Embracing Risk: Hedging Policy for Firms with Real Options*

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December 27, 2013

ABSTRACT

We analyze the dynamic risk management strategy of firms that face a tradeoff between minimizing the costs of financial distress and maximizing financing for future investment. Costly external financing of lumpy investment discourages full hedging because hedging can increase the financing costs and decrease the probability of investment. First, we show that firms with safe assets can choose to hedge more aggressively than firms with risky assets. Second, firms prefer to hedge systematic rather than firm-specific risk, even when hedging technologies for both types of risk are available and equally costly. Third, it is optimal not to hedge when cash savings are low and do not cover investment needs. Therefore, more constrained firms may appear to hedge less aggressively. Our theory generates comprehensive results consistent with actual hedging policies, and without relying on the costs of risk management.

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Contemporary research on determinants of corporate hedging policy has stumbled upon several facts that are seemingly inconsistent with firm optimization. First, hedging activity tends to be concentrated in large mature firms with few growth options (Bartram, Brown, and Fehle (2009), Nance, Smith, and Smithson (1993), Tufano (1996), Haushalter (2000), Graham and Rogers (2002), Mian (1996), Allayannis and Ofek (2001), Gay and Nam (1998)). Second, firms with tighter financing constraints do not appear to hedge more than firms with fewer constraints (Rampini, Sufi, and Viswanathan (2013)). Finally, there is puzzling evidence that riskier firms tend to have lower hedging ratios (see, e.g., Guay (1999)). These facts seem to run counter to the basic prediction that hedging policy is dictated by firm risk.

The goal of this paper is to reconcile these empirical regularities with theory by building a model of real investment and financing. The simple fact that the exercise of investment options typically requires large capital expenditures goes a long way toward explaining the observed hedging policies.[1] Option financing can rationalize, for example, the lack of hedging by the constrained low-net-worth firms since without risk in their cash flows they would never have enough cash to invest. Similarly, it can explain why riskier firms, which inherently have more investment options, hedge less than safer firms.

Intuitively, risk management does not always increase value because it jeopardizes chances to undertake investment for some firms. For example, consider a company with a single profitable fixed-value option, which can only be financed internally. A low-expected-net-worth firm will lack sufficient financing for a large investment project and will abandon it unless its cash flow turns out to be very high. Since hedging effectively moves cash away from the high- to the low-profitability states, it makes the prospect of undertaking investment even more remote and is value-destroying absent any other benefits of hedging. In contrast, a high-expected-net-worth firm will have sufficient financing for a project even with average profitability. Hedging is desirable in this case because it increases the amount of cash in the low-profitability states where investment was previously impossible and at the same time preserves the level of investment in other profitability states.

[1]The assumption of a lump-sum investment is in line with actual experience and can be justified by frictions such as investment indivisibility, fixed costs, and irreversibility.

When the value of investment options increases with profitability, an additional "cash flow correlation" effect appears. If options are in-the-money, the cash flows from the existing assets also tend to be high and thus provide a convenient source of investment funds (e.g., Froot, Scharfstein, and Stein (1993)). Over-hedging decreases the coordination between a firm's investment demand and internal funds, thereby necessitating costly external financing. In our model, a natural positive correlation between cash flows and investment is higher in risky firms and in those with more valuable real options.

The model exhibits parsimony in its assumptions, requiring that investment is larger than operating costs and that external financing is costly. The optimal risk management strategy balances the costs of financial distress with the costs of financing for investment. We provide closed-form solutions for the optimal hedging ratio in the one-period example and then in the full intertemporal model with dynamic cash accumulation. The solution is fairly general and can accommodate cases with common and idiosyncratic risk components, the cost of hedging, and nonlinear hedging strategies.

First, the model shows that the relation between the hedging ratio and asset risk is non-monotonic. In particular, firms with riskier assets may want to leave a larger proportion of their risk unhedged. Intuitively, firms that have riskier cash flows have more valuable real options. Therefore cash flows and the demand for financing tend to have a higher correlation in these firms. In contrast, safer firms exercise fewer growth options and are mainly concerned with eliminating negative cash flow outcomes, which is accomplished by hedging most of their risk exposure. The model shows that the negative relation between asset risk and hedging is stronger when there are fixed costs associated with investment.

Second, and paradoxically, stronger financing constraints can lead to less hedging. In particular, we show that the low-expected-net-worth firms will maximize the value of their investments by leaving their profits unhedged. In a sense, this effect is akin to risk-taking behavior of growth firms, since the value of options increases with underlying risk. The difference, however, is that we are discussing the financing effect that arises because a higher risk in cash savings can increase the probability of investment. Therefore, our results provide a new explanation for the lack of risk management that does not require high costs of hedging or agency problems.

Third, using the model, we can measure the incremental value of the dynamic risk management policy relative to a one-shot policy. In static models, a fixed hedging ratio must balance losses in some regions with gains in others. One must evaluate the firm's payoff function over the whole domain of the underlying risk variable to determine whether this function is overall concave or convex. Under the dynamic policy, the firm can tailor its hedging strategy to specific circumstances, such as the current liquidity position. For example, we show that when the firm's current cash savings are large, the value function is concave and therefore hedging is optimal. In contrast, firm value exhibits convexity when cash savings are relatively low, and it becomes optimal to increase the volatility of cash savings to maximize the value of investment.

Further, we examine how the optimal risk management strategy is affected by the presence of other firms in the economy. The model predicts that the optimal hedging ratio will be higher for firms with more systematic risk (i.e., common risk across firms) and lower for firms with more idiosyncratic risk. To arrive at this result, we rely on the view that firms mainly compete for the common component of their profits and therefore derive less value from investment options that are linked to the overall state of economy.[2] In turn, the limit to growth imposed by the interaction between firms decreases the correlation between cash flows and option value. Risk management policy therefore depends not only on the amount but also on the type of risk that firms are subjected to.

The rest of the paper is organized as follows. The next section offers a brief literature review. Section 2 presents a single-period model, which explains how risk management affects investment. Section 3 lays out a general continuous-time model of investment under financing

[2]The basic idea is that competition erodes the value of a firm's real options (see, e.g., Grenadier (2002)). Competition matters more if the values of assets and future investment options are mainly driven by the common component of risk. For example, if the investment opportunities improve uniformly for all firms in an industry, the increase in competition associated with higher aggregate production and new entry into the market will limit each firm's profits (see, e.g., Caballero and Pindyck (1996) for a discussion of competition effects on real options). However, when a firm's success is *unique*, its real options are likely to increase in value, implying that firms that start with unique assets derive a larger component of their market value from real options than firms with generic assets.

constraints, which allows for cash savings and a dynamic hedging ratio. The last section concludes.

I. Literature Review

Most studies on risk management show how financial hedging can create value. Benefits come from the reduction in expected bankruptcy costs (e.g., Smith and Stulz (1985) and Graham and Smith (1999)); higher debt capacity (Leland (1998) and Graham and Rogers (2002)); convexity in operating costs and/or concavity in the production function (Froot, Scharfstein, and Stein (1993) and Mackay and Moeller (2007)); improvement in contracting terms with firm creditors, customers, and suppliers (Bessembinder (1991)); mitigation of information asymmetry (DeMarzo and Duffie (1995)); and reduction in management overinvestment incentives (Morellec and Smith (2007)).

It is somewhat more difficult to justify insufficient risk management, especially given the recent developments in derivatives trading and reductions in transaction costs. Froot, Scharfstein, and Stein (1993) were the first to argue that firms should not fully eliminate risk exposure when their cash flow is positively correlated with investment options. Our end results differ from those of Froot, Scharfstein, and Stein (1993) because we introduce investment frictions into the model, allow firms to abandon investment if financing is prohibitively expensive, and also consider dynamic policies. A related paper by Adam, Dasgupta, and Titman (2007) shows that incomplete hedging can arise from competition. Their study recognizes that a firm's optimal risk management can depend on its competitors' strategies since the goal of the firm is to increase cash reserves in those states of the world where the competitors lack financing.[3]

Methodologically, we contribute to the new literature on dynamic risk management, cash policies, and investment. Bolton, Chen, and Wang (2011) build a structural investment model with adjustment costs and among other results derive the closed-form expression for the

[3] We also discuss market competition in later parts of the paper. However, unlike Adam, Dasgupta, and Titman (2007), we allow firm entry to depend on the realization of profit shock and also let the firm seek external financing.

optimal hedging ratio. In their model, the firm's investment opportunity set does not depend on the current profitability shock, and the value function is everywhere concave, inducing hedging.[4] Fehle and Tsyplakov (2005) present a dynamic model, in which full hedging is suboptimal when the firm is in deep financial distress or very far from it. Their results rely on leverage, the costs of financial distress, and the fixed costs of hedging.

Similar to us, Rampini and Viswanathan (2010) obtain the result that financially constrained firms can prefer to hedge less, but using a different mechanism. In their model, both financing and risk management involve promises to pay that need to be collateralized, thereby resulting in a tradeoff between a firm's ability to finance current investment and engage in risk management in order to maximize future investment. Since more constrained firms find it more advantageous to use the available cash for current investment, a negative relation between financial constraints and risk management arises. In our setting, there are no collateral constraints, and hence risk management is not limited by the current amount of cash. Nevertheless, firms with low expected-net-worth can prefer to hedge less because this allows them to maximize the probability of future investment.

Our work is also related to studies on investment and cash policies under financing constraints. Bolton, Wang, and Yang (2013) show that the investment threshold is non-monotonic in cash in a real option model with financing constraints. Boyle and Guthrie (2003), Dasgupta and Sengupta (2007) study the implications of cash uncertainty on investment. In these two papers, cash flow risk matters because firms choose investment timing keeping the uncertainty of future funding in mind. Our idea is much simpler: investment increases when more cash is available, which means it must depend on a firm's hedging policy.

II. Investment and Risk Management: Single-Period Analysis

To fix ideas, we develop a model of risk management under financing constraints that builds on the investment literature. We first analyze the general solution for the optimal hedging strategy and then proceed with a discussion of how a firm's net worth, firm risk and its

[4]Bolton, Chen, and Wang (2011) adopt a simplifying modeling assumption that profitability shocks follow an arithmetic Brownian motion.

composition, product market competition, and investment opportunities affect the optimal hedging ratio.

A. Preliminaries

There are three dates in the model corresponding to: (1) choice of the hedging strategy, (2) realization of uncertainty and optimal investment, and (3) the final payoff.[5] There is no discounting. We assume that the firm can hedge its cash flow risk by buying forward contracts and postpone the discussion of nonlinear hedging strategies. If a firm chooses hedging ratio ϕ, the internal funds available at date 2 are

$$w = w_0 + w_1 \left(\phi \bar{\varepsilon} + (1 - \phi) \varepsilon \right), \tag{1}$$

where ε is the primitive uncertainty (the cash flow shock) governed by the probability distribution function $g(\varepsilon)$ with variance σ^2. We assume that $E(\varepsilon) = \bar{\varepsilon}$, which implies that hedging leaves the expected cash flow unchanged.[6] If $\phi = 1$, the firm's internal funds are independent of the profitability shock.

Assets in place require operating costs I_0 (minimum necessary expenses to keep the firm running) at date 2 and will generate a certain profit f_0 at date 3. The assumption of positive operating costs is necessary for a description of the firm's financing problem when it does not invest but keeps its assets idling. It also allows us to model the costs of financial distress when a firm's cash flow decreases below the level of operating costs.

The firm also holds an investment option that allows it to update the existing assets at a fixed cost I payable at date 2. If assets are updated, their profitability increases from f_0 to $\theta f(I)$, with random variable θ capturing the uncertainty in the firm's investment opportunities at date 2. Note that the assumption that new assets will render old ones unproductive (profit f_0 is not available if investment is made) is equivalent to assuming that there are fixed costs of investment. Such costs may, for example, originate from new investment cannibalizing profits associated with assets in place (see, e.g., Hackbarth, Richmond, and Robinson (2012)). To

[5] If the payoff is immediate, the date t=3 is redundant. This notation simply clarifies that the final payoff cash cannot be used for investment.

[6] Hedging could involve a cost (fixed or variable) that could easily be accommodated in the model.

model the fact that firm's investment opportunities are correlated with cash flow shocks, we specify

$$\theta = \alpha\left(\varepsilon - \overline{\varepsilon}\right) + \beta, \tag{2}$$

where $\alpha \geq 0$ captures the correlation and higher β implies better investment opportunities. We also assume that investment technology has decreasing returns to scale, $f_I > 0,\ f_{II} < 0$, and that the amount of investment includes and exceeds operating costs, $I > I_0$.

