The Role of Grandfathering under Simultaneous Market Power in Product and Emission Permit Markets

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ABSTRACT

We analyze the behavior of polluting oligopolistic firms that interact strategically both in the output and the emissions permit markets. We consider a model of two firms competing under different oligopolistic structures while one of them is a dominant firm in the permit market. In this framework, we study and compare the effect of market power and the role of grandfathering in three different situations. We determine that the follower could always make higher profits than the Stackelberg leader if it receives enough more free permits. Under Cournot structure the firm who receives more free permits produces more and make higher profits.

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1. Introduction

A coordinated global agreement to cut down greenhouse gas emissions is far from being reached after many years of formal negotiations since the first big attempt in Kyoto 98. Meanwhile, some market based climate change policies have been put in place at regional levels, the European Trading Scheme (ETS) being the most important experience.

The efficiency of emission permit markets had deserved academic attention well before its practical implementation.\(^1\) The role of market power and the method to do the initial allocation of permits (mainly grandfathering or auctioning) are among the main issues that have attracted the attention of researchers regarding the efficiency of this kind of markets. Montgomery (1972) established that under perfect competition the equilibrium allocation is cost-effective regardless the allocation method. Hahn (1984) considered market power in the permit market for the first time and showed that, in general, the market equilibrium allocation fails to be efficient in the presence of a dominant firm, because such a firm has an incentive to manipulate the permit prices in order to increase its profits. As Hahn noted, if there is a dominant firm operating in the permit market, the resulting allocation is efficient only if the dominant firm is initially allocated with the same amount of permits that it would get in a competitive equilibrium.

In its basic version, both results (Montgomery’s result about the efficiency of a competitive market and Hahn’s result about the lack of efficiency of a non-competitive market) focus on the permit market itself and abstract from the existence of an

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\(^1\) See Baumol and Oates (1971) and Montgomery (1972) as leading examples of academic studies. Early forms of emissions trading took place in USA from 1976 based on emissions reduction credits, but it is the Acid Rain Program, enacted in 1990 and aimed to control SO\(_2\) and NO\(_x\) emissions, the one commonly considered as the first well-established market of emissions. The first phase of this program begun in 1995 (see, for example, Tietenberg 2006).
associated pollutant product market and its corresponding structure. As it has been later acknowledged by several authors, the product market must be taken into account to make a fully-fledged analysis, since the cost associated to the purchase of permits impacts the overall profits that firms aim to maximize, and therefore, a profit-maximizing firm should act in the permit market in accordance with its product market strategy. Misiolek & Elder (1989) extended Hahn’s setting to the product market and concluded that a single dominant firm can manipulate permit prices to drive up the fringe firm’s cost in the product market. Hinterman (2011) found that the threshold of free allocation above which a dominant firm will have an incentive to set the permit price above its marginal abatement costs is below its optimal emissions in a competitive market, and that overall efficiency cannot be achieved by means of the permit allocation alone. Hintermann (2017) pointed out that a dominant net permit buyer may want to increase the permit price, provided that the increase in compliance costs is more than offset by the sum of the revenue increase in the output market and the increase in rents embedded in free allocation.

We aim to address situations in which there is a strong interaction between the strategic behavior in the product and the permit markets. The NOx permit market in California is an important practical example. Nearly 25% of the NOx permits were allocated to facilities that sell power into the California electricity market, which has been recognized for its (unilateral) market power problems. In fact, Kolstad and Wolak (2003) argue that electric utilities used the NOx market to enhance their ability to exercise (unilateral) market power in the electricity market.\(^2\)

The connections between the permit market and the output market are very important, not only at the firm level, but also at the country level. Montero (2009)

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\(^2\) See Fowlie (2010) for the analysis of the effects on permit market efficiency.
argues that some of the large countries in an eventual global carbon market are also big players in energy markets. In this respect, Hagem and Maestad (2006) analyse the optimal strategies of Russia, which is a very relevant player in the international market for emission permits but is also an important fuel exporter. By numerical simulations, they concluded that Russia could have benefited from coordinating its permit exports with its oil and gas exports during the commitment period of the Kyoto Protocol.

In this paper we analyze the behavior of polluting oligopolistic firms (or countries)\(^3\) that interact strategically both in the output and the permit markets, paying particular attention to the market power position of each firm in each market. To this aim we consider a model of two firms competing under different oligopolistic structures while there is a dominant firm in the permit market. In this framework, we study and compare the effect of market power under three different situations, depending on the role that each firm plays in the product market and specifically, if the firm that is dominant in the permit market plays the same role in the product market.

We first set up a simple model of a permit market assuming that there is a dominant firm and another one that behaves as a price taker. We show that the equilibrium of such a market is crucially determined, first, by the equilibrium in the output market and, second, by the initial allocation of permits. We also restate the classical result of Hahn (1984), which implies that, as long as there is any permit trade taking place in the market, the dominant firm is taking an advantage of its position whether it acts as a net buyer or as a net seller of permits. Then, we move on to study the output market and its link with the permit market.

In order to investigate the role of the product market with some detail, we consider three alternative market structures. In the first version we consider a Cournot

\(^3\) Although our model could be interpreted both in terms of firms or countries, for simplicity of exposition, most of the time we will simply refer to “firms”.
oligopoly, so that both firms play a symmetric role, whereas in the other two there are a leader and a follower a la Stackelberg. Specifically, in the second version (Stackelberg 1) we consider a dominant firm in the emission permit market that is also a leader in the product market. In the third version (Stackelberg 2) we consider that the firm that acts as a leader in the output market (Firm 1) is the follower in the permits market.

The aim of the first output model (Cournot) is to study a case in which a firm has a dominant position in the permit market but the firms are otherwise symmetric. This allows us to ask to what extent the leading position in the permit market can break the symmetry in the output market or, the other way around, how the symmetric position can soften the leadership position of one of the firms in the permit market. We are not aware that a model like this (with a Cournot structure in the output market and a dominant firm in the permit market) has been previously addressed in the literature.

