Sovereign Debt Crises and Financial Contagion*

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Abstract

In an integrated global financial system, a sovereign default raises concerns of financial contagion. We develop and estimate a network model of interconnected sovereign borrowers and lenders. Using data on government debt and foreign claims, we estimate the model for the observed borrowing network of 13 European sovereigns over six years. The estimated model implies that credit markets perceive the expected spillover losses resulting from a sovereign default to be relatively small in magnitude. While we find the degree of contagion varies across sovereigns, it does not appear to account for a significant portion of the observed credit spreads in our sample.

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1 Introduction

The recent sovereign debt crisis in Europe has generated concerns of financial contagion. In particular, many policymakers have raised concerns that the default of one sovereign could have spillover effects that result in subsequent defaults or increased borrowing costs for other sovereigns. Given the interconnectedness of global financial markets, where sovereigns are simultaneously borrowing and lending from each other, the potential for spillover effects seems to be a natural concern. The extent to which these effects are quantitatively important, however, remains an open question. In this paper we attempt to empirically evaluate the degree to which credit markets perceive these spillover effects as an economically important component of sovereign credit risk.

Building on the recent theoretical literature of networks, we develop and estimate a network model of sovereign borrowing. We use data from a variety of sources to construct the empirically observed network of borrowing and lending relationships between 13 European sovereigns during the period 2005-2011. In our framework as in the Eurozone, contagion risk arises naturally as a shock to one sovereign can propagate through the network to generate a sovereign debt crisis affecting multiple countries. Using credit default swap spreads on sovereign debt and the network of borrowing and lending relationships, we estimate the magnitude of spillover effects expected by credit markets in the event of a sovereign default.

Our estimates suggest that credit markets perceivethe impact of contagion, transmitted via a balance sheet mechanism of direct spillovers, to be nonzero but relatively small. Specifically, we consider counterfactuals where we simulate the default of one sovereign in the network. We then compute the estimated change in the credit spreads of the other
sovereigns in the network following the counterfactual default. While we find heterogeneity in the effects across sovereigns, the magnitude of the estimated losses resulting from contagion are, for the most part, economically small. Our results imply that credit markets do not demand a significant premium for the potential spillover losses resulting from the observed interconnectedness of European sovereigns.

While there has been much discussion among policymakers and the media highlighting fears of contagion, there remains relatively little empirical analysis on the topic. Due to the long period of low credit risk among developed sovereigns, the concerns of a European sovereign default, and the spillover losses it might induce, are relatively recent. Moreover, the magnitude of these spillover losses is difficult to measure, both due to the nature of these effects as well as a scarcity of available data. We utilize a combination of datasets, many of which are relatively new, along with a framework that builds upon recent theoretical research on networks, to estimate the expected spillover losses perceived by credit markets.

Our paper is related to two existing strands of literature. The first is a relatively new literature that studies financial networks and contagion. The second studies sovereign debt, default, and credit risk, typically in the context of a dynamic general equilibrium model of a small open economy.

The role of networks in financial contagion has been the subject of a growing literature that largely began with the theoretical work of Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000). Both papers model an interbank market where liquidity shocks from consumers in different locations are the reason for interbank deposits and transfers. They consider the possible contagion of insolvency throughout the system if one bank fails, and both models indicate that more connected networks are more resilient against this contagion.
Recent theoretical work has further examined this result in certain stylized environments. Babus (2009) models the formation of a network of interbank deposits in one region with common liquidity shocks, given pre-existing links with banks in another region that has the opposite liquidity shocks. Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) and Elliott, Golub, and Jackson (2012) derive several results based on exogenous networks with canonical topologies (e.g., rings, regular graphs, complete graphs). The empirical literature on financial networks has found evidence of contagion in a variety of settings (see Allen and Babus (2009) for a survey). These analyses typically follow a descriptive statistical approach that treats the network structures as exogenous. Two exceptions which use a model-based approach to assess contagion empirically are Cohen-Cole, Patacchini, and Zenou (2011) and Baral (2012), both of which examine interbank markets.

Relatively little work has considered the role of financial networks in spillovers from a sovereign debt crisis or other international contexts for financial contagion. The most relevant to us is Bolton and Jeanne (2011) and Acharya, Drechsler, and Schnabl (2011), who develop a model that links government debt to the financial sector (where it is used as collateral for interbank loans). Sovereign risk affects the reserves of domestic and foreign banks that hold this debt. With a simple, two-country network, they show that banks diversify their debt portfolios, and so a sovereign default in one country reduces liquidity, and hence investment and output, in both countries. Empirically, a handful of recent papers provide evidence on the international transmission of financial shocks during the recent financial crisis. De Haas and Van Horen (2011) and Giannetti and Laeven (2011) examine

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1 Acemoglu et al. also consider the formation of the network in a particular environment that can allow a ring to emerge.

2 Allen and Gale (2000) note that the different regions in their model could be interpreted as countries.
international lending by large banks, and find that those with greater losses or less access to credit made greater reductions in their cross-border lending. Similarly, Cetorelli and Goldberg (2010) and Popov and Udell (2010) find that the supply of loans fell in emerging markets due to reductions in cross-border lending from international banks and local lending from banks with foreign parents, as well as reductions in local lending by domestic banks affected by the international interbank market.