Whenever the firm is short of internally generated funds for investment or operating costs, it can raise external financing e available at a cost $C(e)$, ($C_e > 0$, $C_{ee} > 0$). As in Kaplan and Zingales (1997) and Hennessy and Whited (2007), the convexity of the external cost function ensures that the optimal investment is both finite and unique.

The firm updates its existing assets if the profit net of investment and financing costs is higher than the profit associated with running existing assets

$$\theta f(I) - I - C(e) \geq F, \tag{3}$$

$$\text{with } F = f_0 - I_0 - C(e_0), \tag{4}$$

where e and e_0 are the financing gaps given, respectively, by $e = I - w$, $e_0 = I_0 - w$. Setting (3) to equality gives the implicit condition for investment threshold ε^*, such that investing in the new technology is optimal only for the cash flow shock above such threshold, i.e., when $\varepsilon > \varepsilon^*$.

In the investment region, the firm chooses the optimal I by solving:

$$\max_I \{\theta f(I) - I - C(e)\}, \tag{5}$$

which results in the first-order condition:

$$\theta f_I\left(I^*\right) - 1 = C_e\left(e\right). \tag{6}$$

To summarize, the firm's profit function in the no-investment (inaction) and investment region is given by

$$\Pi\left(\varepsilon\right) = \left\{ \begin{array}{cc} P = \theta f(I^*) - I^* - C(e), \text{ if } \varepsilon \geq \varepsilon^*, & \text{investment region} \\ P_0 = f_0 - I_0 - C(e_0), \text{ if } \varepsilon < \varepsilon^*, & \text{inaction region} \end{array} \right\}, \tag{7}$$

where the threshold ε^* is determined endogenously.

B. Optimal Hedging Ratio

Before we proceed to analyzing the firm's optimal hedging policy, it is worthwhile to examine how hedging affects investment.

Lemma 1. *Suppose there exists a finite investment threshold ε^*. Then, we have:*

(i) Suppose $\varepsilon^* > \bar{\varepsilon}$. Then hedging increases the investment threshold, $\frac{d\varepsilon^*}{d\phi} > 0$, and decreases the optimal investment level, $\frac{dI^*}{d\phi} < 0$.

(ii) Suppose $\varepsilon^* < \bar{\varepsilon}$. Then hedging decreases the investment threshold, $\frac{d\varepsilon^*}{d\phi} < 0$. The effect of hedging on the optimal investment level is ambiguous.

Proof. All proofs are in the Appendix. □

Intuitively, when a firm decides whether to invest, it internalizes the costs of external financing, which increase with the financing gap. By hedging, the firm effectively transfers cash from the high-profitability states (i.e., from the states above the average, $\varepsilon > \bar{\varepsilon}$) to the low-profitability states ($\varepsilon < \bar{\varepsilon}$). Therefore, investment that was previously undertaken only in the high-profitability states ($\varepsilon^* > \bar{\varepsilon}$) will require a larger amount of external financing when the firm decides to hedge. This will make investment less profitable and lead to an increase in the investment threshold (case (i)). In contrast, if investment was previously undertaken in the low-profitability states ($\varepsilon^* < \bar{\varepsilon}$), it will require a smaller amount of external financing after hedging, which results in a decrease in the investment threshold (case (ii)).

Using these results, we now derive the hedging ratio, which maximizes the expected profit:

$$\max_{\phi} E\left[\Pi(\varepsilon)\right]. \tag{8}$$

The solution to this problem is easiest when the investment payoff is uncorrelated with firm profitability.

Proposition 1. *Suppose $\alpha = 0$ and $\phi \in [0, 1]$. Then, the optimal hedging ratio is given by:*

(i) $\phi^* = 1$ for $\varepsilon^* < \varepsilon_L$,

(ii) $\phi^* = 0$ for $\varepsilon^* \in [\varepsilon_L, \varepsilon_H]$, and

(iii) $\phi^* = 1$ for $\varepsilon^* > \varepsilon_H$,

where the expressions for ε_L and ε_H are given in the Appendix.

Two opposing effects determine the choice of optimal hedging. On the one hand, convexity of costs ($C_{ee} > 0$) and concavity of revenues ($f_{II} < 0$) create an incentive to hedge (as in Froot, Scharfstein, and Stein (1993)). On the other hand, given that the firm has an investment option, there is a disadvantage to hedging. The benefit of hedging is independent of the investment threshold, but the disadvantage is largest when the option value is large, i.e., when the investment option is neither far out-of-the-money nor deep in-the-money. Therefore, a firm will prefer not to hedge for intermediate investment thresholds $\varepsilon^* \in [\varepsilon_L, \varepsilon_H]$.

Since the investment threshold depends on the firm's investment opportunities, β, it follows from the proposition that the hedging ratio is non-monotonic in β. In particular, we show below that hedging ratio can be lower for firms with more valuable investment options.[7]

Corollary 1. *Suppose $\varepsilon^* > \varepsilon_L$. Then, firms with more valuable growth options choose to hedge less, i.e., $\frac{\Delta \phi^*}{\Delta \beta} \leq 0$.*

This result follows from the fact that for firms with very poor investment opportunities (low β), the probability of investment is small. Therefore, the risk management policy in these firms is mostly driven by the desire to minimize the costs of financial distress, which is achieved by choosing high hedging ratios. In contrast, firms with better investment opportunities are concerned about investment financing, which results in choice of lower hedging ratios.

We next show that tighter financing constraints (lower w_0) can lead to less hedging.

Corollary 2. *Suppose $\varepsilon^* < \varepsilon_H$. Then, easing of financing constraints leads weakly to more hedging, i.e., $\frac{\Delta \phi^*}{\Delta w_0} \geq 0$.*

The intuition is that greater savings lead to lower costs of external financing and therefore a lower investment threshold. In this case, it is optimal for a firm to fully hedge its risk exposure since such action guarantees investment and at the same time decreases the costs of external financing when the profitability is low.

[7] The empirical literature generally finds that hedging activity is concentrated in firms with few growth options, as measured by their book-to-market ratios (see, e.g., Bartram, Brown, and Fehle (2009), Mian (1996), Graham and Rogers (2002), Allayannis and Ofek (2001), Haushalter (2000), Gay and Nam (1998). Two papers by Guay (1999) and Geczy, Minton, and Schrand (1997) find the opposite result.

Observing that the effects limiting the value of hedging come from the firm holding investment options, we conjecture that the high volatility firms—i.e., companies holding more valuable options—will be less aggressive in their cash flow risk management.

Corollary 3. *There exists a range of cash flow volatility, σ, where $\Delta\phi^*/\Delta\sigma < 0$.*

To understand why riskier firms may prefer lower hedging ratios, consider the case when $\varepsilon^* > \bar{\varepsilon}$ and the volatility of the profitability shock is close to zero. In this case, the probability of ever reaching the investment threshold is negligibly small, and the hedging ratio is equal to 1.[8] When the volatility increases, however, the probability that the shock crosses the threshold increases. Additionally, the level of investment becomes larger because larger cash flow shocks can be drawn from a distribution with a higher volatility. As a result, we have a lower hedging ratio. Therefore, ϕ^* must decrease in volatility at least in some continuous range of the volatility parameter values.

We next explore how a positive correlation between a firm's investment opportunities and cash flows affects optimal hedging.

Proposition 2. *For all $\alpha > \underline{\alpha}$, the optimal hedging ratio is given by*

$$\phi^* = 1 - \frac{\alpha}{w_1} \frac{\int_{\varepsilon^*}^{\infty} f_I k(e) dG}{\int_{-\infty}^{\varepsilon^*} C_{ee}(e_0)\, dG - \int_{\varepsilon^*}^{\infty} \theta f_{II} k(e) dG}, \tag{9}$$

where

$$k(e) = \frac{C_{ee}(e)}{C_{ee}(e) - \theta f_{II}(e)}, \tag{10}$$

and $\underline{\alpha}$ is the solution of (75) given in the Appendix.

The proposition shows that the natural tendency of real options to become more valuable when the firm is profitable can discourage hedging. In particular, it follows from (9) that firms with a positive correlation between cash flows and investment opportunities ($\alpha > 0$) can use incomplete hedging and that the amount of optimal hedging decreases when the correlation is greater.

[8] More formally, observe that the threshold ε^* is independent of volatility and that $\varepsilon^* > \bar{\varepsilon}$; therefore $\Pr(\varepsilon > \varepsilon^*) \longrightarrow 0$ when $\sigma \to 0$.

C. Comparative Statics

Figure 1 illustrates the effect of hedging on financing of investment and cash shortfall in the distress region. For exposition purposes, we assume here that external financing is prohibitively expensive and investment is possible only out of internal funds. Parameters are chosen in such a way that investment generates a higher profit than running existing assets. Panels A and B are for the cases of $w_0 = 0$ (low net worth) and $w_0 = 30$ (high net worth), respectively. If the low-net-worth firm fully hedges, the investment never occurs. In contrast, the high-net-worth firm always invests when fully hedged. The filled areas show regions of investment (on the right) and distress (on the left) when the firms do not hedge.

In Figure 2, we plot the expected investment level as a function of hedging policy when cash flows and investment opportunities are uncorrelated ($\alpha = 0$). For illustration purposes, we assume a logarithmic payoff function and quadratic costs of external financing. Lemma 1 establishes that the effect of risk management on investment depends on the profitability of investment. If the investment option is out-of-the-money when hedging ratio is zero, then hedging decreases the probability of investment. Indeed, Panel A shows that the expected investment steadily drops with hedging, with the probability of investment eventually going down to zero at high hedging ratios. In contrast, if the investment option is in-the-money when hedging ratio is zero, hedging encourages investment. This effect is observed in Panel B. In fact, when hedging ratio is high, the investment reaches the first-best level, i.e., the investment made by an unconstrained firm.

Figure 3, Panels A and B, provide an illustration of the volatility effect on optimal hedging policy. We plot the optimal investment amount and cash flow realizations for a firm with positive correlation between investment opportunitis and cash flows. The difference between the investment amount and cash represents the "financing gap," which requires external financing. Because of a positive correlation between investment opportunities and cash flow, the optimal investment amount increases with cash flow. Panel A shows what happens when the firm has high expected distress costs in a low profitability state but does not hedge any risk exposure. For comparison we also provide a corresponding function in Panel B, which shows what happens when the firm hedges completely. It is clear that if the firm fully hedges, it minimizes the costs of financial distress where cash is low (since a constant financing gap minimizes convex costs). However, the financing gap of the firm is not constant in the investment region and increases with higher cash flow shocks. Therefore, the firm with a high hedging ratio will incur, in expectation, large costs of raising external financing for investment. In fact, by comparing Panel A and Panel B, we can see that a fully hedged firm will even decrease the amount of state-contingent investment because of financing costs.

The next pair of panels illustrates how firm risk affects hedging. Panels B and C plot the investment demand and cash relation for the case when the volatility of a firm's cash flow is low (Panel C and D). Intuitively, the probability of state-contingent investment is low, and therefore demand for cash is unchanged across states of profitability. It follows that maximum hedging is optimal in this case.

Next, Figure 4 shows how the value of investment and the optimal hedging ratio changes with volatility of the assets. In Panel A of Figure 4, we show the expected investment amount of a constrained firm (solid line). The dashed line displays the expected investment level of an unconstrained firm. It is easy to see that financial constraints affect both the level and probability of investment, and that the expected investment increases in volatility. Finally, in Panel B of Figure 4, we plot the optimal hedging ratio of a firm as a function of volatility. Indeed, we observe that at low volatility, the firm prefers to hedge all of its risk exposure. However, as volatility increases, the optimal hedging ratio drops and can even become negative, which means that a firm may choose to speculate.

Panel A of Figure 5 shows how the optimal hedging ratio changes with financing constraints for a firm with uncorrelated investment opportunities (dashed line) and for a firm with a positive correlation between investment opportunities and cash flows (solid line). The graph shows that the firm with a positive correlation never chooses full hedging and always maintains a lower hedging ratio than the firm with uncorrelated investment opportunities. When financing constraints ease (higher w_0), both types of firms may prefer to hedge more aggressively, but the relation between constraints and hedging is non-monotonic.

In Panel B, we plot the optimal hedging ratio as a function of firm's investment opportunities β. It is clear that firms with higher positive correlation between their cash flows and investment opportunities tend to have lower heddging ratios. Furthermore, the relation between risk management and the value of growth options is non-monotonic, which could explain the mixed empirical results on the relation between hedging ratios and firms' market-to-book ratios.

Finally, Figure 6 shows the relation between the correlation between investment opportunities and cash flows, α, and a firm's optimal hedging ratio. Since larger positive α implies a better coordination between firm's investment needs and internal funds, it is intuitive that the optimal hedging ratio decreases with α. However, the shape of the function crucially depends on the value of investment options, β. For example, when $\beta = 1.5$, the hedging ratio function is flat and equal to zero irrespective of α .

