Synthetic or Real?
The Equilibrium Effects of Credit Default Swaps on Bond Markets *

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Abstract

We develop a model in which credit default swaps (CDS) are non-redundant securities because they are more liquid than the underlying bonds. The introduction of CDS has an ambiguous effect on bond prices: On the one hand, CDSs reduce short selling in bonds and also allow for the endogenous emergence of leveraged basis traders who buy both the bond and the CDS, raising bond prices. On the other hand, CDSs crowd out some bond buyers, decreasing bond prices. We derive testable cross-sectional predictions on the effects of CDS introduction, turnover, and the CDS-bond basis. We show that a policy banning (naked) CDS can raise the issuer’s borrowing costs.

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1 Introduction

Credit Default Swap (CDS) markets, which are essentially insurance markets for the default risk of corporate or sovereign debt, have grown tremendously over the last decade. However, while there is a relatively large literature on the pricing of these securities, much less work has been done on the economic role of CDS markets. In fact, in most pricing models, CDSs are redundant securities. In other words, there is no reason for investors to prefer CDSs over a cash-flow-equivalent position in the underlying bond and a risk-free asset.

In this paper, we develop a theory of non-redundant CDS markets to explore the economic role of CDS markets and their effect on the underlying bond markets. Our model of bond and CDS markets uses a simple, well-documented fact: trading bonds is expensive relative to trading CDSs. Based on this observation, we develop a theory of bond and CDS markets that generates a rich set of predictions regarding the economic role of CDS markets, the relative pricing of the CDS and the underlying bond (the so-called CDS-bond basis), and the effect of the introduction of CDS markets on bond prices.

In our model of bond and CDS markets, investors differ across two dimensions: (i) Investors differ in their beliefs about the bond’s default probability: Optimistic investors view the default of the bond as unlikely, while pessimists think that a default is relatively more likely. (ii) Investors differ in their investment horizon: Investors with longer investment horizons (“buy-and-hold investors”) are unlikely to have to sell their position in the future, while investors with shorter investment horizons are more likely to receive liquidity shocks that force them to sell their position.

If only the bond is traded, relatively optimistic investors with sufficiently long trading horizons buy the bond, whereas relatively pessimistic investors with sufficiently long trading horizons take short positions in the bond. Investors with short investment horizons (relative to their optimism or pessimism about default probabilities) stay out of the market, because for them the transactions costs of trading the bond are too high. The equilibrium bond price reflects the average marginal investor’s
views on default probabilities and the expected future trading costs the average marginal investor incurs over the lifetime of the bond.

When a CDS on the bond is also traded, the equilibrium changes in a number of interesting ways compared to the situation in which only the bond is traded. First, because of their higher transaction costs, bonds are priced cheaply relative to synthetic bonds created with CDSs, a well-documented phenomenon known as the negative CDS-bond basis. Second, the set of investors that hold long or short positions in the bond changes as some investors are drawn into the CDS market. Specifically, our model shows that the introduction of the CDS has three separate effects and, overall, leads to an ambiguous effect on the price of the underlying bond, consistent with the empirical literature that has not found strong average effects of CDS introduction on bond or loan spreads (Hirtle (2009), Ashcraft and Santos (2009)): (1) Some investors who previously held a long position in the bond switch to selling CDS protection, putting downward pressure on the bond price. (2) Because, in equilibrium, the bond trades cheaply relative to the CDS, investors who previously shorted the bond switch to selling CDS protection, putting upward pressure on the bond price. (3) Some investors become “negative basis traders”: they hold a long position in the bond and purchase CDS protection. Hence, the model endogenously generates the negative basis trade, an immensely popular trading strategy in recent years. These basis traders are “price neutral” if they cannot take leverage, but push up the bond price if they can lever their hedged positions. In practice, such trades are highly leveraged.

This last effect documents a novel economic role of CDSs: In our model, CDS markets allow “buy-and-hold” investors who are not very optimistic about the bond’s default probability to hold the bond (using their infrequent trading needs to their comparative advantage) and hedge their exposure in the CDS market, where the average investor is relatively more optimistic about default probabilities but has more frequent trading needs. Hence, the role of CDS markets is liquidity transformation: by allowing to separate the bond’s default risk into a more liquid security, they allow credit risk transfer while, at the same time, catering to the liquidity needs of investors with more frequent trading needs.
Our model generates testable predictions on trading volume in the bond and the CDS markets. First, we show that the introduction of a CDS typically decreases turnover in the underlying bond. This is the case because investors with more frequent trading needs opt for holding and trading the CDS. Second, the model implies that CDS turnover is higher than bond turnover. For relatively illiquid bonds, most credit risk transfer takes place in the CDS market, instead of the bond market, once CDS markets are introduced. This prediction is in line with the stylized facts: proxying for trading volume in the CDS using changes in gross notional CDS amounts as reported by the DTCC (adjusting for trade compression cycles), average CDS turnover is given by 55% per month, while average turnover in the underlying bonds is around 3.5% per month.

Finally, our model provides a framework to study regulatory interventions with respect to CDS markets. For example, a ban on naked CDS, as recently imposed in the European Union on sovereign bonds through EU regulation 236/2012 may, in fact, raise yields for affected issuers. The reason is that if pessimists cannot buy naked CDS, they will short the bond exerting downward price pressure on bonds. We also explore the effects of banning CDS markets altogether and banning both CDS markets and short positions in the underlying bond. We show that the effects of these interventions are also non-trivial and may, in fact, also lead to increases in bond yields for issuers.

Most closely related to our paper are theory papers that explore settings of CDSs (or, more generally, derivatives) as non-redundant securities. Che and Sethi (2011) develop a heterogeneous-beliefs model in which naked CDSs increase a firm’s cost of capital because they crowd out long bond investors. In Gårleanu and Pedersen (2011), derivatives are non-redundant because of differences in margin requirements for the derivative and the underlying security. Banerjee and Graveline (2013) develop a model in which derivatives are non-redundant because they can relax short selling constraints when the underlying security is scarce. Our paper differs from these papers in its specific focus on CDS markets and the source of non-redundancy, which in our framework comes in the form of trading costs in the spirit of Amihud and Mendelson (1986). Our paper is thus also related to models that explore setting in which investors can choose between economically similar markets that involve different

^1 There is also a growing empirical literature on price discovery in in CDS markets (Acharya and Johnson (2007), Hilscher, Pollet, and Wilson (2012)), the effect of CDSs on loan or credit spreads (Hirtle (2009), Ashcraft and Santos (2009)), the effect of CDSs on leverage and maturity (Saretto and Tookes (2013)), the determinants of CDS market existence and CDS positions (Oehmke and Zawadowski (2013)), the CDS-bond basis (Blanco, Brennan, and Marshall (2005), Bai and Collin-Dufresne (2010), Fontana (2011), Li, Kim, and Zhang (2010)), and the effect of CDS markets on bond market liquidity (Das, Kalimipalli, and Nayak (2013)).

2 Model Setup

We consider a financial market with (up to) two assets: a defaultable bond and a credit default swap that references the bond.