Our paper is also related to a literature that studies sovereign borrowing and lending. This strand of literature builds on the seminal work of Eaton and Gersovitz (1981) to study sovereign debt and default in a dynamic general equilibrium model. These papers study sovereign borrowing with endogenous default in dynamic general equilibrium models with incomplete markets. Arellano (2008) uses a quantitative model to study the interaction of sovereign credit risk with output fluctuations and interest rates. A calibrated version of the model is successful in matching Argentina’s business cycles statistics and the country’s 2001 default. Aguiar and Gopinath (2006) use a similar framework to study the quantitative predictions of a model of debt and default in a small open economy. They find a quantitatively important role for a stochastic trend in growth rates for emerging economies. Arellano and Ramanarayanan (2008) extend this framework of a sovereign borrower to allow for the choice of debt maturity. They find a quantitatively important role for a sovereign’s choice of maturity structure as it trades off the hedging benefits of long-term debt against the repayment incentives induced by short-term debt. Borri and Verdelhan (2011) study the importance of a risk averse lender in a model of sovereign borrowing. They show the importance of a risk-averse lender, in this case with habit formation preferences, for matching the sovereign bond yields observed in the data.
Finally, recent work in the finance literature has examined the time series properties and comovement of sovereign credit risk by examining sovereign CDS spreads. Longstaff et al. (2011) find that global factors explain a large portion of the common variation in sovereign CDS prices. Dieckmann and Plank (2011) study CDS spreads for a sample of developed countries and find strong comovement that increased significantly following the financial crisis beginning in 2008. Pan and Singleton (2008) find that the spreads of Mexico, Turkey, and Korea have a high correlation with volatility in the U.S. stock market as measured by the VIX.

The remainder of the paper is organized as follows. Section 2 presents the network model of international borrowing and lending. We discuss our empirical approach to estimating the model in Section 3 and the data used in the estimation in Section 4. In Section 5 we present the results from the estimated model. Finally, in Section 6 we use the estimated model to simulate counterfactual sovereign defaults and examine the contagion effects of these shocks propagating through the network of borrowing and lending relationships.

2 Theoretical Framework

In this section we describe the model framework that we use to estimate the degree of contagion in European sovereign debt markets. The goal is to empirically assess the degree to which credit markets perceive contagion effects, via a balance sheet mechanism of direct spillovers, as an economically important aspect of sovereign credit risk. We do not attempt to explain the levels of borrowing or distribution of borrowing and lending connections between sovereigns observed in the data. Rather, we take these observed relationships as given and ask how much of the observed sovereign credit spreads are due to credit markets pricing a
direct contagion channel.\textsuperscript{3}

Our starting point is the recent theoretical literature that studies networks of connected borrowers and lenders. In the network, each sovereign is a distinct entity and sovereigns are connected through their borrowing and lending with each other. The empirical analysis will consider a network of European sovereigns for which we have sufficient data to estimate contagion effect.

The network consists of countries $i = 1, \ldots, N$ and time is discrete. Sovereign $i$’s output at date $t$, denoted $Y_{it}$, is stochastic and taken to be exogenous. Each period $t$, sovereigns have claims on each other that were established in the previous period. We denote sovereign $i$’s claims on sovereign $j$ at date $t$ as $l_{ijt}$. Additionally, we allow a sovereign to borrow from a financial entity outside of the network. Thus, sovereign $i$’s total debt, denoted $D_{it}$, equals the sum of claims on $i$ held by all other sovereigns $j \neq i$, plus $i$’s obligations to creditors that are outside the network of countries.

Given the possibility of default, a sovereign may not receive the full return from its claims on other countries. We denote the solvency of each sovereign with the indicator $s_{it}$. If a sovereign defaults, its creditors receive a portion of their claims, based on an exogenous recovery rate $\delta$. Thus the total returns received from other countries is

$$
R_{it} \equiv \sum_{j \neq i} l_{ijt}[\delta + (1 - \delta)s_{jt}].
$$

(1)

We specify a sovereign’s solvency condition as a function of its output ($Y_{it}$), debt obliga-

\textsuperscript{3}Note that while the interconnectedness of sovereign borrowing and lending might result in a contagion premium in sovereigns’ cost of borrowing, we are not suggesting that this interconnectedness is suboptimal. Indeed, there may be benefits to this interconnectedness and the evaluation of these benefits is outside the scope of this paper. Given policymakers’ stated fears of the costs of contagion, our goal is simply to empirically evaluate these costs.
tions ($D_{it}$), portfolio of loans to other sovereigns ($R_{it}$), and a financial shock ($X_{it}$). Specifically, sovereign $i$’s solvency at date $t$ is determined by the condition:

$$s_{it} = \mathbb{1} \{\beta Y_{it} + \gamma R_{it} - \alpha D_{it} + X_{it} > \pi\}$$  \hspace{1cm} (2)

where $\pi$ is the threshold required for solvency (which could be positive or negative). The presence of $s_{jt}, j \neq i$ in $R_{it}$ establishes the interdependencies among countries and means that solvency is jointly determined in the network as the solution to the system of solvency equations. There may be multiple solutions to this system, as we discuss in the empirical approach.

To summarize this framework, each period $t$ unfolds as follows:

0. Countries are endowed with bilateral claims ($l_{ijt}$) and total debts ($D_{it}$), which were established in the previous period.

1. Output ($Y_{it}$) and financial shocks ($X_{it}$) are realized.

2. Solvency ($s_{it}$) is jointly determined among the network of countries.

3. The process is repeated the following period for the set of countries that remain solvent.

We now turn to the discussion of how we implement estimation of the system of solvency conditions defined by equation (2).

3 Empirical Approach

The goal is to estimate equation (2) and use it to assess the potential for contagion among the countries in our network. The equation generates a predicted solvency probability for
each country, which we will match to data on CDS prices, suitably transformed. Specifically, we suppose that the CDS data provide information on rational beliefs in period \( t-1 \) about solvency probabilities in period \( t \), denoted \( p_{it} \). These observable beliefs should equal the expectation of the solvency indicators from (2), conditional on the information available in step 3 of period \( t-1 \):

\[
p_{it} = E[s_{i,t}|(Y_{jt-1}, l_{jkt}, D_{jt}, X_{j,t-1}), j, k = 1 \ldots N] \tag{3}
\]

This expectation can be found using the distributions of period \( t \) output \((Y_{jt})\) and shocks \((X_{jt})\) conditional on their previous values and then solving for the joint solvency outcomes \((s_{i,t})_{i=1}^{N}\) based on (2).\(^4\) (We describe the specifications of these forecasts below.)