D. Economy with Multiple Firms

In the benchmark model we consider the firm's decisions in isolation. However, both investment and risk management strategies must be determined in conjunction with strategies of other firms.[9] It is intuitive that real options are less valuable in a competitive economy since increased production by other firms can depress the price of output and therefore impose a limit to profitability. Since the profitability of new investment is reduced by competition, firms invest less and are also less concerned with financing investment. Our model then predicts a higher optimal hedging ratio.

The relation between real options and competition is well known in the real options literature; however most of the related studies are concerned with the exercise timing. For example, Grenadier (2002) argues that competition decreases the value of real options and the advantage of waiting to invest. In contrast, Leahy (1993) and Caballero and Pindyck (1996) argue that despite the fact that the option to wait is less valuable in a competitive environment, irreversible investment is still delayed because upside profits are limited by new entry. By

[9] Zhu (2012) empirically analyzes the relation between hedging policies and competition and concludes that the hedging strategy a firm chooses affects the probability of the exit.

focusing on nonlinear production technology, Novy-Marx (2007) shows that firms in a competitive industry may delay irreversible investment longer than suggested by a neoclassical framework. None of these studies, however, analyze hedging incentives.

We formalize the intuition by representing the total cash flow of the firm as the sum of two parts—the component of the profit common across all firms and a firm-specific profit component. We assume that firms are identical aside from the differences in their cash flow composition. The required adjustment to the previous section's model is as follows: we supply the profitability shock with a firm-index "i" and separate it into common and idiosyncratic components

$$\varepsilon_i = \beta_i v_m + \sqrt{1-\beta_i^2} v_i, \tag{11}$$

where we denote $0 < \beta_i < 1$ the sensitivity of the total cash flow to the common shock. It is related but not necessarily equal to the market "beta." We assume that the two components have the same mean $\overline{v}$ and are drawn from an identical probability distribution with density function $G(v)$. This assumption allows us to vary the mix of firm-specific risk (by changing β_i) without changing the volatility of total cash flow.

With competition among different firms, the optimal investment strategy turns out to be a function of both the total cash flow shock ε_i and the common component of profit v_m. Since competitors are more likely to invest with high shock v_m, a particular firm's investment is more profitable when v_m is relatively low and the idiosyncratic component v_i is relatively high.

It is important that in our model firms compete to the extent that their profit shocks contain the common component. Therefore, firms in the economy that have a larger proportion of the systematic profit risk are less valuable and have fewer real options. The following proposition summarizes these facts.

Proposition 3. *In the economy with multiple firms:*

1. *The optimal hedging ratio ϕ^* increases with competition.*
2. *The optimal hedging ratio ϕ^* increases with β_i.*

The proposition states that the optimal hedging ratio increases with the systematic risk exposure because the probability of exercise decreases. Even if the total cash flow is currently high, firms may not invest heavily because they expect new entry to reduce profitability in the future. In contrast, firms with a high proportion of unique risk possess valuable investment options and expect to invest in the future. Therefore, the correlation between their cash flows and investment is large and leads to their adoption of a lower hedging ratio.[10]

E. Hedging with State-Contingent Securities

Here we extend the model allowing a firm to hedge a different amount of risk—given a particular profitability state—by trading Arrow-Debreu securities. Of course, a state-contingent hedging strategy may be infeasible because markets are inherently incomplete, some risks are unhedgeable, or firms cannot accurately identify ahead of time the full set of future profitability states. Nevertheless, it is valuable to consider what type of hedging policy the firm would adopt if it were allowed to freely redistribute cash flows.

Assuming that profitability-contingent securities are fairly priced and there are no transaction costs, the expected cash amount must be independent of hedging. Therefore, if by hedging the firm redistributes its cash in such a way that in each financial state, ε, it has cash $w(\varepsilon)$, we must have

$$\int w(\varepsilon)\, dG(\varepsilon) = \overline{w}. \tag{12}$$

The optimal policy maximizes firm value subject to the above fair-pricing constraint

$$\max_{w(\varepsilon)} \int \Pi(\varepsilon, w(\varepsilon))\, dG(\varepsilon). \tag{13}$$

From the above maximization problem, the sensitivity of the firm's cash to shock ε must be state-independent and equal to the Lagrangian multiplier λ on the constraint, i.e.,

$$\Pi_w = \lambda. \tag{14}$$

[10]Mello and Ruckes (2005), Adam, Dasgupta, and Titman (2007) analyze optimal hedging in models with product market competition. In their setting, firms choose a hedging policy simultaneously with their rivals in anticipation of the opponent's strategy and with the purpose to increase the chances of its own survival in competition. The mechanism in our model is different. The ex-post competition decreases the value of investment options that are not unique to the firm. This approach is more similar to that in the real options literature (see, e.g., Grenadier (2002), Novy-Marx (2007)).

The previous expression essentially defines optimal state-contingent policy $w^*(\varepsilon)$; since λ is a constant independent of ε, we can use the implicit function theorem to show

$$\frac{dw^*(\varepsilon)}{d\varepsilon} = \left\{ \begin{array}{c} -\frac{\alpha f_I}{\theta f_{II}} > 0, \text{ for } \varepsilon > \varepsilon^* \\ 0, \text{ for } \varepsilon \leq \varepsilon^* \end{array} \right\}. \tag{15}$$

Intuitively, (15) says that the optimal amount of cash, $w^*(\varepsilon)$, is monotonically increasing in ε in the investment region and is constant in the non-investment region. The optimal hedging policy in the investment region will therefore call for purchases of non-linear derivatives (e.g., out-of-the money call options), while in the inaction region hedging is linear and can be accomplished with purchases of forward contracts.

Recall that the probability and expected amount of investment increases with volatility. Therefore, the following two predictions immediately follow. First, firms engage in more non-linear hedging strategies as firm risk increases. Second, hedging becomes less aggressive on average with volatility. Specifically, the average hedging ratio, defined by analogy with formula (1), which implied $dw/d\varepsilon = (1-\phi)\, w_1$, is given by

$$\overline{\phi} = 1 + \frac{\alpha}{w_1} \int_{\varepsilon^*}^{\infty} \frac{f_I}{\theta f_{II}} dG(\varepsilon), \tag{16}$$

and decreases with volatility.

The one-period model provides the basic intuition for the static problem. However, it assumes that firms can only finance investment out of their cash flows or external financing. This may be unrealistic because liquidity issues and a persistent wedge between internal and external financing costs forces firms to save. We now consider a dynamic model with the accumulation of cash and dynamic risk management.

III. Dynamic Model with Cash Accumulation

This section presents and solves the multi-period model with an investment option, cash savings, and dynamic risk management policy.

A. Model Setup

Firms are price takers, and at every point in time they maximize their instantaneous profit with respect to the quantity of produced goods, q. The firm's profit flow is given by

$$\Pi_t dt = \max_q \left(q_t p_t - m q_t^{\frac{\gamma}{\gamma-1}} \right) dt, \tag{17}$$

where $\gamma > 1$, and p_t is the price of output that follows a geometric Brownian motion

$$\frac{dp_t}{p_t} = \mu_p dt + \sigma_p dB_p. \tag{18}$$

The first term in the profit function is the revenue received from the sale of output at the current market price, $q_t p_t$. The second term captures the convex operating costs incurred during the course of production. The firm's instantaneous optimization with respect to quantity produced gives the optimal output flow

$$q_t^* (p_t) = \left(\frac{p_t (\gamma - 1)}{m\gamma} \right)^{\gamma - 1}, \tag{19}$$

Substituting this expression in (17), we can write the firm's profit flow as

$$\Pi_t dt = p_t^\gamma \delta dt, \tag{20}$$

where

$$\delta = \frac{1}{\gamma} \left(\frac{\gamma - 1}{m\gamma} \right)^{\gamma - 1}. \tag{21}$$

The output flow depends on the current value of p_t^γ and is therefore locally deterministic.

The firm is initially endowed with a stock of cash C_0 and can invest it in a risk-free security earning a riskless rate of return r. In addition, the firm accumulates cash from its retained earnings and is subject to shocks that can unpredictably increase or decrease its savings. The cash dynamic of the firm evolves according to

$$dC_t = rC_t dt + \delta p_t^\gamma dt + \sigma_c C_t dB_c. \tag{22}$$

For convenience, we assume that the error terms are uncorrelated, i.e., $E[dB_c dB_p] = 0$. The last term in (22) captures the uncertainty in the firm's environment and guarantees that the cash process is not locally deterministic. For example, cash savings can change because of unanticipated expenses or revenues and after the settlement of lawsuits.

The firm has no leverage and never finds it optimal to terminate production. To model the incentives to hedge and also to avoid the singularity of solution when the cash variable approaches zero from above, we simply assume that the firm has to satisfy the minimum working capital requirement. We impose the minimum requirement on savings as

$$C \geq \underline{c} p^{\gamma}, \tag{23}$$

with constant $\underline{c} > 0$.[11] Once the cash level falls below this threshold, the firm must raise more cash at the marginal cost k_d. One could interpret such cost as a proportional cost of financial distress that is incurred in the low-cash states. At the threshold $\underline{c}$, we have

$$V_C \left(p, \underline{c} p^{\gamma}\right) = 1 + k_d, \tag{24}$$

where V_C is the first-order derivative of firm value with respect to cash savings.

In addition to receiving the profit flow, the firm may have an opportunity to invest in a new project at time τ. In particular, if a Poisson shock with intensity λ arrives at time τ, the firm can invest an amount ip^{γ} and obtain an instantaneous payoff of $\theta\left(p\right) p^{\gamma}$, where $\theta\left(p\right)$ is a weakly increasing function reflecting better investment opportunities for higher profitability. Notice that the investment amount and the payoff from investment are both made proportional to p^{γ} to ensure that the firm's cash flows do not outgrow investment.

If the firm does not have a sufficient cash balance to make investments, it needs to raise external financing and pay the associated financing costs $k\left(ip^{\gamma} - C\right)^{+}$. We allow for the possibility that $k \neq k_d$ because the costs of raising external financing in distress can, in principle, differ from costs incurred at other times. In summary, there is a single option, which arrives stochastically and can require an amount of external financing correlated with the firm's profit. The option is worthless if the costs of financing exceed the benefit of the investment.

[11] We assume that the firm cannot optimize over the choice of $\underline{c}$. In our setup with no financial leverage, the lowest possible value of $\underline{c}$ would always minimize the capital requirement cost.

For tractability purposes, we assume that after the investment the firm pays a dividend and is immediately liquidated. The value of the firm at date τ (equal to the final payout) is therefore

$$f(C,p,\tau) = C + \max\left(0, \theta(p)\, p^{\gamma} - ip^{\gamma} - k\left(ip^{\gamma} - C\right)^{+}\right), \tag{25}$$

at any other time, the value of the firm is equal to the present value of cash flows, including the interest on cash, plus the expected liquidation value

$$V(p_t, C_t) = \int_t^{\tau} \left(rC_t + \delta p_t^{\gamma}\right) dt + \int_t^{\tau} \sigma_c C_t dB_c + e^{-r\tau} f(C,p,\tau). \tag{26}$$

Using Ito's lemma and expression (22), we can describe the value function as a solution to the Hamilton-Jacoby-Bellman equation

$$\begin{aligned}(r+\lambda) V(p,C) &= \frac{\sigma_p^2 p^2}{2} V_{pp} + \frac{\sigma_c^2 C^2}{2} V_{CC} + \mu_p p V_p \\ &\quad + \left(rC + \delta p^{\gamma}\right) V_C + \lambda C + \lambda \max\left(0, \theta(p)\, p^{\gamma} - ip^{\gamma} - k\left(ip^{\gamma} - C\right)^{+}\right).\end{aligned} \tag{27}$$

We therefore obtain a two-dimensional partial differential equation (PDE) with respect to the profitability shock p and amount of cash C. In general, this equation does not have the analytical solution. In the next section, we discuss a particular case of uncorrelated investment option, for which the PDE can be reduced to the ordinary differential equation.

B. The Case of Uncorrelated Investment

Our model admits a closed-form solution (as an Ordinary Differential Equation with two boundary conditions) for the case when the value of the investment option is uncorrelated with the cash flow. In particular, we consider the case when $\theta(p) = \theta$, with $\theta > i$. With this assumption, the value of a firm's assets, including cash, is proportional to p^{γ}. One can think of the scaling parameter p^{γ} as a new numeraire or a new currency, in terms of which all values, such as firm value, cash holdings, investment, and payoff, will be computed. Because of this scaling property, the model is identical to a much simpler one, where all values are scaled by a factor of p^{γ}.