The second version (Stackelberg 1) depicts a situation in which a single firm has the ability to lead both markets at the same time, which a priori is the most favorable situation for the firm itself but, arguably, the less favorable for the other firm and also for the sake of market efficiency. Similar versions of this model have been addressed in the literature in the form of a Stackelberg leader-follower setting or a leader-fringe structure. Apart from Hinterman (2011), which has been mentioned above, Chevalier (2008) considers a permit market with both spatial and intertemporal trading, a big dominant firm and a large number of small firms who are nonstrategic but forward looking. The equilibrium is characterized by for the monopoly case and for intermediate cases. Tanaka & Chen (2012) consider a Cournot-fringe model with market power in both product and permits market to simulate the California electricity market and they show that Cournot firms can significantly raise both power price and permit price, which results in a great loss in social surplus.
As far as we know, our third case, (Stackelberg 2) has not been analyzed in the literature yet. The motivation for this version is to look for conditions under which being a leader in one of these markets represents a competitive advantage or, in other words, which is the best place to exercise market power. With this version of the model, we try to get some insight about the relative importance of having a leadership position in the output market and in the permit market. One possible way to motivate the interest of this approach has to do with Russia’s role in the Kyoto Protocol. It has been argued that USA rejection to enter this Protocol resulted from the fact that its least costly way to implement the required targets would have involved large purchases of emission credits from Russia (see Bernard et.al, 2003). This situation can be captured in a simple stylized way by stating that Russia was (or the USA thought it was) a leader in the permit maker. Simultaneously, it can be argued that USA has a stronger leadership position than Russia in the output market. One can be interested in assessing the relative strength of both leaderships when it comes to get a better economic outcome. This question is what we aim to address in a simplified way with our third model.

The remainder of this article has the following structure: In the next section 2 we describe the basic elements of the model. Section 3 analyses the equilibrium in the emissions permit market. In the next three sections we explore the implications of grandfathering in the firms output and profit within three different oligopolistic structures: Cournot (Section 4) and the two versions of the Stackelberg model that we have discussed above (Sections 5 and 6). Section 7 states our conclusions.

2. The Model. Basic Elements

We consider a model with two firms indexed by \( i = 1, 2 \). Both firms enjoy an initial firm-specific free allocation of permits, \( S_i (i = 1, 2) \). In the output market we
state a duopolistic framework. We denote individual output as \( x_i \) and total output as \( X \) and the inverse demand function as \( P(X) = a - bX \). On the production side we assume that the firms face a constant marginal cost equal to \( c \). Gross emissions are assumed to be proportional to the firms’ output \( (rx_i) \) where \( r \) is the pollution intensity, which is common for both firms. The firms can reduce emissions by either reducing output or making some abatement effort. Abated quantities are denoted by \( q_i \) \((i = 1, 2)\) and net emissions are given by gross emissions minus abatement, i.e., \( e_i = rx_i - q_i \). We assume the same abatement technology for both firms with abatement cost given by the quadratic function \( C(q_i) = d + tq_i \), where \( d \) and \( t \) are technological parameters.\(^4\)

Finally both firms trade permits. We denote permits purchased by firm \( i \) as \( y_i \), so that \( y_i < 0 \) means that firm \( i \) sells rather than buys permits. The market equilibrium condition implies \( y_1 = -y_2 \).

We also assume that the emissions generated by both firms can be perfectly monitored without cost by the regulatory authorities and firms cannot emit more than the number of permits they hold.\(^5\)

### 3. A dominant firm in the emission permits market

As explained above, the emissions game has two sub-stages. It the last one, Firm 2 choses its emissions (or, equivalently, abatement) by solving the following problem:

\[
\begin{align*}
\min_{\{q_2\}} & \quad q_2 (d + tq_2) + py_2 \\
\text{s.t.} & \quad S_2 + q_2 + y_2 = rx_2
\end{align*}
\]

\(^4\) The same function has been used by Sartzetakis (1997) or André and de Castro (2017).

\(^5\) Alternatively, we can interpret that a high enough penalty has to be paid to ensure that there is no room for moral hazard.
We can substitute the constraint into the objective function and arrive at the familiar first-order condition, which states that marginal abatement cost equals the permit price. By solving such condition for $q_2$ we obtain firm 2’s demand for abatement and combining this expression with the constraint we get optimal demand for permits of firm 2 as a function of the permit price:

$$q_2(p) = \frac{p - d}{2r}$$  \hspace{1cm} (2)

$$y_2 = rx_2 - \frac{p - d}{2r} - S_2.$$  \hspace{1cm} (3)

The dominant firm minimizes its own costs anticipating the reaction of the follower and taking into consideration the permit market-clearing condition given by

$$y_1 = rx_1 - q_1 - S_1 = -rx_2 + q_2(p) + S_2 = -y_2.$$  \hspace{1cm} (4)

Solving the cost-minimizing problem yields the optimal abatement of the dominant firm (details can be found in the Appendix):

$$q_i = \frac{r(2x_i + x_d) - 2S_i}{3} - S_2.$$  \hspace{1cm} (5)

Combining (2), (3), (4) and (5) we obtain firm 2’s optimal abatement and demand for permits as a function of output and the initial allocation of permits:

$$q_2 = \frac{r(2x_2 + x_1) - 2S_2}{3} - S_1,$$  \hspace{1cm} (6)

$$y_2 = -y_1 = \frac{(S_1 - S_2) - r(x_1 - x_2)}{3} = q_2 - q_1.$$  \hspace{1cm} (7)

Equation (7) shows that the dominant firm will be a net buyer of permits ($y_1 > 0$) when its abatement exceed the follower’s. In such a case, the dominant firm buys permits at a price that is lower than its marginal cost of abatement, as can be easily proof using equation (2):
\[ p = d + 2tq_2 < d + 2tq_1 \] 

In the same way it can be stated that if the dominant firm is a net seller of permits the price will exceed its marginal cost of abatement. In both cases the dominant position is an instrument that firm 1 can use to reduce its cost and increase its profit. This conclusion is in the classical line of Hahn (1984).