Depending on the values of \( Y_{it} \) and \( X_{it} \) across all countries, there may be multiple solutions to (2). When this occurs, we select the “best-case” outcome in which the fewest countries default.\(^5\) For example, suppose that given the loans and debts established in the previous period and a realization of the current output and financial shocks, there are two solutions for \( s_{i,t} \) and \( s_{j,t} \): either both countries default or both are solvent. We always select the equilibrium where \( i \) and \( j \) remain solvent and pay each other back. This would be the result if there were some coordination process, as all countries are at least weakly better off when there are fewer defaults. The best-case solution can be found with a simple iterative procedure: start with repayment amounts as though all countries were solvent; use (2) to

\[^4\text{To spell this out, equation (3) can be written as}

\[
(p_{it})_{i=1}^{N} = \int_{(Y_{t}, X_{t})}^{1} \{\beta Y_{it} + \gamma R_{it} - \alpha D_{it} - X_{it} > \pi\}_{i=1}^{N} dF[(Y_{t}, X_{t})|(Y_{t-1}, X_{t-1})],
\]

where \( Y_{t}, X_{t}, \) etc., are vectors across all countries, and the solvency indicators are embedded in \( R_{it} \).

\[^5\text{Elliott, Golub, and Jackson (2013) makes the same assumption.}\]
determine which countries would in fact default; reduce the repayment amounts based on these defaults; use (2) to determine if any additional countries would default; repeat this process until no further countries would default.

We specify the forecasted distribution of output \( (Y_{it}) \) as a function of its previous level and growth rate. To capture co-movements across countries we partition the growth rate into a common component and country-specific residuals using a principal components analysis of growth rates among the countries in our network. The common component of the growth rate in country \( i \), denoted \( \Delta Y_{it}^c \), is its actual growth rate \( (\Delta Y_{it}) \) projected onto the first principal component, and the residual is denoted \( \Delta Y_{it}^r = \Delta Y_{it} - \Delta Y_{it}^c \). As the notation indicates, \( \Delta Y_{it}^c \) varies across countries because we use the projection onto the first PC rather than the first PC itself. This allows some countries to be more integrated or dependent on the global economy than others. The mean forecast for \( Y_{it} \) is specified as a linear combination of its previous level and these variables: \( \beta_1 Y_{i,t-1} + \beta_2 \Delta Y_{i,t-1}^c + \beta_3 \Delta Y_{i,t-1}^r \). The distribution assumed for this forecast is a normal with variance \( \sigma_Y^2 \). Thus the forecasted distribution of output for country \( i \) in period \( t \) is:

\[
Y_{it} | (Y_{i,t-1}, \Delta Y_{i,t-1}^c, \Delta Y_{i,t-1}^r) \sim N(\beta_1 Y_{i,t-1} + \beta_2 \Delta Y_{i,t-1}^c + \beta_3 \Delta Y_{i,t-1}^r, \sigma_Y^2).
\]

These are the market expectations in period \( t - 1 \) for output in period \( t \).

The shock \( X_{it} \) is also assumed to follow a normal distribution, with mean zero and variance \( \sigma_X^2 \). In the current implementation this shock is IID across countries and over time. However we plan to allow for correlation across countries in a future version. (Identification with correlated shocks is discussed at the end of this section.)
Applying these specifications, the solvency probabilities in (3) are

\[
(p_{it})^N_{i=1} = \int 1 \left\{ \beta_1 Y_{i,t-1} + \beta_2 \Delta Y_{i,t-1}^c + \beta_3 \Delta Y_{i,t-1}^r + \gamma R_{it} - \alpha D_{it} - X_{it} > \pi \right\}^N_{i=1} \cdot \prod_{j=1}^N \frac{1}{\sigma_Y} \phi \left( \frac{\tilde{Y}_{jt}}{\sigma_Y} \right) \frac{1}{\sigma_X} \phi \left( \frac{X_{jt}}{\sigma_X} \right) d\tilde{Y}_{jt} dX_{jt},
\]

where \( \tilde{Y}_{it} \) is the deviation of \( Y_{it} \) from its conditional mean and \( \phi \) is the standard normal density. This is a system of equations for the vector of solvency probabilities; the interdependencies arise from the solvency indicators \( s_{jt} \) that are embedded in \( R_{it} \). To simplify this expression we combine the shocks \( \tilde{Y}_{it} \) and \( X_{it} \) as \( \epsilon_{it} = \tilde{Y}_{it} - X_{it} \) and normalize the parameters so that \( \epsilon_{it} \) has unit variance. Also, because \( \beta \) is not separately identified from \( \beta_1, \beta_2, \) and \( \beta_3, \) we set \( \beta = 1. \) As a consequence the parameters \( \beta_1, \beta_2, \beta_3 \) are interpreted as the combination of the forecast for future output and the relationship between output and solvency. Finally we include a time trend in the threshold so that \( \pi \) is specified as \( \pi_0 + \pi_1 t. \) This yields the specification which we take to the data:

\[
(p_{it})^N_{i=1} = \int 1 \left\{ \beta_1 Y_{i,t-1} + \beta_2 \Delta Y_{i,t-1}^c + \beta_3 \Delta Y_{i,t-1}^r + \gamma R_{it} - \alpha D_{it} + \epsilon_{it} > \pi_0 + \pi_1 t \right\}^N_{i=1} \cdot \prod_{i=1}^N \phi(\epsilon_{it}) d\epsilon_{it}
\]

This expression is computed via Monte Carlo integration. For each vector of draws, \( (\epsilon_{it})^N_{i=1} \), we solve (2) for the vector of solvency indicators. The average of these indicators across all draws provides an approximation of the solvency probabilities.