2.1 Bonds

We assume that a defaultable bond with positive net supply of $S > 0$ is traded. The bond’s equilibrium price is given by $p$. The bond has random Poisson maturity with arrival rate $\lambda$. Before maturity, the bond pays a continuous coupon rate $r$. At maturity, the bond pays back its face value of 1 with probability $1 - \pi$. With probability $\pi$, the bond defaults and pays 0. Reflecting the widely documented

^2 None of our results depend on maturity being random. We make this assumption because it ensures stationarity, as will become clear below.
illiquidity of the bond market, we assume that the bond is costly to trade. Moreover, we allow these trading costs to differ depending on whether investors take long or short positions (we want to allow short positions to be costlier than long positions to reflect costs of locating and borrowing the bond involved in a short sale). We model these trading costs in the spirit of Amihud and Mendelson [1986]. Specifically, we assume that investors incur a trading cost $c_l$ when selling a long position in the bond before maturity (they do not incur a trading cost if they hold the bond to maturity). Investors who short the bond incur a trading cost $c_s$, irrespective of whether they hold the short position until maturity. This last assumption reflects the fact that investors who short the bond have to buy back the bond even if the bond matures.

2.2 Credit default swaps

In addition to the bond, a credit default swap that references the bond is available. The credit default swap is an insurance contract on the bond’s default risk: it pays off 1 if the bond defaults at maturity and zero otherwise. For simplicity we assume that the CDS is perpetual: once entered, it provides default protection over the entire lifetime of the bond. We denote the CDS’s equilibrium price by $q$.

The CDS differs from the bond in two important ways. First, the CDS, a derivative contract, is in zero net supply. Second, the CDS has lower trading costs than the bond. Specifically, we assume that the CDS (derivative) has no transaction cost if held to maturity (since it does not entail locating the physical asset). If a CDS investor has to liquidate his position before maturity of the bond, he incurs a (relatively low) transaction cost $c_d$, which captures the cost of selling the CDS prematurely or entering an offsetting CDS contract. Overall, we assume that trading costs in the CDS market are lower than in the bond market, and that shorting is (weakly) more expensive than going long:

\[ c_s \geq c_l \geq c_d \geq 0. \] (1)

In practice, CDS contracts have fixed maturities (tenors), the most common being 1, 5 and 10 years. We use the perpetual setup to accommodate, in the easiest possible way, a CDS on a bond with random maturity. Our setup is thus comparable to one in which investors match maturities of finite-maturity bonds and CDSs.
In much of our analysis, we assume for simplicity that the CDS market involves no transaction costs (i.e., $c_d = 0$).

2.3 Investors

There is a mass of competitive investors who can trade in the bond and the CDS. Investors are heterogeneous across two dimensions: (i) expected holding periods and (ii) beliefs on default probabilities.

Regarding holding periods, we assume that investor $i$ has to liquidate his position with Poisson intensity $\mu_i \geq 0$. Investors with low $\mu_i$ can be interpreted as buy-and-hold investors, whereas investors with high $\mu_i$ are investors that receive more frequent liquidity shocks and are thus more active traders. To preserve stationarity, we assume that if one investor exits, another investor with the same characteristics enters. Investors agree to disagree on the bond’s default probability. Specifically, investor $i$ believes that the bond defaults at maturity with probability $\pi_i \in [\bar{\pi} - \frac{\Delta}{2}, \bar{\pi} + \frac{\Delta}{2}]$. To bound probabilities between 0 and 1 we assume that $1 - \frac{\Delta}{2} \geq \bar{\pi} \geq \frac{\Delta}{2}$. An investor type is thus characterized by an expected trading frequency $\mu_i$ and a subjective default probability $\pi_i$, and we assume that investors are distributed in the two-dimensional plane according to the joint distribution function $F(\pi, \mu)$.

Many of our results can be proved by imposing only weak conditions on $F(\pi, \mu)$. In particular, we will make two main assumptions on $F$:

(A1): $F$ is symmetric around $\bar{\pi}$ with respect to the disagreement about the default probability.

(A2): $F$ satisfies the “stretch property”

These two assumptions imply:

$$F(\mu, \pi) = F\left(\mu, \frac{\left|\pi - \bar{\pi}\right|}{\Delta}\right)$$

Note that (A1) is a simple symmetry assumption. The stretch property (A2) guarantees that an increase in the dispersion parameter $\Delta$ stretches out the distribution symmetrically on both sides. These assumptions are satisfied by many common distributions.
While essentially all of our results hold under A(1) and A(2), occasionally we will use a particularly simple distribution to calculate closed-form solutions for prices and quantities. For this example distribution, we assume that there is a mass one of “buy-and-hold” investors with infrequent trading needs \( \mu_L \geq 0 \), and a mass \( K_H \) of “speculators” with more frequent trading needs \( \mu_H > \mu_L \). The parameter \( K_H \) allows us to vary the number of speculators relative to buy-and-hold investors. Both buy-and-hold investors’ and speculators’ beliefs regarding the bond’s default probability are distributed uniformly on \([\bar{\pi} - \frac{\Delta}{2}, \bar{\pi} + \frac{\Delta}{2}]\). Below, we will refer to this specific distribution as the “double-uniform” distribution (i.e., two classes of investors whose beliefs are uniformly distributed on \([\bar{\pi} - \frac{\Delta}{2}, \bar{\pi} + \frac{\Delta}{2}]\)). The double-uniform distribution is illustrated in Figure 1. Note that the double-uniform distributions satisfies A(1) and A(2).

Figure 1: The double-uniform distribution
The Figure illustrates the double-uniform distribution used to calculate closed-form solutions. There is a mass one of “buy-and-hold” investors with trading frequency \( \mu_L \) and a mass \( K_H \) of investors with more frequent trading needs \( \mu_H \). For both classes of investors, the beliefs about the bond’s default probability are uniformly distributed on \([\bar{\pi} - \frac{\Delta}{2}, \bar{\pi} + \frac{\Delta}{2}]\).
Investors are risk neutral and, for simplicity, we abstract away from time discounting. Investors are subject to portfolio restrictions that allow them to hold up to one unit of credit risk. Hence, each investor can either go long one bond, short one bond, buy one CDS, or sell one CDS. Moreover, investors can buy hedged portfolios. One option is to buy a bond and insure it by also buying a CDS (a so-called negative basis trade). The other hedged trade is to short sell a bond and also sell a CDS (a so-called positive basis trade). Because these basis trades do not involve credit risk, we assume that investors can leverage them up to a maximum leverage of $L \geq 1$ (where $L = 1$ implies that they can take no leverage). Finally, investors can always hold cash, which can be seen as an outside option with zero return.

3 Benchmark: No CDS

Before introducing the CDS, we briefly consider, as a benchmark case, the equilibrium without a CDS market. First, we consider the case in which only long positions in the bond are possible, then the case in which investors can take both long and short positions in the bond. All proofs throughout the paper are relegated to the Appendix.

3.1 Only long positions allowed

Investor $i$’s net valuation of a long position in bond is given by

$$V_{l,i} = -p + \frac{r}{\mu_i + \lambda} + \frac{\mu_i}{\mu_i + \lambda}(p - c_l) + \frac{\lambda}{\mu_i + \lambda}(1 - \pi_i).$$

An investor who takes a long position in the bond pays $p$ today. He then receives the coupon until he sells the bond or the bond matures (note that $\mu_i + \lambda$ is the sum of the intensities with which investor $i$ has to sell the bond and with which the bond matures). If the investor has to sell the bond before maturity (which happens with probability $\frac{\lambda}{\mu_i + \lambda}$), he receives $p - c_l$. If the bond matures (which
happens with probability $\frac{\lambda}{\mu_i+\lambda}$, the investor receives an expected payoff of $1 - \pi_i$. Investor $i$ thus buys the bond if $V_{t,i} \geq 0$.

Figure 2: **Equilibrium without shorting**

The figure illustrates the equilibrium when only the bond is trading and only long positions in the bond are allowed. The bond is purchased by investors who are sufficiently optimistic about the bond’s default probability and who have long enough holding horizons, as illustrated by the grey triangle.