Equation (7) also reveals that the equilibrium in the permit market is driven by two main elements: first, the equilibrium in the output market, and more specifically, the difference in both firms' output, and second, the difference in the initial permit allocation. If we consider the particular case where both firms initially receive the same amount of permits, Equation (7) shows that the net demand for permits is proportional to the difference in output which means that the firm that produces more (less) output acts as a net buyer (seller) of permits. If, apart from receiving the same amount permits, both firms produced the same output, in equilibrium both firms would abate the same amount of emissions and there would not be any trade of permits. In the next section we show that this is the case under Cournot competition.

To have a full picture, in the following sections we investigate the equilibrium of the output market and how such equilibrium is influenced by the initial allocation. We consider two possibilities: first, both firms compete simultaneously in output a la Cournot, and second, there is a follower and a leader, a la Stackelberg. In the second case, in turn, we consider two possibilities depending on whether the leader in the output market is the same firm that has a similar position in the permit market or not.
4. A Cournot Model in the output market

In this section we assume that, in the first stage of the game, both firms choose their output simultaneously maximizing their individual profit a la Cournot and anticipating that in the emissions permit market firm 1 is a dominant firm while firm 2 acts as a price taker. The profit maximization problem of firm 2 is:

$$\text{Max} \left[ a - b(x_1 + x_2) \right] x_2 - cx_2 - q_2\left(d + t q_2\right) - p\left(r x_2 - q_2 - S_2\right)$$

s.t. Eq(2) and (6)  

and, from the first order condition, we derive the reaction function of firm 2, which is given by

$$x_2 = \frac{9(a - c - dr) + 2rt(8S_2 + S_1)}{18b + 16r^2t} \cdot \frac{9b + 2r^2t}{18b + 16r^2t} x_1.$$  

(9)

In a similar way we obtain firm 1’s reaction curve:

$$x_1 = \frac{9(a - c - dr) + 6rt(2S_1 + S_2)}{18b + 12r^2t} - \frac{9b + 6r^2t}{18b + 12r^2t} x_2.$$  

(10)

Solving the system given by equations (9) and (10) we obtain the optimal output of both agents in terms of the parameters of the model:

$$x_2 = \frac{(a - c - dr)}{3b + 2r^2t} + \frac{rtS_2(26b + 20r^2t) - 8brtS_1}{(9b + 10r^2t)(3b + 2r^2t)},$$  

(11)

$$x_1 = \frac{(a - c - dr)}{3b + 2r^2t} + \frac{rtS_1(22b + 20r^2t) - 4brtS_2}{(9b + 10r^2t)(3b + 2r^2t)}. $$  

(12)

Note that the reaction functions are not equal and, therefore, output is not necessarily equal for both firms in equilibrium, as it would be the case in the standard Cournot model with symmetric firms. This is due to the different role that the firms play in the emissions game. To have a more accurate understanding of the difference between both firms’ output, we combine (11) and (12) to get
\[ x_1 - x_2 = \frac{10rt(S_1 - S_2)}{(9b + 10r^2t)}. \]  

(13)

This result may seem surprising to some extent. If both firms initially receive the same amount of permits, firm 1’s dominant position in the emission permits market does not lead to any advantage in practical terms as both firms will produce the same amount of output \( (x_1 = x_2) \). As can be seen from equations (5) and (6), abatement will be the same for both firms and, as a consequence, there will not be any permit trade and both firms will also make the same profit.

Plugging equations (11) and (12) into equation (5) and (6) and rearranging we obtain the optimal abatement of both firms in terms of the parameters of the model:

\[ q_1 = \frac{r(a - c - dr) - b(2S_1 + S_2)}{(3b + 2r^2t)}, \]  

(14)

\[ q_2 = \frac{r(a - c - dr) - 2bS_2(9b + 8r^2t) + bS_1(9b + 14r^2t)}{(3b + 2r^2t)(9b + 10r^2t)}, \]  

(15)

and combining both expressions we can compute the difference in the abatement made by both firms, which in turn, according to (7), provides the amount of permits traded in the market equilibrium:

\[ q_1 - q_2 = \frac{3b(S_2 - S_1)}{9b + 10r^2t}. \]  

(16)

In order to provide some further understanding of the interaction between the permit allocation and the position of each firm in the market, consider as a sensitivity analysis that the initial allocation of one of the two firms is increased in one permit. This change can be understood as a marginal cost reduction. In such a case, the firm could react by 1) marginally increasing output (and hence, gross emissions) to make use of the additional permit, 2) reducing abatement (to save abatement cost) or 3) a
combination of both. We ask how differently both firms react to a change in their allocation.

From equations (11) and (12) it can be easily proved that firm 2’s output is more sensitive to its permit allocation than firm 1, i.e., when firm 2 receives one more permit, its output increase is higher than the firm 1’s output increase when this firm receives one more permit. Formally

$$\frac{\partial x_2}{\partial S_2} - \frac{\partial x_1}{\partial S_1} = \frac{4brt}{(9b + 10r^2t)(3b + 2r^2t)} > 0.$$  

On the other hand, when Firm 1 receives one more permit, it decreases abatement in a higher amount that firm 2 does, when the latter receives one more permit. Formally

$$\frac{\partial q_2}{\partial S_2} - \frac{\partial q_1}{\partial S_1} = \frac{4br^2t}{(9b + 10r^2t)(3b + 2r^2t)} > 0.$$  

Therefore, when facing an increase in its permit allocation, firm 2 is relatively more prone to react by increasing output and firm 1 by decreasing abatement. The reason why firm 1’s adjustment is more biased towards abatement relates to its dominant position in the permit market, as it can buy permits at a price lower than its marginal abatement cost or sell permits at a price higher than its marginal abatement cost.

The main findings for the Cournot case can be summarized in the following proposition.

**PROPOSITION 1**

a) When firm 1 receives more (less) free permits than firm 2, it produces more (less) output and consequently its gross emissions are higher (lower).

b) The firm that receives more free permits makes a higher profit.
c) When initially endowed with an additional permit, Firm 1’s output increases less and firm 1’s abatement decreases more than firm 2’s output and abatement when endowed with an additional permit.