It is therefore convenient to define the scaled cash variable, which will be the main state variable in the model

$$c = Cp^{-\gamma}. \tag{28}$$

The value of the firm at date τ (equal to the final payout) is

$$f(c,p) = cp^{\gamma} + p^{\gamma}\left(\theta - i - k(i-c)^{+}\right)^{+}, \tag{29}$$

where the first term is accumulated cash and the second term is the option payoff. When the low cash level triggers significant costs of external financing at a particular "trigger" level, the option is optimally abandoned. We define such a trigger level of scaled cash by

$$c^{*} \equiv i - \frac{\theta - i}{k}. \tag{30}$$

Finally, if a firm receives an option to invest when its cash savings are high enough, i.e.,

$$c \geq \overline{c} \equiv i, \tag{31}$$

it does not need to raise any external financing and will not incur costs. To avoid the confusion with the case of an unconstrained firm discussed later, note that even if $c \geq \overline{c}$ the firm remains potentially constrained. This is because cash c can fall below $\overline{c}$ before an option arrives and therefore firm value is lower than that of the unconstrained firm by the amount of expected financing costs.

C. The Solution for Firm Value Without Hedging

We next proceed to describing the value function. For uncorrelated investment, $\theta(p) = \theta$, the Hamilton-Jacoby-Bellman equation for the value function can be simplified from (27) and using (28) as

$$\begin{aligned}(r+\lambda)V(p,C) \quad &= \quad \frac{\sigma_p^2 p^2}{2}V_{pp} + \frac{\sigma_c^2 c^2 p^{2\gamma}}{2}V_{CC} + \mu_p p V_p \\ &\quad + (rC + \delta p^{\gamma})V_C + \lambda p^{\gamma}\max\left(c, c + \theta - i - k(i-c)^{+}\right).\end{aligned} \tag{32}$$

We conjecture that firm's value function can be written in the following separable form

$$V(p,C) \equiv p^{\gamma} v\left(\frac{C}{p^{\gamma}}\right) = p^{\gamma} v(c), \tag{33}$$

where $v(c)$ is the scaled value function. Using definitions in (28) and (33), it is easy to show

$$V_p(p,C) = \gamma p^{\gamma-1}\left(v(c) - cv'(c)\right), \tag{34}$$

$$V_C(p,C) = v'(c),\ V_{CC}(p,C) = v''(c)\,p^{-\gamma}, \tag{35}$$

$$V_{pp}(p,C) = \gamma(\gamma-1)\,p^{\gamma-2}\left(v(c) - cv'(c)\right) + c^2\gamma^2 p^{\gamma-2} v''(c). \tag{36}$$

We can then write the second-order ordinary differential equation (ODE) for value function $v(c)$, where now c is the only variable

$$\left(\lambda + r - \gamma\mu_p - \frac{\sigma_p^2}{2}\gamma(\gamma-1)\right) v(c) = \left(\sigma_p^2\gamma^2 + \sigma_c^2\right)\frac{c^2 v''(c)}{2} \tag{37}$$
$$+v'(c)\left(\delta + rc - \gamma\mu_p c - \frac{\sigma_p^2 c}{2}\gamma(\gamma-1)\right) + \lambda \max\left(c, c+\theta-i-k(i-c)^+\right).$$

To properly characterize the solution, we also need to specify two boundary conditions. The first condition comes from the working capital requirement (23), which we can rewrite as

$$v'(\underline{c}) = 1 + k_d. \tag{38}$$

The second condition applies when cash savings are large, $c \to \infty$. At this point, the firm is not constrained and does not incur any costs of raising external financing. Therefore, the firm is worth its cash holdings, plus the value of fixed assets and the investment opportunity

$$v(c) \to c + g, \tag{39}$$

where constant g can be determined endogenously by substituting (39) into (37)

$$(r+\lambda)(c+g) = \gamma g\mu_p + \frac{\sigma_p^2}{2}\gamma(\gamma-1)\,g + rc + \delta + \lambda(c+\theta-i). \tag{40}$$

Solving this equation for g gives

$$g = \frac{\delta + \lambda(\theta - i)}{r + \lambda - \gamma\mu_p - \frac{\sigma_p^2}{2}\gamma(\gamma-1)}. \tag{41}$$

Note that the value of future cash flows, g, increases with the profitability of investment $\theta - i$, the probability of option arrival λ, and firm profitability δ. Additionally, the value of the firm increases with volatility σ_p^2 since for $\gamma > 1$ the firm's payout is convex in p. At this point, we can describe the scaled value function $v(c)$ as a solution to the ODE (37), subject to the boundary conditions (38) and (39). Since the function $v(c)$ concavity/convexity determines the incentive to hedge the uncertainty of cash flows, we proceed to finding the second derivative of this function.

First, a simple argument shows that $v(c)$ must be concave at least in some range of the cash variable. The upper boundary condition requires that the function is linear at high values of cash, implying that $v' \to 1$ as $c \to \infty$. Further, the lower boundary condition at $\underline{c}$ requires that $v' = 1 + k_d \geq 1$. Therefore, we at least know that the function is concave in some region. Furthermore, the Appendix proves that function $v(c)$ which satisfies the ODE must be concave in the whole region above $\overline{c}$ (that is, where the option is exercised using firm's own cash savings only). It follows then that it is always optimal to reduce cash flow risk when $c \geq \overline{c}$.

Second, in the region where $c < \overline{c}$, the value function can be concave or convex, depending on the parameters. We prove in the Appendix that if the costs of financial distress are not too large compared to the costs of financing the investment, there will exist a convexity region for lower values of c. This result is explained by the fact that the firm holds an option to invest, which has a convex payoff in the cash variable. For example, when cash is just below investment threshold c^*, it can pay off for a firm to increase the volatility in cash savings since such behavior increases the probability of cash exceeding the threshold c^*, and therefore also increases the value of the option. However, when cash savings are very low and the probability of exercise is small, a firm may actually prefer to hedge because this allows it to avoid distress and the costly external financing associated with it. It is important that the additional value of risk in cash flows comes from the option to abandon the investment if the financing is insufficient. If it were *always* required to exercise the option, cash flow risk would be value-destroying because it would necessitate large financing costs when cash is very low.

In Figure 7, Panel A, we show the shape of function $v(c)$ for a given set of parameters. The solution to ODE (37) is obtained numerically. The upper dashed line in the graph gives the value of the firm if it were completely unconstrained (i.e., $k_d = k = 0$), in which case the firm would always exercise its investment option when the option arrives. It is clear that firm value approaches this line as the accumulated cash savings increase beyond $\bar{c}$. The lower dashed line shows the value of the firm that does not have an investment option and is not subject to the costs of financial distress (i.e., $\lambda = 0$ and $k_d = 0$). As predicted, the function exhibits concavity in the region of the high values of cash and convexity in the region of the low values. Panels B helps to evaluate how the first derivative of $v(c)$ change with cash.

Having analyzed the shape of the value function that gives us guidance on where it is optimal to increase or decrease risk of cash savings, we now turn to determining the optimal amount of hedging.

D. Hedging Policy

To reduce the volatility of cash savings, the firm can buy financial securities (e.g., forward contracts) that are correlated with the firm's profitability. We first consider a case in which the firm can optimize dynamically over its hedging policy. In particular, at any point the firm can choose to enter into $H_t = p^{\gamma}\phi_t$ futures contracts. The futures price has no drift under the risk-neutral measure and evolves as follows

$$\frac{dF_t}{F_t} = \sigma_F dB_F. \tag{42}$$

Hence, the dynamics of the portfolio, which includes the cash savings and hedging security is given by

$$dC_t = rC_t dt + \delta p_t^{\gamma} dt + \sigma_c C_t dB_c - p^{\gamma}\phi_t F_t \sigma_F dB_F - \pi p_t^{\gamma}\phi_t dt, \tag{43}$$

where the last term captures the cost of hedging. We assume the following correlation structure:

$$E[dB_c dB_F] = \rho_c dt, \tag{44}$$

$$E[dB_p dB_F] = \rho_p dt. \tag{45}$$

The costs associated with hedging can be thought of as either transaction fees or the cost of holding cash in a margin account and posting collateral. Note that complete hedging in this framework would imply that dC is a locally deterministic function and would be achieved if there is a perfect correlation between profit uncertainty and hedging security ($\rho_c = -1$ or $\rho_c = 1$) and if

$$\phi_t = \frac{\rho_c \sigma_c c_t}{F \sigma_F}. \tag{46}$$

Using the expression for the cash evolution specified in (43) and also using the previously obtained derivatives (34)-(36) and $V_{Cp} = -\gamma v''(c)\, cp^{-1}$, we can write the Hamilton-Jacoby-Bellman equation for the scaled firm value $v(c)$

$$\begin{aligned}(r+\lambda)\, v(c) \quad &= \quad \max_{\phi} \left(\gamma\mu_p + \gamma(\gamma-1)\frac{\sigma_p^2}{2}\right)\left(v - cv'\right) + \frac{v''c^2}{2}\left(\gamma^2\sigma_p^2 + \sigma_c^2\right) \\ &\quad + v'\left(rc + \delta - \pi\phi\right) + \frac{v'' F \sigma_F}{2}\left(\phi^2 F\sigma_F - 2c\phi\sigma_c\rho_c + 2\gamma c\phi\sigma_p\rho_p\right) \\ &\quad + \lambda \max\left(c, c + \theta - i - k\,(i-c)^+\right).\end{aligned} \tag{47}$$

Differentiating with respect to hedging policy ϕ gives the optimal choice of hedging

$$\phi_t^* = \frac{\rho_c \sigma_c c_t - \gamma\rho_p\sigma_p c_t}{F\sigma_F} - \frac{\pi}{\left(F\sigma_F\right)^2}\frac{v'}{\left(-v''\right)}. \tag{48}$$

Note that the second-order condition is satisfied as long as function $v(c)$ is concave. It is worth examining the expression (48) with care. First, observe that the second term is negative, and therefore optimal hedging is less than complete (compare (48) to (46)). Second, observe that the optimal hedging ratio decreases with the volatility of the output price, σ_p, particularly if the convexity of the firm's profit (γ) is high. Third, ϕ_t decreases with the cost of hedging.

When $v(c)$ is convex, there is no interior solution for hedging and the firm chooses between corner solutions $\phi_{\min}$ and $\phi_{\max}$. In particular, if there are no costs of hedging ($\pi = 0$), the firm will choose $\phi^* = \phi_{\min}$ when

$$\frac{\phi_{\max} + \phi_{\min}}{2} < \frac{\sigma_c\rho_c c - \gamma\sigma_p\rho_p c}{F\sigma_F}. \tag{49}$$

In any case, the firm will leave its cash exposed to shocks. Interestingly, the firm's hedging policy can remain the same for extended periods of time even as its liquidity position or the value of the hedging asset changes. However, it is also possible for the firm to radically change its hedging position from time to time (and even go from hedging to speculation). For example, if σ_p is relatively small ($\sigma_p < \frac{\sigma_c\rho_c}{\gamma\rho_p}$), the firm will switch from $\phi_{\min}$ to $\phi_{\max}$ as its scaled cash savings increase.

IV. Conclusion

In this study, we analyze the relation between optimal risk management policy and investment under financing constraints. In particular, we recognize that hedging policy can affect the probability of option exercise and also the cost of financing. The optimal amount of financial hedging balances the benefits of lower expected financial distress costs with the better ability to finance investment. The model demonstrates importance of real frictions, such as irreversibility of investment or fixed costs.

The predictions of the model are consistent with the empirical findings: firms with less financing constraints operate with higher hedging ratios, and firms with more risky cash flows operate with lower hedging ratios. The hedging ratio is linked theoretically to the value of growth options, the ratio of firm-specific to systematic risk, and the costs of forming a hedging portfolio. Therefore, the model generates additional empirical predictions for future work. Our results offer an alternative explanation for the observed hedging policies that does not rely on the cost of hedging.

The analytical solution for the one-period model and the dynamic model with cash accumulation can be used in other applications. For example, it would be relatively easy to extend the model to study optimal dividend policy and the implications of a minimum cash balance requirement.

V. Appendix A: Proposition Proofs

Proof of Lemma 1. Using the implicit function theorem for (3), we obtain

$$\frac{d\varepsilon^*}{d\phi} = \frac{\left(C_e\left(e\right) - C_e\left(e_0\right)\right)\left(\varepsilon^* - \overline{\varepsilon}\right)}{\frac{\alpha f}{w_1} + \left(C_e\left(e\right) - C_e\left(e_0\right)\right)\left(1 - \phi\right)}. \tag{50}$$

Our assumptions imply that $I^* > I_0$, $\alpha \geq 0$, $C_{ee} > 0$, and $e > e_0$.