We estimate the parameters in (4) by minimizing the squared error between the empirical solvency probabilities that are derived from CDS rates and the predicted solvency probabilities from the above model. For consistent estimation this assumes that the error term in
period $t$ is not correlated with output or the claims and debts established in period $t - 1$.

We estimate the parameters in (4) by minimizing the squared error between the empirical solvency probabilities that are derived from CDS rates and the predicted solvency probabilities from the above model. To consider identification, our model can be viewed as an extension of the model in Krauth (2006), which has simple, group-based interactions. Our model features a similar information structure (i.e., the shocks are common knowledge) but applies to a context where economic interactions take place on a weighted, directed graph. To our knowledge, general results on identification are not available for nonlinear models with networks that are more complex than a simple group-based structure. However based on the results for linear network models in Bramoullé, Djebbari, and Fortin (2009) it seems that the richer network structure would only further facilitate the identification arguments developed for nonlinear group-based models in Brock and Durlauf (2007). More importantly, we speculate that a contemporaneous correlation in the shocks could be accommodated. Specifically if $\epsilon_{it}$ were decomposed into a common and idiosyncratic component, as in $u_t$ and $v_{it}$, we believe that the variance of $u$ could be identified separately from the parameters in (4). This follows from similar logic as for the identification of a nonlinear panel model with random effects. All the variables in (4) exhibit variation across countries at a point in time, including the claims that influence $R_{it}$. Hence the effect of a common unobservable should be identifiable, at least in terms of its distribution. Finally we note, more intuitively, that identification requires that the error term ($\epsilon_{it}$) is not correlated with any of the observable variables. This would be violated if, for example, the underlying shocks $\tilde{Y}$ or $X$ influence

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6The relevant features have only been considered separately. Brock and Durlauf (2007) establish the generic identification of nonlinear models with group-based interactions. Bramoullé, Djebbari, and Fortin (2009) give conditions for the identification of linear models with network interactions.
the values of the observable variables in the current period and are correlated over time.\footnote{Serial correlation in these shocks seems likely. We plan to assess the magnitude of any resulting bias by using lags of the observable variables to instrument for their current values. The bias should diminish with longer lags, as any correlation with the current shocks would be diminished. The change in the parameter estimates when longer lags are used could indicate the magnitude of the bias.}

4 Data

In this section we discuss the data used in estimating the market’s perception of contagion in the previously described borrowing and lending network of European sovereigns. We begin by collecting data on international financial relationships provided by the Bank for International Settlements (BIS) and sovereign debt from the IMF. We supplement these data with macroeconomic and financial variables from the OECD and IMF. Finally, we collect data on sovereign credit default swaps (CDS) from CMA.

The IMF reports the dollar amount of a sovereign’s debt held by foreign creditors. While this gives the amount of a sovereign’s debt held abroad, it is an aggregated measure that does not provide the nationalities of the various foreign creditors holding a given sovereign’s debt. To construct the empirical network of borrowing and lending, we supplement the IMF data with data from the BIS consolidated banking statistics database.

The central banks of BIS member countries collect data on the balance sheet composition of the banks in their jurisdiction. They aggregate these data and report to the BIS the breakdown of banks’ assets according to the country of the issuer of the security. For the BIS member countries, this provides a network of the claims held by the banks of one country on another. These represent all financial claims, not just sovereign debt. To construct our network of sovereign debt claims, we weight the external sovereign debt amounts reported...
by the IMF according to the shares reported by the BIS.\textsuperscript{8} Both the BIS and IMF data are reported over our sample period at a quarterly frequency. Using these data, we construct a network of lending relationships consisting of the 13 countries for the period starting in the first quarter of 2005 to the third quarter of 2011. Table I lists the countries included in our sample.

Figure 1 gives a representation of the lending network in 2011Q3, the last period in our data. The arrows represent financial claims that one country has on another. These amounts are normalized by the size of the economy of the lender country, using a country’s 2004 GDP, to reflect their relative exposure. Darker arrows indicate larger proportional amounts, and claims worth less than one percent of the lender’s 2004 GDP are not shown. The darkest arrows represent claims worth over 50\% of the lender’s 2004 GDP. Nearly all countries have claims on each other, and so arrows can be bidirectional such as between Austria (AT) and Italy (IT). Yet there are many unidirectional arrows because often one country out of any pair has a relatively small amount of claims on the other.

The algorithm places more strongly connected countries in the center and more weakly connected countries in the periphery.\textsuperscript{9} The UK is a very large debtor when its claims are put relative to its creditors’ economies, so it is made quite central. France (FR) is both

\textsuperscript{8}Note that this assumes the foreign sovereign debt holdings of a country’s financial institutions are proportional to the total foreign asset holdings. For a concrete example, suppose the BIS data report that 40\% of the total financial claims issued by entities located in country A are held by institutions located in country B and 60\% are held by institutions located in country C. Additionally, suppose that the IMF reports that of the debt issued by the government of sovereign A, $50\text{ }\text{billion} is held by foreign creditors. Our construction of the network would then assume that $20 billion of sovereign A’s debt is held by country B and $30\text{ }\text{billion} is held by country C. Note that there are several BIS reporting countries that are not included in the network we consider and so we are not making the assumption that all of the 13 sovereigns’ debt is held by the other sovereigns in our network.

\textsuperscript{9}However this is not a unique representation of the network, as it is a projection of an $N\times N$ matrix into two dimensions. Different algorithms produce different visual representations, although the qualitative features are reasonably stable.
a moderately large lender and debtor in the normalized amounts, and Germany (DE) is a fairly large debtor. Other countries such as the Netherlands (NL) may serve as important “bridges” in the network structure.

In addition to the lending relationships, we collect data on countries’ GDP, investment, yields on government-issued long-term bonds, and spreads on sovereign credit default swaps. The GDP and investment series for each country come from the OECD’s Quarterly National Accounts dataset. Specifically, we use quarterly data on GDP that is annualized, seasonally adjusted, and measured in fixed PPP. Additionally, we collect yields on 10-year government bonds from the OECD’s Monthly Monetary and Financial Statistics database.