5. A Stackelberg Model with the same Leader in Both Markets

In this section we assume that firm 1 is not only a dominant firm in the emission permits market but also in the product market. To some extent, our approach is similar to the followed by Hinterman (2011) but introducing several significant differences. First, Hinterman considers a dominant firm and a competitive fringe in the output while we are analyzing a duopoly. In our model the follower that acts strategically, by placing itself in its reaction curve, while the Hinterman's approach the firms in the fringe are price takers i.e.: price equals their marginal cost in the optimum. A second depart from Hinterman's paper is the fact that we are setting a separable cost function considering a particular abatement function and a constant ratio between output and emissions. Finally we consider a linear demand function.

Now the timing of the game is the following: first, Firm 1 set its output acting as a Stackelberg leader and simultaneously decides its level of abatement and the price of permits anticipating the reaction of the price taker. Second, Firm 2 set its output and abatement level as a follower.

Both firms solve a profit maximization problem to determine their optimal levels of abatement, taking into account the cost of buying permits. The follower’s profit maximization problem is

\[
\text{Max} \left[ a - b \left( x_1 + x_2 \right) \right] x_2 - cx_2 - q_2 \left( d + tq_2 \right) - py_2
\]

\[s.t. \quad y_2 = rx_2 - q_2 - S_2\]  

(17)
We can substitute the constraint into the objective function and arrive at the familiar first-order conditions that marginal abatement costs equal the permit price, and the reaction curve for the output.

\[
FOC(x_2) \implies x_2 = \frac{1}{2b}(a - c - rp) - \frac{1}{2}x_1, \quad (18)
\]

\[
FOC(q_2) \implies q_2 = \frac{p - d}{2t}. \quad (19)
\]

The dominant firm takes equations (18) and (19) into account when maximizing its own profits. It also incorporates the permit market-clearing condition, which is given by

\[
y_1 = rx_1 - q_1 - S_1 = -y_2 = -rx_2(x_1) + q_2(p) + S_2. \quad (20)
\]

Equation (20) lets us find the permit price as a function of the leader’s output and abatement:

\[
p = \frac{bt(rx_1 - 2q_1) + bd - 2bt(S_1 + S_2) + rt(a - c)}{b + r^2t}. \quad (21)
\]

The permit price is increasing in leader’s output and decreasing in leader’s abatement. If we combine Equation (21) with Equation (18), and applying the chain rule, we find the slope of the reaction curve of the follower:

\[
\frac{\partial x_2}{\partial x_1} = \frac{-rt}{2(b + r^2t)} - \frac{1}{2}. \quad (22)
\]

Equation (22) captures the double effect of an increase in the leader’s output: The standard effect due to the dominant position in the output market, and the indirect effect due to the permit price increase that further reduce the follower’s output. This effect can be identified as the so called “raising rival’s cost” that can be found in the related literature. In short, a dominant firm can improve its position in the product market indirectly via manipulation of input prices (in our case, the price of emission permits).
The leader's profit maximization problem on output and abatement is:

\[
\text{Max} \left( a - b \left( x_i + x_2(x_i, p(x_i, q_i)) \right) \right) x_i - cx_i - q_i \left( d + t q_i \right) - p \left(x_i, q_i \right) y_i \left(x_i, q_i \right)
\]  

(23)

The resulting FOC’s are:

\[
a - 2 bx_i - x_2 - x_i \frac{\partial x_2}{\partial x_i} - c - p \frac{\partial y_i}{\partial x_i} - y_i \frac{\partial p}{\partial x_i} = 0 \quad x_i > 0
\]  

(24)

\[
(-b x_i) \frac{\partial x_2}{\partial p} \frac{\partial p}{\partial q_i} - d - 2 t q_i - y_i \frac{\partial p}{\partial q_i} - p \frac{\partial y_i}{\partial q_i} = 0 \quad q_i > 0
\]  

(25)

Combining (24) and (21) the solution to the leader´s problem is

\[
x_i = \frac{3 \left( a - c - dr \right) + 2rt \left( 2S + S_2 \right)}{6b + 4r^2t}
\]  

(26)

\[
q_i = \frac{2r \left(a - c - dr\right) - 2b \left( 2S + S_2 \right)}{6b + 4r^2t}
\]  

(27)

And the follower output comes from plugging equation (26) in (18)

\[
x_2 = \frac{a - c - dr + 2rt S_2}{4 \left( b + r^2 t \right)}
\]  

(28)

Once we have solved the model we analyze whether a particular allocation of free permits can alter the Stackelberg model standard results, with the following finding:

**PROPOSITION 2**

*In any interior solution, the leader’s output is always higher than the follower’s regardless the initial allocation of free permits.*

We have checked that, for some range of parameter values, although the leader always produces more than the follower, the follower can still make a higher profit than the leader if the follower’s allocation of permits is large enough as compared to the leader’s.
6. A Stackelberg Model with a Different Leader in Each Market

In this section we analyze another situation where one firm dominates the product market and the other dominates the permit market. The rest of the elements of the model are the same as in the previous section.

With such an approach we aim at determining the potential advantages of being a leader in each of these markets. For consistency with the previous sections, we denote as “Firm 2” the one that acts as a leader in the output market (and as a price taker in the permit market) while “Firm 1” is still the dominant firm in the permit market and act as a follower in the output market. Initially, we consider that both firms enjoy an equal initial free allocation of permits, i.e. \( S_1 = S_2 = S \). Afterwards, we relax this assumption.

The game has four stages that develop as follows:

1. Firm 2 sets its output acting as a Stackelberg leader
2. Firm 1 sets its output as a follower
3. Firm 1 decides its abatement level (and thus its demand for permits) and the price of permits anticipating the reaction of Firm 2.
4. Firm 2 decides its level of abatement and demand for permits acting as a price taker.