Therefore (i) $\frac{d\varepsilon^*}{d\phi} > 0$ for $\varepsilon^* > \overline{\varepsilon}$, and (ii) $\frac{d\varepsilon^*}{d\phi} < 0$ for $\varepsilon^* < \overline{\varepsilon}$. Additionally, using condition (6) yields

$$\frac{dI^*}{d\phi} = -\frac{w_1\left(\varepsilon - \overline{\varepsilon}\right)C_{ee}}{C_{ee} - \theta f_{II}}. \tag{51}$$

It is clear that $\frac{dI^*}{d\phi} < 0$ for all states ε in the investment region if $\varepsilon^* > \overline{\varepsilon}$. □

Proof of Proposition 1. If $\alpha = 0$, the optimization function does not have an interior maximum, so the optimal hedging ratio is either a minimum $\phi^* = 0$ or a maximum $\phi^* = 1$.

If the firm hedges completely, its cash savings at date 2 are independent of the state and the profit is given by either $P_0\left(\overline{\varepsilon}\right)$ or $P\left(\overline{\varepsilon}\right)$ depending on whether investment is optimal at $\overline{\varepsilon}$, where

$$P_0\left(\overline{\varepsilon}\right) = f_0 - I_0 - C\left(I_0 - w_0 - w_1\overline{\varepsilon}\right), \tag{52}$$

$$P\left(\overline{\varepsilon}\right) = \beta f\left(I^*\right) - I^* - C\left(I^* - w_0 - w_1\overline{\varepsilon}\right). \tag{53}$$

Note that since $\alpha = 0$, the optimal investment I^* also does not vary with the profitability state in the event of full hedging.

If the firm does not hedge, its cash savings w vary with the state ε and there exists a threshold ε^* at which the firm starts to invest. To determine under which ϕ the expected profit is maximized, let us consider two cases: (1) $\varepsilon^* < \overline{\varepsilon}$ (the firm would invest at the average profitability, $P\left(\overline{\varepsilon}\right) > P_0\left(\overline{\varepsilon}\right)$); and (2) $\varepsilon^* > \overline{\varepsilon}$ (the firm would not invest at the average profitability, $P\left(\overline{\varepsilon}\right) < P_0\left(\overline{\varepsilon}\right)$).

If $\varepsilon^* < \overline{\varepsilon}$, the difference between the expected profit in the case of full hedging and the case of no hedging is

$$
\begin{aligned}
& P(\bar{\varepsilon}) - \int_{-\infty}^{\varepsilon^*} P_0(\varepsilon)\, dG(\varepsilon) - \int_{\varepsilon^*}^{\infty} P(\varepsilon)\, dG(\varepsilon) \\
= \; & P(\bar{\varepsilon}) - E(P(\varepsilon)) + \int_{-\infty}^{\varepsilon^*} (P(\varepsilon) - P_0(\varepsilon))\, dG(\varepsilon).
\end{aligned} \tag{54}
$$

Since the profit function is concave it must be that $P(\bar{\varepsilon}) > E(P(\varepsilon))$ and the first term above is positive. However, the second term is negative and captures the option value lost by hedging. Since the first term is independent of ε^* and the second term increases in magnitude with ε^*, there must exist such ε_L that for $\varepsilon^* > \varepsilon_L$ hedging destroys value ($\phi^* = 0$) and for $\varepsilon^* < \varepsilon_L$ hedging creates value ($\phi^* = 1$). Such ε_L can be found by setting expression (54) to zero and plugging ε_L in place of ε^*.

Similarly, if $\varepsilon^* > \bar{\varepsilon}$, the difference between the expected profit under full hedging and no hedging is

$$
\begin{aligned}
& P_0(\bar{\varepsilon}) - \int_{-\infty}^{\varepsilon^*} P_0(\varepsilon)\, dG(\varepsilon) - \int_{\varepsilon^*}^{\infty} P(\varepsilon)\, dG(\varepsilon) \\
= \; & P_0(\bar{\varepsilon}) - E(P_0(\varepsilon)) - \int_{\varepsilon^*}^{\infty} (P(\varepsilon) - P_0(\varepsilon))\, dG(\varepsilon).
\end{aligned} \tag{55}
$$

The first term is positive because of the concavity of the profit function, whereas the second term is negative and captures the value of the investment option lost by hedging. Since the first term is independent of ε^* and the second one decreases with ε^*, there must exist ε_H such that for any $\varepsilon^* > \varepsilon_H$ hedging creates value ($\phi^* = 0$) and for $\varepsilon^* < \varepsilon_H$ hedging destroys value ($\phi^* = 0$). The value of ε_H is found by setting (55) to zero at $\varepsilon^* = \varepsilon_H$. □

Proof of Corollary 1. From (3) we obtain

$$
\frac{d\varepsilon^*}{d\beta} = -\frac{f}{\alpha f + w_1(1-\phi)(C_e(e) + C_e(e_0))} < 0,
$$

which implies that firms with more valuable options choose to invest at the lower threshold. Therefore, from Proposition 1 it follows that $\frac{\Delta \phi^*}{\Delta \beta} \leq 0$. □

Proof of Corollary 2. Using the definition of the investment threshold (3), we obtain the comparative statics with respect to the firm's initial cash position w_0

$$
\frac{d\varepsilon^*}{dw_0} = -\frac{C_e(e) - C_e(e_0)}{\alpha f + w_1(1-\phi)(C_e(e) - C_e(e_0))} < 0. \tag{56}
$$

Therefore, as w_0 increases and the firm becomes less constrained, the investment threshold ε^* decreases. Proposition 1 shows that the optimal hedging ratio depends on the investment threshold.

If initially $\varepsilon^* < \varepsilon_L$, then full hedging remains optimal as w_0 increases since the firm remains in the same region (see Proposition 1). If initially $\varepsilon^* \in [\varepsilon_L, \varepsilon_H]$, then the hedging ratio either remains unchanged or increases to $\phi^* = 1$. Therefore, an increase in the firm's internal cash reserves can result in more hedging. $\square$

Proof of Corollary 3. Recall from the proof of Proposition 1 that when $\varepsilon^* < \overline{\varepsilon}$, the difference between the expected profit in the case of full hedging and the case of no hedging is

$$P\left(\overline{\varepsilon}\right) - E\left(P\left(\varepsilon\right)\right) + \int_{-\infty}^{\varepsilon^*} \left(P\left(\varepsilon\right) - P_0\left(\varepsilon\right)\right) dG\left(\varepsilon\right). \tag{57}$$

Note that the investment threshold ε^* is independent of volatility. When the volatiliy is small, the firm never reaches the region below ε^*. Therefore, the second term in the formula above disappears, while the first term is positive and induces hedging. A firm with low volatility and $\varepsilon^* < \overline{\varepsilon}$ chooses $\phi^* = 1$. As volatility increases, the second term starts to create a greater disadvantage to hedging, with a resulting decrease in the hedging ratio.

Similarly, if $\varepsilon^* > \overline{\varepsilon}$, the difference between the expected profit under full hedging and no hedging is

$$P_0\left(\overline{\varepsilon}\right) - E\left(P_0\left(\varepsilon\right)\right) - \int_{\varepsilon^*}^{\infty} \left(P\left(\varepsilon\right) - P_0\left(\varepsilon\right)\right) dG\left(\varepsilon\right). \tag{58}$$

If the volatility is small, the second term is zero and thus $\phi^* = 1$. As volatility increases, the second term becomes more important and hedging ratio drops. $\square$

Proof of Proposition 2. From (8) we can use Leibniz's rule to obtain the first order condition with respect to ϕ as

$$\int_{\varepsilon^*}^{\infty} P_w \frac{\partial w}{\partial \phi} dG\left(\varepsilon\right) + \int_{-\infty}^{\varepsilon^*} P_{0w} \frac{\partial w}{\partial \phi} dG\left(\varepsilon\right) + \frac{d\varepsilon^*}{d\phi}\left(P_0\left(\varepsilon^*\right) - P\left(\varepsilon^*\right)\right) g\left(\varepsilon^*\right) = 0, \tag{59}$$

and by $P_0\left(\varepsilon^*\right)$ we denote the profit when the firm does not invest ($\varepsilon^* > \varepsilon$). Because the profit function is continuous at ε^*, the last term is zero, and the condition (59) becomes

$$E\left[\Pi_w \frac{\partial w}{\partial \phi}\right] = 0. \tag{60}$$

By applying (1), we can simplify the first order condition (60) to

$$cov\left(\Pi_w,\varepsilon\right)=0. \tag{61}$$

Using Stein's lemma for normally distributed profitability shocks, $g\left(\varepsilon\right)\sim N\left(\overline{\varepsilon},\sigma^2\right)$, and using the expression (61), we have

$$E(\Pi_{w\varepsilon})\sigma^2=0. \tag{62}$$

Alternatively, if the distribution is not normal, the same expression can be obtained from the second-order Taylor expansion around $\overline{\varepsilon}$. In the investment region, $\varepsilon>\varepsilon^*$, we obtain

$$\Pi_{w\varepsilon}=P_{w\varepsilon}=\left(\alpha f_I+\theta f_{II}I_\varepsilon\right)I_w+\left(\theta f_I-1-C_e\right)I_{w\varepsilon}-C_{ee}\left(I_\varepsilon-w_1\left(1-\phi\right)\right)\left(I_w-1\right), \tag{63}$$

which simplifies using the first order condition for investment (6) to

$$\Pi_{w\varepsilon}=P_{0w\varepsilon}=\left[\alpha f_I+\theta f_{II}I_\varepsilon-C_{ee}I_\varepsilon+C_{ee}w_1\left(1-\phi\right)\right]I_w+C_{ee}\left[I_\varepsilon-w_1\left(1-\phi\right)\right]. \tag{64}$$

Differentiating implicitly equation (6),

$$\begin{aligned} I_\varepsilon &= \frac{\alpha f_I+C_{ee}w_1\left(1-\phi\right)}{C_{ee}-\theta f_{II}}, && (65)\\ I_w &= \frac{C_{ee}}{C_{ee}-\theta f_{II}}, && (66)\end{aligned}$$

and substituting these expressions in (64) we obtain

$$P_{w\varepsilon}=\frac{C_{ee}\alpha f_I+\theta f_{II}C_{ee}w_1\left(1-\phi\right)}{C_{ee}-\theta f_{II}}. \tag{67}$$

In the inaction region, $\varepsilon<\varepsilon^*$, we have

$$P_{0w\varepsilon}=-C_{ee}\left(e_0\right)w_1\left(1-\phi\right). \tag{68}$$

Substituting (64), rewrite (62) as

$$E\left[\Pi_{w\varepsilon}\right]=0=\int_{\varepsilon^*}^{\infty}P_{w\varepsilon}dG\left(\varepsilon\right)+\int_{-\infty}^{\varepsilon^*}P_{0w\varepsilon}dG\left(\varepsilon\right). \tag{69}$$

Finally, solving this equation for the optimal hedging ratio ϕ^* yields

$$\phi^*=1-\frac{\alpha}{w_1}\frac{\int_{\varepsilon^*}^{\infty}\frac{f_IC_{ee}}{C_{ee}-\theta f_{II}}dG\left(\varepsilon\right)}{\int_{-\infty}^{\varepsilon^*}C_{ee}\left(e_0\right)dG\left(\varepsilon\right)-\int_{\varepsilon^*}^{\infty}\frac{\theta f_{II}C_{ee}}{C_{ee}-\theta f_{II}}dG\left(\varepsilon\right)}. \tag{70}$$

The second order condition with respect to ϕ is

$$\int_{\varepsilon^*}^{\infty} P_{ww} \left(\frac{\partial w}{\partial \phi}\right)^2 dG(\varepsilon) + \int_{-\infty}^{\varepsilon^*} P_{0ww} \left(\frac{\partial w}{\partial \phi}\right)^2 dG(\varepsilon) \tag{71}$$
$$+\frac{d\varepsilon^*}{d\phi} \left(P_{0w}(\varepsilon^*) - P_w(\varepsilon^*)\right) w_0 (\overline{\varepsilon} - \varepsilon^*) g(\varepsilon^*) < 0.$$

Since

$$P_{ww} = \frac{\theta f_{II} C_{ee}}{C_{ee} - \theta f_{II}} < 0, \tag{72}$$

$$P_{0ww} = -C_{ee} < 0, \tag{73}$$

the first two terms in (71) are negative. The last term in (71) is positive and is equal to

$$\frac{\left(C_e(e) - C_e(e_0)\right)^2 (\varepsilon^* - \overline{\varepsilon})^2 w_1 g(\varepsilon^*)}{\frac{\alpha f}{w_1} + \left(C_e(e) - C_e(e_0)\right)(1 - \phi)} > 0. \tag{74}$$

