Performance Share Plans: Valuation and Optimal Design*

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Abstract

Performance share plans are an increasingly important component of executive compensation packages. A performance share plan is an equity-based, long-term incentive plan where the number of shares to be awarded is quasi-linear function of a performance measure over a fixed time period. A special case is a performance-vested share plan, which provides a fixed number of shares whenever a performance measure exceeds a threshold goal. We derive closed-form formulas for the value of a performance share plan or a performance-vested share plan when the performance measure is: (1) a non-traded measure following an Arithmetic Brownian Motion (e.g., earnings per share), (2) a non-traded measure following a Geometric Brownian Motion (e.g., revenue), or (3) the price of a traded asset following a Geometric Brownian Motion (e.g., a stock price). In a principal-agent setting, we solve for the optimal design of a performance share plan that maximizes outside shareholder wealth while accounting for the incentive effect on executive effort. We find that the optimal performance share plan is uncapped (has no upper bound), that the optimal slope of the payoff function balances the marginal incentive effect against marginal cost, and that performance-vested share plans are not optimal. Using actual performance share data of S&P 500 firms, we find that value of performance shares using our model is different from what firms report as the grant date fair value of awards.

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1 Introduction

A performance share plan is equity-based, long-term incentive component of executive compensation in which the number of shares to be awarded is based on a performance result. A plan can be based on a variety of alternative performance measures, such as earnings per share, revenue, stock price, return on invested capital, return on equity, etc. Once a performance measure is chosen, then three design parameters determine the plan's payoff structure. First is the threshold goal, which is a low level of performance below which the plan pays zero shares. Second is the stretch goal, which is a high level of performance above which the plan pays a maximum number of shares. Intermediate values of performance between the threshold goal and the stretch goal are called the incentive zone. In the incentive zone, the number of shares awarded increases linearly in performance and the third parameter is the slope of the payoff function (i.e., the change in shares awarded per change in performance).

A special type of performance share plan is a performance-vested share plan. A performance-vested share plan provides a fixed number of shares whenever performance exceeds a threshold goal and zero shares otherwise. In other words, a performance-vested share plan is performance share plan in which stretch goal is equal to threshold goal, or equivalently, in which the width of the incentive zone is zero.

The number of US firms using performance share plans for their top executives has increased dramatically in recent years. According to Frederick W. Cook, a compensation consulting firm, the percentage of firms that include performance share plans in their compensation packages for CEOs increased from 34% in 1997 to 62% in 2010.[1] Given the increased use of performance share plans by US public firms, the economics of performance share plans in executive compensation is an important topic for researchers.

We develop a theory of the valuation and optimal design of a performance share plan.

[1]Please see Figure 1, which is adapted from *The Top 250 Survey* by Frederick W. Cook from 1997 to 2010. Similar findings are also reported in *2011 CEO Pay Strategies Report* by Equilar.

We begin by deriving closed-form formulas for the value of a performance share plan or a performance-vested share plan when the performance measure is: (1) a non-traded measure following an Arithmetic Brownian Motion (e.g., earnings per share), (2) a non-traded measure following a Geometric Brownian Motion (e.g., revenue), or (3) the price of a traded asset following a Geometric Brownian Motion (e.g., a stock price). The value depends on the three design parameters: the threshold goal, stretch goal, and the slope of the payoff function. It also depends on environmental factors, such as the volatility of the performance measure, the beginning stock price, the beginning level of the performance measure, the length of the performance period, and the risk-neutral growth rate of the performance measure.[2]

Using a principal-agent model, we solve for the optimal design of performance share plans. We find that the optimal performance share plan is uncapped (i.e., the stretch goal goes to infinity), that the optimal slope of the payoff function balances the marginal incentive effect against marginal cost, and that performance-vested share plans are not optimal. We show that the optimal threshold goal is decreasing in the cost of effort and the optimal slope of the payoff function is sometimes increasing and sometimes decreasing in the cost of effort.

Using actual compensation data of S&P 500 firms, we find that the value of performance shares are different from what is reported as the grant date fair value of the grants. We find that the value of performance share plans using EPS as the performance measure are on average 20% to 27% higher than what firms reported as the fair value. For the plans using revenue as the performance measure, we find that the value of performance share plans are on average 34% to 45% lower than what firms reported as the fair value.

Our paper is related to three streams of literature within the larger literature on executive compensation.[3] One stream of literature analyzes the optimal design of CEO compensation

[2]For a non-traded performance measure, the risk-neutral growth rate is the nominal growth rate less the risk premium for the systematic risk of the performance measure. For a traded performance measure (e.g., a stock price), the risk neutral growth rate is the risk-free rate.

[3]Excellent surveys of the executive compensation literature include Abowd and Kaplan (1999), Murphy (1999), Prendergast (1999), Core, Guay, and Larcker (2003), and Hall and Murphy (2003).

in a canonical principal-agent setting. DeMarzo and Sannikov (2006) and DeMarzo and Fishman (2007) analyze the optimal dynamic contract including the dynamic reoptimization of effort over time. These studies have contributed to our knowlege of the optimal *unconstrained* design of CEO compensation, where compensation may take any functional form and may be renegotiated at any time. However, their great generality abstracts away from many real-world compensation components, such as options, bonuses, performance shares, etc.

A second stream of literature takes the functional form of real-world compensation compenents as given and perform a theoretical analysis of the up-front[4] executive compensation decision. Representative of this approach are Hall and Murphy (2000, 2002), Dittmann and Maug (2007), and Dittmann, Maug, and Spalt (2010), which take the functional form of stock options as given and perform a theoretical analysis of the up-front executive compensation decision in the presence of stock options. Analogously, we take the functional form of real-world performance shares as given and perform a theoretical analysis of the up-front executive compensation decision in the presence of performance shares.

A third stream of literature analyzes performance shares in particular. Martellini and Urosevic (2005) value performance shares when the performance measure is the stock price. This is a no-arbitrage result, because the underlying asset is traded. By contrast, we expand the scope to value performance shares when the performance measure is a non-traded measure following either an Arithmetic or a Geometric Brownian Motion. These two cases are equilibrium results, precisely because the underlying assets are not traded.[5] Bettis, Bizjak, Coles, and Kalpathy (2010) empirically investigate stock or option grants with performance-based vesting provisions. They find that these provisions provide meaningful incentives to execu-

[4]In practice, renegotiation typically takes place when the executive contract comes up for renewal and seldom within the life of a contract.

[5]A stream of accounting literature studies different performance measures. See Lambert and Larcker (1987), Bushman and Indjejikian (1993), Kim and Suh (1993), Sloan (1993), Lambert (1993), and Core, Guay, and Verrecchia (2003).

tives and document that the firms with performance-based vesting provisions significantly outperform the control firms. Bizjak, Kalpathy, and Thompson (2012), a concurrent paper to ours, develops *approximate* present value formulas for equity awards with performance-based vesting provisions. By contrast, we develop closed-form formulas both for the value of performance share plans and performance-vested share plans and we analyze their optimal design.

The remainder of the paper is organized as follows. Section 2 derives closed-form formulas for the value of performance share plans and performance-vested share plans. Section 3 solves a principal-agent model to determine the optimal design of performance share plans. Section 4 concludes. Appendix A provids background on performance share plans and compares the value of performance share plans using our model to the value actually reported by firms. Appendix B contains proofs.

2 Valuation

2.1 Payoff At Maturity

When designing a performance share plan, the board of directors first chooses the performance measure. In our sample (see Appendix A), the most widely-used performance measures are stock return, earnings per share, revenue, return on invested capital, and return on equity.

Next, the board of directors designs the share reward function, which is illustrated in Figure 2(a). The x-axis is the level of performance at maturity (M_T). The share reward function is the thick, three-segment line, which shows the number of shares awarded as a function of performance. It is a quasi-linear function of performance that is capped at a lower bound (L) and an upper bound (H). If performance is at or below L, then zero shares are awarded. If performance is between L and H, then the number of shares awarded is a linear function of performance with a slope λ.[6] If performance is at or above H, then a maximum number of shares (N_H) are awarded, where N_H is related to the other three contract parameters as follows

$$N_H = \lambda(H - L). \tag{1}$$

Based on this structure, we can express the number of shares (N_T) that are awarded at maturity under a performance share plan as

$$N_T = \begin{cases} 0 & \text{for } M_T \leq L & \text{(2a)} \\ \lambda\,(M_T - L) & \text{for } L < M_T \leq H, & \text{(2b)} \\ N_H & \text{for } H < M_T. & \text{(2c)} \end{cases}$$

The monetary payoff of a performance share plan (PSP_T) is the number of shares awarded multiplied by the stock price at maturity (S_T)

$$PSP_T = N_T S_T. \tag{3}$$

Figure 2(b) shows that the monetary payoff of a performance share plan depends on *two* random variables: (1) the performance measure at maturity (M_T) and (2) the stock price at maturity (S_T). The influence of the performance measure at maturity is easily seen by looking along the upper-left edge, where the monetary payoff is flat below L, increasing linearly between L and H, and then flat above H. The influence of the stock price at maturity is easily seen by looking along the upper-right edge, where the monetary payoff increases linearly in the stock price at maturity.

A bull spread is an option trading strategy involving two options on the same underlying

[6] In practice, the slope of the payoff function is typically specified using two parameters: a target goal (Z) and the target number of shares (N_Z) to be awarded if performance exactly equals the target goal ($M_T = Z$). The slope can then calculated as $\lambda = \frac{N_Z}{Z-L}$.

asset with the same maturity date, but with different exercise prices. Specifically, it is a long position in a call (put) option with a lower exercise price and a short position in a call (put) option with a higher exercise price. The share reward function in Figure 2(a) resembles the monetary payoff diagram of a bull spread. However, Figure 2(b) makes clear that a performance share plan depends on two random variables. By contrast, a bull spread only depends on one random variable, the stock price at maturity, and does not depend on a performance measure at maturity.[7]

But, there is an interesting analogy. Like a bull spread, a performance share plan can be shown to be a long position and a short position in a simpler component. We call this simpler component an *uncapped performance share plan*. Figure 3 shows the share reward function of two different uncapped performance share plans plotted against performance at maturity (M_T). The first uncapped performance share plan has a threshold goal of L. It awards zero shares when performance is at or below L and awards $\lambda(M_T - L)$ shares when performance is above L. The second uncapped performance share plan has a threshold goal of H. It awards zero shares when performance is at or below H and awards $\lambda(M_T - H)$ shares when performance is above H. An uncapped plan can be thought of as the limiting case of a regular performance share plan as the upper bound goes to infinity - thus, the name *uncapped*.

The table below shows that the share reward function of two uncapped plans sum to a performance share plan. Specifically, the last three columns show the share rewards for the three regions of performance: (1) below L, (2) between L and H, and (3) above H. The first row shows the share reward function of a long position in an uncapped plan with a threshold L and the second row shows the same for a short position in an uncapped plan

[7]Holmström (1979) argues that any additional information about the managerial effort, regardless of how imprecise it is, can improve the utility of both the principal and the agent. An implication of this insight is that performance shares, which pay the manager based on stock price at maturity and another performance measure at maturity, could potentially improve shareholder utility above the stock options, which pay the manager solely based on the stock price at maturity.

with a threshold H.

Table 1: Two uncapped plans sum to a performance share plan

The share reward function of a long position in an uncapped performance share plan with a threshold L and a short position in an uncapped performance share plan with a threshold H sum to the share reward function of a performance share plan with threshold L and stretch H.

	$M_T \leq L$	$L < M_T \leq H$	$H < M_T$
Long an Uncapped Perf. Share Plan with threshold L	0	$\lambda(M_T - L)$	$\lambda(M_T - L)$
Short an Uncapped Perf. Share Plan with threshold H	0	0	$-\lambda(M_T - H)$
Total = Perf. Share Plan with threshold L and stretch H	0	$\lambda(M_T - L)$	$\lambda(H - L) = N_H$

The final row shows the total of these two positions. It is identical to the share reward function of a performance share plan with threshold L and stretch H shown above in equation 2. The tight connection between a performance share plan and the two corresponding uncapped plans will carry over their valuation.

2.2 Stochastic Processes

We observe that performance share plans are based on a wide variety of performance measures that have different attributes. Some performance measures have strictly non-negative realizations (e.g., revenue). However, other performance measures can be either positive or negative (e.g., earnings per share, return on equity). Most performance measures are not traded assets. However, an exception is when the performance measure is the firm's stock price.