In Table II we report the time series mean and standard deviation of each country’s GDP growth and bond yield during the sample period. All statistics are computed at a quarterly frequency over our sample period of 2005Q1 to 2011Q3. The first column of the table indicates that all countries in our sample have a positive average GDP growth over this period, however, there is significant heterogeneity in the average growth rate and volatility of growth across countries. The third and fourth columns of the table display the time series mean and standard deviation of the change in each country’s investment. Again, we see significant heterogeneity in these values across the sovereigns in our sample. The last two columns display the mean and standard deviation of each country’s 10-year government bond.

We collect CDS prices from CMA for each country in our sample. The prices are all for 5-year CDS contracts referencing the sovereign entity, with all contracts denominated in US dollars. Table III displays summary statistics for the time series of CDS prices for each of the sovereigns in our sample. We report the time series mean and standard deviation, as well as
the serial correlation of the monthly sovereign CDS prices. Comparing the first and second columns of Table III, we see that the time series standard deviation exceeds the mean for many sovereigns in our sample. In addition, the table shows that the sovereign CDS prices exhibit a high degree of serial correlation.

Figure 2 plots the time series of the cross-sectional mean and quartiles of the sovereign CDS prices in our sample. For each date, we compute the cross-sectional mean, median, 25th, and 75th percentiles of the sovereign CDS prices. Panel A displays the time series of the mean and Panel B presents the quartiles. Beginning in 2008, we observe a substantial rise in both the level and cross-sectional dispersion of CDS prices and this pattern continues through the remainder of our sample.

We next investigate the comovement of the sovereign CDS prices in our sample by estimating the first two principal components for our CDS price series. Panel A of Table IV reports the first two principal components estimated for the sovereign CDS prices for the countries in our sample. The first principal component explains 64% of the variation in the sovereign CDS prices and the first two principal components together explain 89% of the variation. As a point of reference, we estimate the first two principal components for the government long-term bond yields and GDP growth rates in our sample. These values are reported in Panels B and C of Table IV. The first principal component explains 60% and 63% of the variation in government bond yields and GDP growth rates, respectively.

From these data, we construct the variables used in equation (4) as follows. Using the 5-year sovereign CDS spreads and the U.S. Treasury yield curve, we compute the time series of quarterly solvency probabilities for each sovereign.\footnote{Note that this transformation of CDS spreads to solvency probabilities assumes a 40% recovery rate and a discount factor derived from the current Treasury yield.} For the gross returns on loans, $R_{i,t}$,
we directly use the yields on 10-year government bonds. For the amount of loans from one country to another, $l_{ijt}$, we directly use the BIS data on foreign claims on an ultimate risk basis. While these claims mostly have maturities longer than three months, the existence of secondary markets means that claims do not have to be held until maturity. Lastly, as noted before, we fix the exogenous recovery rate at $\delta = 0.4$.

5 Estimates and Model Fit

The parameters recovered by our estimation procedure are listed in Table V. The effect of foreign debts on a sovereign’s predicted solvency probability is substantial. To give a magnitude we consider a difference of 0.34 in the normalized debt load, which is the standard deviation of this variable. This is a large amount: a difference in total foreign debt equal to one-third of a country’s annual GDP in 2004. However several countries see their foreign debt range by more than this amount over our time period. The average marginal effect of the normalized debt load, scaled by this amount, is 43 bps. This is quite large relative to the average quarterly default probability within our estimation sample of 130 bps. The average marginal effect of the expected returns on financial claims is smaller: a one standard deviation (0.33) increase in the normalized measure of these expected returns would reduce the probability of default by 12 bps.

The two components of quarterly GDP growth rates also have substantial effects on the model’s predicted solvency probabilities. A one percentage-point increase in the common component reduces the sovereign’s probability of default in the next quarter by 60 bps. A one percentage-point increase in the residual component reduces the probability of default by 47
The effect of the normalized GDP level is smaller. Scaling the average marginal effect of this variable by its standard deviation (0.035) gives a change of 13 bps in the quarterly default probability.

Figure 3 plots the observed and predicted solvency probabilities to illustrate the dispersion in the data and the model’s fit. In particular, we plot each sovereign’s “observed” solvency probability, derived from its 5-year CDS spread, against the model’s predicted solvency probability for each quarter in our sample. Taking the model literally these would be interpreted as the observed and predicted market expectations for the probability that a sovereign does not default in the next quarter. We can see that for most of the observations, the observed and predicted solvency probabilities are both close to 1. However, the figure illustrates clear exceptions to this, most notably Greece, Ireland, and Portugal. Generally, the model predictions match their counterpart empirical values relatively well, as seen from the fact that most observations fall relatively close to the 45° line. The model appears to overestimate the solvency probability of Portugal and Ireland, relative to their empirical values, while it somewhat underestimates the solvency probability of Italy. The predictions for Greece fall closely on both sides of the 45° line and fit the variation for that country quite well.

In Table VI, we present additional results on the distribution of the “observed” solvency probabilities as well as the fit of the model’s predicted probabilities with the data from the 293 observations used in estimation. Panel A presents quartiles as well as the 5th and 95th percentiles of the distribution of quarterly solvency probabilities. The first column reports the empirical probabilities computed from the sovereign CDS spreads and the second column

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11 The standard deviations of these components of GDP growth rates are 0.9 (common) and 0.7 (residual), so the use of a one-unit change is reasonable relative to the variation in the data.
reports the counterpart values predicted by the model under the estimated parameter values
given in Table V. These results indicate that the observed and predicted distributions match
closely. Panel B reports the correlation of the observed and predicted solvency probabilities,
as well as the sum of squared residuals. These values are 0.839 and 0.05, respectively.