### 3.2 Long and short positions allowed

#### 3.2.1 General results

Now consider the case in which also short sales of the bond are possible. Investor $i$’s net valuation of a short position in the bond is given by

\[
V_{s,i} = p - \frac{r}{\mu_i + \lambda} - \left( \frac{\mu_i}{\mu_i + \lambda} p - \frac{\lambda}{\mu_i + \lambda} (1 - \pi_i) - c_s \right).
\]  

(4)

An investor with a short position receives $p$ today, is required to pay the coupon $r$ either until the bond matures or the short position is covered in response to a liquidity shock. If the short position
is covered, the investor has to purchase the bond for \( p \), whereas if the bond matures the investor has to cover his short position at an expected cost of \( 1 - \pi_i \). Finally, the short seller incurs the shorting cost \( c_s \). Investor \( i \) takes a short position if \( V_{s,i} \geq 0 \), which is the case if he is sufficiently pessimistic (\( \pi \) sufficiently large).

The resulting demand for long and short positions is illustrated in Figure 3 (under the assumption that parameters are such that there is demand for short positions in equilibrium). Note that there is a gap between the long triangle and the shorting triangle, even for true buy-and-hold investors who never need to liquidate their position (\( \mu = 0 \)), since the shorting fee has to be paid irrespective of whether the short position is sold early or the bond matures. Market clearing implies that the overall amount bought by long investors is equal to the amount shorted plus bond supply \( S \).

Figure 3 illustrate the impact of allowing short selling: Pessimistic investors with sufficiently long holding horizons choose to short the bond. Hence, for markets to clear, the triangle of buyers must get larger. This makes the effective marginal investor less optimistic and more likely to be hit by a liquidity shock, reducing the price of the bond relative to the case in which no shorting is allowed. We summarize this benchmark case in the following proposition.

**Proposition 1. Benchmark case: Bond market without CDS.**

*Introducing short sales weakly decreases the bond price and weakly increases bond turnover.*

- if \( c_s \geq \bar{c}_s \), no shorting occurs in equilibrium and price and turnover coincide with the case in which just long positions are allowed

- if \( c_s < \bar{c}_s \), there is (weakly) positive short interest in equilibrium. The bond price is weakly lower and bond turnover weakly higher than in the absence of shorting.

**3.2.2 Double-uniform example**

Using the double uniform distribution discussed above, we now calculate the bond price for the two benchmark cases above (under some parameter restrictions). When only long positions are possible,
Figure 3: Equilibrium with shorting but without CDS
The figure illustrates the equilibrium when only the bond is trading and both long and short positions in the bond are possible. Given that $c_s$ is not too large, investors who are pessimistic about the bond’s default probability and have sufficiently long holding horizons short the bond. For markets to clear, this means that the triangle of long bondholders has to grow relative to the case in which only long positions are possible (illustrated by the dotted line). The average marginal investor is less optimistic and has a shorter holding horizon than when only long positions are possible, leading to a decrease in the bond price.

The bond price is given by

$$p_{\text{long}} = 1 - \bar{\pi} + \frac{\Delta}{2} + \frac{r - c_l \mu_L}{\lambda} - S \Delta. \quad (5)$$

The bond price is thus given by the valuation of the most optimistic buy-and-hold investor ($\mu = \mu_L$) minus a term that captures the reduction in the marginal buyer’s valuation as supply increases, since the bond supply has to be absorbed by moving to increasingly less optimistic buyers. (For simplicity we have assumed here that in equilibrium none of the bond supply is purchased by $\mu_H$-investors.)
When both long and short positions are possible and $c_s$ is such shorting occurs in equilibrium, the bond price is given by

$$p_{\text{long&short}} = \frac{1}{\lambda} \left[ \bar{\pi} + \frac{r}{\lambda} - \frac{c_l \mu_L}{\lambda} - \frac{S \Delta}{2} + \frac{c_s}{2} + \frac{c_s + c_l \mu_L}{\lambda} \right].$$  (6)

The first three terms in (6) reflect the median buy-and-hold valuation. The remaining terms reflect the effects of trading costs and supply. First, the larger the outstanding supply, the lower the valuation of the marginal investor, reflected by the term $\frac{S \Delta}{2}$. Note, however, that in comparison to (5) the slope of this effect has halved. This is because an increase in supply lowers the equilibrium price, which drives out some short sellers, thus attenuating the effect of variations in supply on the price. The second term, $\frac{c_s}{2}$, reflects the “fixed costs” of short selling relative to taking long positions. In Figure 3, this effect is captured by the gap between the long triangle and the shorting triangle. The distance between the two triangles is exactly $c_s$, which means that the gap increases the valuation of the effective marginal investor by $c_s^2$. Finally, the term $\frac{c_s + c_l \mu_L}{\lambda}$ captures short sellers’ attenuation of the bond’s lifetime trading costs. When $c_l$ increases, the price of the bond drops because investors become more reluctant to purchase the bond. However, as the bond price drops, short selling becomes less attractive, such that part of the effect is offset by a drop in short selling. Similarly, an increase in $c_s$ makes short selling less attractive, thus raising the price of the bond. However, part of this effect is offset since, as the bond price rises, some long investors drop out of the market.

4 Introducing the CDS

We now introduce a CDS contract to the analysis. Analogously to before, we determine the demand for long and short CDS positions by considering the net valuations to investors of taking positions in the CDS (in addition to the net valuations of going long or short in the bond, which we derived in equations (3) and (4) above.) The net payoff to investor $i$ of purchasing a CDS on the bond is given
by
\[
V_{\text{buyCDS},i} = -q + \frac{\mu_i}{\mu_i + \lambda} (q - c_d) + \frac{\lambda}{\mu_i + \lambda} \pi_i, \tag{7}
\]
and the net payoff to investor \(i\) of selling a CDS on the bond is given by
\[
V_{\text{sellCDS},i} = q + \frac{\mu_i}{\mu_i + \lambda} (-q - c_d) - \frac{\lambda}{\mu_i + \lambda} \pi_i. \tag{8}
\]

Hence, investor \(i\) is willing to buy a CDS \((V_{\text{buyCDS},i} \geq 0)\) if he believes the default probability is sufficiently high, \(\pi_i \geq q + \frac{c_d \mu_i}{\lambda}\), whereas investor \(i\) is willing to sell CDS protection \((V_{\text{sellCDS},i} \geq 0)\) if he believes that the bond’s default probability is sufficiently low, \(\pi_i \leq q - \frac{c_d \mu_i}{\lambda}\).

To determine whether investors will take positions in the CDS, we now compare payoffs of CDS positions to the payoffs from taking positions directly in the underlying bond. Optimistic investors (low \(\pi_i\)) switch to selling CDS instead of buying the bond if \(V_{\text{sellCDS},i} > V_{l,i}\), which holds for optimistic investors that are sufficiently likely to be hit by a liquidity shock. Similarly, pessimistic investors (high \(\pi_i\)) switch from shorting the bond to purchasing a CDS if \(V_{\text{buyCDS},i} > V_{l,i}\), which is the case if they are sufficiently likely to be hit by a liquidity shock. Finally, investors switch to a negative basis trade if \(L \cdot (V_{\text{buyCDS},i} + V_{l,i}) > \max[V_{\text{buyCDS},i}, V_{l,i}]\).

### 4.1 Frictionless CDS

#### 4.1.1 General results

For simplicity, we first focus on the case in which the CDS can be traded costlessly (i.e., \(c_d = 0\)). After establishing our main results under the assumption of frictionless CDS markets (which is particularly tractable), we then also discuss the case in which the CDS market is relatively more liquid than the bond market, but not frictionless \((c_l > c_d > 0)\).