To encompass all of these cases, we will value performance share plans under three alternative modeling assumptions: (1) a non-traded performance measure following an Arithmetic Brownian Motion, (2) a non-traded performance measure following a Geometric Brownian Motion, or (3) the price of a traded asset following a Geometric Brownian Motion. We begin by analyzing the first case and then turn to the latter two cases after that.

Let M_t be the performance measure at time t at any time during the performance period $[0, T]$. Assume that the performance measure is not traded, but evolves continuously based on an Arithmetic Brownian Motion as given by

$$dM = \alpha_M dt + \sigma_M dW_1, \tag{4}$$

where α_M is the instantaneous drift of performance, σ_M is the instantaneous standard deviation of performance, and dW_1 is the increment of a standard Wiener process. An Arithmetic Brownian Motion can have negative realizations, so this would be a good model to represent performance measures that can go negative (e.g., earnings per share, free cash flow, operating income, etc.).

Let S_t be the firm's stock price at time t. Assume that the stock price follows a Geometric Brownian Motion as given by

$$\frac{dS}{S} = hdM + \alpha_S dt + \sigma_S dW_2, \tag{5}$$

where h is the sensitivity of the stock price to the performance measure, α_S is the instantaneous drift of the stock, σ_M is the instantaneous standard deviation of the stock, and dW_2 is the increment of a standard Wiener process which is independent of dW_1. Substituting (4) into (5), we obtain

$$\frac{dS}{S} = (h\alpha_M + \alpha_S)\, dt + h\sigma_M dW_1 + \sigma_S dW_2. \tag{6}$$

We value performance shares using the risk-neutral valuation methodology of Cox and Ross (1976).[8] To do so, we transform the processes above to their corresponding risk-neutral processes. For a non-traded asset this is done by reducing the instantaneous growth rate by

[8]The Cox and Ross (1976) risk-neutral method values derivative securities as if agents are risk neutral, but it does not require that agents actually are risk neutral.

the market price of risk times the corresponding instantaneous standard deviation (σ_M).[9] Let ν be the market price of risk for this particular type of risk. Let $\hat{M}_t$ be the performance measure under the following risk-neutral process

$$d\hat{M} = (\alpha_M - \nu\sigma_M)\, dt + \sigma_M dW_1. \tag{7}$$

For a traded asset, such as a stock, the instantaneous drift is adjusted to be the instantaneous riskfree rate r. Let $\hat{S}_t$ be the stock price under the following risk-neutral process

$$\frac{d\hat{S}}{\hat{S}} = rdt + \sigma dz, \tag{8}$$

where $\sigma \equiv \sqrt{h^2\sigma_M^2 + \sigma_S^2}$ and dz is an increment of a standard Wiener process. Based on these processes, the terminal value of the risk-neutral performance measure ($\hat{M}_T$) is normally distributed and the terminal value of risk-neutral stock price ($\hat{S}_T$) is log-normally distributed. For simplicity, let $\hat{Y}_T$ be the natural log of the risk-neutral stock return $\ln\left(\hat{S}_T/S_0\right)$, which is normally distributed. Thus, the distributions of $\hat{M}_T$ and $\hat{Y}_T$ are given by:

$$\hat{M}_T \sim \mathcal{N}\left(M_0 + (\alpha_M - \nu\sigma_M)\, T, \sigma_M^2 T\right) \text{ and} \tag{9}$$

$$\hat{Y}_T \sim \mathcal{N}\left(\left(r - \frac{1}{2}\sigma^2\right) T, \sigma^2 T\right). \tag{10}$$

[9]See, for example, Hull (2012), page 767.

Finally, the correlation between $\hat{M}_T$ and $\hat{Y}_T$ is[10]

$$\rho = \frac{\sigma_M}{\sqrt{\sigma_M^2 + \frac{\sigma_S^2}{h^2}}}. \tag{11}$$

2.3 Date 0 Value

2.3.1 When A Non-Traded Measure Follows An Arithmetic Brownian Motion

Let $PSP_0^i(L, \lambda, H)$ be the date 0 value of a performance share plan with threshold goal L, slope λ, stretch goal H, and the superscript $i \in \{A, G, P\}$ identifies one of the three types of performance measures. Let $UPS_0^i(L, \lambda)$ be the date 0 value of an uncapped performance share plan, where the threshold goal is the first argument (L in this case or H in another case), a slope λ, and the superscript $i \in \{A, G, P\}$ identifies one of the three types of performance measures. In this subsection, an A superscript is used to identify variables in the case when a non-traded performance measure follows an Arithmetic Brownian Motion (e.g., earnings per share).

Proposition 1 *When a non-traded performance measure follows an Arithmetic Brownian Motion, the date 0 value of a performance share plan is*

$$PSP_0^A(L, \lambda, H) = UPS_0^A(L, \lambda) - UPS_0^A(H, \lambda) \tag{12}$$

[10] The correlation between $\hat{M}_T$ and $\hat{Y}_T$ can be derived as follows:

$$\rho = \frac{\sigma_{\hat{M}\hat{Y}}}{\sigma_{\hat{M}}\sigma_{\hat{Y}}} = \frac{h\sigma_M^2 T}{\sigma_M\sqrt{T}\sigma\sqrt{T}} = \frac{h\sigma_M}{\sigma} = \frac{h\sigma_M}{\sqrt{h^2\sigma_M^2 + \sigma_S^2}} = \frac{\sigma_M}{\sqrt{\sigma_M^2 + \frac{\sigma_S^2}{h^2}}}.$$

where

$$UPS_0^A(L,\lambda) = S_0\lambda\left[\{M_0 + (\alpha_M - \nu\sigma_M)T - L\}N\left(d_1^A\right) + \sigma_M\sqrt{T}n\left(d_1^A\right)\right] \tag{13}$$

$$UPS_0^A(H,\lambda) = S_0\lambda\left[\{M_0 + (\alpha_M - \nu\sigma_M)T - H\}N\left(d_2^A\right) + \sigma_M\sqrt{T}n\left(d_2^A\right)\right], \tag{14}$$

and where $d_1^A = \frac{M_0 - L + \left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$, $d_2^A = \frac{M_0 - H + \left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$, *and* $N(\cdot)$ *and* $n(\cdot)$ *are the cumulative distribution and density functions of the standard normal.*

Proof See Appendix B.

Intuitively, equation (12) shows that the value of a performance share plan is equal to the value of a long position in an uncapped performance share plan with a threshold goal of L and the value of a short position in an uncapped performance share plan with a threshold goal of H. Intuitively, equation (13) shows that the value of an uncapped performance share plan is the product of the current stock price S_0, the slope of the payoff function λ, and the term in square brackets, which is the expected value of the performance measure given that it is above L times the probability that it is above L.[11]

Figures 4(a) and 4(b) illustrate the value of a performance share plan. In Figure 4(a), the solid curve is the ex-ante value of a performance share plan, which rises rapidly from slightly below the threshold goal, continues rising in the incentive zone, slows down as current performance (M_0) approaches the stretch goal. The value asymptotically approaches S_0N_H. By analogy to the options literature, the dotted line represents the *intrinsic value* of the performance share and the vertical gap between the date 0 value of a performance share and the intrinsic value represents the *time value* of the performance share plan. The time value is positive over most of the incentive zone, but turns slightly negative near the stretch goal H. Figure 4(b) shows how the value of a performance share varies with the current level

[11]This formula is qualitatively different from the Black-Scholes option pricing model in that it involves a performance measure component $(M_0 + (\alpha_M - \nu\sigma_M)T)$, in addition to the stock price.

of performance (M_0) and the current stock price (S_0). In the upper-left edge, we observe the value curve, which increases with performance in the incentive zone. In the upper-right edge, we observe that the value of performance share increases linearly with current stock price.

2.3.2 When A Non-Traded Measure Follows A Geometric Brownian Motion

Now we consider the case in which a non-traded performance measure follows a Geometric Brownian Motion. A Geometric Brownian Motion never goes negative, so this would be a good model to represent performance measures that never go negative (e.g., revenue). Specifically, we assume that

$$\frac{dM}{M} = \alpha_M dt + \sigma_M dW_1, \tag{15}$$

and

$$\begin{aligned} \frac{dS}{S} &= h\frac{dM}{M} + \alpha_S dt + \sigma_S dW_2 & (16) \\ &= (h\alpha_M + \alpha_S)\, dt + h\sigma_M dW_1 + \sigma_S dW_2, & (17) \end{aligned}$$

where h is the sensitivity of the stock price to change in the performance measure, and dW_1 and dW_2 are increments of independent standard Wiener processes. A G superscript is used to identify variables in the case when a non-traded performance measure follows a Geometric Brownian Motion.

Proposition 2 *When a non-traded performance measure follows a Geometric Brownian Motion, the date 0 value of a performance share plan is*

$$PSP_0^G(L, \lambda, H) = UPS_0^G(L, \lambda) - UPS_0^G(H, \lambda) \tag{18}$$

where

$$UPS_0^G(L,\lambda) = S_0\lambda\left[M_0 e^{\left(\alpha_M-\nu\sigma_M+h\sigma_M^2\right)T} N\left(d_1^G\right) - LN\left(d_2^G\right)\right], \tag{19}$$

$$UPS_0^G(H,\lambda) = S_0\lambda\left[M_0 e^{\left(\alpha_M-\nu\sigma_M+h\sigma_M^2\right)T} N\left(d_3^G\right) - HN\left(d_4^G\right)\right], \tag{20}$$

and where $d_1^G = \frac{\ln\frac{M_0}{L}+\left(\alpha_M-\nu\sigma_M+\left(h+\frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$, $d_2^G = d_1^G - \sigma_M\sqrt{T}$, $d_3^G = \frac{\ln\frac{M_0}{H}+\left(\alpha_M-\nu\sigma_M+\left(h+\frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$, *and* $d_4^G = d_3^G - \sigma_M\sqrt{T}$.

Proof See Appendix B.

2.3.3 When A Traded Asset Price Follows A Geometric Brownian Motion

Now we consider the case in which the performance measure is the price of a traded asset following a Geometric Brownian Motion (e.g., a stock price). Specifically, we assume that

$$\frac{dM}{M} = \frac{dS}{S} = \alpha_S dt + \sigma_S dW_1. \tag{21}$$

When the underlying asset (the stock) is a traded asset, a performance share plan can be valued by the no arbitrage approach of Black and Scholes (1993). Here we obtain the Martellini and Urosevic (2005) result as a special case of our more general Geometric Brownian Motion result. A P superscript is used to identify variables in the case where the performance measure is the price of a traded asset following a Geometric Brownian Motion.

Proposition 3 (Martellini and Urosevic) *When the performance measure is the price of a traded asset following a Geometric Brownian Motion, the date 0 value of a performance share plan is*

$$PSP_0^P(L,\lambda,H) = UPS_0^P(L,\lambda) - UPS_0^P(H,\lambda) \tag{22}$$

where

$$UPS_0^P(L,\lambda) = S_0\lambda\left[S_0 e^{\left(r+\sigma_S^2\right)T} N\left(d_1^P\right) - LN\left(d_2^P\right)\right], \tag{23}$$

$$UPS_0^P(H,\lambda) = S_0\lambda\left[S_0 e^{\left(r+\sigma_S^2\right)T} N\left(d_3^P\right) - HN\left(d_4^P\right)\right], \tag{24}$$

and where $d_1^P = \frac{\ln\frac{S_0}{L}+\left(r+\frac{3}{2}\sigma_S^2\right)T}{\sigma_S\sqrt{T}}$, $d_2^P = d_1^P - \sigma_S\sqrt{T}$, $d_3^P = \frac{\ln\frac{S_0}{H}+\left(r+\frac{3}{2}\sigma_S^2\right)T}{\sigma_S\sqrt{T}}$, *and* $d_4^P = d_3^P - \sigma_S\sqrt{T}$.

Proof This follows immediately from Proposition 2 by changing performance measure values to stock price values: $M_0 = S_0$, $h = 1$, and $\sigma_M = \sigma_S$. Since the underlying asset is the price of a traded asset, the performance share plan can be valued by the no arbitrage approach. In this case, the risk neutral growth rate becomes the riskfree rate ($\alpha_M - \nu\sigma_M = r$). **Q.E.D.**

This formula is similar to that of Proposition 2, but it is fundamentally different in one respect from those of both Propositions 1 and 2. Unlike other performance measures which are not traded securities, a stock is a traded security, and thus the result in Proposition 3 is a result of no arbitrage. In this case, there are no preference parameters in the formula. In contrast, the results for performance measures other than the stock price are equilibrium results, and these models involve the preference parameters α_M and ν.