The model is solved by backward induction. Stages 3 and 4 have already been solved in Section 3. Due to the assumption \( S_1 = S_2 = S \), equations (5) and (6) become

\[
q_1 = \frac{r(2x_1 + x_2 - 3S)}{3} \tag{29}
\]

\[
q_2 = \frac{r(2x_2 + x_1 - 3S)}{3} \tag{30}
\]

and the associated permit price is

\[
p = d - 2dS + \frac{2tr(2x_1 + x_2)}{3}. \tag{31}
\]
From (29) and (30) we get the number of traded permits as a proportion of the difference between the agent’s output quantities, and also as the difference between the abated quantities.

\[ y_i = \frac{r(x_1 - x_2)}{3} = q_1 - q_2 \]  

which is a particular case of (7) when both firms receive the same amount of free permits.

In the first two stages of the game the agents solve its profit maximization problem sequentially as it is common in the Stackelberg models. As in a standard Stackelberg model, the product leader (firm 2) considers the follower’s reaction curve and closes the market. After some algebra we get optimal outputs in terms of the parameters (detailed calculations are given in the Appendix):

\[ x_2 = \frac{(6b + 6r^2)t(a - c - dr + 2rtS)}{(6b + 4r^2t)(2b + 3r^2t)} > 0 \]  

\[ x_1 = \frac{(3b + 6r^2t)(a - c - dr + 2rtS)}{(6b + 4r^2t)(2b + 3r^2t)} > 0 \]

Throughout our analysis, we assume \((a-c-dr) > 0\), to make sure that output is always positive even in the case of null allocation of free permits \((S = 0)\).

Using expressions (33) and (34), we can immediately compare the output of both firms:

\[ x_2 - x_1 = \frac{3b(a - c - dr + 2rtS)}{(6b + 4r^2t)(2b + 3r^2t)} > 0 \]
Equation (35) shows that as long as both firms produce positive amounts, the leader’s output is always greater than the follower’s, which is a standard result in the Stackelberg model. Actually, in the standard Stackelberg model with linear demand and constant marginal cost, the leader produces exactly double as much as the follower. In our case, the ratio between the outputs of both firms equals

\[
\frac{x_2}{x_1} = \frac{2b + 2r^2t}{b + 2r^2t} \in (1,2)
\]

and so the leader produces less than double except if \( r = 0 \) or \( t = 0 \), which would lead us to the standard Stackelberg model. The higher the value of \( r \) and \( t \), the more similar the outputs of both firms are. The interpretation of this result is that, the more important the environmental problem (as determined by the pollution intensity and the convexity of the abatement cost function) the more able firm 1 is to overcome the leadership position of firm 2 in the output market, although it will never be able to leapfrog firm 2 in terms of output.

To find out how firm 1 takes advantage of its leadership in the permit market, note the fact that \( x_2 > x_1 \), combined with (29) and (30) implies that \( q_2 > q_1 \), i.e., firm 2 makes more abatement than firm 1, and since the abatement cost function is strictly convex, we conclude that, in equilibrium, the marginal abatement cost of firm 2 is higher than that of firm 1 and this result, combined with equation (2), implies that firm 1 exerts its market power in the permit market by setting a price that is above its marginal abatement cost (and equal to that of firm 2). The main features of the equilibrium are summarized in the following proposition.

**PROPOSITION 3**

*In equilibrium, the following results hold:*
a) Firm 2 produces more and abates more than firm 1.

b) Firm 2 is a net buyer and firm 1 is a net seller of permits.

c) The permit price is above firm 1’s marginal cost of abatement.

The proof of this proposition is straightforward. Equation (35) and equation (32) directly imply a) and b). If we take these equations combined with equation (2) we obtain the following inequality that proves c):

\[ p = d + 2tq_1 > d + 2tq_2. \]

Plugging equations (33) and (34) into equations (29) and (30), abatement and the amount of traded permits can also be stated in terms of the parameters of the model as follows

\[ q_2 = \frac{r(5b + 6r^2t)(a - c - dr + 2trS)}{(6b + 4r^2t)(2b + 3r^2t)} - S \tag{37} \]

\[ q_1 = \frac{r(4b + 6r^2t)(a - c - dr + 2trS)}{(6b + 4r^2t)(2b + 3r^2t)} - S \tag{38} \]

It is trivial to see that both firms output and abatement are increasing in the demand intercept (parameter \( a \)) and in the number of allocated free permits (parameter \( S \)) and decreasing in marginal output cost (parameter \( c \)) and the marginal cost of the first unit of abatement (parameter \( d \)). The difference of outputs and abatements follows the same rule, meaning that all changes have a greater impact on the product market leader’s variables.

\[ q_2 - q_1 = \frac{r(x_2 - x_1)}{3} = \frac{br(a - c - dr + 2trS)}{(6b + 4r^2t)(2b + 3r^2t)} = y_2 \tag{39} \]

Regarding profit, we come up with the following Proposition:
PROPOSITION 4

a) The profit of the output market leader (Firm 2) is always greater than the profit of the emissions permit market leader (Firm 1).

b) The difference between Firm 2’s and Firm 1’s profits is increasing in parameters “a” and “S” and decreasing in parameters “c” and “d”.

We prove Proposition 4 in the Appendix. The economic interpretation of the results regarding parameters S and c is straightforward. As long as the number of free permits is increasing, less abatement is needed for a fixed production and the product market leader is taking a bigger advantage. On the other hand, if the marginal product cost is increasing firm 1 is suffering less impact in its profits since its production level is always lower than the production level of firm 2.

Asymmetric free allocation of permits

In this subsection we relax the assumption that both firms receive the same amount of initial permits by allowing for S₁ being different from S₂. For the sake of completeness, we consider, not only the case that firm 2 receives more permits than firm 1, which could be justified if the regulator allocates permits in proportion to production amounts, but also the opposite case, which could explain the dominant position of firm 1 in the permit market. For comparison purposes we also assume that the regulator sets the same total cap as in the symmetric case. That means \(2S = S₁ + S₂\).