6 Simulations and Contagion

Using the estimated version of equation (4) we can simulate the short-run effect of the default
of one sovereign on the risk of default of other sovereigns in our network. This provides one
assessment of the potential for contagion. We can also construct a measure of the expected
spillover losses due to this increased probability of default of other sovereigns. Our measure,
which is described further below, can then be combined across countries to give a picture of
the total potential for contagion in the network over time.

To simulate the default of some country \( j \) in period \( t \), we fix the solvency indicator for
that country at zero \( (s_{jt} = 0) \) and recompute the solvency probabilities for all other countries
according to (4). The four panels in Figures 4 and 5 respectively plot the increase in the
default probability for selected sovereigns given a default in Greece, Portugal, Italy, and
Spain. These simulations should be considered separately for each period, as there are no
cumulative or long-run effects expressed in the model. There are notable differences in the
impact of a default of one of these four sovereigns. Greece and Portugal in Figure 4 each
pose a small threat to one large country: the risk of default in France or Spain in 2011 rises
by 3-4 bps following a default by Greece or Portugal, respectively. However toward the end
of the sample period Greece poses a larger threat to Portugal that is not reciprocated. This
can be explained by the fact that Portugal held a relatively large amount of Greek debt at
at that time: in 2011Q1 the normalized measure of Portugal’s claims on Greek debt is 0.14 (i.e., an amount equal to 0.14 times Portugal’s GDP in 2004). By contrast, the measure of Greek claims on Portugal’s debt is negligible. Between the two large economies in Figure 5, a default in Italy has substantially larger effects, particularly on France which sees increases in default risk of up to 20 bps (note the different scale for this plot). The overall difference between Italy and Spain is not surprising given that Italy’s foreign debt was three times larger than Spain’s in 2010 and 2011.

To account for the large differences across sovereigns in the amount of foreign debt, we construct a measure of the expected spillover in losses due to additional defaults, per dollar of foreign debt of the country with the original default. This measure is defined as follows. Given a default in country \( j \) in period \( t \), we use the above simulations to calculate the change in solvency probabilities among the other countries in the network. Let \( \hat{p}_{it} \) be the original predicted solvency probability for country \( i \) in period \( t \) using the estimated model, and let \( \tilde{p}_{it}(j) \) be the simulated solvency probability under the counterfactual that country \( j \) defaults. The expected loss from the increased probability of default at country \( i \) due to the default of country \( j \) would be \( [\hat{p}_{it} - \tilde{p}_{it}(j)]D_{it} \). The total expected spillover losses due to the default of country \( j \) is the sum of this across all countries \( i \neq j \). Finally, to normalize for the amount of foreign debt in the original default, our measure divides these expected losses by the total foreign debt of country \( j \), yielding

\[
\lambda_{jt} \equiv \frac{1}{D_{jt}} \sum_{i \neq j} [\hat{p}_{it} - \tilde{p}_{it}(j)]D_{it}
\]

This measure expresses the contagiousness of the foreign debt of country \( j \), per dollar of that debt. It captures differences in the potential for contagion that are due to a country’s
position in the network—i.e., who its creditors are, and how sensitive those creditors are to 
losses—rather than the amount of its foreign debt.

Figure 6 plots $\lambda_{jt}$ for the most at-risk sovereigns (Panel A) and for five large European 
economies (Panel B). The magnitudes of these spillovers are not large: for each $1$ of debt 
directly lost in default, the expected losses from additional defaults at other countries are 
less than 0.5 cents. The levels and trends are quite similar among all the countries in both 
panels.\footnote{The United Kingdom (GB) has lower levels than other countries because a relatively large proportion of its debt is held outside Europe (e.g., by the United States). This debt is included in the normalization but cannot be counted toward spillover losses.} Figure 7 shows $\lambda_{jt}$ for smaller European economies (Panel A) and the weighted 
average among all the sovereigns in our sample (Panel B). Austria’s foreign debt has the 
highest potential for contagion, with expected spillover losses per dollar of debt that are 
roughly double those from any other sovereign (note the different scale for this plot). This 
turns out to be the case because Italy holds a relatively large share of Austria’s debt, and 
Italy is more sensitive to any losses on its claims because it has a somewhat higher risk of 
default.

Finally, to assess how sovereign default risk and the overall potential for contagion has 
changed over time, we consider the total expected amounts of direct losses and of spillover 
losses. For the former we simply multiply total debt ($D_{it}$) by the predicted default probability 
($1 - \hat{p}_{it}$) and add across countries. For the latter we multiply the expected spillover losses 
from each single-country default by the predicted default probability for that country, and 
add across countries.\footnote{In other words, the total expected spillover losses are: $\sum_j (1 - \hat{p}_{jt}) \sum_{i \neq j} [\hat{p}_{it} - \tilde{p}_{it}(j)] D_{it}$.} These series are plotted in Figure 8. The total expected losses 
exceed $500$ billion in 2011 (as before, each quarter should be considered independently). 
The expected losses due to contagion are a small portion of the total (about $1/500$). They
rise rapidly starting in the second half of 2010, to just under $2 billion at the end of our sample period in 2011.

7 Conclusion

The recent sovereign debt crisis in the Eurozone has renewed concerns of financial contagion. With interconnected borrowing and lending relationships between Eurozone sovereigns, the default of one sovereign can lead to heightened spreads and potentially subsequent defaults among other sovereigns. In this paper we evaluate these concerns empirically. We construct a network model of cross-holdings of sovereign debt among Eurozone countries. Using the cross-holdings of sovereign debt from the data, we estimate parameters of the network model and conduct counterfactual experiments to quantify the effects of contagion in the network. Our estimates suggest a role for a direct contagion effect resulting from a sovereign default. In some cases we find a significant increase in borrowing costs as the result of contagion in the network.
References


Table I:
Country List

This table lists the countries included in our sample.