2.3.4 Factors affecting the value of performance shares

Figures 5(a)-5(c) show how the value of a performance share plan is affected by the three contract parameters. Figure 5(a) shows the value of a performance share plan for different threshold goals, while keeping the slope the same and the width of the incentive zone $(H-L)$ constant. L (and H) are for the low threshold goal plan and L' (and H') are for the high threshold goal plan. A low threshold goal plan is more valuable at all levels of current performance

Figure 5(b) shows the value of a performance share plan for different slopes (λ) but with the same target and stretch goals. The valuation formulas are all multiplied by λ, so doubling λ doubles the value of a performance share plan.

Figure 5(c) shows the value of a performance share plan for different widths of incentive zone. L and H represent a narrow incentive zone and L' and H' represent a wide incentive zone. A narrow incentive zone plan is more valuable at low levels of current performance and a wide incentive zone plan is more valuable at high levels of current performance.

Figures 5(d)-5(f) shows how the value of a performance share plan is affected by various environmental factors. Figure 5(d) shows the value of a performance share for different volatilities of the performance measure. For high (low) values of current performance, higher volatility decreases (increases) the value of a performance share plan because there are limited potential gains (losses) on the upside (downside) and greater potential losses (gains) on the downside (upside).

Figure 5(e) shows the value of a performance share plan for different current stock prices. As the stock price increases, the value of a performance share increases, simply because each share is worth more. The valuation formulas all begin with the stock price S_O, so doubling the stock price doubles the value of a performance share plan.

Finally, Figure 5(f) shows the value of a performance share plan for different times to maturity (or different lengths of a performance period). Analogous to options, more time increases the value of a performance share plan, because it increases the time value component (the extra value above the intrinsic value).

2.4 Performance-Vested Share Plans

A special case of a performance share plan is a performance-vested share plan. A performance-vested share plan provides a fixed number of shares (N_H) whenever performance exceeds a threshold goal ($M_T > L$) and zero shares otherwise. In other words, a performance-vested share plan is performance share plan in which stretch goal is equal to threshold goal ($H = L$), or equivalently, in which the width of the incentive zone is zero ($H - L = 0$).

Let PVS_0^i be the date 0 value of a performance-vested share plan, where $i = A$ is when a non-traded performance measure follows an Arithmetic Brownian Motion, $i = G$ is when a non-traded performance measure follows an Geometric Brownian Motion, and $i = P$ is when the performance measure is the price of a traded asset following an Geometric Brownian Motion. The following proposition values performance-vested share plans in these three cases.

Proposition 4 *Under three alternative assumptions about the performance measure, the date 0 value of a performance-vested share plan is*

$$PVS_0^A = S_0 N_H N\left(d_2^A\right), \tag{25}$$

$$PVS_0^G = S_0 N_H N\left(d_4^G\right), \tag{26}$$

$$PVS_0^P = S_0 N_H N\left(d_4^P\right), \tag{27}$$

where $d_2^A = \frac{M_0 - H + \left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$, $d_4^G = \frac{\ln\frac{M_0}{H} + \left(\alpha_M - \nu\sigma_M + \left(h + \frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M\sqrt{T}} - \sigma_M\sqrt{T}$, *and* $d_4^P = \frac{\ln\frac{S_0}{H} + \left(r + \frac{3}{2}\sigma_S^2\right)T}{\sigma_S\sqrt{T}} - \sigma_S\sqrt{T}$.

Proof The performance-vested share plan formulas are obtained by starting with the analogous performance share plan formula and then evaluating the limit as $H \to L$. Starting from the Proposition 1 formula, $\lim_{H \to L} d_1^A = d_2^A$, so the first and third terms vanish and the second term divided by $H - L$ goes to $N\left(d_2^A\right)$, which yields equation (25). Starting from the Proposition 2 formula, $\lim_{H \to L} d_1^G = d_3^G$, so the first term vanishes and $\lim_{H \to L} d_2^G = d_4^G$, so the second term divided by $H - L$ goes to $N\left(d_4^G\right)$, which yields equation (26). Starting from the Proposition 3 formula, $\lim_{H \to L} d_1^P = d_3^P$, so the first term vanishes and $\lim_{H \to L} d_2^P = d_4^P$, so the second term divided by $H - L$ goes to $N\left(d_4^P\right)$, which yields equation (27). **Q.E.D.**

All three performance-vested share plan formulas have the intuitive interpretation of being the current stock price times the fixed number of shares times the probability that performance exceeds a threshold goal ($M_T > L$) under the risk neutral process.

3 Optimal Design

In this section, we embed the valuation models derived above in a principal-agent model to determine the optimal design of a performance share plan. Our model follows the classic principal-agent model. The firm hires a manager with a compensation plan at the beginning of the period (time 0) and compensates the manager at the end of the period (time T). We assume that the manager is compensated via a fixed salary (b) paid at the end of the period and a performance share plan with threshold goal L, slope λ, and stretch goal H. During the performance period, the manager exerts effort to maximize her expected utility given the compensation contract. At the end of the period, the manager receive her compensation.

3.1 The Manager's Problem

The manager's initial endowment is assumed to be zero. Thus, her end of the period wealth is solely her compensation. The manager's end-of-period compensation C_T is

$$C_T = b + S_T \left(\max\left\{0, \min\left\{\left(M_T - L\right)\lambda, N_H\right\}\right\}\right), \tag{28}$$

where the second term is the payoff of the performance share plan.

The performance of the firm is assumed to depend on both the manager's realized effort a and other random factors. Let the end-of-period performance M_T be given by

$$M_T = M_B + a + \int_0^T dM, \tag{29}$$

where M_B is the hypothetical base level of firm performance that would result if the manager put in zero effort and $\int_0^T dM$ is the influence of other random factors over time.

We assume that on date 0, immediately after the manager is hired, the compensation contract is publically disclosed and the market forecasts the manager's effort $\bar{a}$ under the contract. Also on date 0, the market updates its forecast firm performance by incorporating its forecast of the manager's effort $M_0 = M_B + \bar{a}$. Finally, the date 0 stock price updates to $S_0 = RM_0 = R(M_B + \bar{a})$, where R is a stock price / performance ratio.

Analogously, the terminal stock price S_T depends on the manager's realized effort a and other random factors as follows

$$S_T = R\left(M_B + a\right) + \int_0^T dS, \tag{30}$$

where $\int_0^T dS$ is the influence of other random factors over time and we assume that the stock does not pay any dividends.

The manager's utility $U_M\left(C_T(a), a\right)$ is assumed to depend on terminal compensation, which is influenced by the manager's effort, and on the disutility of effort. Specifically, we assume that the manager's utility is risk neutral in terminal compensation and suffers disutility as a cubic funtion of effort[12]

$$U_M\left(C_T(a), a\right) = C_T(a) - ka^3, \tag{31}$$

where k is the manager's utility cost of effort.

The manager chooses a non-negative effort level to maximize her expected utility of

[12]Examining the performance share plan formulas in Propositions 1 and 2, we see that compensation is linear in the current stock price S_0 (which is linear in effort), has a term that is linear in current performance M_0 (which is linear in effort), and includes the cumulative normal terms $N\left(\right)$ (which are influenced by effort). So overall compensation is greater than a quadratic function of effort, but less than a cubic function of effort. Thus, the disutility of effort must be at least cubic in effort in order to produce a well-defined concave funtion with a unique optimum.

terminal wealth given the compensation contract.

$$\max_{a\in[0,\infty)} E\left(C_T(a) - ka^2\right), \tag{32}$$

$$\text{s.t. } E\left(C_T(a) - ka^2\right) \geq \overline{U}, \tag{33}$$

where $\overline{U}$ is the reservation utility of the manager. Equation (32) is the incentive compatibility condition and equation (33) is the participation constraint of the manager. Let a^* denote optimal managerial effort, which maximizes her expected utility above. In a rational expectations equilibrium, the market's forecast of managerial effort must turn out to be correct ($\bar{a} = a^*$).

3.2 The Outside Shareholders' Problem

Shareholders are assumed to be risk neutral. Outside shareholders[13] are assumed to maximize outside shareholder value (i.e., the value of the firm net of compensation paid to the manager). The essential trade-off is that a compensation contract can create an incentive for the manager to increase effort, which increases outside shareholder value, but the cost of managerial compensation decreases outside shareholder value. Let C_0 be the date 0 value of the manager's compensation. Discounting (28) back to present, we get

$$C_0 = be^{-rT} + PSP_0^i(L, \lambda, H). \tag{34}$$

where $i \in \{A, G, P\}$ represents the three alternative performance measure cases.

Let N be the number of shares outstanding. It is assumed that shareholders understand the manager's problem above and thus can correctly forecast the optimal managerial effort

[13]In practice, the board of directors designs the compensation contracts and negotiates with the manager. It is assumed that the board acts in the best interests of outside shareholders.

a^* that will result from a given compensation plan. The outside shareholders' problem is

$$\max_{L,\lambda,H} NS_0\left(a^*\right) - C_0, \tag{35}$$

$$\text{s.t. } a^* = \arg\max_{a\in[0,\infty)} E\left(U_M\left(C_T(a), a\right), \right) \tag{36}$$

$$L, \lambda, H \geq 0, and \tag{37}$$

$$H \geq L. \tag{38}$$

Equation (35) has outside shareholders choosing optimal compensation plan parameters L, λ, and H, which combined with the resulting optimal managerial effort, maximizes outside shareholder value. Equation (37) is the non-negativity constraints on threshold goal, slope, and stretch goal. Equation (38) states that the stretch goal needs to be at least as high as the threshold goal.

3.3 Numerical Solution

We solve the principal-agent model by dynamic programming. Since the manager moves last, the manger's problem is solved first. Given that the manager's problem is nonlinear, we solve it numerically using a standard hill-climbing technique for constrained optimization. The result is an optimal managerial effort for a given triplet of contract parameters $a^*\left(L, \lambda, H\right)$.

Next, we turn to the outside shareholders' problem. Outside shareholders wish to determine the optimal contract parameters out of the set of all feasible contract parameters. In order to analyze the outside shareholders objective function we need to know the optimal managerial effort $a^*\left(L, \lambda, H\right)$ for any set of contract parameters that we wish to consider. We could use brute force to determine the optimal managerial effort and outside shareholder objective function for thousands or even tens of thousands of contract parameter triplets and then select the highest outside shareholder objective function, but that would be inefficient.

Instead, we devise a method to approximate the optimal managerial effort over a reasonable range of contract parameters and then steadily improve the accuracy of the approximation to any arbitrary degree of accuracy.

Here are the steps:

1. Select an upper bound (subscript U) and a lower bound (subscript L) for each of the three contract parameters. Figure 6 shows how these bounds specify a three-dimensional space in the shape of a cube, where the triplet of parameters (L, λ, H) are all within the corresponding bounds $L \in [L_L, L_U]$, $\lambda \in [\lambda_L, \lambda_U]$, and $H \in [H_L, H_U]$.

2. Solve the manager's problem using a standard hill-climbing technique for constrained optimization to obtain the optimal managerial effort for the eight corners of the cube. Figure 6 shows the optimal managerial effort a^* as a function of the triplet of contract parameters at each corner: $a^*(L_L, \lambda_L, H_L)$, $a^*(L_U, \lambda_L, H_L)$, ..., $a^*(L_U, \lambda_U, H_U)$.

3. In the outside shareholder's problem, approximate the optimal managerial effort for any point in the cube space using a weighted-average of the eight corners as given by $a^*(L, \lambda, H) = \sum_{i=L,U} \sum_{j=L,U} \sum_{k=L,U} w_i^L w_j^\lambda w_k^H a^*(L_i, \lambda_j, H_k)$, where the weights in the λ dimension are $w_L^\lambda = \frac{\lambda_U - \lambda}{\lambda_U - \lambda_L}$, and $w_U^\lambda = 1 - w_L^\lambda$ and the weights in the L and H dimensions are analogous. For any point in the cube space, this function *interpolates* an approximate optimal effort from the precise optimal effort values of the eight corners.

4. Solve the outside shareholders' problem, subject to the constraints in step 1, using a standard hill-climbing technique for constrained optimization to obtain a provisional optimal contract triplet (L^*, λ^*, H^*).

5. Branch depend the following conditions:

   (a) If a provisional optimal contract parameter is on an upper (lower) bound, then raise (lower) that upper (lower) bound and repeat from step 2.

(b) If all three provisional optimal contract parameters are on the interior of their respective ranges, then shrink both the upper and lower bounds towards the interior point (usually about halfway on each side) on one or more dimensions and repeat from step 2.

(c) When the upper and lower bounds have become sufficiently close to the interior point on all three dimensions such that the optimal effort values for the eight corners are identical to an arbitrary number of digits (i.e., the approximation in step 3 achieves an arbitrary degree of accuracy), then stop.