The abatement quantities and the demand for permits follow from equations (5), (6) and (7). Proceeding in a similar way as in the symmetric case we obtain the following corresponding expressions for the output quantities.
\[ x_2 = \frac{(6b + 6r^2 t)(a - c - dr) + 4rt\left[b(4S_2 - S_1) + 3r^2 tS_2\right]}{6b + 4r^2 t}\left(2b + 3r^2 t\right) \]  

(33.a)

\[ x_1 = \frac{(3b + 6r^2 t)(a - c - dr) + 2rt\left[b(5S_1 - 2S_2) + 6r^2 tS_1\right]}{6b + 4r^2 t}\left(2b + 3r^2 t\right) \]  

(34.a)

We compute the difference as:

\[ x_2 - x_1 = \frac{3b(a - c - dr) + 2brt(10S_2 - 7S_1) + 12rt(r^2 t)(S_2 - S_1)}{6b + 4r^2 t}\left(2b + 3r^2 t\right) \]  

(35.a)

And now the sign is ambiguous. The free allocation of permits directly impact both firm’s outputs and consequently firm’s profits.

Consider first \( S_2 > S_1 \). By simple differentiation it can be immediately seen that firm 2 increases production and decreases abatement as the difference is increasing, while firm 1’s output decreases and abatement increases.

From Equation (23.a) we immediately obtain

\[ \frac{\partial (x_2 - x_1)}{\partial S_2} = \frac{4rt(5b + 3r^2 t)}{6b + 4r^2 t}\left(2b + 3r^2 t\right) > 0. \]  

(36.a)

From Equations (7) and (23.a) we conclude

\[ \frac{\partial (q_2 - q_1)}{\partial S_2} = \frac{-4b^2 - 2br^2 t}{6b + 4r^2 t}\left(2b + 3r^2 t\right) < 0. \]  

(37.a)

As we know from equation (7) the difference in abatement determines the amount of sold and bought permits. Therefore, (37.a) implies that the number of permits demanded by firm 2 is decreasing as a consequence of the reduction in the difference of abated quantities. And the permit price is also decreasing because Firm 2 lower abatement means a decrease in its marginal abatement cost and therefore in the permit price. The number of permits sold reach zero at a certain value of the difference as it is shown in the following proposition (which is proved in the Appendix).
PROPOSITION 5

a) There is a particular allocation of free permits, with \( \hat{S}_2 > \hat{S}_1 \); \( 2S = \hat{S}_2 + \hat{S}_1 \), such that the market is closed without transactions. At this point both firms abate the same amount of emissions. Specifically,

\[
\hat{S}_2 = \frac{r(a - c - dr) + 8S(b + r^2t)}{2(4b + 3r^2t)}
\]  \hspace{1cm} (40)

b) For any allocation between the symmetric one (\( S_1 = S_2 \)) and the one given by (40), Firm 2 is a net buyer of permits and the permit price exceeds firm 1’s marginal cost of abatement.

c) For any allocation above the threshold value \( \hat{S}_2 \) firm 2 becomes a net seller of permits. The permit price is below firm 1’s marginal cost of abatement.

It trivially follows that whenever \( S_2 \geq S_1 \) firm 2 produces 2 more and makes a higher profit than firm 1.

Consider now the case \( S_1 > S_2 \). By simple differentiation, as in the previous case, it can be shown that, as the difference \( S_1 - S_2 \) increases, firm 1 increases output and decreases abatement while firm 2 decreases output and increases abatement. The number of permits sold is increasing as it is the permit price because the marginal cost of abatement of firm 2 is also increasing. At a certain value of the difference, both firms produce exactly the same amount of output.

PROPOSITION 6

a) There is a particular allocation of free permits \( S_1^*, S_2^* \) (satisfying \( S_1^* > S_2^* \)) such that both firms produce the same amount of output. Specifically,
\[ S_1^* = \frac{3b(a - c - dr) + 4rtS(10b + 6r^2t)}{2rt(17b + 12r^2t)} = 2S - S_2^* \]

(41)

b) For any allocation between the symmetric one \((S_1 = S_2)\) and the one defined by (41), firm 1 produces less than firm 2. Beyond this threshold firm 1 produces more than firm 2.

c) At the threshold point and beyond, Firm 1’s profits are higher than firm´s 2 profit.

We prove the proposition in the Appendix. The importance of this proposition is to show that the dominant position that firm 2 enjoys in the output market can be offset if the cost advantage enjoyed by firm is reinforced by assigning it a larger number of free permits. In such a case, firm 1 can make higher profits because marginal product cost is the same for both firms and the dominant position in the emissions permit market makes that firm 1 is selling permits at a price exceeding its marginal cost of abatement.

7. Concluding Remarks

In this article we have considered three different oligopolistic structures in the product market under the common assumption of imperfect competition in the related permit market. The main focus of our analysis has been to analyze the role of grandfathering in the outcome of firms in terms of output and profits. Our preliminary conclusions are as follows.

For the Cournot model, we have shown that if both firms initially receive the same amount of permits, there will not be any permit trade. Both firms will produce the same quantities and make the same profit and therefore firm 1’s dominant position in the emission permits market does not lead to any competitive advantage. If the
allocation is not symmetric, the firm who receives more free permits produces a higher quantity and makes higher profits.

When there is only one leader for both markets, and regardless the allocation of free permits, the leader is always producing more than the follower, although the follower can still make a higher profit than the leader if the follower’s allocation of permits is large enough as compared to the leader’s.

For the Stackelberg model with two different leaders, one in each market, the Stackelberg leader produces more and make more profits than the dominant firm in the permit market under a symmetric allocation of free permits but as soon as we consider that the permit market leader is receiving more permits, both firms output tends to equalize first and it comes to a point where the Stackelberg leader in the product market produces less and makes less profits than the follower. This results suggest than exerting market power in the output market is more advantageous for a firm than doing it in the permit market.