Figure 4 depicts the holding regions after the introduction of a frictionless CDS contract, assuming for now that basis traders cannot take leverage \((L = 1)\). When the CDS is frictionless, the equilibrium in the CDS market becomes particularly simple: all investors for whom \(\pi_i < q\) are willing to sell CDS,
while all investors with $\pi_i > q$ are willing to purchase CDS. Assuming that there are infinitely many investors with frequent trading needs (which we assume in the rest of this section), this means that the CDS price reflects the investors’ median belief about the default probability

$$q = \bar{\pi}.$$  

(9)

To derive the bond price, we need to consider how the availability of the CDS affects the incentives of investors with less frequent trading needs. Here, Figure 4 shows that the introduction of the CDS contract affects the bond market equilibrium in three ways. First, when a CDS is available, investors with relatively frequent trading needs who, in the absence of the CDS, used to purchase the bond now prefer to sell the CDS. In the figure, this results in the triangle of long bondholders being cut off at the top (for ease of comparison, the triangle of long bond positions in the absence of CDS is depicted by the dashed line). The resulting reduction on demand for the bond exerts downward pressure on the bond price. Second, the introduction of the CDS drives out short selling: The triangle of investors that formerly shorted the bond (depicted by the dotted line) vanishes, because those investors now prefer to purchase the CDS instead of shorting the bond. The reason why investors prefer to use the CDS market to take bearish bets works through the equilibrium price: Because trading costs for the bond exceed trading costs for the CDS, the bond trades at a discount relative to the CDS. Hence, investors who wish to take a bearish bet on the bond prefer to do this through the CDS market rather than through the bond market. The elimination of short sellers through the introduction of the CDS exerts upward pressure on the bond price. Third, Figure 4 shows that the introduction of the CDS leads to a new class of investors. Some investors, who in the absence of the CDS would simply purchase the bond, now purchase both the bond and the CDS. These investors thus become so-called negative basis traders: They hold a hedged position in the bond and the CDS, thus locking in the price difference between the bond and the CDS. Rather than taking bets on credit risk, these traders now act as pure arbitrageurs.
Figure 4: Equilibrium with CDS, basis traders cannot take leverage

This figure illustrates the equilibrium when both the bond and the CDS are trading and basis traders cannot take leverage ($L = 1$). The introduction of the CDS has three effects: (i) Some investors who absent the CDS would purchase the bond now choose to sell CDS protection. (ii) Because of the negative CDS-bond basis, all former short sellers prefer to purchase a CDS, which eliminates the shorting triangle. (iii) Investors who formerly bought the bond but whose beliefs about the bond’s default probability are below the median $\bar{\pi}$ become basis traders who purchase the bond and buy CDS protection. The dotted lines illustrate the long and short triangles in the absence of the CDS.

When basis traders do not take leverage ($L = 1$), as in Figure 4, their presence does not affect the bond price. The reason is that, absent leverage, basis traders would purchase the bond even in the absence of the CDS. When basis traders can take leverage ($L > 1$), on the other hand, they exert upward pressure on the bond price. Specifically, the ability of basis traders to take leverage raises the equilibrium bond price in two ways. First, when basis traders can take on leverage, their presence raises the demand for the bond. In Figure 4, each basis trader now purchases more than one bond (keeping the number of basis traders as before). Second, the ability to take leverage makes the basis trade more profitable and thus expands the number of basis traders.
These results on basis traders highlight a novel economic role of CDSs in our model. Specifically, the CDS allows buy-and-hold investors with relatively infrequent trading needs to purchase the bond and then lay off the credit risk to more optimistic investors with more frequent trading needs via the CDS. Hence, by transferring credit risk from buy-and-hold investors to speculators in the CDS market, CDSs are vehicles for liquidity transformation. This is possible, because the CDS allows investors separate the bond’s credit risk from the bond’s illiquidity.

Figure 5: Equilibrium with CDS, basis traders can take leverage
The figure illustrates the equilibrium when both the bond and the CDS are trading and basis traders can take leverage ($L > 1$). The ability to take leverage makes the basis trade more attractive, such that the basis trade triangle expands compared to Figure 4. Because of the increased demand from basis traders, the average marginal investor who buys the bond becomes more optimistic and has a longer holding horizon than when $L = 1$. For ease of comparison, the dashed line illustrates the rectangle of investors who purchase the bond when basis traders cannot take leverage ($L = 1$).

Based on the effects discussed above, Proposition 2 summarizes the effects of CDS introduction on the equilibrium in the bond market.

**Proposition 2.** The effect of introducing a CDS on the bond price.

*Compared to the equilibrium in which only the bond trades (and investors can take long and short
positions in the bond), the introduction of the CDS can increase or decrease the bond price, depending on distribution of investors. Furthermore, the bond price always increases in $L$, the ability of basis traders to take leverage.

The ambiguous effect of CDS introduction on the price of the bond follows directly from the three demand effects discussed above. To see this, assume for simplicity that basis traders cannot take leverage ($L = 1$), such that their price effect is neutral. In this case, the effect of CDS introduction on the bond price depends on whether more investors switch from a long position in the bond to selling CDS, or from a short position in the bond to buying CDS. This, in turn, depends on the distribution of investors on the two-dimensional plane illustrated in Figure 4. Note that the ambiguous effect of CDS introduction on bond yields may explain why the empirical literature has had a hard time documenting that the availability of CDSs reduces bond yields. For example, Ashcraft and Santos (2009), Hirtle (2009), and Saretto and Tookes (2013) find mixed evidence that, on average, the introduction of CDSs lowers firms’ costs of capital. Our analysis thus helps guide empirical research on what kind of bond issuers are more likely to benefit from the introduction of a CDS: see our discussion in case of the double-uniform distribution.

In addition to the results on bond price, our model has implications for turnover in the bond and the CDS market. The effect of CDS introduction on bond turnover can be seen directly from Figure 4. As a result of CDS introduction, investors with relatively frequent trading needs, that used to hold the bond, switch to selling the CDS, reducing bond turnover. In addition, short sellers switch to buying the CDS, also reducing bond turnover.

Bond turnover, defined as the amount of CDS trading divided by the notional amount of outstanding CDSs, is larger than bond turnover because CDS investors have higher average trading frequencies than bond investors. The triangle of investors who sell the CDS is on top of the region of long bond investors in Figure 4. Because there is no short selling when the CDS trades, this direct comparison of average trading frequencies of investors suffices. This prediction is confirmed in the data used by

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4The size of the two effects will differ even if investors are completely symmetrically distributed on the $\pi-\mu$ plane.
Oehmke and Zawadowski (2013): proxying for trading in CDSs using changes in the gross notional outstanding reported by the DTCC (adjusting for trade compression cycles), average CDS turnover is given by 55% per month, while turnover in the underlying bonds is around 3.5% per month. The following proposition summarizes the discussion:

**Proposition 3. Turnover in the bond and the CDS market.**

When CDS market is introduced, the turnover in the underlying bond market decreases (given some assumptions about the distribution of investors). In equilibrium, the turnover is higher in the CDS market than in the bond market.