The beauty of this approach is that the solution to the outside shareholders' problem is on the interior of the cube, so the upper and lower bound constraints are *not* binding, and the cube becomes arbitrarily small, so the solution to the manager's problem can achieve any arbitrary degree of accuracy. In other words, both problems are solved at the same time without any binding constraints and to any degree of accuracy.

3.4 Optimal Design Results

3.4.1 The Unconstrained Case

One case that we analyze the following parameter values: salary b=\$1,000,000, cost of effort k=1,000,000, performance with zero effort M_B=2, mean performance change α_M=2, standard deviation of performance change σ_M=2, mean stock return α_S=12%, standard deviation of stock return σ_S=20%, market price of risk ν=.2, sensitivity to stock price to performance h=.01, stock price to performance ratio R=10, time to maturity T=1 year, riskfree rate r=1%, number of outstanding shares N=1,000,000, and type of performance measure is a non-traded performance measure following a Geometric Brownian Motion (e.g., revenue).

For this case we find that one set of provisional optimal contract parameters is $L = 589.94$, $\lambda = 78,634$, and $H = 100,589.94$. Given that $M_0 = 3.21$ and $\sigma_M = 2$, then L is 293 standard deviations above the mean. Thus, the odds of a manager getting a positive payoff under such a plan are astronomical. But if you do get a positive it could be huge. In this case, the maximum number of shares N_H works out to be 7,863,414,573 shares. In other words, these provisional optimal contract parameters describe a "lottery ticket" – it rarely pays off, but the payoff could be large.

There is a close relationship between the three parameters, such that manager compensation stays in a reasonable relationship to outside shareholder value. In this case, manager compensation C_0 =$6.1 million and outside shareholder value is $26.1 million. However, these are *provisional* parameters. All three parameters increase with each iteration of the numerical solution process. In effect, an even rarer lottery ticket with an even larger slope increases outside shareholder value. There is no upper limit to these provision solutions. Thus, there is no well-defined solution.

3.4.2 The Constrained Case

Given these extreme results, we have examined how to modify the problem to bring it back into a realistic perspective. We do this by modifying the shareholders' problem to add the following constraint

$$L \leq M_0. \tag{39}$$

Equation (39) states that threshold goal must be no higher than the current value of the performance measure. This constraint holds true in practice most of the time.

Let the $*$ superscript designate an optimal design parameter or the value of a performance share plan with optimal design parameters. Adding the constraint above, we find that the optimal threshold goal is always equal to the upper bound value $L^* = M_0$.

Next we find that there is an unique optimal slope parameter λ^*. The intuition for this result is shown in Figure 7. It shows the outside shareholder value for different values of the slope λ, where the optimal managerial effort $a^*(L, \lambda, H)$ is updated for each value of λ. In this figure, the other two design parameters (L and H) are kept at fixed values. At one extreme as the slope λ is reduced down to zero, then optimal managerial effort drops to zero. In this case, even though the cost of performance-based compensation drops to zero, outside shareholder value drops to a low level because there is zero managerial effort. At the other extreme, as λ is increased to a high level, optimal managerial effort continues to rise. But outside shareholders are worse off on a net basis, because the terms of the compensation contract become so costly that the entire value of the firm is paid to the manager and outside shareholder value drops to zero. In between these two extremes, outside shareholder value is hump-shaped. This leads to a unique interior optimum λ^* that precisely balances the marginal incentive effect against marginal cost.

Next we find that the optimal stretch goal H^* is unbounded. Said differently, the optimal performance share plan is an *uncapped* performance share plan. At each iteration of the numerical solution process, the provisional stretch goal becomes larger. By a certain point in the process, the provisional stretch goal is hundreds of standard deviations above the mean of the performance measure. As a result the value of a performance share plan under the provision optimal parameters becomes identical (down to penny accuracy) to the value of an uncapped performance share plan. Thus, an optimal performance share plan *is* an uncapped performance share plan.

Recall that performance-vested share plans can be thought of as a performance share plan in the limiting case as $H \to L$. A direct implication of H^* being unbounded is that performance-vested share plans are *not* optimal.

Incorporating the results above, the following proposition shows that the performance share plan formulas become much simpler under the optimal constrained design parameters.

Proposition 5 *Under three alternative assumptions about the performance measure, the* *date 0 value of a performance share plan with optimal constrained design parameters is*

$$PSP_0^{A*} = UPS_0^A\left(M_0, \lambda^*\right), \tag{40}$$

$$PSP_0^{G*} = UPS_0^G\left(M_0, \lambda^*\right), \tag{41}$$

$$PSP_0^{P*} = UPS_0^P\left(M_0, \lambda^*\right), \tag{42}$$

where

$$UPS_0^A\left(M_0, \lambda^*\right) = S_0\lambda^*\left[\left\{\left(\alpha_M - \nu\sigma_M\right)T\right\}N\left(d_1^A\right) + \sigma_M\sqrt{T}n\left(d_1^A\right)\right], \tag{43}$$

$$UPS_0^G\left(M_0, \lambda^*\right) = S_0M_0\lambda^*\left[e^{\left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T}N\left(d_1^G\right) - N\left(d_2^G\right)\right], \tag{44}$$

$$UPS_0^P\left(M_0, \lambda^*\right) = \left(S_0\right)^2\lambda^*\left[e^{\left(r+\sigma^2\right)T}N\left(d_1^P\right) - N\left(d_2^P\right)\right], \tag{45}$$

and where $d_1^A = \frac{\left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$, $d_1^G = \frac{\left(\alpha_M - \nu\sigma_M + \left(h+\frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$, $d_2^G = d_1^G - \sigma_M\sqrt{T}$, $d_1^P = \frac{\left(r+\frac{3}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$, *and* $d_2^P = d_1^P - \sigma\sqrt{T}$.

Proof The optimal performance share plan formulas are obtained by starting with the analogous performance share plan formula, substituting $L = M_0$ and $\lambda = \lambda^*$, and then evaluating the limit as $H \to +\infty$. Starting from the Proposition 1 formula, $\lim_{H\to+\infty} N(d_2^A) = 0$ and $\lim_{H\to+\infty} n(d_2^A) = 0$, so $UPS_0^A\left(H, \lambda^*\right)$ vanishes, which yields equation (40). Starting from the Proposition 2 formula, $\lim_{H\to+\infty} N(d_3^G) = 0$ and $\lim_{H\to+\infty} N(d_4^G) = 0$, so $UPS_0^G\left(H, \lambda^*\right)$ vanishes, which yields equation (41). Starting from the Proposition 3 formula, $\lim_{H\to+\infty} N(d_3^P) = 0$ and $\lim_{H\to+\infty} N(d_4^P) = 0$, so $UPS_0^P\left(H, \lambda^*\right)$ vanishes, which yields equation (42). **Q.E.D.**

3.4.3 Comparative Statics

Finally, we examine some comparative statics for the optimal constrained performance share plan. Figures 8(a)-8(d) show the impact of the cost of effort on optimal compensation design parameters. Figure 8(a) shows the very intuitive result that a higher cost of effort leads to lower optimal managerial effort a^*. Figure 8(b) shows that firms with a higher cost of effort choose a lower optimal threshold goal L^*. The intuition is that a higher cost of effort leads to lower effort, which leads to a lower date 0 value of performance M_0 (incorporating the market's effort forecast) and this constrains L^* to be lower. Figure 8(c) shows that firms with a higher cost of effort have a lower current stock price S_0, which follows immediately from the lower date 0 value of performance M_0. Figure 8(d) shows a non-monotonic, humped-shape relationship between the cost of effort and the optimal slope λ^*. So the optimal slope λ^* is sometimes increasing and sometimes decreasing in the cost of effort.

These comparative static results are for a non-traded performance measure following a Geometric Brownian Motion. Qualitatively we get the same results as Figures 8(a)-8(c) in the other two performance measure cases. However, the optimal slope λ is strictly decreasing in the cost of effort in the other two performance measure cases.

4 Conclusion

Performance share plans are an important addition to the picture of executive compensation. We derive closed-form formulas for the value of a performance share plan or a performance-vested share plan when the performance measure is: (1) a non-traded measure following an Arithmetic Brownian Motion (e.g., earnings per share), (2) a non-traded measure following a Geometric Brownian Motion (e.g., revenue), or (3) the price of a traded asset following a Geometric Brownian Motion (e.g., a stock price). In a principal-agent setting, we solve for the optimal design of a performance share plan that maximizes outside shareholder wealth while accounting for the incentive effect on executive effort. We find that the optimal performance share plan is uncapped (has no upper bound), that the optimal slope of the payoff function balances the marginal incentive effect against marginal cost, and that performance-vested share plans are not optimal.

A Background on Performance Share Plans

Compensation to CEOs of public firms in US has increased dramatically since 1990s. In this context, the Internal Revenue Service (IRS) adopted regulation IRS 162(m). It aimed to restrain the growth of executive compensation by taxing compensation expenses that are 1) above one million dollars per executive and 2) not linked to performance (e.g. salary). By the law of unintended consequences, stock options in executive compensation skyrocketed as US corporations granted compensation in excess of one million dollars as an *incentive pay.* By the early 2000s, most of the largest US corporations used stock options in their executive compensation package.

In early 2000s, several corporate scandals involving stock options emerged – most notably, the Enron scandal. Furthermore, Heron and Lie (2007) document possible option backdating fraud. For both reasons, granting stock options to the executives as an incentive pay was heavily criticized and this resulting in some firms replacing/reducing their stock option grants. In 2005, the Financial Accounting Standard Board (FASB) mandated expensing stock option grants. This lead to the search for an alternative compensation scheme that would both: (1) satisfy IRS 162(m) tax-deduction eligibility and (2) relieve public concern on excessive stock option grants.

US corporations increasingly turned to performance share plans in their executive compensation package from mid-2000s forward. As we can see from Figure 1, the percentage of firms granting performance share plans almost doubled in five years. The sharp increase contrasts the sharp decline in the percentage of firms granting stock options from 99% to 70% during the same period. The pros of performance share plans are: (1) performance share plans are eligible for tax deduction,[14] (2) performance share plans can measure performance over longer performance periods (the typical performance period is three years) such that the award is much less subject to short-term manipulation of performance, and (3) the award is made in company shares such that the executives are incentivized to improve even longer-term performance.

To document how important performance share plans have became in executive compensation package, we collected data on executive compensation from the proxy statements of the firms that were included in the S&P 500 index as of January 2006. The sample period is from fiscal years ending on or after December 15, 2006 to fiscal years ending on or before November 30, 2012. We also included statistics for S&P MidCap and SmallCap firms by inferring from ExecuComp database in the *Plan-based awards* table.

Table 2 describes the compensation structure of S&P LargeCap firms and separately S&P MidCap and SmallCap firms. Columns under *Hand-collected* shows statistics from hand-collected data from proxy statements and columns under *From ExecuComp* shows statistics from the ExecuComp database. Under *S&P LargeCap*, we show statistics for S&P 500 firms from both data sources for comparison. As is evident from the table, statistics from the two data sources are relatively close to each other. However, when compared to *S&P MidCap and SmallCap* firms, we can see that S&P LargeCap firms pay more compensation to their CEOs and more in the form of incentive pay. Specifically, S&P LargeCap firms pay 77% in the form of incentive pay (performance cash, performance shares, performance options, performance-vested restricted stocks, and stock options) while S&P MidCap and SmallCap firms pay only 44% in the form of incentive pay.

Panel A presents number of firms granting each compensation component and the mean and median target amount conditional on that component being granted. The amount

[14] Restricted stock grants, another alternative to the stock options, are not eligible for tax deduction under IRS 162(m).

of compensation granted in the form of performance shares is sizeable. 1,210 firms used performance share plans; granting a mean of \$3.66 million and a median of \$2.66 million. The mean amount is significantly larger than average salary (\$1.10 million) and performance cash (\$2.22 million).

Panel B reports the unconditional breakdown of the compensation components (i.e, not conditioning on whether that component is offered). Performance shares represent 17% of S&P LargeCap firms and 9% of S&P MidCap and SmallCap firms. This approaches the size of stock options and performance cash, which represents 24% and 23% of S&P LargeCap firms and 15% and 19% of S&P MidCap and SmallCap firms, respectively.

Panel C presents the breakdown between performance shares and stock options. Out of 2,577 firm-years in the S&P LargeCap sample, 2,143 firm-years granted stock options, performance shares, or both. Among these 2,143 firm-years, 44% granted stock options only, 16% granted performance shares only, and 40% granted both. Among the 40% of firm-years that granted both, 52% is granted as performance shares when they valued heuristically at the target performance goal.