For a sufficiently large α (i.e., for $\alpha > \underline{\alpha}$), the condition (71) is satisfied, where $\underline{\alpha}$ is a solution of the following equation

$$\int_{\varepsilon^*}^{\infty} P_{ww} (\varepsilon - \overline{\varepsilon})^2 dG + \int_{-\infty}^{\varepsilon^*} P_{0ww} (\varepsilon - \overline{\varepsilon})^2 dG = \frac{\left(C_e(e) - C_e(e_0)\right)^2 (\varepsilon^* - \overline{\varepsilon})^2 g(\varepsilon^*)/w_1}{\overline{\alpha} f + \left(C_e(e) - C_e(e_0)\right) w_1 (1 - \phi^*)}. \tag{75}$$

Note that when $\alpha \to 0$, from (70), we have $\phi^* \to 1$. The denominator in (74) is linear in α, and therefore the last term in the second order condition is infinite for a very small α. Therefore, when $\alpha \to 0$ the solution is "bang-bang". □

Proof of Proposition 3. Similarly to the steps in Proof of Proposition 2, we obtain the first order condition for the optimal hedging ratio as

$$E\left[\Pi_{w\varepsilon}(\phi^*)\right] = 0. \tag{76}$$

In the investment region, we have

$$P_{w\varepsilon} = \frac{C_{ee} \alpha f_I + \theta f_{II} C_{ee} w_1 (1 - \phi)}{C_{ee} - \theta f_{II}}, \tag{77}$$

whereas in the region where the firm operates its existing assets, we obtain

$$P_{0w\varepsilon} = -C_{ee}(e_0) w_1 (1 - \phi). \tag{78}$$

Substituting these expressions into the first-order condition (76) and rewriting the expectation yields

$$
\begin{aligned}
E\left[\Pi_{w\varepsilon}\right] &= 0=\int_{-\infty}^{\widehat{v}_m}\left(\int_{\frac{\varepsilon_i^*-\beta_i v_m}{\sqrt{1-\beta_i^2}}}^{\infty} P_{w\varepsilon} dG\left(v_i\right)\right) dG\left(v_m\right) \\
&+\int_{-\infty}^{\widehat{v}_m}\left(\int_{-\infty}^{\frac{\varepsilon_i^*-\beta_i v_m}{\sqrt{1-\beta_i^2}}} P_{0w\varepsilon} dG\left(v_i\right)\right) dG\left(v_m\right)
\end{aligned} \tag{79}
$$

Finally, solving this equation for the optimal hedging ratio ϕ^* yields

$$
\phi^*=1-\frac{\alpha}{w_1} \frac{\int_{-\infty}^{\widehat{v}_m}\left(\int_{\frac{\varepsilon_i^*-\beta_i v_m}{\sqrt{1-\beta_i^2}}}^{\infty} \frac{C_{ee} f_I dG(\varepsilon_i)}{C_{ee}-\theta f_{II}}\right) dG\left(v_m\right)}{\int_{-\infty}^{\widehat{v}_m}\left(\int_{-\infty}^{\frac{\varepsilon_i^*-\beta_i v_m}{\sqrt{1-\beta_i^2}}} C_{ee}\left(e_0\right) dG\left(v_i\right)-\int_{\frac{\varepsilon_i^*-\beta_i v_m}{\sqrt{1-\beta_i^2}}}^{\infty} \frac{\theta f_{II} C_{ee} dG(v_i)}{C_{ee}-\theta f_{II}}\right) dG\left(v_m\right)} \tag{80}
$$

To show that $\frac{d\phi^*}{d\beta_i}>0$, note that ϕ^* depends on β_i only through the limits of the integration

$$
v_i^*\left(v_m\right)=\frac{\varepsilon_i^*-\beta_i v_m}{\sqrt{1-\beta_i^2}}, \tag{81}
$$

which has the meaning of the minimum idiosyncratic shock v_i which warrants new investment at the current realized value of the systematic shock v_m. Note that $v_i^*\left(v_m\right) \rightarrow +\infty$ when $\beta_i \rightarrow 1$, i.e., an infinitely large idiosyncratic shock is required to trigger investment if almost all risk comes from the systematic component. Therefore, it follows from (80) that $\phi^* \rightarrow 1$ as $\beta_i \rightarrow 1$, and $\phi^*<1$ if $\beta_i<1$. □

Proof of Formula (15). Differentiating explicitly expression (7) in the investment region and using the first order condition for investment we obtain

$$
\begin{aligned}
P_{ww} &= \frac{\theta f_{II} C_{ee}}{C_{ee}-\theta f_{II}}, \quad (82)\\
P_{w\varepsilon} &= \frac{\alpha f_I C_{ee}}{C_{ee}-\theta f_{II}}. \quad (83)
\end{aligned}
$$

Hence, the optimal sensitivity of cash to shock in the investment region is

$$
\frac{dw^*(\varepsilon)}{d\varepsilon}=-\frac{P_{w\varepsilon}}{P_{ww}}=-\frac{\alpha f_I}{\theta f_{II}}>0. \tag{84}
$$

Similarly, in the region where the firm operates its existing assets, i.e., if $\varepsilon < \varepsilon^*$, we obtain

$$P_{0ww} = -C_{ee}, \tag{85}$$

$$P_{0w\varepsilon} = 0. \tag{86}$$

Therefore, the optimal sensitivity of cash to shock in the non-investment region is equal to zero, which corresponds to full hedging

$$\frac{dw^*(\varepsilon)}{d\varepsilon} = 0. \tag{87}$$

$\square$

Proof of concavity of $v(c)$ function when $c \geq \bar{c}$.. We have shown that function $v(c)$ must have at least some region of c values where it is concave. The following argument shows that the value function contains no convex region when $c \geq \underline{c}$. The ODE for the value function is

$$\begin{aligned}(r+\lambda)v(c) &= \left(\gamma v(c) - \gamma c v'(c)\right)\mu_p + \frac{\sigma_p^2}{2}\gamma(\gamma-1)\left(v(c) - cv'(c)\right) \\ &\quad + \frac{\sigma_p^2}{2}c^2\gamma^2 v''(c) + v'(c)(rc+\delta) + \lambda f(c),\end{aligned} \tag{88}$$

with function $f(c)$ being concave.

We proceed with the proof by contradiction. Suppose function $v(c)$ is convex at point c_2. Then it should be possible to pick such values c_1 and c_3, with $c_1 < c_2 < c_3$ and $c_2 = \alpha c_1 + (1-\alpha)c_3$, that

$$v'(c_1) = v'(c_2) = v'(c_3) = b, \tag{89}$$

and

$$v''(c_1) < 0,\ v''(c_2) > 0,\ v''(c_3) < 0. \tag{90}$$

Using these conditions and letting $v_1 = v(c_1)$, $v_2 = v(c_2)$ and $v_3 = v(c_3)$, we can then write the following inequalities

$$(r+\lambda)v_1 < \gamma\left(\mu_p + \frac{\sigma_p^2}{2}(\gamma-1)\right)(v_1 - c_1 b) + b(rc_1+\delta) + \lambda f(c_1), \tag{91}$$

$$(r+\lambda)v_3 < \gamma\left(\mu_p + \frac{\sigma_p^2}{2}(\gamma-1)\right)(v_3 - c_3 b) + b(rc_3+\delta) + \lambda f(c_3), \tag{92}$$

$$(r+\lambda)v_2 > \gamma\left(\mu_p + \frac{\sigma_p^2}{2}(\gamma-1)\right)(v_2 - c_2 b) + b(rc_2+\delta) + \lambda f(c_2). \tag{93}$$

Let $\widehat{v}_2 \equiv \alpha v_1 + (1-\alpha) v_3$. If the function $v(c)$ is convex at point c_2, then it must be that $\widehat{v}_2 > v_2$. Using this fact, we can rewrite (93) as

$$\begin{aligned} (r+\lambda)\widehat{v}_2 \quad < \quad & \gamma\left(\mu_p + \frac{\sigma_p^2}{2}(\gamma-1)\right)(\widehat{v}_2 - c_2 b) + b(rc_2+\delta) \\ & + \lambda\left[\alpha f(c_1) + (1-\alpha) f(c_3)\right]. \end{aligned} \tag{94}$$

Taking the difference, we obtain

$$\begin{aligned} (r+\lambda)(\widehat{v}_2 - v_2) \quad < \quad & \left[\gamma\mu_p + \frac{\sigma_p^2}{2}\gamma(\gamma-1)\right](\widehat{v}_2 - v_2) \\ & + \lambda\left[f(c_2) - \alpha f(c_1) - (1-\alpha) f(c_3)\right]. \end{aligned} \tag{95}$$

Since

$$r+\lambda > \gamma\mu_p + \frac{\sigma_p^2}{2}\gamma(\gamma-1)\,, \text{ and } f(c_2) > \alpha f(c_1) + (1-\alpha) f(c_3)$$

we get a contradiction.

□

Proof of existence of convexity of $v(c)$ function when $c < \overline{c}$. Suppose the distress costs are small ($k_d \to 0$). Then, the slope of the value function $v(c)$ is the same at the lower boundary and the upper boundary, i.e.,

$$v'(\underline{c}) = v'(c \to \infty) = 1,$$

At the lower boundary, $c \to \underline{c}$, the investment option is far out-of-the-money and hence the value of the firm is the same as of a firm without option. However, at the upper boundary, the firm always exercises the option, that is the firm value approaches the value of the unconstrained firm *with* the investment option. To have the same slope at both boundaries, but a higher value at the right boundary, the function $v(c)$ must have convexity on the left and concavity on the right. The inflection point may or may not coincide with the exercise threshold ε^*. □

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Figure 1. Investment and Distress Regions

We assume the firm can only invest out of its internal funds and payoff from investment is given by $f(I) = b \log I$. The parameters are set as follows: $w_1 = 40$, $I_0 = 30$, $b = 100$, $\alpha = 0$, $f_0 = 350$, $\bar{\varepsilon} = 2$, $\sigma = 1$. Note that $I^* = 100$ and is marked with a dashed line, and $f(I) - I > f_0 - I_0$. The solid flat (upward-sloping) line shows the firm's internal funds under full hedging (no hedging). The filled areas show regions of investment and distress when the firm does not hedge. Panels A and B are for the cases of $w_0 = 0$ and $w_0 = 30$, respectively.