<table>
<thead>
<tr>
<th>Country</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>AT</td>
</tr>
<tr>
<td>Belgium</td>
<td>BE</td>
</tr>
<tr>
<td>Finland</td>
<td>FI</td>
</tr>
<tr>
<td>France</td>
<td>FR</td>
</tr>
<tr>
<td>Germany</td>
<td>DE</td>
</tr>
<tr>
<td>Greece</td>
<td>GR</td>
</tr>
<tr>
<td>Ireland</td>
<td>IE</td>
</tr>
<tr>
<td>Italy</td>
<td>IT</td>
</tr>
<tr>
<td>Netherlands</td>
<td>NL</td>
</tr>
<tr>
<td>Portugal</td>
<td>PT</td>
</tr>
<tr>
<td>Spain</td>
<td>ES</td>
</tr>
<tr>
<td>Sweden</td>
<td>SE</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>GB</td>
</tr>
</tbody>
</table>
Table II: Summary Statistics for Macroeconomic Variables

This table reports the mean, $\mu(\cdot)$, and standard deviation, $\sigma(\cdot)$, for each country’s quarterly GDP growth ($\Delta y$) and long-term government bond yield ($r$) over the sample period 2005Q1 - 2011Q3. Further details regarding the data are provided in Section 4.

<table>
<thead>
<tr>
<th>Country</th>
<th>$\mu(\Delta y)$</th>
<th>$\sigma(\Delta y)$</th>
<th>$\mu(r)$</th>
<th>$\sigma(r)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>0.45</td>
<td>0.88</td>
<td>3.78</td>
<td>0.49</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.34</td>
<td>0.75</td>
<td>3.87</td>
<td>0.44</td>
</tr>
<tr>
<td>Finland</td>
<td>0.35</td>
<td>1.75</td>
<td>3.68</td>
<td>0.53</td>
</tr>
<tr>
<td>France</td>
<td>0.23</td>
<td>0.62</td>
<td>3.71</td>
<td>0.47</td>
</tr>
<tr>
<td>Germany</td>
<td>0.40</td>
<td>1.16</td>
<td>3.47</td>
<td>0.58</td>
</tr>
<tr>
<td>Greece</td>
<td>0.06</td>
<td>1.22</td>
<td>6.25</td>
<td>3.65</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.14</td>
<td>2.01</td>
<td>5.13</td>
<td>2.03</td>
</tr>
<tr>
<td>Italy</td>
<td>0.02</td>
<td>0.94</td>
<td>4.28</td>
<td>0.48</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.37</td>
<td>0.85</td>
<td>3.66</td>
<td>0.52</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.06</td>
<td>0.82</td>
<td>4.9</td>
<td>1.90</td>
</tr>
<tr>
<td>Spain</td>
<td>0.29</td>
<td>0.74</td>
<td>4.16</td>
<td>0.58</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.55</td>
<td>1.32</td>
<td>3.47</td>
<td>0.55</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.19</td>
<td>0.94</td>
<td>4.19</td>
<td>0.62</td>
</tr>
<tr>
<td>Average</td>
<td>0.28</td>
<td>1.08</td>
<td>4.18</td>
<td>0.98</td>
</tr>
</tbody>
</table>
Table III: Summary Statistics for Sovereign CDS Prices

This table presents summary statistics for prices of 5-year Credit Default Swaps referencing the sovereign entities in our sample. The columns labeled ‘Mean’ and ‘Std Dev’ display the time series mean and standard deviation of the CDS price series. The autocorrelation of the monthly CDS prices is given in the column labeled ‘AC(1)’. The final column reports the number of monthly observations available for each sovereign’s CDS price series. All CDS prices are taken from CMA.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std Dev</th>
<th>AC(1)</th>
<th>Min</th>
<th>Max</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>47.8</td>
<td>58.9</td>
<td>0.95</td>
<td>1.5</td>
<td>235</td>
<td>100</td>
</tr>
<tr>
<td>Belgium</td>
<td>61.6</td>
<td>83.2</td>
<td>0.97</td>
<td>1.2</td>
<td>307.3</td>
<td>100</td>
</tr>
<tr>
<td>Finland</td>
<td>39</td>
<td>20.6</td>
<td>0.86</td>
<td>7.3</td>
<td>85.3</td>
<td>48</td>
</tr>
<tr>
<td>France</td>
<td>48.5</td>
<td>57.4</td>
<td>0.98</td>
<td>1.5</td>
<td>214.7</td>
<td>81</td>
</tr>
<tr>
<td>Germany</td>
<td>24</td>
<td>28.3</td>
<td>0.95</td>
<td>0.9</td>
<td>110.8</td>
<td>81</td>
</tr>
<tr>
<td>Greece</td>
<td>427.8</td>
<td>1020.3</td>
<td>0.92</td>
<td>5</td>
<td>6499.8</td>
<td>97</td>
</tr>
<tr>
<td>Ireland</td>
<td>175.5</td>
<td>253.3</td>
<td>0.99</td>
<td>2</td>
<td>882.1</td>
<td>105</td>
</tr>
<tr>
<td>Italy</td>
<td>97.6</td>
<td>127.3</td>
<td>0.97</td>
<td>5.6</td>
<td>485.4</td>
<td>100</td>
</tr>
<tr>
<td>Netherlands</td>
<td>36.6</td>
<td>35.9</td>
<td>0.91</td>
<td>1.2</td>
<td>120.8</td>
<td>80</td>
</tr>
<tr>
<td>Portugal</td>
<td>208.4</td>
<td>363.9</td>
<td>0.98</td>
<td>2</td>
<td>1601</td>
<td>100</td>
</tr>
<tr>
<td>Spain</td>
<td>115.6</td>
<td>134.4</td>
<td>0.97</td>
<td>2</td>
<td>462.5</td>
<td>85</td>
</tr>
<tr>
<td>Sweden</td>
<td>29.2</td>
<td>29.9</td>
<td>0.80</td>
<td>1.4</td>
<td>140</td>
<td>105</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>63.8</td>
<td>31.7</td>
<td>0.92</td>
<td>7</td>
<td>147.3</td>
<td>54</td>
</tr>
<tr>
<td>Average</td>
<td>105.8</td>
<td>172.7</td>
<td>0.94</td>
<td>2.97</td>
<td>868.6</td>
<td>88.8</td>
</tr>
</tbody>
</table>
Table IV: Principal Components Analysis

This table presents the first two principal components for the CDS prices (Panel A), 10-year bond yields (Panel B), and GDP growth rates (Panel C), for the sovereigns in our sample. CDS prices are taken from CMA, bond yields are from the International Financial Statistics tables from the IMF, and GDP growth rates are computed from the OECD’s quarterly national accounts.