APPENDIX

Permit Market Leader Optimal Solution

The leader solves the following problem

\[
\begin{align*}
\min_{\{q, y\}} & \quad q_1(d + t q_1) + p y_1 \\
\text{s.t.} & \quad S_1 + q_1 + y_1 = rx_1 \\
& \quad p = d + 2t q_2 = d + 2t(rx_2 - S_2 - y_2)
\end{align*}
\]

(A1)

Solving for \( p \) in the above equation leads to a single variable problem
\[ p = d + 2t (rx_2 - S_2 + y_1) = d + 2t (rx_2 + rx_1 - q_1 - S_2 - S_1) \]
\[
\min_{q_1} q_1 \left( d + tq_1 \right) + (rx_1 - S_1 - q_1) \left[ d + 2t (rx_2 + rx_1 - q_1 - S_2 - S_1) \right] \quad (A2)
\]

The FOC of this problem is

\[ d + 2tq_1 - 2t (rx_1 - q_1 - S_1) - d - 2t (rx_2 + rx_1 - q_1 - S_2 - S_1) = 0 \]

Solving for \( q_1 \)

\[ 6tq_1 = 2t (2rx_1 + rx_2 - 2S_1 - S_2) \Rightarrow q_1 = \frac{2rx_1 + rx_2 - 2S_1 - S_2}{3} \]

**Proof of Proposition 1**

Let us consider the function

\[ \Pi = \Pi_1 - \Pi_2 = (P - c)(x_1 - x_2) - dy_1 - t\left(q_1^2 - q_2^2\right) - 2py_1 \]
\[ \Pi = \Pi_1 - \Pi_2 = (P - c)(x_1 - x_2) - dy_1 - t\left(q_1 + q_2\right)(q_1 - q_2) - 2py_1 \quad (A3) \]

This function shows the difference between profits. We have taken into account equation (7) and we denote the product price as \( P \). By simple algebraic manipulation we obtained the following expression

\[ \Pi = \Pi_1 - \Pi_2 = (P - c)(x_1 - x_2) - y_1 \left( d + 2p + t\left[r(x_1 + x_2) - (S_1 + S_2)\right]\right) \quad (A4) \]

We have considered equations (5) and (6) to arrive at the following equation

\[ q_1 + q_2 = r\left(x_1 + x_2\right) - (S_1 + S_2) \quad (A5) \]

The sum of both firm’s abatement must be non-negative as long as any firm abatement is non-negative. If firm 1 is a net buyer of permits \( (y_1 > 0) \), the second term of the function is positive while the first one is negative \( (x_1 - x_2 < 0) \). Therefore the profit made by firm 2 exceeds the profit made by firm 1. The opposite case can be proved in a similar way.

**Model 2 Firm 1 Problem in the Output Market**

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Firm 1 solves the following problem

\[
\max \left[ a - b \left( x_1 + x_2 \right) \right] x_1 - c x_1 - (d + t q_1) q_1 - p y_1
\]  

\[(A6)\]

FOC lead us to the reaction curve after some algebraic operations

\[
a - b x_2 - 2 b x_1 - c - d \frac{\partial q_1}{\partial x_1} - 2 t q_1 \frac{\partial q_1}{\partial x_1} - \frac{\partial p}{\partial x_1} y_1 - p \frac{\partial y_1}{\partial x_1} = 0
\]

\[
a - b x_2 - 2 b x_1 - c - \frac{2 dr}{3} - \frac{4 tr \left( 2 r x_1 + r x_2 - 3 S \right)}{3} - \frac{2 tr \left( r \left( x_1 - x_2 \right) \right)}{3} - \left( 3 d - 6 t S + 2 tr \left( 2 x_2 + x_1 \right) \right) \frac{r}{3} = 0
\]

\[
x_i = \frac{3 \left( a - c - dr \right) + 6 tr S}{6 b + 4 tr^2} - \frac{1}{2} x_2
\]  

\[(A7)\]

**Model 2 Firm 2 Problem in the Output Market**

The leader in the output market solves the following problem

\[
\max \left[ a - b \left( x_1 + x_2 \right) \right] x_2 - c x_2 - (d + t q_2) q_2 - p y_2
\]  

\[(A8)\]

The FOC for optimal output is

\[
a - 2 b x_2 - b x_1 - b x_2 \frac{\partial x_1}{\partial x_2} - c - d \frac{\partial q_2}{\partial x_2} - 2 t q_2 \frac{\partial q_2}{\partial x_2} - \frac{\partial p}{\partial x_2} y_2 - p \frac{\partial y_2}{\partial x_2} = 0
\]

The partial derivatives take the following values

\[
\frac{\partial y_2}{\partial x_2} = r + \frac{r}{3} = \frac{r}{2}; \frac{\partial q_2}{\partial x_2} = \frac{2 r}{3} - \frac{r}{6} = \frac{r}{2}; \frac{\partial p}{\partial x_2} = \frac{4 tr}{3} - \frac{tr}{3} = tr
\]

Plugging these values on the FOC and taking into account the reaction curve of the follower, lead us to the following expression

\[
a - 2 b x_2 - \frac{1}{2} b x_2 - b \left( \frac{3 \left( a - c - dr \right) + 6 tr S}{6 b + 4 tr^2} - \frac{1}{2} x_2 \right) - c - \frac{dr}{2} - tr \left( \frac{2 rx_2 + r x_1 - 3 S}{3} \right) - \left( 3 d - 6 t S + 2 tr \left( 2 x_2 + x_1 \right) \right) \frac{r}{2} - tr \left( \frac{rx_2 - r x_1}{3} \right) = 0
\]
Now some algebraic operations yields the optimal output value of the leader as it is shown in Equation (21)

\[
x_2 = \frac{(3b + 3tr^2)(a - c - dr + 2trS)}{(3b + 2tr^2)(2b + 3tr^2)} = \frac{(6b + 6tr^2)(a - c - dr + 2trS)}{(6b + 4tr^2)(2b + 3tr^2)}
\]

(A9)

Plugging this value in the reaction curve of the follower yields Equation (22)

**Proof of Proposition 4**

We define the function of the difference between firms profit as:

\[
\Pi = \Pi_2 - \Pi_1 = (P - c)(x_2 - x_1) - d(q_2 - q_1) - t(q_2^2 - q_1^2) - 2py_2
\]

(A10)

Considering Eq (20) we compute the function as

\[
\Pi = (a - bx_1 - bx_2 - c)(x_2 - x_1) - dy_2 - ty_2(q_1 + q_2) - 2(d + 2y_2)t_2
\]

(A11)

We plug equations (33), (34), (35), (37), (38), and (39) in (A11) and after some tedious calculations we obtain the following expression

\[
\Pi = \frac{b^2}{4} \frac{(a - c - dr + 2trS)^2(9b + 13r^2t)}{(2b + 3r^2t)(3b + 2r^2t)}
\]

(A12)

The expression is obviously strictly positive and proves the first part of this Proposition.