Table 3 reports on the specific performance measures that are used in performance share plans. We hand-collected this information for the S&P LargeCap sample. Out of 500 firms, 401 unique firms reported 3,117 observations of performance measures. They reported using stock return 21% of the time, earnings per share (EPS) 17% of the time, revenue 10%, return on invested capital (ROIC) 7%, and return on equity (ROE) 5%, and other measures 41% of the time. There are many other measures used including profit measures (e.g., operating income, net income, and profit before tax) and cash flow measures (e.g., free cash flow, cash flow from operations, and economic value added).

Table 4 reports the design of performance share plans and performance-vested share plans with earnings per share (EPS) or revenue as a performance measure. Panel A reports the threshold (L) and stretch (H) goals divided by the performance level in the previous year. It also reports the slopes of the plans. The slope is in thousand shares per \$1 for EPS measures, and thousand shares per \$BN for revenue measures. Panel B reports the threshold goal for performance-vested share plans divided by the performance level in the previous year. The threshold goals are generally higher than the previous year's performance level implying that firms do intend to reward performance improvements.

Table 5 reports the valuation of performance share plans and performance-vested share plans using our Propositions 1, 2, and 4. We are only able to value the plans with earnings per share (EPS) or revenue as a performance measure due to data availability and ambiguity of performance measures across different firms (*e.g. while the definitions of EPS and revenue are standard across firms, the definitions of ROIC and ROE are quite different across firms*). To value the plans, we need to have all threshold, stretch, and slope data for performance share plans, and the threshold goal data for performance-vested share plans. We were able to identify 228 performance share plans and 38 performance-vested share plans with all available data. For EPS measure firms, we use price-to-earnings ratio as the sensitivity of stock price to performance improvement (h), and for revenue measures, we use return-to-revenue growth (previous year's stock return divided by previous year's revenue growth) as h. We present valuation results using various value of premium on non-traded performance measure (ν) ranging from zero to one. As we can see from the table, valuation of plans with EPS as a performance measure is generally higher than reported grant date fair value of performance share plans. However, the value of plans using revenue as a performance measure is generally lower than the value reported by firms.

B Proofs

B.1 Proof of Proposition 1

The current value of a performance share plan is the expected value of the payoff at maturity under the risk-neutral growth rate discounted back to date 0 at the riskfree rate as given by

$$\begin{aligned} PSP_0^A &= e^{-rT}\int_{-\infty}^{L}\int_{-\infty}^{+\infty} 0\times S_0 e^{Y_T} f(\cdot) dY_T dM_T \\ &+ e^{-rT}\int_{L}^{H}\int_{-\infty}^{+\infty} \lambda\left(M_T - L\right) S_0 e^{Y_T} f(\cdot) dY_T dM_T \\ &+ e^{-rT}\int_{H}^{+\infty}\int_{-\infty}^{+\infty} \lambda\left(H - L\right) S_0 e^{Y_T} f(\cdot) dY_T dM_T. \end{aligned} \tag{B.1.1}$$

By adding and subtracting $e^{-rT}\int_{H}^{+\infty}\int_{-\infty}^{+\infty}\lambda\left(M_T - H\right) S_0 e^{Y_T} f(\cdot) dY_T dM_T$,

$$\begin{aligned} PSP_0^A &= S_0 e^{-rT}\int_{L}^{+\infty}\int \lambda\left(M_T - L\right) e^{Y_T} f(\cdot) dY_T dM_T \\ &- S_0 e^{-rT}\int_{H}^{+\infty}\int \lambda\left(M_T - H\right) e^{Y_T} f(\cdot) dY_T dM_T && \text{(B.1.2)} \\ &= UPS_0^A\left(L,\lambda\right) - UPS_0^A\left(H,\lambda\right), && \text{(B.1.3)} \end{aligned}$$

which is equation (12). The value of an uncapped performance share plan can be split into two terms

$$\begin{aligned} UPS_0^A(L,\lambda) &= S_0 e^{-rT}\lambda\int_{L}^{+\infty}\int_{-\infty}^{+\infty} M_T e^{Y_T} f(\cdot) dY_T dM_T \\ &- L S_0 e^{-rT}\lambda\int_{L}^{+\infty}\int_{-\infty}^{+\infty} e^{Y_T} f(\cdot) dY_T dM_T. \end{aligned} \tag{B.1.4}$$

Denote the PDF of conditional distribution of Y_T given M_T as $f(Y_T|M_T)$ and the PDF of M_T as $f(M_T)$. Then the value of an uncapped performance share plan is given by

$$\begin{aligned}UPS_0^A(L,\lambda) &= S_0e^{-rT}\lambda\int_L^{+\infty}\int_{-\infty}^{+\infty}M_Te^{Y_T}f(M_T)f(Y_T|M_T)dY_TdM_T\\ &- LS_0e^{-rT}\lambda\int_L^{+\infty}\int_{-\infty}^{+\infty}e^{Y_T}f(M_T)f(Y_T|M_T)dY_TdM_T. \quad \text{(B.1.5)}\\ &\equiv A-B. \quad \text{(B.1.6)}\end{aligned}$$

Let's first work with the latter term (B). Conditional distribution of Y given M is

$$Y_T|M_T \sim \mathcal{N}\left(\mu_Y+\frac{\sigma_Y}{\sigma_M}\rho\left(M_T-\mu_M\right),\left(1-\rho^2\right)\sigma_Y^2\right), \quad \text{(B.1.7)}$$

where ρ is the correlation coefficient between Y_T and M_T given in Equation (11).[15]

$$e^{Y_T}f(Y_T|M_T) = \frac{1}{\sqrt{2\pi\left(1-\rho^2\right)\sigma_Y^2}}\exp\left(Y_T-\frac{\left(Y_T-\left(\mu_Y+\frac{\sigma_Y}{\sigma_M}\rho\left(M_T-\mu_M\right)\right)\right)^2}{2\left(1-\rho^2\right)\sigma_Y^2}\right) \quad \text{(B.1.8)}$$

$$= \exp\left(\frac{\left(1-\rho^2\right)\sigma_Y^2}{2}+\mu_Y-h\mu_M\right)\exp\left(hM_T\right)f(Y_T+\left(1-\rho^2\right)\sigma_Y^2|M_T). \quad \text{(B.1.9)}$$

where $h=\frac{\sigma_Y}{\sigma_M}\rho$. Thus,

$$\begin{aligned}B &= LS_0e^{-rT}\lambda C_1\int_L^{+\infty}e^{hM_T}f(M_T)\int_{-\infty}^{+\infty}f(Y_T+\left(1-\rho^2\right)\sigma_Y^2|M_T)dY_TdM_T \quad \text{(B.1.10)}\\ &= LS_0e^{-rT}\lambda C_1e^{\left(h\mu_M+\frac{1}{2}h^2\sigma_M^2\right)}\int_L^{+\infty}f(M_T+h\sigma_M^2T)dM_T, \quad \text{(B.1.11)}\end{aligned}$$

because $\int_{-\infty}^{+\infty}PDF = CDF(+\infty) = 1$.[16]

[15] Please see Greene (2003, pp. 868).

[16] We followed the same steps from Equations (B.1.8) to (B.1.9) to derive (B.1.11).

Plugging in $\mu_Y = rT - \frac{1}{2}\sigma_Y^2$, we have $C_1 e^{\left(h\mu_M + \frac{1}{2}h^2\sigma_M^2\right)} = 1$.

$$B = LS_0 e^{-rT} \int_L^{+\infty} f(M_T + h\sigma_M^2 T) dM_T. \tag{B.1.12}$$

We can rewrite Equation (B.1.12) by standardizing $M_T + h\sigma_M^2 T$.

$$B = LS_0\lambda \int_{-d_1}^{+\infty} f\left(Z_1\right) dZ_1 \tag{B.1.13}$$
$$= LS_0\lambda \int_{-\infty}^{d_1} f\left(-Z_1\right) d\left(-Z_1\right) \tag{B.1.14}$$
$$= LS_0\lambda N\left(d_1^A\right), \tag{B.1.15}$$

where Z_1 is a standardized variable of $M_T + h\sigma_M^2 T$, $d_1^A = \frac{M_0 - L + \left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T}{\sigma_M\sqrt{T}}$, and $N(d_1^A)$ is the value of CDF of standard normal distribution evaluated at d_1^A.

Now let's work on the first part (A) of the valuation. Because A and B is mostly similar except for minor differences, we can use most of the result from above.

$$A = S_0\lambda \int_L^{+\infty} M_T f(M_T + h\sigma_M^2 T) dM_T \tag{B.1.16}$$
$$= S_0\lambda \int_L^{+\infty} \left(M_T + h\sigma_M^2 T\right) f(M_T + h\sigma_M^2 T) dM_T - S_0\lambda h\sigma_M^2 T \int_L^{+\infty} f(M_T + h\sigma_M^2 T) dM_T \tag{B.1.17}$$
$$= S_0\lambda \int_L^{+\infty} \left(M_T + h\sigma_M^2 T\right) f(M_T + h\sigma_M^2 T) dM_T - S_0\lambda h\sigma_M^2 T N\left(d_1^A\right). \tag{B.1.18}$$

The following Lemma from Johnson et. al. (1994, Section 10.1) is useful.

Lemma 1 *Given a normally distributed random variable $X \sim \mathcal{N}\left(\mu, \sigma^2\right)$ and a lower bound L, the following is true:*

$$E\left(X|X > L\right) Pr\left(X > L\right) = \mu N\left(\frac{\mu - L}{\sigma}\right) + \sigma n\left(\frac{\mu - L}{\sigma}\right), \tag{B.1.19}$$

where $n(\cdot)$ is the density function and $N(\cdot)$ is the cumulative distribution function of a standard normal distribution.

Using this Lemma, we have

$$A = S_0\lambda\left(\left(\mu_M + h\sigma_M^2 T\right)N\left(d_1^A\right) + \sigma_M\sqrt{T}n\left(d_1^A\right)\right) - S_0\lambda h\sigma_M^2 TN\left(d_1^A\right) \tag{B.1.20}$$

$$= S_0\lambda\left(\mu_M N\left(d_1^A\right) + \sigma_M\sqrt{T}n\left(d_1^A\right)\right), \tag{B.1.21}$$

where $n\left(d_1^A\right)$ is the PDF of a standard normal variable evaluated at d_1^A. The value of an uncapped performance share with threshold goal of L and slope of λ is

$$UPS_0(L,\lambda) \equiv A - B = S_0\lambda\left[\left\{M_0 + \left(\alpha_M - \nu\sigma_M\right)T - L\right\}N\left(d_1^A\right) + \sigma_M\sqrt{T}n\left(d_1^A\right)\right], \tag{B.1.22}$$

which is equation (13). Substitute H in place of L above to get equation (14) and the expression for d_2^A. **Q.E.D.**

B.2 Proof of Proposition 2

The current value of a performance share plan is the expected value of the payoff at maturity under the risk-neutral growth rate discounted back to date 0 at the riskfree rate as given by

$$\begin{aligned} PSP_0^G = {} & e^{-rT}\int_{-\infty}^{L}\int_{-\infty}^{+\infty} 0 \times S_0 e^{Y_T} f(\cdot)dY_T dM_T \\ & + e^{-rT}\int_{L}^{H}\int_{-\infty}^{+\infty} \lambda\left(M_T - L\right)S_0 e^{Y_T} f(\cdot)dY_T dM_T \\ & + e^{-rT}\int_{H}^{+\infty}\int_{-\infty}^{+\infty} \lambda\left(H - L\right)S_0 e^{Y_T} f(\cdot)dY_T dM_T. \end{aligned} \tag{B.2.1}$$

By adding and subtracting $e^{-rT}\int_H^{+\infty}\int_{-\infty}^{+\infty}\lambda\left(M_T-H\right)S_0e^{Y_T}f(\cdot)dY_TdM_T$,

$$\begin{aligned}
PSP_0^G &= S_0e^{-rT}\int_L^{+\infty}\int\lambda\left(M_T-L\right)e^{Y_T}f(\cdot)dY_TdM_T \\
&\quad - S_0e^{-rT}\int_H^{+\infty}\int\lambda\left(M_T-H\right)e^{Y_T}f(\cdot)dY_TdM_T \qquad \text{(B.2.2)} \\
&= UPS_0^G\left(L,\lambda\right)-UPS_0^G\left(H,\lambda\right), \qquad \text{(B.2.3)}
\end{aligned}$$

which is equation (18).