### 4.1.2 Double-uniform example

Using the double uniform distribution, we now calculate the bond price when a CDS is available and basis traders can take leverage \( L \geq 1 \). For simplicity, we focus on the case \( K_H \to \infty \) (i.e., there is a large mass of “speculators” with more frequent trading needs \( \mu_H \)). This assumption ensures that the CDS is priced according to the median belief about default probability at \( q = \bar{\pi} \). Then, the bond price in the presence of CDS is given by

\[
\hat{p}_{\text{withCDS}} = 1 - \bar{\pi} + \frac{r}{\lambda} - \frac{c_l \mu L}{\lambda} - \frac{(S - \frac{1}{2})^+ \Delta}{1 + 2(L - 1)L}. \tag{10}
\]

Equation (10) shows that, in the presence of the CDS, we can distinguish two cases. First, if the supply of the bond is relatively small, \( S \leq 1/2 \), the bond price is simply given by the valuation of the median buy-and-hold investor (\( \mu = \mu_L \)). When \( S > 1/2 \), an additional term appears which captures the effect of supply \( S \) on the bond price. For a given supply \( S > 1/2 \), this effect is smaller, the higher the leverage \( L \) that basis traders can take.
We can also explicitly calculate the change in the bond yield that results from CDS introduction. Assuming that $S > \frac{1}{2}$:

$$
dy = \frac{(c_s - S \Delta) \lambda + (c_l + c_s) \mu L}{2} + \frac{(S - \frac{1}{2}) \lambda \Delta}{1 + 2(L - 1)L}.
\tag{11}
$$

Equation (11) illustrates the price effects discussed above. In our double-normal example, we have set parameters such that, in the absence of the CDS, the entire supply of the bond is absorbed by buy-and-hold ($\mu = \mu_L$) investors. These investors continue to demand the bond even once the CDS is introduced. Since none of the investors with frequent trading needs hold the bond to start out with, the introduction of the CDS does not eliminate any demand for the bond. In the absence of leverage for basis traders ($L = 1$), the sole effect of introducing the CDS is thus the elimination of short sellers and bond yields drop unambiguously. Equation eq:dylev shows the effect of basis traders: If basis traders can take leverage, the presence of the CDS allows basis traders to absorb more of the bond supply and lay off credit risk in the fairly priced CDS market. This results in a further reduction in the bond yield, as illustrated by the second term in (11).

Based on the discussion above, we can summarize our results on the impact of CDS introduction on bond yields:

**Corollary 1. CDS introduction and the cost of capital.**

For the double-uniform distribution, the introduction of the CDS reduce bond yields by more if

- basis traders can take high leverage $L$ (liquidity transformation)
- disagreement $\Delta$ about default probability is high
- bond trading costs $c_l$ and $c_s$ are low

---

20
4.2 The CDS-bond basis

4.2.1 General results

One quantity that has generated considerable attention, in particular since the financial crisis of 2007-2009 is the so-called CDS-bond basis. The CDS-bond basis is defined as the difference between the spread of a synthetic bond composed of a long position in the CDS and a risk-free bond (of the same maturity and coupon as the risky bond) and the spread of the risky bond. Intuitively speaking, when the CDS-bond basis is negative, the CDS spread is smaller than the bond spread, meaning that the bond is cheaper than the payoff-equivalent synthetic bond constructed from the CDS and a risk-free bond.

Absent frictions, the CDS-bond basis should be approximately zero. The reason is that a portfolio consisting of a long bond position and a CDS that insures the default risk of the bond should yield the risk-free rate. While no arbitrage implies that the CDS-bond basis should be exactly equal to zero only if certain assumptions hold (see [Duffie, 1999]), absent frictions it should be approximately zero in practice. Since the financial crisis, the CDS-bond basis has been consistently negative for most reference entities as documented, for example, by [Bai and Collin-Dufresne, 2010] and [Fontana, 2011]. Moreover, widening in the size of the negative basis seems to be correlated with the funding conditions of arbitrageurs (i.e., basis traders).

In our model, the negative basis between bonds and CDSs arises endogenously from the difference in trading costs of the bond and the CDS. To calculate the basis, note that, in our setting, the price of a risk-free bond with the same coupon and maturity as our risky bond is given by \( p_f = 1 + \frac{r}{\lambda} \). Hence, we can calculate the spread of the risky bond above the risk-free bond as \( \lambda (p_f - p) \). Given price \( q \) of the CDS, we can calculate the per unit time price of CDS protection, the CDS spread, as \( \lambda q \). Hence, the CDS-bond basis can be expressed as:

\[
\text{basis} = \lambda q - \lambda (p_f - p) = \lambda \left( q + p - 1 - \frac{r}{\lambda} \right)
\]  

(12)
Proposition 4. \textit{Negative CDS-bond basis.}

When trading costs for the bond are higher than trading costs for the CDS ($c_l > c_d$), the CDS-bond basis is negative. The CDS-bond basis is more negative when

- the leverage of basis traders $L$ decreases
- bond trading cost $c_l$ are high
- when disagreement $\Delta$ about default probability is high

The result on basis trader leverage $L$ comes from the fact that, all else equal, higher leverage for basis traders increases the demand for the bond, pushing up the bond price and thus reducing the negative basis. Hence, at times when basis traders can take a lot of leverage, the model predicts a less negative basis, whereas, during times of tough financing conditions for basis traders, the basis will be more negative. This mechanism was likely a major contributor to the widening of the negative basis during the financial crisis, when the leverage of basis traders fell from around 25 to 5 (see Mitchell and Pulvino (2012); Bai and Collin-Dufresne (2010)).

While the result on leverage is mostly a time-series prediction, the remaining two comparative statics in Proposition 4 are cross-sectional predictions. First, bonds with high trading costs (relative to trading costs in the associated CDS) are predicted to have more negative CDS-bond bases. Second, the model predicts that bonds for which investors disagree significantly on default probabilities will have more negative bases. The reason for this result is that an increase in disagreement reduces the number of basis traders, but leaves the number of long bond investors unchanged. As a result, the bond price drops, widening the basis. Notice that in reality, bonds with higher default risk are more likely to have high disagreement and face high trading costs. Thus our results imply that lower rated bonds should have a more negative basis, a result that is true during the financial crisis (see Bai and Collin-Dufresne (2010)).

\footnote{This can be seen from Figure 4. Holding prices fixed, increasing $\Delta$ leaves the number of agents that go long the bond unchanged, but reduces the number of investors in the basis-trade triangle. For markets to clear, the bond price must drop.}
4.2.2 Double-uniform example

While the results above are essentially distribution-free, the double-uniform distribution, combined with frictionless CDS, also provides a simple way to calculate the CDS-bond basis. Under the double-uniform distribution, we can calculate the CDS-bond basis as

$$\text{basis} = -c_l \mu_L - \lambda \Delta \frac{(S - \frac{1}{2})^+}{1 + 2(L - 1)L}. \quad (13)$$

The first term in (13) captures the difference in expected trading costs between the bond and the CDS and is independent of the supply of the bond. This term captures the effect of bond trading costs on the basis, as discussed in Proposition 4. The second term (which under the double-normal distribution is present if $S > 1/2$) captures the effect of bond supply, disagreement, and basis traders on the CDS-bond basis. As the bond supply increases, the marginal unit of the bond is held by less optimistic investors, leading to a more negative CDS-bond basis. This effect, however, is attenuated by basis traders, who purchase the bond and lay off the credit risk in the CDS market. The more leverage basis traders can take, the smaller this second effect. Hence, as discussed in Proposition 4, through their arbitrage trade, basis traders compress the basis and reduce the borrower’s cost of capital. Finally, (13) illustrates the effect of disagreement on the basis: More disagreement (higher $\Delta$) increases the second term, leading to a more negative basis.