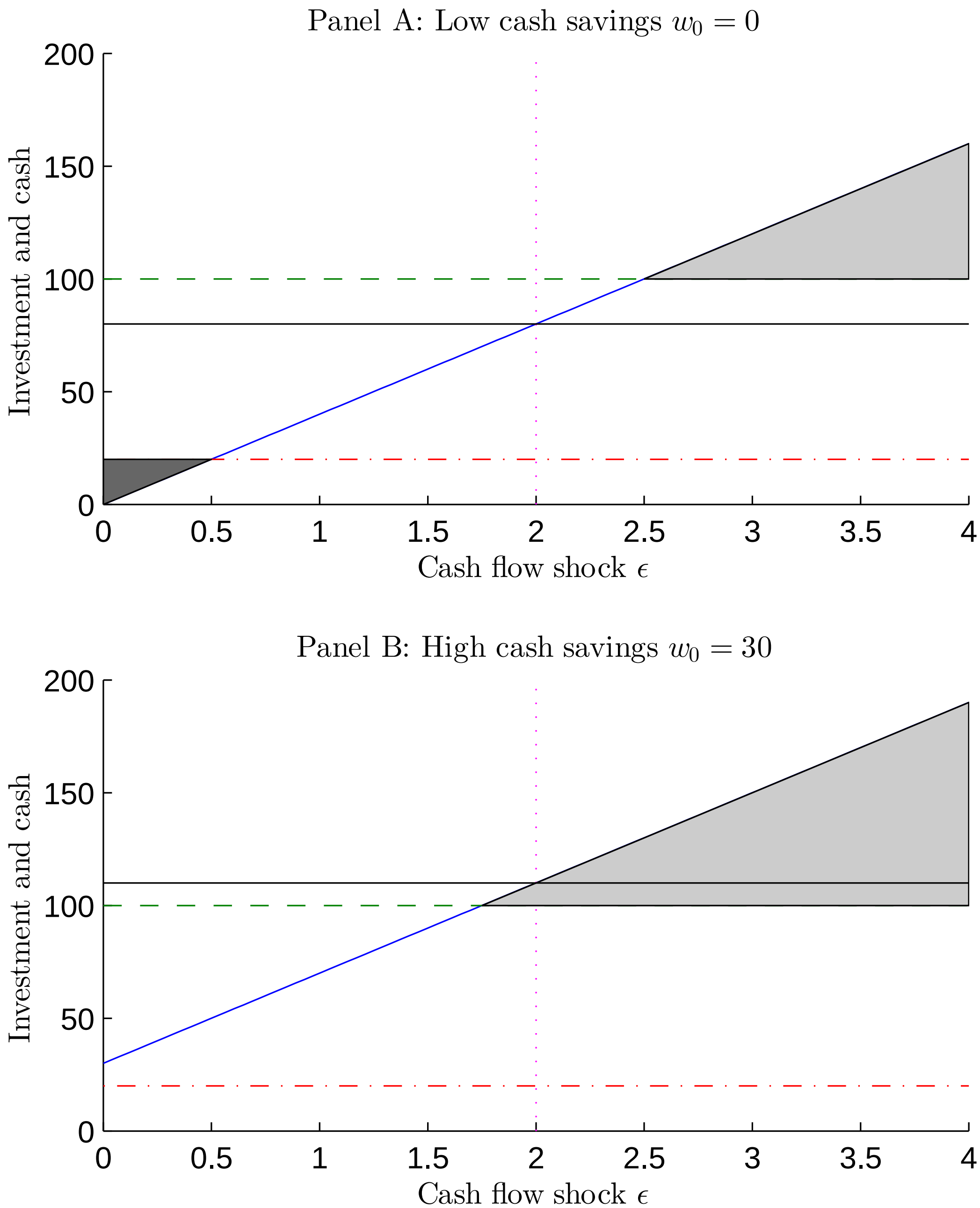
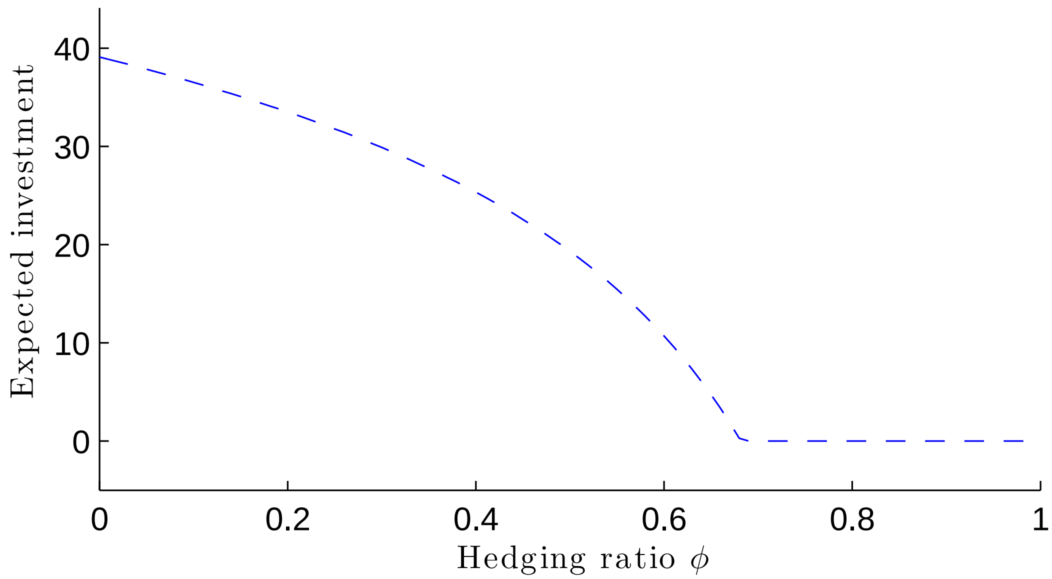


Figure 2. Expected Investment and Hedging

This figure shows the expected investment level as a function of hedging ratio ϕ. We assume $f(I) = b \log I$ and $C(e) = \frac{ke^2}{2}$ and set parameters as follows: $w_1 = 24$, $w_0 = 0$, $I_0 = 50$, $b = 300$, $\alpha = 0$, $f_0 = 1900$, $k = 0.08$, $\bar{\varepsilon} = 2$. Panels A is for the case when the investment option is out-of-the-money at zero hedging, $\beta = 1.45$. Panel B is for the case when the investment option is in-the-money at zero hedging, $\beta = 1.5$.

Panel A: Option is out-of-the-money with no hedging, $\epsilon* > \bar{\varepsilon}$



Panel B: Option is in-the-money with no hedging, $\epsilon* < \bar{\varepsilon}$

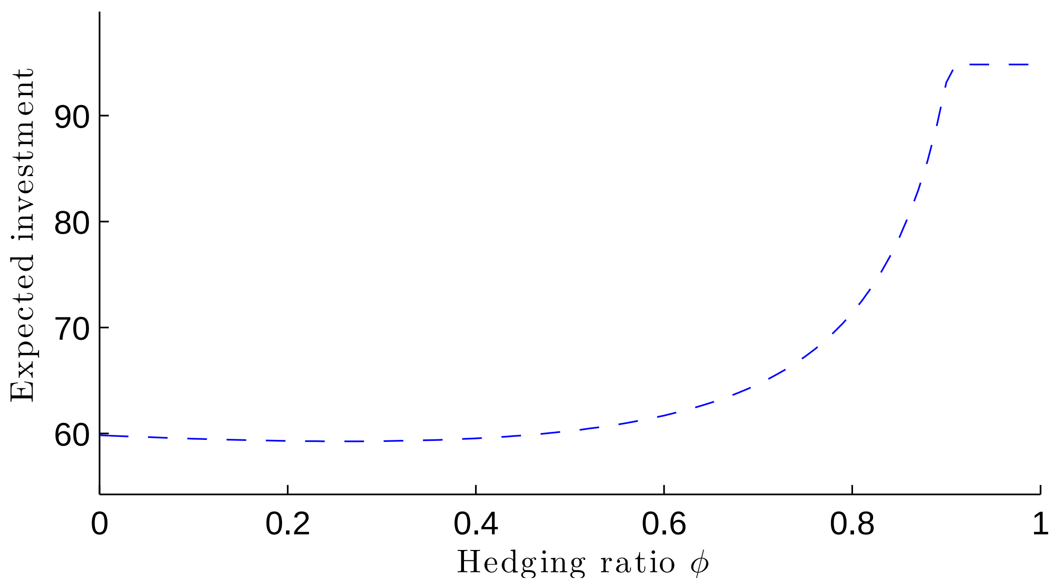


Figure 3. Financing Gap and Hedging

This figure shows a firm's cash flow (solid line) and optimal investment level (dashed line) as a function of the primitive uncertainty shock ε. We assume $f(I) = b \log I$ and $C(e) = \frac{ke^2}{2}$ and set parameters as follows: $w_1 = 24$, $w_0 = 0$, $I_0 = 50$, $b = 300$, $\alpha = 1$, $\beta = 1$, $f_0 = 1900$, $k = 0.08$, $\bar{\varepsilon} = 2$. Panels A and B are for high volatility $\sigma = 1.2$; Panels C and D are for low volatility $\sigma = 0.2$.

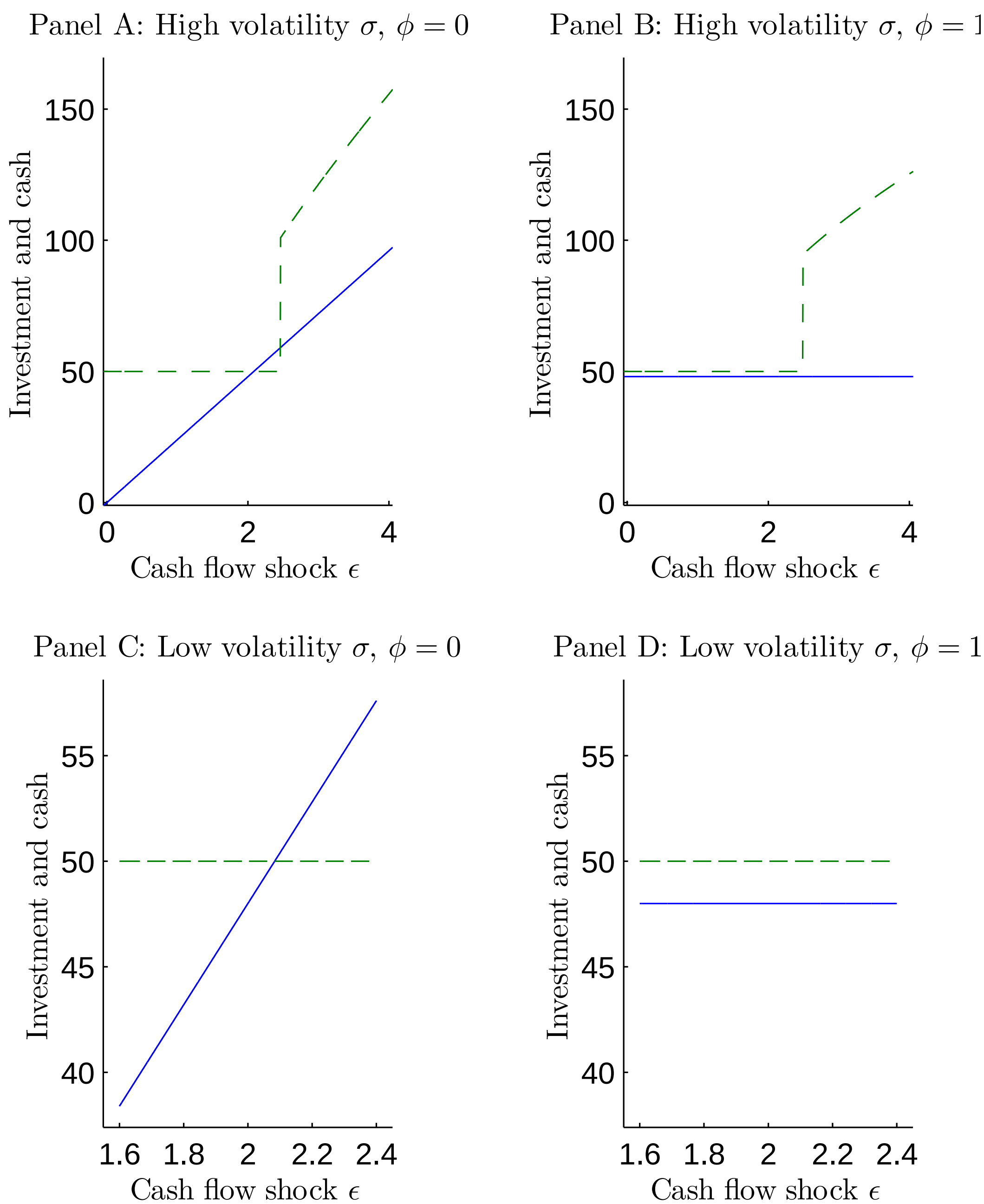


Figure 4. Expected Investment and Optimal Risk Management

Panel A shows the expected investment level of a constrained firm (solid line) and an unconstrained firm (dashed line) as a function of volatility σ. Panel B displays the optimal hedging ratio φ (solid line) as a function of volatility σ. Hedging ratios below zero (dashed line) indicate speculation. Whenever investment exceeds cash flow, the firm raises external financing. We assume $f(I) = b \log I$ and $C(e) = \frac{ke^2}{2}$ and set the parameters as follows: $w_1 = 24$, $w_0 = 0$, $I_0 = 50$, $b = 300$, $\alpha = 1$, $\beta = 1$, $f_0 = 1900$, $k = 0.08$, $\bar{\varepsilon} = 2$.