Introduce a change of variables for the terminal value $\hat{M}_T$. Specifically, define $\hat{X}_T = ln\left(\frac{\hat{M}_T}{M_0}\right)$. Then $\hat{X}_T$ is normally distributed as follows

$$\hat{X}_T \sim \mathcal{N}\left(\left(\alpha_M-\nu\sigma_M-\frac{1}{2}\sigma_M^2\right)T,\sigma_M^2T\right). \qquad \text{(B.2.4)}$$

Denote the PDF of conditional distribution of Y_T given X_T as $f(Y_T|X_T)$ and the PDF of X_T as $f(X_T)$. Then the value of an uncapped performance share plan is given by

$$\begin{aligned}
UPS_0^G(L,\lambda) &= S_0M_0e^{-rT}\lambda\int_{\ln\frac{L}{M_0}}^{+\infty}\int_{-\infty}^{+\infty}e^{X_T}e^{Y_T}f(X_T)f(Y_T|X_T)dY_TdX_T \\
&\quad - LS_0e^{-rT}\lambda\int_{\ln\frac{L}{M_0}}^{+\infty}\int_{-\infty}^{+\infty}e^{Y_T}f(X_T)f(Y_T|X_T)dY_TdX_T. \qquad \text{(B.2.5)} \\
&\equiv A-B. \qquad \text{(B.2.6)}
\end{aligned}$$

Following same steps from Equations (B.1.8)-(B.1.15), we obtain

$$B = LS_0\lambda N\left(d_2^G\right), \qquad \text{(B.2.7)}$$

where $d_2^G = \frac{\ln\frac{M_0}{L}+\left(\alpha_M-\nu\sigma_M+\left(h-\frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M\sqrt{T}} = d_1^G-\sigma_M\sqrt{T}$ and where d_1^G is derived below.

Now let's work on the first part (A) of the valuation.

$$A = S_0 M_0 \lambda \int_{\ln \frac{L}{M_0}}^{+\infty} e^{X_T} f(X_T + h\sigma_X^2) dX_T \tag{B.2.8}$$

$$= S_0 M_0 e^{\left(h+\frac{1}{2}\right)\sigma_M^2 T + \mu_X} \lambda \int_{\ln \frac{L}{M_0}}^{+\infty} f(X_T + (h+1)\,\sigma_X^2) dX_T. \tag{B.2.9}$$

We know that $\mu_X = \left(\alpha_M - \nu\sigma_M - \frac{1}{2}\sigma_M^2\right) T$, so $e^{\left(h+\frac{1}{2}\right)\sigma_M^2 T + \mu_X} = e^{\left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T}$, and by standardizing $X_T + (h+1)\,\sigma_X^2$,

$$A = S_0 M_0 e^{\left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T} \lambda N\left(d_1^G\right), \tag{B.2.10}$$

where $d_1^G = \frac{\ln \frac{M_0}{L} + \left(\alpha_M - \nu\sigma_M + \left(h+\frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M \sqrt{T}}$.

Thus, the value of an uncapped performance share plan is:

$$UPS_0^G(L, \lambda) \equiv A - B = S_0 \lambda \left[M_0 e^{\left(\alpha_M - \nu\sigma_M + h\sigma_M^2\right)T} N\left(d_1^G\right) - L N\left(d_2^G\right)\right], \tag{B.2.11}$$

which is equation (19), and where $d_1^G = \frac{\ln \frac{M_0}{L} + \left(\alpha_M - \nu\sigma_M + \left(h+\frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M \sqrt{T}}$ and $d_2^G = \frac{\ln \frac{M_0}{L} + \left(\alpha_M - \nu\sigma_M + \left(h-\frac{1}{2}\right)\sigma_M^2\right)T}{\sigma_M \sqrt{T}}$. Substitute H in place of L above to get equation (20) and the expressions for d_3^G and d_4^G.

Q.E.D.

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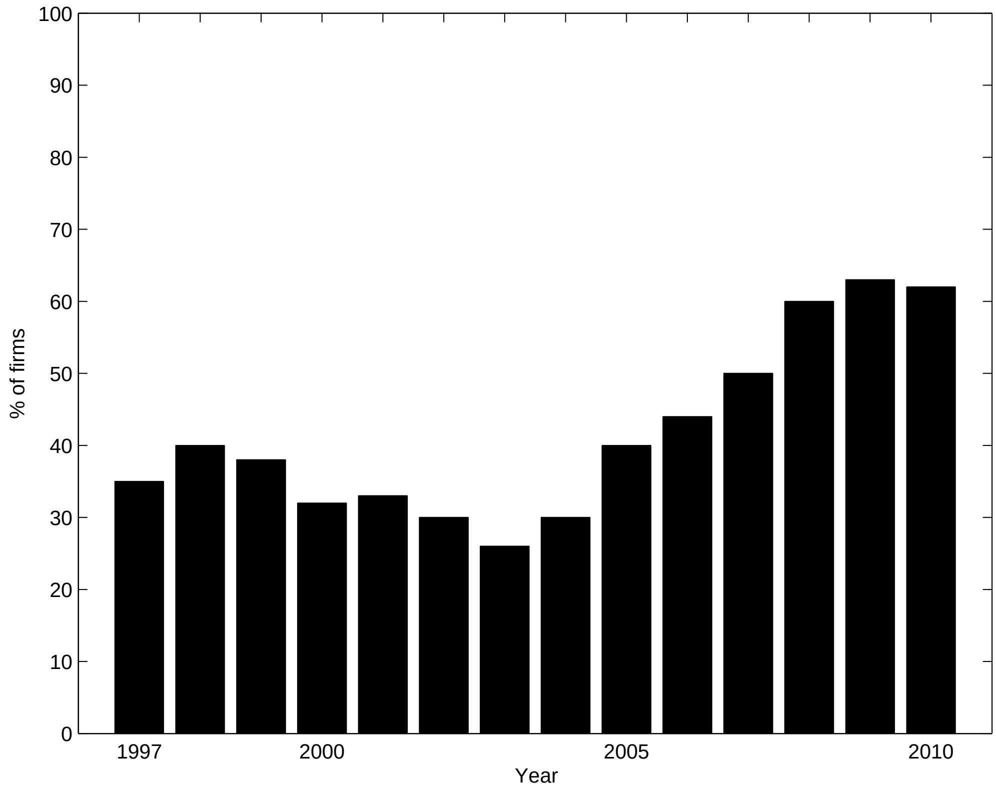
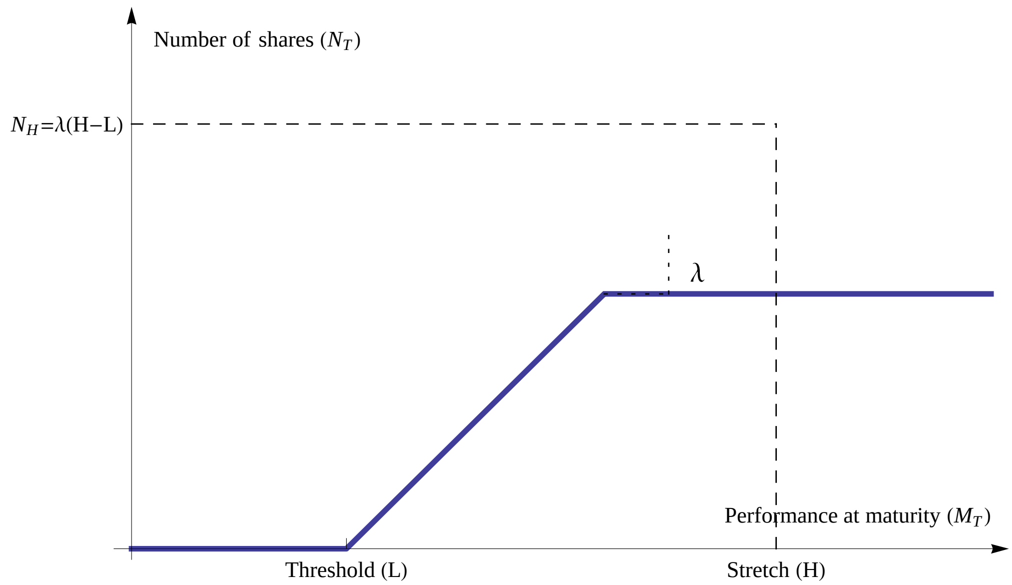
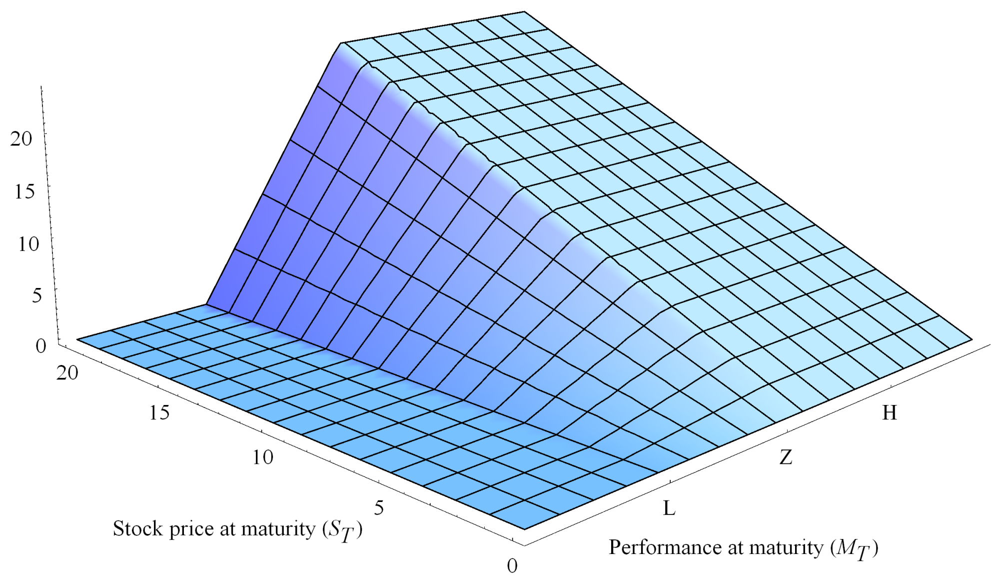


Figure 1: Firms granting performance share plans

This figure shows the percentage of firms that grant performance share plans to their executives. The sample period is from 1997 to 2010 and the top 250 Forbes firms are included. The data is collected from annual *The Top 250 Survey* by C. King of Frederick W. Cook.



(a) Share reward function for a performance share plan



(b) Monetary payoff of a performance share plan

Figure 2: Payoff at maturity of a performance share plan

These figures show the payoff at maturity of a performance share plan. Figure 2(a) shows the number of shares awarded under a performance share plan by performance at maturity. Figure 2(b) shows the monetary payoff of a performance share by performance at maturity and by stock price at maturity.

