi} + \tilde{r}I$ . Then, we can write the first-order conditions for the investment decision as

$$\begin{aligned} & B_1 \left( q \left( g_H(\pi_1) \left( \frac{\partial \pi_H}{\partial x_h} - \tilde{r} \right) \right) + (1-q) g_L(\pi_1) \left( \frac{\partial \pi_L}{\partial x_h} - \tilde{r} \right) \right) \\ & + B_2 \left( q g_H(\pi_2) \left( \frac{\partial \pi_H}{\partial x_h} - \tilde{r} \right) + (1-q) g_L(\pi_2) \left( \frac{\partial \pi_L}{\partial x_h} - \tilde{r} \right) \right) = 0, \end{aligned}$$

which we can write as

$$q \frac{(B_1 g_H(\pi_1) + B_2 g_H(\pi_2))}{(B_1 g_L(\pi_1) + B_2 g_L(\pi_2))} \frac{\partial \pi_H}{\partial x_h} + (1-q) \frac{\partial \pi_L}{\partial x_h} = \left[ q \frac{(B_1 g_H(\pi_1) + B_2 g_H(\pi_2))}{(B_1 g_L(\pi_1) + B_2 g_L(\pi_2))} + (1-q) \right] \tilde{r}$$

and symmetrically for  $x_l$ , we obtain that

$$q \frac{(B_1 g_H(\pi_1) + B_2 g_H(\pi_2))}{(B_1 g_L(\pi_1) + B_2 g_L(\pi_2))} \frac{\partial \pi_L}{\partial x_l} + (1-q) \frac{\partial \pi_H}{\partial x_l} = \left[ q \frac{(B_1 g_H(\pi_1) + B_2 g_H(\pi_2))}{(B_1 g_L(\pi_1) + B_2 g_L(\pi_2))} + (1-q) \right] \tilde{r}.$$

Now, define  $y = \frac{(B_1 g_H(\pi_1) + B_2 g_H(\pi_2))}{(B_1 g_L(\pi_1) + B_2 g_L(\pi_2))}$  as the weighted average likelihood ratio and note that

any changes with respect to  $y$  are going to be equivalent to changes in  $\pi_1$  and  $\pi_2$  given our assumption of increasing likelihood ratio. Then, we have that

$$\begin{aligned} qy \frac{\partial \pi_H}{\partial x_h} + (1-q) \frac{\partial \pi_L}{\partial x_h} &= [qy + (1-q)] \tilde{r} \\ qy \frac{\partial \pi_L}{\partial x_l} + (1-q) \frac{\partial \pi_H}{\partial x_l} &= [qy + (1-q)] \tilde{r}. \end{aligned}$$

Now, we can first observe that if  $y = 1$ , then  $\tilde{r} = r$  will induce first-best investment decisions. However, we know that for the equilibrium contract,  $y > 1$  because it will tolerate some excess aggressiveness and thus if we can show that  $\frac{\partial(x_h+x_l)}{\partial y} > 0$ , we know that the manager will, in equilibrium, exhibit empire-building preferences and that distortion will become increasingly severe as we increase the performance threshold (i.e. increase the likelihood ratio at the bonus threshold). Since  $\frac{\partial \pi_H}{\partial x_h} > \frac{\partial \pi_L}{\partial x_h}$  and  $\frac{\partial \pi_L}{\partial x_l} < \frac{\partial \pi_H}{\partial x_l}$ , we can immediately see that increasing  $y$  increases the value of investing in  $x_h$  by  $q \left( \frac{\partial \pi_H}{\partial x_h} - \tilde{r} \right)$  while it decreases the marginal value of investing in  $x_l$  by  $q \left( \tilde{r} - \frac{\partial \pi_L}{\partial x_l} \right)$ . Note also that the distortion will disappear if  $q \rightarrow 1$  because then all uncertainty will disappear. From here, we then have that