To prove the second part, we take into account that a necessary condition for producing and selling positive quantities is:

\[a - c - dr > 0,\]

We differentiate (A12) and we find the following results:
\[
\frac{\partial \Pi}{\partial S} = \frac{b^2 \text{tr} (a-c-dr+2trS)(9b+13tr^2)}{(2b+3tr^2)^2 (3b+2tr^2)^2}
\]

\[a-c-dr+2trS > 0 \Rightarrow \frac{\partial \Pi}{\partial S} > 0\]

\[
\frac{\partial \Pi}{\partial d} = \frac{-1}{2} \frac{b^2 \text{tr} (a-c-dr+2trS)(9b+13tr^2)}{(2b+3tr^2)^2 (3b+2tr^2)^2}
\]

\[a-c-dr+2trS > 0 \Rightarrow \frac{\partial \Pi}{\partial d} < 0\]

\[
\frac{\partial \Pi}{\partial c} = \frac{-1}{2} \frac{b^2 \text{tr} (a-c-dr+2trS)(9b+13tr^2)}{(2b+3tr^2)^2 (3b+2tr^2)^2}
\]

\[a-c-dr+2trS > 0 \Rightarrow \frac{\partial \Pi}{\partial c} < 0\]

\[
\frac{\partial \Pi}{\partial a} = \frac{1}{4} \frac{b^2 \text{tr} (a-c-dr+2trS)(9b+13tr^2)}{(2b+3tr^2)^2 (3b+2tr^2)^2}
\]

\[a-c-dr+2trS > 0 \Rightarrow \frac{\partial \Pi}{\partial a} > 0\]

**Proof of Proposition 5**

Based on equation (7) the condition for a positive or null demand of permits is:

\[r(x_2-x_1)-(S_2-S_1) \geq 0\]  \hspace{1cm} (A13)

Plugging equation (35a) in (A13) yields

\[3br(a-c-dr)-(12b^2 + 6br^2t)S_2 + (12b^2 + 12br^2t)S_1 \geq 0\]  \hspace{1cm} (A14)

Solving (A14) in the equality case for $S_2$, and taking into account that $S = S_1 + S_2$, we obtain Equation (40), which proves the first part of the proposition.

Firm 2 is a net buyer of permits if

\[r(x_2-x_1)-(S_2-S_1) > 0\]  \hspace{1cm} (A15)

which is the case when both firms receive the same amount of free permits, because in this case firm 2’s output exceeds firm 1’s, as can be seen in Equation (35.a). When the
value of $S_2$ is above the threshold in Equation (40), the expression (A14) is strictly positive and this fact proves the second part of the proposition.

The last part of the proposition follows immediately when we consider

$$r(x_2 - x_1) - (S_2 - S_1) < 0 \quad (A16)$$

which is the condition for Firm 2 being a net seller of permits. In that case equation (A14) becomes

$$3br(a - c - dr) - (12b^2 + 6br^2)S_2 + br(t(12b^2 + 12br^2)S_1 < 0 \quad (A17)$$

and $S_2$ satisfies the required condition of being beyond the threshold.

**Proof of Proposition 6**

To prove the first part of the proposition we simply set equation (35.a) to zero and solve for $S_1$ to find the threshold value. Since the output functions are continuous and we have already proved that $x_2 > x_1$ at the symmetric allocation of permits, the second part of the proposition is trivially proved.

Now consider the function

$$\Pi = \Pi_1 - \Pi_2 = (P - c)(x_1 - x_2) - d(q_1 - q_2) - t(q_1^2 - q_2^2) - 2p(y_1 - y_2) \quad (A18)$$

At the threshold point, both outputs are equal $x_1 = x_2$ and Firm 1 is a seller of permits ($y_1 < 0$). The above equation can be reduced to

$$\Pi = \Pi_1 - \Pi_2 = -dy_1 - 2ty_1(q_1 + q_2) - 2py_1 > 0 \quad (A19)$$

Beyond the threshold point the first term on the right side of (A18) is positive and the rest of the terms of that expression is (A19). Both equations are positive and the proposition is proved.

**Proof of Proposition 2**
Based on Equations (26) and (28) we see that the difference between outputs is increasing in $S_1$ and decreasing in $S_2$.

$$
\frac{\partial (x_i - x_2)}{\partial S_1} = \frac{4rt}{(6b + 4r^2t)} > 0; \frac{\partial (x_i - x_2)}{\partial S_2} = \frac{2rt}{(6b + 4r^2t)} - \frac{2rt}{(4b + 4r^2t)} < 0
$$

We consider that the maximum number of free permits that can be allocated is covering total gross emissions. The minimum difference is obtained when all the free permits go to the follower, therefore $S_2 = r(x_1 + x_2)$. Plugging this result into equations (26) and (28), both firms output are given by

$$
x_i = \frac{3(a - c - dr) + 2r^2t(x_1 + x_2)}{6b + 4r^2t}; \quad x_2 = \frac{(a - c - dr) + 2r^2t(x_1 + x_2)}{4b + 4r^2t} \quad (A20)
$$

The solution for the above system is given by

$$
x_i = \frac{3b(a - c - dr) + 2r^2t(a - c - dr)}{b(6b + 5r^2t)}; \quad x_2 = \frac{3b(a - c - dr) + 4r^2t(a - c - dr)}{2b(6b + 5r^2t)} \quad (A21)
$$

And the difference between both outputs (in this limit case) shows that the leader is always producing more than the follower

$$
x_i - x_2 = \frac{3(a - c - dr)}{2(6b + 5r^2t)} > 0 \quad (A23)
$$

References


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