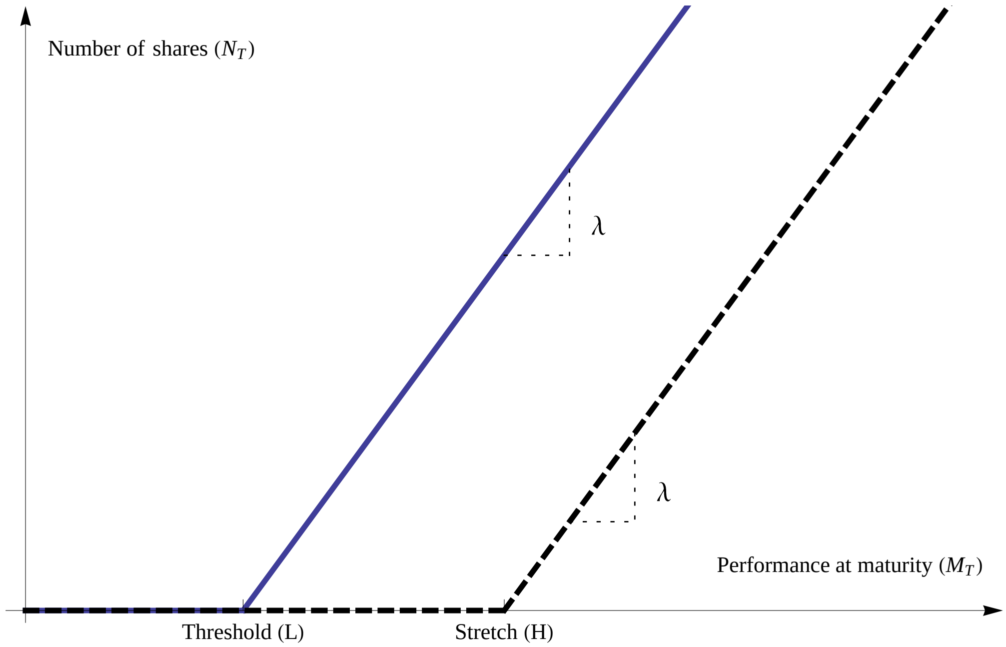
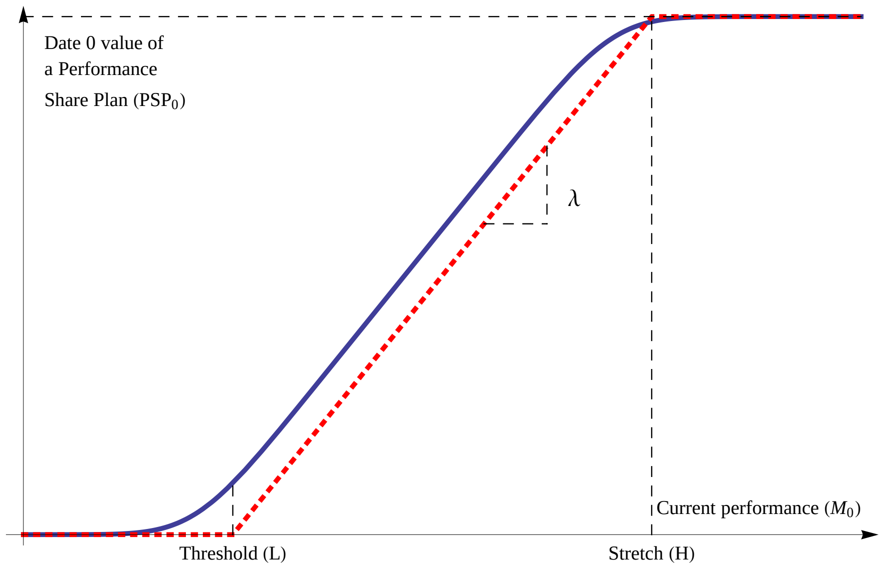
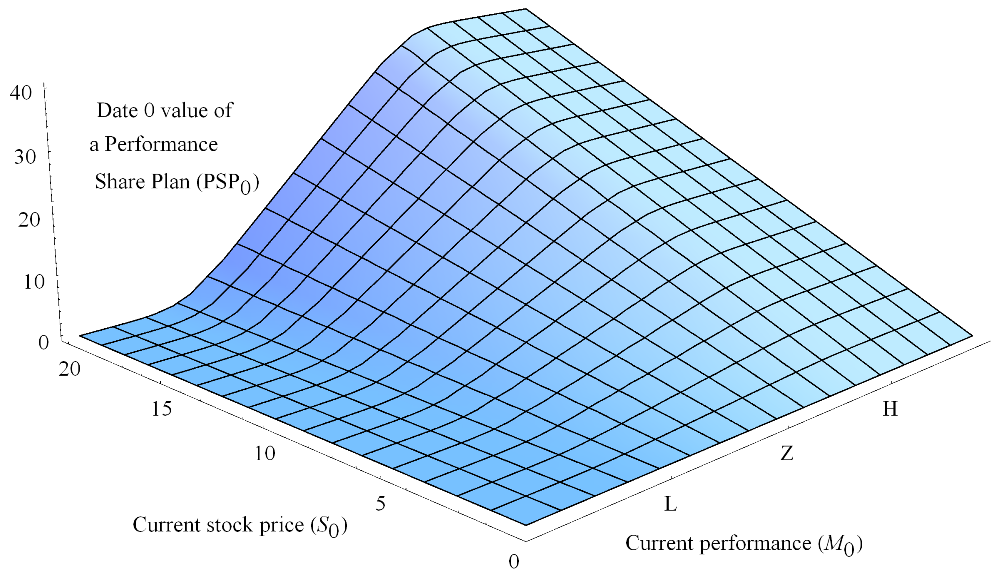


Figure 3: Share reward function for two uncapped performance share plans

This figure shows the share reward function of two uncapped performance share plans plotted against performance at maturity (M_T). The bold line shows the one uncapped performance share plan with a threshold goal of L. The dotted line shows the second uncapped performance share plan with a threshold goal of H. An uncapped plan can be thought of as the limiting case of a regular performance share plan as the upper bound goes to infinity.



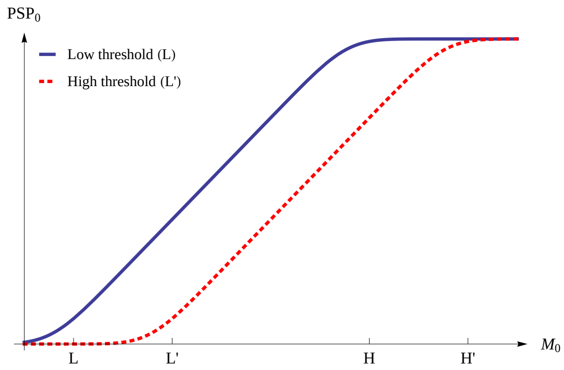
(a) Date 0 value vs. intrinsic value of a performance share plan by current performance measure



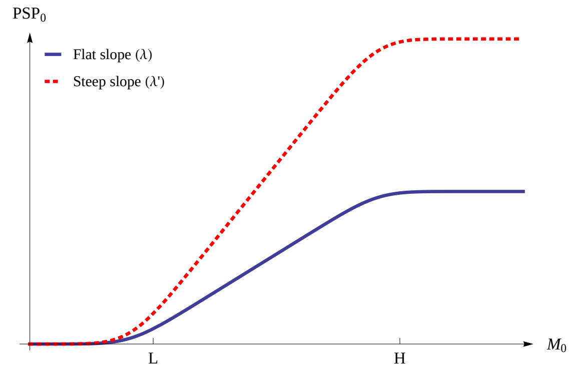
(b) Date 0 value of a performance share plan by current performance measure and current stock price

Figure 4: Value of a performance share plan

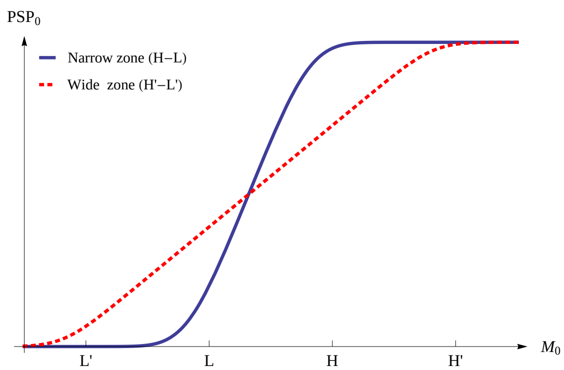
These figures show the date 0 value of a performance share. The bold line in Figure 4(a) shows how the date 0 value of a performance share changes with the current level of performance. The dotted line represents the intrinsic value of the performance share plan and the vertical gap between the date 0 value of a performance share plan and the intrinsic value represents the time value of the performance share plan. Figure 4(b) shows how the value changes with the current performance measure and the current stock price.



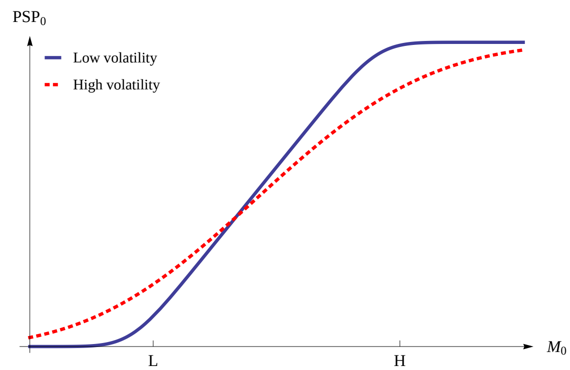
(a) Threshold goal (L)



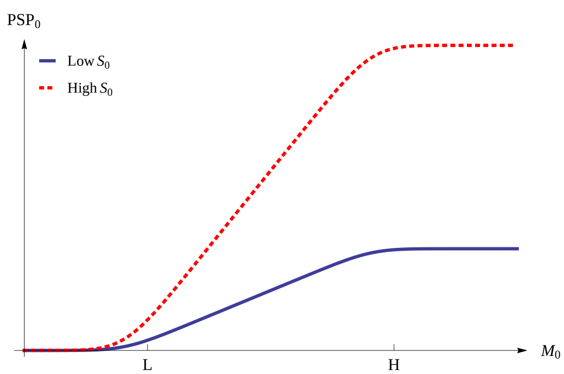
(b) Slope of payoff function (λ)



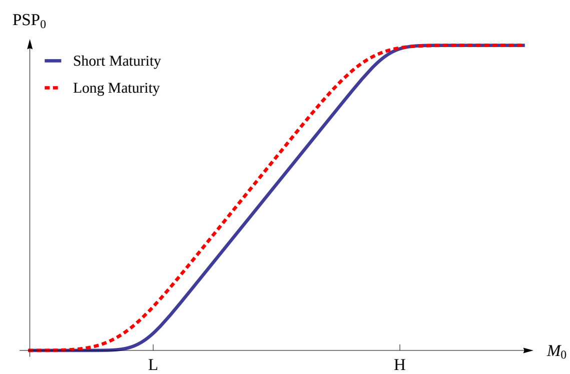
(c) Width of incentive zone ($H - L$)



(d) Volatility of performance measure (σ_M)



(e) Current stock price (S_0)



(f) Performance period (T)

Figure 5: Factors affecting the value of a performance share plan

These figures show how different factors affect the value of a performance share plan. The figures show the value for current level of performance measure (M_0). Figures 5(a), 5(b), and 5(c) show the value of a performance share plan (PSP_0) by the three contract parameters: the threshold goal (L), the slope of payoff function (λ), and the width of incentive zone ($H-L$). Figures 5(d), 5(e), and 5(f) show the value of a performance share plan (PSP_0) by various environmental factors: the volatility of performance measure (σ_M), the current stock price (S_0), and the performance period (T).

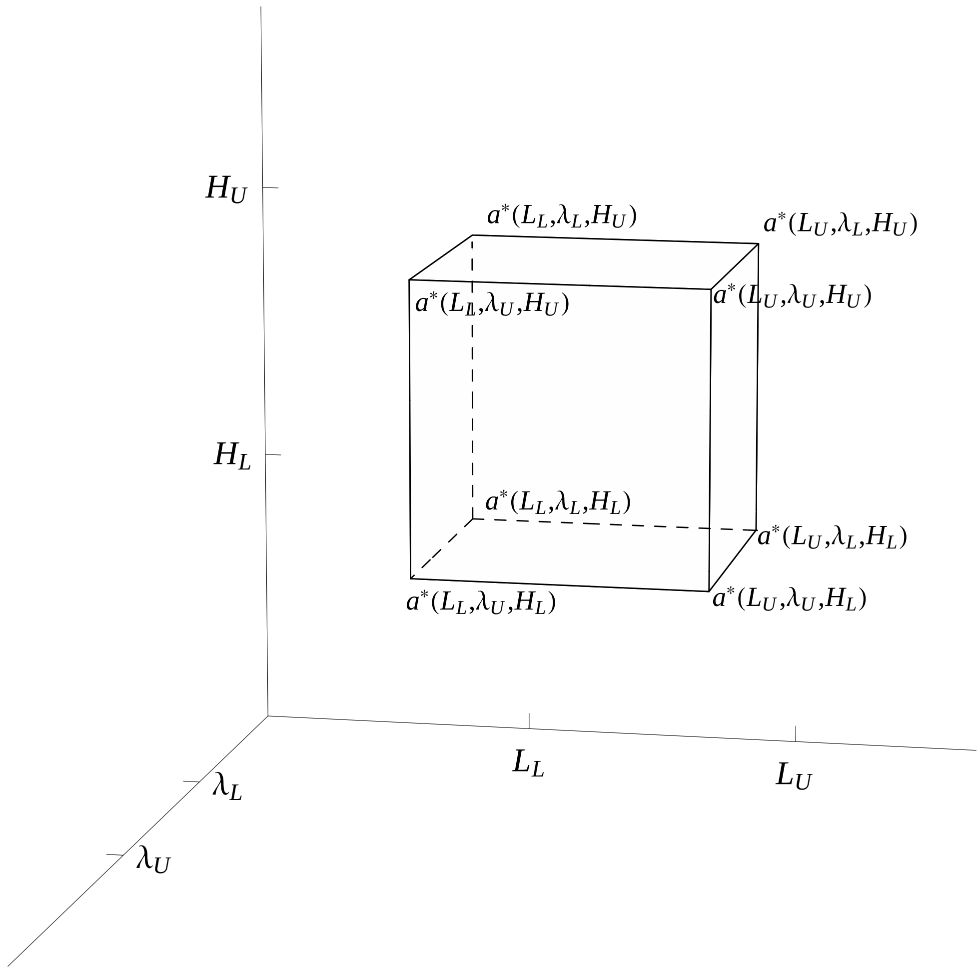


Figure 6: Upper and lower bounds specify a three-dimensional cube space

This figure shows shows how upper and lower bounds for each of the three contract parameters specify a three-dimensional space in the shape of a cube. In other words, at every point in the cube the triplet of contract parameters (L, λ, H) are within the corresponding bounds $L \in [L_L, L_U]$, $\lambda \in [\lambda_L, \lambda_U]$, and $H \in [H_L, H_U]$. We find the optimal managerial effort a^* as a function of the triplet of contract parameters at each of the eight corners.