<table>
<thead>
<tr>
<th>Panel A: CDS Prices</th>
<th>Proportion</th>
<th>Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component 1</td>
<td>0.640</td>
<td>0.640</td>
</tr>
<tr>
<td>Component 2</td>
<td>0.248</td>
<td>0.888</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Bond Yields</th>
<th>Proportion</th>
<th>Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component 1</td>
<td>0.599</td>
<td>0.599</td>
</tr>
<tr>
<td>Component 2</td>
<td>0.246</td>
<td>0.845</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: GDP Growth Rates</th>
<th>Proportion</th>
<th>Cumulative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component 1</td>
<td>0.630</td>
<td>0.630</td>
</tr>
<tr>
<td>Component 2</td>
<td>0.079</td>
<td>0.709</td>
</tr>
</tbody>
</table>

Table V: Parameter Estimates

This table presents our main estimates of equation (4). The data and construction of the variables are described in Section 4.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>1.293</td>
<td>GDP level (normalized by 2004 GDP)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.204</td>
<td>Common component of GDP growth rate</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td>0.159</td>
<td>Residual component of GDP growth rate</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.433</td>
<td>Foreign debt x 1+r (normalized by 2004 GDP)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.122</td>
<td>Foreign claims x 1+r (normalized by 2004 GDP)</td>
</tr>
<tr>
<td>$\pi_0$</td>
<td>2.726</td>
<td>Threshold</td>
</tr>
<tr>
<td>$\pi_1$</td>
<td>-0.075</td>
<td>Time trend</td>
</tr>
</tbody>
</table>
Table VI:
**Observed and Model Predicted Solvency Probabilities**

This table compares the observed empirical solvency probabilities, derived from 5-year CDS spreads, to those predicted from the estimated model. Panel A reports percentiles from the observed and model-predicted distribution of solvency probabilities. Panel B reports the correlation of the observed and predicted probabilities, the sum of squared residuals from a regression of the observed on the predicted solvency probabilities, and the number of observations in our estimation sample.

<table>
<thead>
<tr>
<th>Percentile</th>
<th>Observed</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th</td>
<td>0.946</td>
<td>0.949</td>
</tr>
<tr>
<td>25th</td>
<td>0.987</td>
<td>0.983</td>
</tr>
<tr>
<td>50th</td>
<td>0.995</td>
<td>0.994</td>
</tr>
<tr>
<td>75th</td>
<td>&gt; 0.999</td>
<td>&gt; 0.999</td>
</tr>
<tr>
<td>95th</td>
<td>&gt; 0.999</td>
<td>&gt; 0.999</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho(\text{Observed, Predicted}) )</td>
<td>0.839</td>
<td></td>
</tr>
<tr>
<td>SSR</td>
<td>0.050</td>
<td></td>
</tr>
<tr>
<td>( N \text{ obs.} )</td>
<td>293</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: Network Graph. This figure displays the network structure of lending relationships in the first quarter of 2011. Countries are represented by their two letter abbreviation in Table I. Arrows represent claims that one country has on another, and darker arrows indicate larger amounts in proportion to the creditor country’s GDP in 2004. For further description, see Section 4.
Figure 2: Mean and Quartiles for CDS Prices. This figure plots the mean, median, and first and third quartiles of the cross-section of the CDS prices for our sample of sovereigns. In Panel (A) the time series of the cross-sectional mean is plotted for January 2003-April 2012. Panel (B) plots the time series of the cross-sectional median CDS price (solid black line) along with the 25th and 75th percentiles (blue dotted lines). All CDS prices are for 5-year CDS contracts referencing the sovereign entity. The sovereigns comprising our sample are listed in Table I.
Figure 3: Predicted and Observed Solvency Probabilities. This figure plots the predicted and observed quarterly solvency probabilities for each country at each quarter in our sample. The observed solvency probabilities are obtained with a transformation of 5-year CDS contract prices, as described in Section 4. The predicted solvency probabilities are from the estimated version of equation (4). Country abbreviations are listed in Table I.
Figure 4: Simulated Default Probabilities: Greece and Portugal. This figure shows the change in the probability of default for selected sovereigns assuming a default in Greece (A) or Portugal (B), in simulations using the estimated equation (4). See Section 6 for details.
Figure 5: Simulated Default Probabilities: Italy and Spain. This figure shows the change in the probability of default for selected sovereigns assuming a default in Italy (A) or Spain (B), in simulations using the estimated equation (4). See Section 6 for details.
Figure 6: Expected Spillover Losses: At-Risk Sovereigns and Large Economies. Measure of expected losses due to increased risk of default at other countries, conditional on a default at the single country indicated in each series. See equation (5) for definition.
Figure 7: Expected Spillover Losses: Medium or Small Economies, and Weighted Average. Measure of expected losses due to increased risk of default at other countries, conditional on a default at the single country indicated in each series. See equation (5) for definition. Weighted average is among all countries in the sample, weighted by each country’s total foreign debt.
Figure 8: Total Expected Losses and Expected Losses Due to Contagion of Default. Amount of expected losses from all sovereigns in the sample, based on predicted solvency/default probabilities, and amount of expected spillover losses. See Section 6 for further description.