4.3 CDS markets with frictions: $c_d > 0$

We now briefly discuss the case in which the CDS market is relatively more liquid than the bond market, but not frictionless, $c_l > c_d > 0$. In this case, the CDS price will reflect trading frictions in the CDS market and will no just reflect the actuarially fair default probability. In fact, using the symmetric of $F(\cdot)$, one can show that in this case

$$q > \bar{\pi}. \quad (14)$$
The main difference compared to the case with frictionless CDS is illustrated in Figures 6 and 7. The most significant difference is that, when also the CDS market has trading costs, it is no longer the case that all investors with \( \pi_i < \bar{\pi} \) are willing to CDS and all investors with \( \pi_i > \bar{\pi} \) are willing to purchase CDS. Hence, rather than being rectangles, the “sell CDS” and “buy CDS” regions also become triangles, as some investors now prefer to stay out of the market altogether and hold cash (the white region).

All the qualitative results derived in the frictionless setting above remain valid in the case where the CDS market also involves trading frictions. For example, as before, the introduction of a CDS affects the bond markets through the same three effects: the CDS eliminates some long bondholders, eliminates short sellers, and it introduces hedged, potentially leveraged basis traders. As before, when \( L = 1 \) basis traders are price neutral (Figure 6), whereas basis traders exert downward pressure on the bond yield when they can take leverage (Figure 7).

While most of the main economic results of our model do not change when the CDS market is also subject to trading frictions, this case is useful in gauging the effect of policy interventions in Section 5.

4.4 Illiquid CDS market: \( c_d > c_l \)

4.4.1 General results

The main driving assumption behind our model is that trading costs in the CDS market are lower than trading costs in the bond market. While the assumption that the CDS market is more liquid than the underlying bond is probably a good one for most corporate (and potentially also sovereign) bonds, there may be a few examples where the reverse assumption may make sense. For example, in the case of U.S. Treasuries it may be reasonable to assume that trading costs in the (generally very liquid) Treasury are lower than in the market for CDSs on U.S. Treasuries. By flipping our trading costs assumption to \( c_d > c_l \), our model may also generate interesting implications to study these cases.
Figure 6: **Equilibrium with CDS when \( c_d > 0 \), basis traders cannot take leverage**

The figure illustrates the equilibrium when also the CDS involves trading frictions (\( c_d > 0 \)) and basis traders cannot take leverage (\( L = 1 \)). Compared to Figure 4 where \( c_d = 0 \), the sell CDS and buy CDS regions are now triangles, reflecting higher CDS trading costs for investors as \( \mu \) increases. As in the case with frictionless CDS, the introduction of the CDS has three effects: (i) Some investors who absent the CDS would purchase the bond now choose to sell CDS protection. (ii) Because of the negative CDS-bond basis, all former short sellers prefer to purchase a CDS, which eliminates the shorting triangle. (iii) Investors who formerly bought the bond but whose beliefs about the bond’s default probability are below the median \( \bar{\pi} \) become basis traders who purchase the bond and buy CDS protection. For ease of comparison, the dotted lines are the buy bond and short bond triangles when no CDS is available.

Our model allows us to explore this case by simply flipping our trading cost assumption to \( c_d > c_l \). The resulting equilibrium is illustrated in Figure 8. When the CDS is more costly to trade than the bond, investors with relatively infrequent trading needs (“buy-and-hold investors”) choose to sell the CDS instead instead of buying the bond.

If the bond and CDS market were completely symmetric, one would expect the basis to become positive and positive basis traders to appear. For this to happen, the basis would need to be high enough to offset the cost of shorting a bond. However, such a high basis would drive out CDS buyers.
Figure 7: **Equilibrium with CDS when** $c_d > 0$, **basis traders can take leverage**

The figure illustrates the equilibrium when also the CDS involves trading frictions ($c_d > 0$) and basis traders can take leverage ($L > 1$). Relative to Figure 6, the triangle of basis traders expands because the ability to take leverage increases the profitability of the basis trade. The resulting increase in demand for the bond pushes up the bond price, analogously to the arguments in the frictionless CDS ($c_d = 0$) case. For ease of comparison, the dotted lines are the **buy bond** and **short bond** triangles when no CDS is available.

In a similar way as the negative basis drives out bond shorters but in that case this is not an issue since the bond is in positive net supply. The following proposition formalizes this intuition:

**Proposition 5. Illiquid CDS market.**

If the CDS market is less liquid than the bond market ($c_d > c_l$):

- the CDS-bond basis is weakly positive,
- there are no basis traders in equilibrium (neither positive, nor negative).

One can also see that the turnover in the bond market is likely to be higher than that in the bond market. While this is not the case for average corporations, there might be special cases in which this prediction holds.
4.4.2 Double-uniform example

In case of the double-uniform distribution, assuming buy&hold investors hold CDS, others the bond:

$$\text{basis} \bigg|_{c_d > c_l} = \frac{1}{2} \left( c_s (\lambda + \mu_H) - \frac{S\Delta \lambda}{K_H} - c_l \mu_H \right)$$

which does not depend on $c_d$ because of the specific distribution we chose: there is no margin of adjustment along the dimension of expected holding period. One can show that for $K_H \geq 1$, the introduction of the CDS market always raises the price of the bond.

5 Policy Implications

In this section we use our model to analyze a number of policy interventions regarding CDS markets. Specifically, we consider (i) banning naked CDS positions (as recently implemented in the EU), (ii)
banning all CDSs altogether, and (iii) banning both CDSs and short positions in bonds. In all three cases, our model shows that the effects of these interventions on bond yields (which is usually the motivation for policymakers) are non-trivial and can go potentially in the “wrong” direction, meaning that they may increase financing costs for issuers of bonds (i.e., firms or sovereigns).

5.1 Banning naked CDS positions

EU regulation No 236/2012 currently bans purchasing CDS protection as a means speculation on sovereign bonds. The regulation allows CDS purchases for market participants who own the underlying bond (or have other significant exposure to the sovereign). Moreover, short selling of the bond is allowed as long as the short position is covered (i.e., the seller borrows the bond before shorting).

Banning naked CDS positions has two main effects (see Figure 5.1): First, some investors that previously purchased naked CDS protection switch to shorting the bond. Hence, as a result of banning naked CDS positions, short sellers reappear, putting downward pressure on the bond price. Second, some investors that formerly held naked CDS protection now switch to holding covered CDS positions by entering the basis trade. This increases demand for the bond and thus exerts upward pressure on the bond price. Overall, the effect of banning naked CDS positions (while still allowing short selling) can either or decrease the financing costs for bond issuers. Thus the current EU regulation may, contrary to its goal, raise financing costs for affected sovereigns.

5.1.1 Double-uniform example

For the double-uniform distribution, we can calculate the effect of a ban on naked CDS positions on bond prices explicitly (given parameter restrictions). Assuming for simplicity, that \( L = 1, K_H \to \infty \) and \( S > 0 \), we can calculate the change in the bond price and the CDS price as

\[
\begin{align*}
p_{\text{no naked CDS}} - p_{\text{withCDS}} &= \frac{2}{3} \cdot \left( \left( S - \frac{1}{2} \right) \Delta + \left( c_l + c_s \right) \frac{\mu_L}{\lambda} + c_s - c_d \frac{\mu_H + \mu_L}{2\lambda} \right) \\
q_{\text{no naked CDS}} - q_{\text{withCDS}} &= -\frac{\Delta}{2} + \frac{c_d \mu_H}{\lambda}
\end{align*}
\]
Figure 9: **Banning naked CDS when $c_d > 0$ and $L > 1$**

The figure illustrates the equilibrium when naked CDS positions are banned. Compared to Figure 7, which depicts the same setup except that naked CDS positions are allowed, there are two major changes. Some investors who used to purchase CDS protection now short the bond, exerting downward pressure on the bond price. Some investors who used to buy naked CDS protection now become basis traders, which exerts upward pressure on the bond price when $L > 1$. The overall effect on the bond price is thus ambiguous.