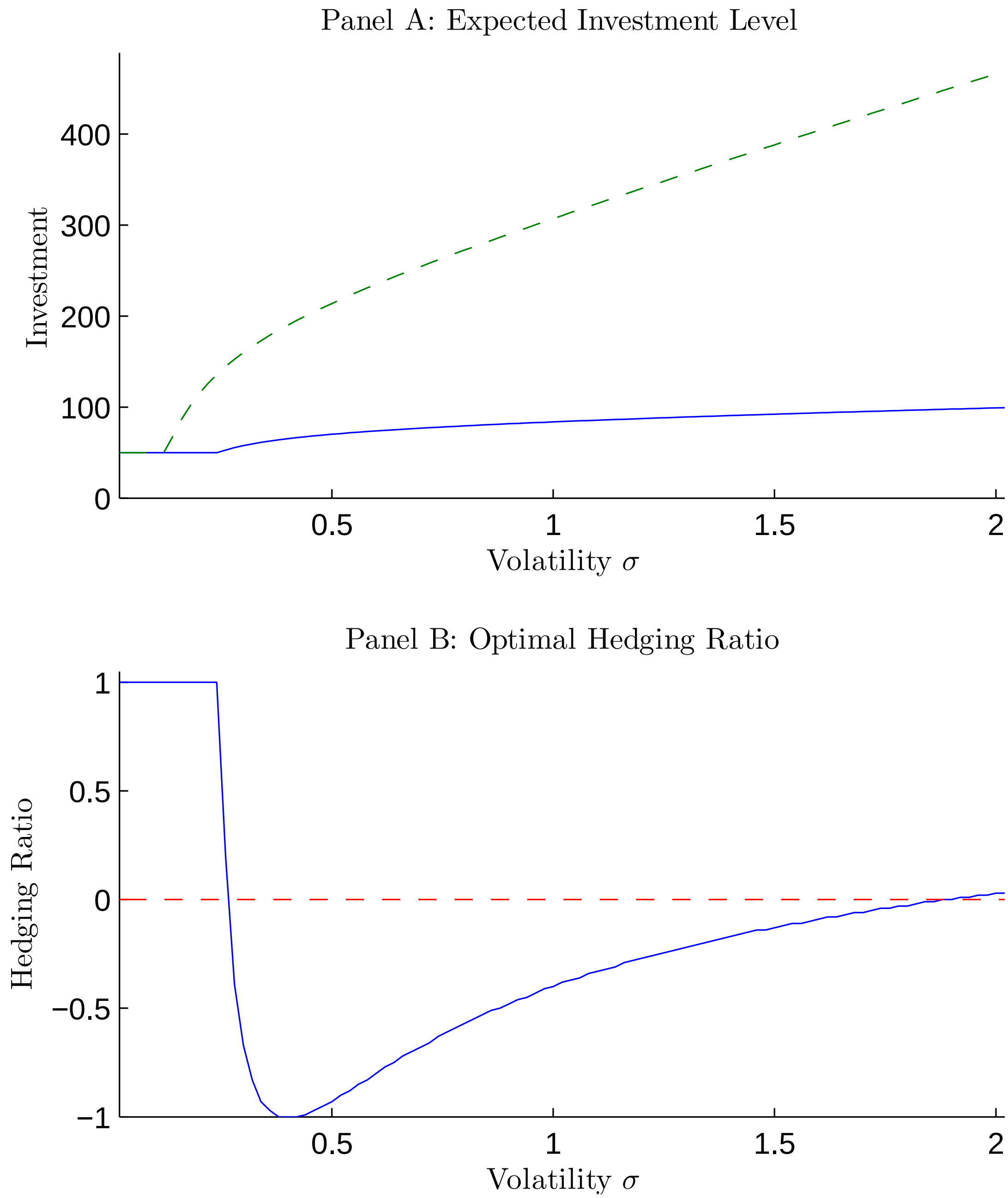


Figure 5. The Effect of Financing Constraints and Growth Options on Hedging Ratio

This figure shows the optimal hedging ratio for a firm with zero correlation between investment opportunities and cash flows (dashed line) and for a firm with positive correlation (solid line). Panel A plots the hedging ratio as a function of financing constraints w_0, and Panel B as a function of a firm's investment opportunities β. We assume $f(I) = b \log I$ and $C(e) = \frac{ke^2}{2}$ and set the parameters as follows: $w_1 = 24$, $w_0 = 0$, $I_0 = 50$, $b = 300$, $\beta = 1$, σ=0.8, $f_0 = 1900$, $k = 0.08$, $\bar{\varepsilon} = 2$.

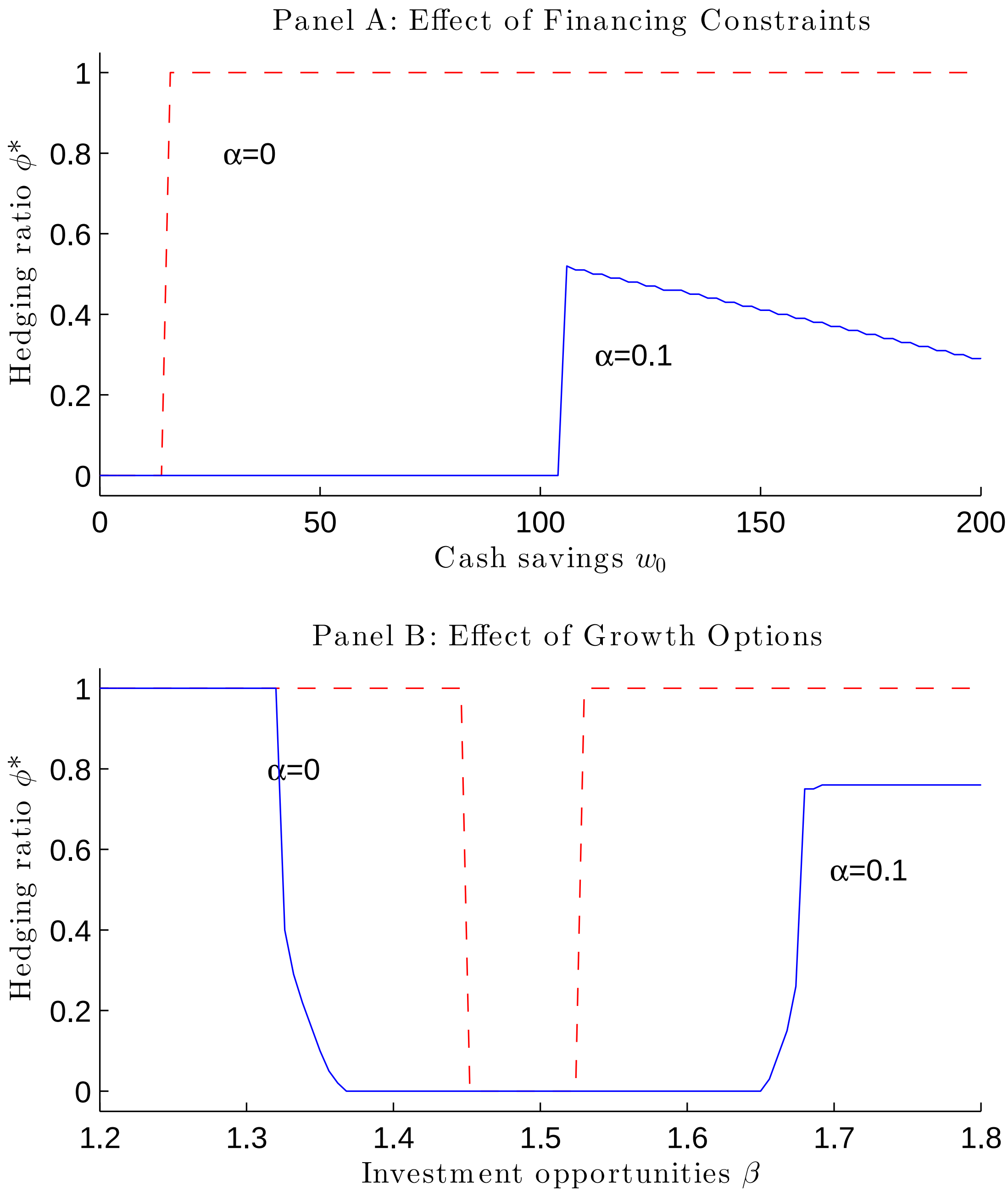


Figure 6. The Effect of Positive Correlation on Hedging Ratio

This figure shows the optimal hedging ratio as a function of correlation between investment opportunities and cash flows, α. We assume $f(I) = b \log I$ and $C(e) = \frac{ke^2}{2}$ and set the parameters as follows: $w_1 = 24$, $w_0 = 0$, $I_0 = 50$, $b = 300$, σ=0.8, $f_0 = 1900$, $k = 0.08$, $\bar{\varepsilon} = 2$.

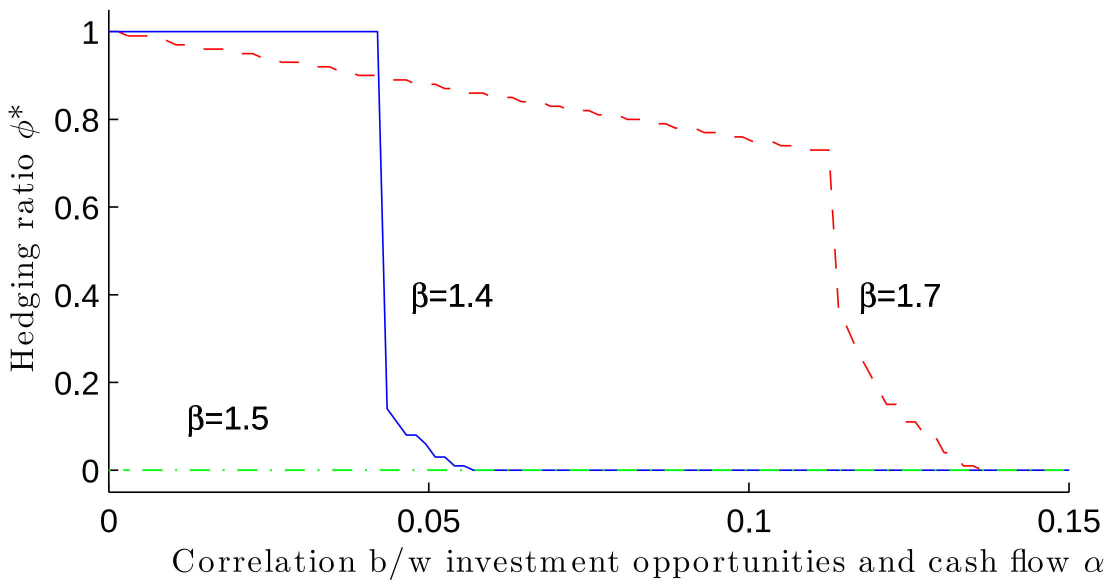
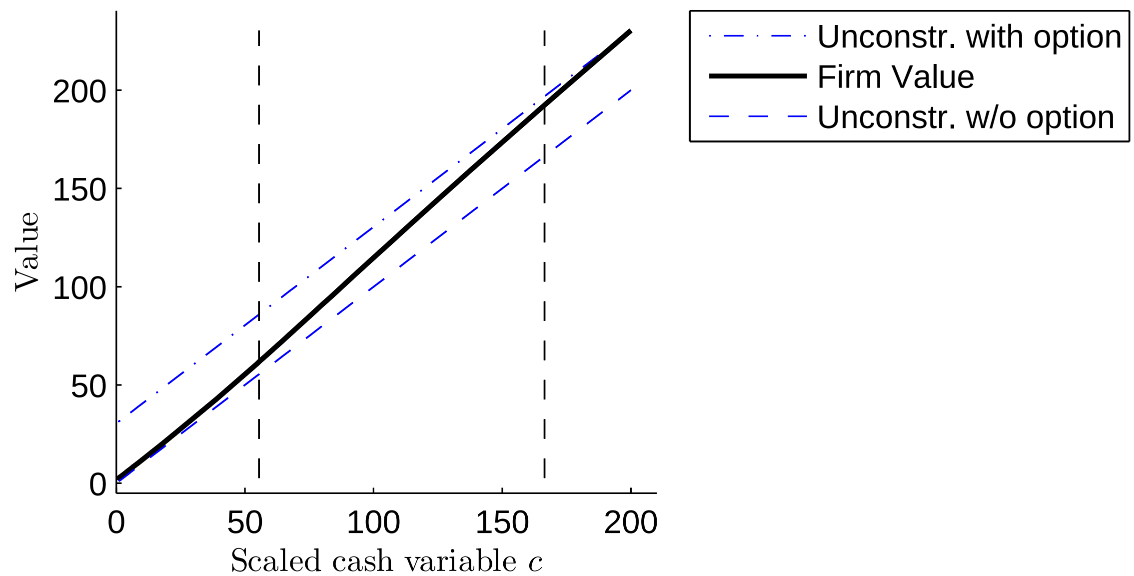


Figure 7. Value function $v\,(c)$ and its first derivative

This figure shows the shape of value function $v\,(c)$ (Panel A) and its first derivative (Panel B). The solution to ODE (37) is obtained numerically. The upper dashed line in Panel A gives the value of the firm if it were completely unconstrained (i.e., $k_d = k = 0$). The lower dashed line shows the value of the firm that does not have an investment option and is not subject to the costs of financial distress (i.e., $\lambda = 0$ and $k_d = 0$). Vertical lines denote the option exercise threshold and the value of c at which the firm can finance the exercise internally.

Panel A: Value function $v(c)$ and its asymptotes



Panel B: First derivative of $v(c)$