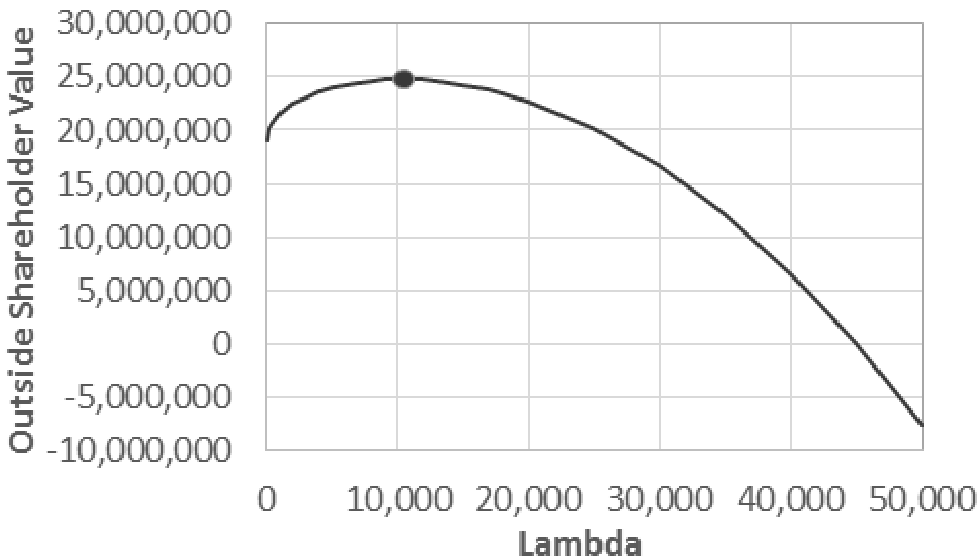
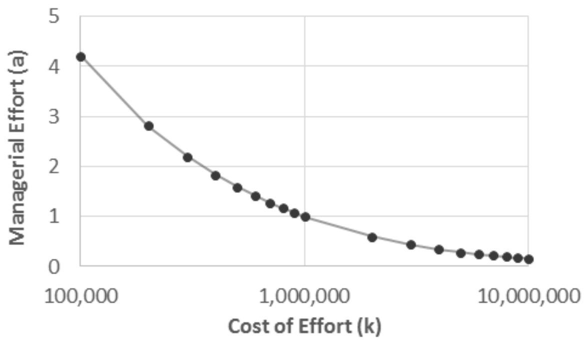
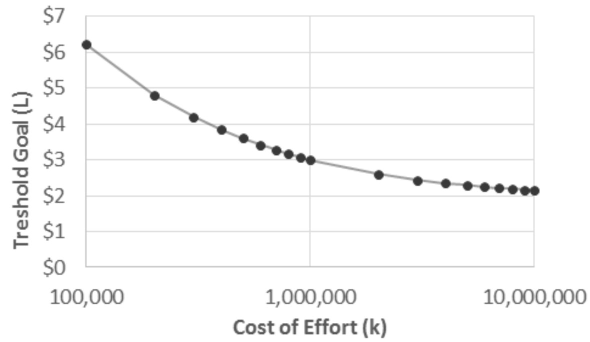


Figure 7: Outside shareholder value by lambda

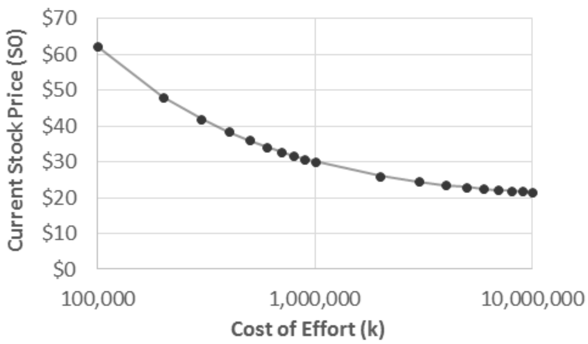
This figure shows outside shareholder value (the value of the firm net of compensation to the CEO) by the slope of the performance share plan (λ), where the optimal managerial effort (a^*) varies as λ varies. The other two design parameters (L and H) are kept at fixed values.



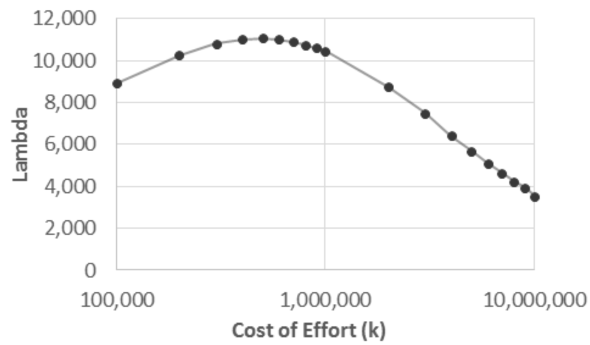
(a) Optimal managerial effort (a^*)



(b) Threshold goal (L^*)



(c) Stock price (S_0)



(d) Slope of payoff structure (λ^*)

Figure 8: Optimal design parameters by the cost of effort

These figures show optimal design parameters by the cost of effort. Figures 8(a)-8(d) show the optimal managerial effort (a^*), optimal threshold goal (L^*), current stock price (S_0), and the optimal slope (λ^*) by the cost of effort.

Table 2: Compensation structure

This table shows the compensation structure of CEOs over fiscal years ending from December 15^{th}, 2006 to November 30^{th}, 2012. *S&P LargeCap* are S&P 500 firms, and *S&P MidCap & SmallCap* are S&P 1,500 firms excluding S&P 500 firms. *Hand-collected* are data collected from proxy statements and *From ExecuComp* are data from the ExecuComp database. Panel A shows conditional mean and median of target amount of each compensation component. Panel B shows unconditional mean and median breakdown of compensation components. *Performance Cash* includes cash-based annual and long-term incentive pay. *Performance Share Plans*, *Performance Options*, and *Performance-Vested Share Plans* are equity-based compensations in which the number of shares or options awarded is tied to the performance of pre-specified measures. Panel C presents data on firms that use stock options only, performance shares only, or both. w_P is the weight of performance shares.

	S&P LargeCap						MidCap & SmallCap		
	Hand-collected			From ExecuComp			From ExecuComp		
Components	Obs	Mean	Med.	Obs.	Mean	Med.	Obs.	Mean	Med.
Panel A. Conditional mean of target compensation amount by compensation component ($ MM)									
Salary	2,567	1.10	1.00	2,594	1.10	1.00	5,546	0.67	0.64
Restricted Stocks	1,237	2.86	1.91	1,279	3.21	2.06	2,754	1.28	0.72
Incentive Pay									
Perf. Share Plans	1,210	3.66	2.66	1,347	3.88	2.90	1,494	1.40	0.89
Performance Cash	2,202	2.22	1.55	2,111	2.22	1.54	3,840	0.84	0.64
Performance Options	40	2.86	2.10	57	2.75	1.85	111	1.16	0.63
Perf.-Vest. Share Plans	283	3.32	2.54	74	3.29	2.63	97	1.16	0.86
Stock Options	1,808	3.44	2.43	1,790	3.60	2.55	2,647	1.17	0.75
Panel B. Breakdown of total target compensation by compensation component (%)									
Salary	2,577	18.58	13.87	2,669	19.30	13.64	5,570	39.59	29.49
Restricted Stocks	2,577	14.02	0.00	2,693	15.33	0.00	5,570	16.04	0.00
Incentive Pay									
Perf. Share Plans	2,577	16.85	0.00	2,668	18.84	4.62	5,570	8.81	0.00
Performance Cash	2,577	22.60	18.94	2,668	20.28	17.86	5,570	19.44	19.05
Performance Options	2,577	0.43	0.00	2,668	0.51	0.00	5,570	0.52	0.00
Perf.-Vest. Share Plans	2,577	3.85	0.00	2,668	0.97	0.00	5,570	0.56	0.00
Stock Options	2,577	23.66	23.09	2,689	23.30	22.48	5,570	15.05	0.00
Total Incentive Pay		67.40			63.90			44.37	
Panel C. Breakdown between performance shares and stock options (%)									
Components	Obs.	w_p	% obs	Obs.	w_p	% obs	Obs.	w_p	% obs
Only stock options	2,143	0	43.5	2,190	0	38.5	3,427	0	56.4
Mixed	2,143	51.6	40.8	2,190	52.6	43.2	3,427	47.4	20.8
Only perf. shares	2,143	100.0	15.6	2,190	100.0	18.3	3,427	100.0	22.8

Table 3: Performance measures used in performance share plans

This table documents the performance measures used in performance share plans of S&P 500 firms over fiscal years ending from December 15^{th}, 2006 to November 30^{th}, 2012. 401 unique firms used performance share plans at least once during the sample period. *Shareholder return* is the sum of the capital gain and the percentage dividends paid.

Performance measures	# of obs.	% of total
Shareholder Return	656	21.05
Earnings Per Share (EPS)	539	17.29
Revenue	308	9.88
Return on Invested Capital	205	6.58
Return on Equity	145	4.65
Other Performance Measures	1,264	40.55
Total	3,117	100.00

Table 4: Design of performance share plans and performance-vested share plans

This table illustrates the design of performance share plans and performance-vested share plans of the firm-years with available data. Among the firm-years that use either earnings per share (EPS) or revenue as a performance measure, there are in total 228 performance share plans and 37 performance-vested share plans with threshold, stretch, target number of shares data. *Threshold* is the threshold goal divided by the previous year's performance level, *Stretch* is the stretch goal divided by the previous year's performance level, and *Slope* is the incremental number of shares (in thousands) for a unit ($1 for EPS measures and $BN for revenue measures) increase in the level of performance.

	Earnings Per Share (EPS)			Revenue		
	Threshold	Stretch	Slope	Threshold	Stretch	Slope
Panel A. Performance Share Plans						
Mean	0.98	1.18	557	1.05	1.14	254
Median	1.03	1.15	273	1.03	1.11	93
Observations	163	163	165	64	64	64
Panel B. Performance-Vested Share Plans						
Mean	1.01		∞	1.15		∞
Median	1.10			1.06		
Observations	26			11		

Table 5: Value of performance share plans and performance-vested share plans

This table illustrates the value of performance share plans and performance-vested share plans that use either earnings per share (EPS) or revenue as a performance measure. We first calculate the value of the performance share plans and performance-vested share plans using Propositions 1, 2, and 4. We then divide the value by the *Grant date fair value of equity awards* in the table of *Grants of plan-based awards* in the proxy statements. For plans with EPS as a performance measure, we use price-to-earnings for h, and for plans with revenue as a performance measure, we use return-to-revenue growth for h. We show the values for different levels of ν, the premium on non-traded performance measure, ranging from zero to one.

	Performance Share Plans			Performance-Vested Share Plans		
	All	EPS	Revenue	All	EPS	Revenue
ν=0.0						
Mean	1.10	1.27	0.66	0.99	1.03	0.88
Median	0.90	1.11	0.59	1.00	1.00	0.98
ν=0.2						
Mean	1.09	1.26	0.64	0.98	1.03	0.86
Median	0.89	1.11	0.59	1.00	1.00	0.97
ν=0.4						
Mean	1.07	1.24	0.62	0.98	1.03	0.83
Median	0.88	1.11	0.55	1.00	1.00	0.96
ν=0.6						
Mean	1.05	1.23	0.60	0.97	1.03	0.81
Median	0.86	1.07	0.51	1.00	1.00	0.95
ν=0.8						
Mean	1.04	1.21	0.57	0.96	1.02	0.79
Median	0.85	1.05	0.45	1.00	1.00	0.94
ν=1.0						
Mean	1.02	1.20	0.55	0.95	1.02	0.77
Median	0.85	1.03	0.43	0.99	1.00	0.92