While the effect on bond price is ambiguous, the price of CDS decreases unless CDS trading costs $c_d$ are high. Thus the fact that CDS spreads decrease when the policy is introduced does not mean that bond prices rise. Set for example $\mu_L = 0$ and $S$ close to $\frac{1}{2}$. When is banning naked shorts likely to decrease the issuer’s borrow cost? From (16), it is clear that more liquid bond markets (low $c_l$ and $c_s$) are more likely to benefit and also ones with more disagreement (high $\Delta$). If the CDS market is less liquid (higher $c_d$), the ban is also more likely to decrease borrowing costs. The following proposition formalizes these statements:

**Proposition 6. Banning naked CDS.**

For the double-uniform distribution, the effect of banning naked CDS has an ambiguous effect on the yield of the underlying bond. A naked CDS ban is more effective in raising the bond price if
• bond trading costs $c_l$ and especially $c_s$ is high,

• CDS trading cost $c_d$ is low.

5.2 Banning CDS market altogether

The second policy intervention we consider is an outright ban of the CDS market. Within the context of our model, this is a simple comparison between the equilibrium with a CDS and the equilibrium without CDS. From Proposition 2, we know that relative to the case where only long and short positions in the bond are allowed, the effect of the CDS on the bond yield is ambiguous. This immediately implies that banning CDS markets altogether may raise yields for issuers.

More specifically, our model predicts that banning CDS is more likely to increase yields under certain conditions. First, banning the CDS is more likely to increase yields if absent the CDS there is significant short selling of the bond. This is the case if there is a large mass of pessimistic buy-and-hold investors and if the costs of short selling $c_s$ are low. Second, banning CDS is likely to increase yields when basis traders can take significant leverage ($L$ is large). This is because banning the CDS eliminate basis traders and thus their ability to act as liquidity transformers for the bond’s credit risk.

5.3 Banning CDS and shorting the bond

Finally, we briefly consider the intervention of banning both the CDS and short positions in the bond. This takes us back to the long-only case discussed in Proposition 1. Interestingly, even this intervention is not guaranteed to lower bond yields. While restricting short positions concurrently with banning CDS positions prevents the reemergence of short sellers in response to elimination of the CDS market, the tradeoff is now one between (i) increased demand from investors who formerly sold the CDS and now purchase the bond and (ii) the reduction in demand for the bond that results from the elimination of basis traders. Since basis traders are price-neutral when they cannot take leverage, the banning the CDS and short selling always lowers the bond yield then $L = 1$. When $L > 1$, on
the other hand, the intervention may increase bond yields, and is more likely to do so the higher the leverage of basis traders $L$ and the larger the supply of the bond $S$.

6 Conclusion

This paper provides a liquidity-based model of CDS markets, bond markets and their interaction. CDSs are non-redundant in our framework because they have lower trading costs than bonds. Our model shows that when investors are heterogeneous in their trading horizon and beliefs regarding the bond’s default probability, the introduction of a CDS affects the underlying bond market in a non-trivial way. In particular, the effect of CDS introduction on the bond price is ambiguous and depends on the amount of short selling absent the CDS, the crowding out of long bond investors through the CDS, and the ability of basis traders, who emerge endogenously, to take leverage. Beyond characterizing the impact of CDS introduction on the pricing of the underlying bond, the model also generates testable empirical predictions regarding trading volume in bond and CDS markets, as well as the cross-sectional and time-series properties of the CDS-bond basis.
References


Appendix A: Proofs

Proof of Proposition 1

Long only positions — Investor $i$ thus buys the bond if $V_{l,i} \geq 0$, which yields

$$\pi_i \leq 1 - p + \frac{r}{\lambda} - \frac{c_l}{\lambda} \mu_i.$$  \hfill (A1)

Hence, investors must be sufficiently optimistic and patient in order to buy the bond. The demand implied by (A1) is illustrated in Figure 2. Market clearing implies that total demand is equal to outstanding bond supply $S$, or

$$\int_0^{\pi} \int_{\hat{\mu}}^{1-p+\frac{r}{\lambda}+\frac{c_s}{\lambda}} F(\mu, \pi) d\pi d\mu = S.$$  \hfill (A2)

Long and short positions — Investor $i$ takes a short position if $V_{s,i} \geq 0$, which is the case if

$$\pi_i \geq 1 - p + \frac{r}{\lambda} + c_s + \frac{c_s}{\lambda} \mu_i.$$  \hfill (A3)

Thus short sellers are pessimistic and patient, located in the lower right triangle of the $(\mu, \lambda)$ plane (see Figure 3). Shorting can only be present in equilibrium only if

$$c_s \leq \bar{c}_s = -1 + p + \frac{\Delta}{2} \frac{r}{\lambda} + \bar{\pi}. \hfill (A4)$$

Note that this is not a sufficient condition because even if the shorting triangle has a non-zero area, the distribution $F$ of investors still may be such that a zero measure of agents are located in the triangle and thus there is no shorting in equilibrium. If there is no shorting in equilibrium, the bond price $p$ is the same as in the case where only long positions are allowed. If $c_s \leq \bar{c}_s$, the following market clearing conditions pins down the price of the bond:

$$\int_0^{\pi} \int_{\hat{\mu}}^{1-p+\frac{r}{\lambda}+\frac{c_s}{\lambda}} F(\mu, \pi) d\pi d\mu = S + \int_0^{\bar{\pi}} \int_{1-p+\frac{r}{\lambda}+\frac{c_s}{\lambda}}^{\frac{r}{\lambda}+\frac{c_s}{\lambda}} F(\mu, \pi) d\pi d\mu \hfill (A5)$$

If there is non-zero short selling in equilibrium, the price of the bond $p$ has to be lower in the presence of shorting. Assume the counterfactual that when shorting is allowed, $p$ remains the same as if shorting is not allowed. Short sellers create an excess supply of bond, thus the market cannot clear. The bond price $p$ has to
decrease to increase the buying triangle and thus increase demand. Thus the bond price \( p \) is weakly lower in the presence of shorting if \( c_s \leq \bar{c}_s \).

The result that bond turnover weakly increases the bond turnover comes form the following observations. If there is no short selling in equilibrium, the turnover does not change. If there is positive short selling, the original holders of the bonds (absent short selling) are still holding the bond and trading at frequency \( \mu_i \) as they exit and a new agent enters. The new short sellers and the buyers absorbing the supply Since the amount of bonds outstanding remained unchanged, turnover weakly increases when shorting is allowed.

**Proof of Proposition 2**

*CDS positions* — We first explain the regions in Figure 3. To determine whether investors will take positions in the CDS, we now compare payoffs of CDS positions to the payoffs from taking positions directly in the underlying bond. Optimistic investors (low \( \pi_i \)) switch to selling CDS instead of buying the bond if \( V_{\text{sellCDS},i} > V_{l,i} \), which holds for optimistic investors that are sufficiently likely to be hit by a liquidity shock:

\[
\mu_i \geq \frac{\lambda}{c_l - c_d} \left( \frac{r}{\lambda} + 1 - p - q \right)
\]  

(A6)

which is a horizontal line in the \((\mu, \pi)\) plane. Since the bond supply is strictly positive \( S > 0 \), one needs a bond buyers to be present for bond markets to clear. Thus for any investor to buy the bond instead of selling CDS, because of \( c_l - c_d > 0 \) one needs \( \frac{r}{\lambda} + 1 - p - q \geq 0 \). Equality can only be sufficient if there is at least a mass of \( S \) of agents at \( \mu_i = 0 \). In Equation (A10) we show that this condition is also necessary for the presence of negative basis traders, who also buy the bond. Similarly, pessimistic investors (high \( \pi_i \)) switch from shorting the bond to purchasing a CDS if \( V_{\text{buyCDS},i} > V_{l,i} \), which is the case if they are sufficiently likely to be hit by a liquidity shock:

\[
\mu_i \geq \frac{\lambda}{c_s - c_d} \left( -\frac{r}{\lambda} + q - 1 + p - c_s \right)
\]  

(A7)

The previous condition \( \frac{r}{\lambda} + 1 - p - q \geq 0 \) and \( c_s > c_d > 0 \) means that the right hand side of this inequality is always negative: thus there are no short-sellers in equilibrium because \( \mu_i \geq 0 \) by definition.