$$\frac{\partial x_h}{\partial y} = -\frac{q \left( \frac{\partial \pi_H}{\partial x_h} - \tilde{r} \right)}{dD_H} \quad \text{and} \quad \frac{\partial x_l}{\partial y} = -\frac{q \left( \frac{\partial \pi_L}{\partial x_l} - \tilde{r} \right)}{dD_L},$$

but we also know that

$$\frac{\partial x_h}{\partial q} = -\frac{\left( \frac{\partial \pi_H}{\partial x_h} - \frac{\partial \pi_L}{\partial x_h} \right) - (y-1)\tilde{r}}{dD_H} \quad \text{and} \quad \frac{\partial x_l}{\partial q} = -\frac{\left( \frac{\partial \pi_L}{\partial x_l} - \frac{\partial \pi_H}{\partial x_l} \right) - (y-1)\tilde{r}}{dD_L},$$

so that 
$$\frac{\partial x_h}{\partial y} + \frac{\partial x_l}{\partial y} = \frac{q \left( \frac{\partial \pi_H}{\partial x_h} - \tilde{r} \right)}{\left( \frac{\partial \pi_H}{\partial x_h} - \frac{\partial \pi_L}{\partial x_h} \right) - (y-1)\tilde{r}} \frac{\partial x_h}{\partial q} + \frac{q \left( \frac{\partial \pi_L}{\partial x_l} - \tilde{r} \right)}{\left( \frac{\partial \pi_L}{\partial x_l} - \frac{\partial \pi_H}{\partial x_l} \right) - (y-1)\tilde{r}} \frac{\partial x_l}{\partial q}.$$

The rest is then just manipulation of the first-order conditions to simplify the expressions. First, note that

$$qy \frac{\partial \pi_H}{\partial x_h} + (1-q) \frac{\partial \pi_L}{\partial x_h} = [qy + (1-q)] \tilde{r}$$

can be rearranged as

$$\left( y \frac{\partial \pi_H}{\partial x_h} - \frac{\partial \pi_L}{\partial x_h} \right) - [y-1] \tilde{r} = \frac{1}{q} \left( \tilde{r} - \frac{\partial \pi_L}{\partial x_h} \right)$$

and similarly for  $qy \frac{\partial \pi_L}{\partial x_l} + (1-q) \frac{\partial \pi_H}{\partial x_l} = [qy + (1-q)] \tilde{r}$ , giving

$$\left( y \frac{\partial \pi_L}{\partial x_l} - \frac{\partial \pi_H}{\partial x_l} \right) - [y-1] \tilde{r} = \frac{1}{q} \left( \tilde{r} - \frac{\partial \pi_H}{\partial x_l} \right),$$

which allows us to simplify the above to

$$q^2 \left[ \frac{\left( \frac{\partial \pi_H}{\partial x_h} - \tilde{r} \right)}{\left( \tilde{r} - \frac{\partial \pi_L}{\partial x_h} \right)} \frac{\partial x_h}{\partial q} + \frac{\left( \frac{\partial \pi_L}{\partial x_l} - \tilde{r} \right)}{\left( \tilde{r} - \frac{\partial \pi_H}{\partial x_l} \right)} \frac{\partial x_l}{\partial q} \right].$$

Similarly, we can rearrange the first-order conditions to

$$\frac{\left[ \frac{\partial \pi_H}{\partial x_h} - \tilde{r} \right]}{\left[ \tilde{r} - \frac{\partial \pi_L}{\partial x_h} \right]} = \frac{(1-q)}{qy} \quad \text{and} \quad \frac{\left( \frac{\partial \pi_L}{\partial x_l} - \tilde{r} \right)}{\left( \tilde{r} - \frac{\partial \pi_H}{\partial x_l} \right)} = \frac{[(1-q)]}{qy},$$

so that in equilibrium,

$$\frac{\partial x_h}{\partial y} + \frac{\partial x_l}{\partial y} = \frac{q(1-q)}{y} \left[ \frac{\partial x_h}{\partial q} + \frac{\partial x_l}{\partial q} \right],$$

and so the impact on total investment is proportional to the impact of information on the total level of investment desired.