*Basis traders* — Investors who perform the negative basis trade have to prefer the basis trade above buying the bond if \( \pi_i < \bar{\pi} \) and have to prefer it above buying CDS if \( \pi_i < \bar{\pi} \). Investors switch from a long position in
the bond to a negative basis trade if

\[ \pi_i \geq Lq - (L - 1) \left( \frac{r}{\lambda} + 1 - p \right) + \frac{\mu_i}{\lambda} \left( Lc_d + (L - 1)c_l \right) \]  \hspace{1cm} (A8)

and switch from buying a CDS to a negative basis trade if

\[ \pi_i \leq L \left( \frac{r}{\lambda} + 1 - p \right) - (L - 1)q - \frac{\mu_i}{\lambda} \left( (L - 1)c_d + Lc_l \right) \]  \hspace{1cm} (A9)

This investor’s performing the basis trade form a triangle in the \((\mu, \pi)\) plane, the height of which is given by the following, which simplifies to the expression on the right hand side in case of \(q = \bar{\pi}\) and \(c_d = 0\):

\[ \frac{L \left( r - (-1 + p + q)\lambda \right) + \lambda (q - \bar{\pi})}{(-1 + L)c_d + Lc_l} = \frac{\lambda}{c_l} \left( 1 + \frac{r}{\lambda} - p - q \right). \]  \hspace{1cm} (A10)

Thus the height of the basis triangle in this case for \(c_d = 0\) is the same as the height of the bond buying region expressed in Equation A6.

**Effect of leverage** — Let us analyze the effect of increasing basis traders’ leverage \(L\) for fixed prices \(p, q\).

First, increasing \(L\) increases the amount of bonds basis traders buy. Second, increasing \(L\) broadens the base of the basis triangle without affecting the height (Equation A10). In Equation A8 if \(\frac{r}{\lambda} + 1 - p - q > 0\) the left side of the basis triangle moves more to the left as \(L\) increases (the limit of \(\pi_i\) decreases for any given \(\mu_i\)). In Equation A9 if \(\frac{r}{\lambda} + 1 - p - q > 0\) the right side of the basis triangle moves more to the right as \(L\) increases (the limit of \(\pi_i\) increases for any given \(\mu_i\)). This second effect also means that more investors become basis traders as \(L\) increases. Both effects increase the demand for bonds, thus the market can only clear if the bond price \(p\) increases (note that \(q = \bar{\pi}\) is fixed).

**Proof of Proposition 3**

The trading frequency of all agents selling the CDS is higher than the trading frequency of any agent buying the bond either long only or in basis trade (see Figure 4). Thus the turnover of CDS must be unambiguously higher than that of the bond. Trades from CDS buyers further add to the higher turnover of CDS.

The introduction of a CDS market changes the bond holding regions in three ways, all of the pointing towards a lower bond turnover (see Figure 4). The elimination of the shorting triangle unambiguously decreases bond trading. Also, the remaining bond buyers (including the basis traders) with the highest trading frequency
have a lower trading frequency than the lowest trading frequency bond buyers that have been eliminated (upper 
half of buying triangle). [TO BE COMPLETED]

\[\square\]

**Proof of Corollary 1**

Bond yields are reduced by CDS introduction if bond prices increase. All results follow from taking deriva-
tives of \( dy \) w.r.t. the variables of interest (see Equation 11).

\[\square\]

**Proof of Proposition 4**

The necessary condition for equilibrium in the proof of Proposition 2 is that \( \frac{r}{\lambda} + 1 - p - q \geq 0 \). Since by 
definition, basis = \( \lambda \left( q + p - 1 - \frac{r}{\lambda} \right) \) and \( \lambda > 0 \), the condition for equilibrium implies basis \( \leq 0 \).

**Effect of disagreement** — Assume that disagreement \( \Delta \) increases but prices \( p, q \) remain unchanged. The 
bond buying rectangle will be stretched horizontally but given the stretch property of \( F \) this does not change 
the mass of agents purchasing only the bond. The basis triangle does not change if \( p, q \) are fixed even if the 
distribution \( F \) is stretched horizontally. By the stretch property of \( F \), less agents will be distributed in any 
fixed area of the \((\mu, \pi)\) plane as \( \Delta \) increases. Thus there are less basis traders and less demand for the bond, 
meaning the bond market can only clear if \( p \) drops which yields a more negative CDS-bond basis.

**Effect of trading costs** — Assume that bond trading cost \( c_l \) increases but prices \( p, q \) remain unchanged. The 
bond buying region becomes lower since the height is

\[
\frac{\lambda}{c_l - c_d} \left( \frac{r}{\lambda} + 1 - p - q \right)
\]

which is decreasing in \( c_l \). The same is true for the height of the basis triangle. However, the horizontal basis of 
the bond buying region and the basis triangle does not change. Thus there is a smaller mass of agents willing 
to buy bonds, consequently \( p \) must decrease, therefore making the basis more negative.

**Effect of basis trader leverage** — Follows immediately from the proof of Proposition 2 showing that \( p \) 
increases if \( L \) increases.

\[\square\]

**Proof of Proposition 5**

A necessary condition for there to be an equilibrium is for the CDS market to clear. For this one needs the 
CDS buyer area to be non-zero:

\[
\frac{r - (-1 + p + q)\lambda + \lambda c_s}{c_d - c_s} > 0
\]

(A12)
that is \( c_s \) cannot be very high. Furthermore that there are CDS sellers iff:

\[
-\frac{r + (-1 + p + q)\lambda}{c_d - c_l} > 0
\]  

(A13)

The two together yield that the basis needs to be weakly positive but cannot be too positive.

\[
\begin{align*}
\lambda c_s \geq & \quad \text{basis} \\
\left|c_d - c_l\right| & \geq 0
\end{align*}
\]  

(A14)

Note that positive basis trading is impossible in equilibrium since the following condition is needed for the positive basis trade area to exist at \( \mu_i = 0 \) one would need basis \( > \lambda c_s \).

\[\square\]

**Proof of Proposition 6**

The closed form for general \( L \) in case of the double-uniform distribution.

\[
P_{\text{no naked CDS}} - P_{\text{with CDS}} =
\]

\[
\frac{\left(-S + L \left(L - 3L^2 + 2L^3 + S\right)\right) \Delta \lambda + (1 + 2(-1 + L)L)((1 + L)\lambda c_s + (1 - 2L)c_d \mu_H) + (L(-3 + 2L)c_d + (1 - L + 2L^3)(c_l + c_s)) \mu_L}{(2 + L(-5 + 2L(4 + L(-3 + 2L))))}\lambda
\]  

(A15)

\[
q_{\text{no naked CDS}} - q_{\text{with CDS}} = -\frac{\Delta}{2} + \frac{c_d \mu_H}{\lambda}
\]  

(A16)

One can show that the first derivative of the price difference w.r.t. \( c_s \) and \( c_l \) is positive. If \( L > 1 \), the derivative w.r.t. \( c_d \) is negative.

\[\square\]