Economic growth, tourism and environmental policy

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Abstract

In this paper a dynamic model of economic growth and tourism is studied. The model considers tourists in the economy as optimizing agents, interacting with domestic consumers. The number of tourists arriving in the country follows the Tourism Are Life-cycle model with a tourism carrying capacity dependent on environmental quality. Sustainable tourism implies controls on inflows and also pollution abatement policies to keep the economy growing. The paper studies the effects of tourism taxes on growth and welfare.

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1 Introduction

Tourism has been regarded as an important source of economic growth and wealth for many countries. However, tourism promotion usually relies on the destination original attractions such as beaches, scenery and culture (Butler, 2009), which depreciate as tourism grows. During the sixties and seventies tourism businesses grew at an uneven pace in many European cities and in some early resorts. Over time, however, both politicians and citizens realized the dangers from unending growth and a new concept has emerged, sustainable tourism, which focuses on the environmental, social and economic aspects in this sector.

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Sustainable tourism implies not only controls on inflows (Inskeep, 1994) but also investment in tourist attractiveness (Albaladejo and Martínez-García, 2017). Over the last 15 years there has been a general increase in the number and scope of tourism-related taxes, fees and charges (OCED, 2014). These tourism taxes can be divided in two main categories: general and specific taxes (Durbarry and Sinclair, 2001). In the first group we can find taxes that fall on both tourists and residents, as the VAT, while in the second group there are others as visas, taxes associated with air travel and taxes on hotel stays that affect tourist activities.

Specific taxes on hotel stays are being implemented in many countries, to the national or regional level. Many popular destinations, as New York, Paris, Berlin, Rome, Amsterdam or Barcelona have taxes of these type, and others, as London, are considering its introduction (London Finance Commission, 2017).

The establishment of these type of taxes is always under debate. There are pros and cons to use them. The hotel sector usually has an unanimous position against it considering that a tax, even small, can discourage guests from staying overnight and can reduce the spending in other services. It also consider that it reduces the competitiveness of the destination against others, and move people from hotels to other places as virtual renting platforms that offer accommodation that it is not under the levy. On the other side, these taxes revenues are usually used for promotion and improvement of the touristic sector,
and the development of facilities connected to tourism. They can be seen, in that sense, as a mean to compensate the impact of tourism on the environment and public services of the area to promote a sustainable model of the touristic sector.¹

In this paper, the controversy about the taxes is developed in a formal framework by linking the effects of fiscal policy on tourism development and growth. It may contribute to the debate clarifying the possible effects of different policies. The main questions addressed are: for an economy based on tourism exploitation, under what conditions are sustained economic growth and the preservation of environmental quality compatible? In such case, which are the taxes necessary to reach this outcome?

An important feature of tourism is that the bundle of goods and services purchased by a tourist is consumed jointly with unpriced natural amenities, such as climate and scenery. Several authors have studied the connection between environment and tourism. Cerina (2007) focuses on tourism based on natural resources. In this model tourists are crowding-averse and gain utility from environmental quality. Assuming that the tourists are pollution-adverse, Giannoni (2008) analyses long-term growth and sustainability of tourism. Preservation of the environment can be understood as a kind of public good. Rigall-i-Torrent (2008) concludes that correct supply of public goods may contribute to sustainable development overtime in tourism economies. Some authors like Gómez et al. (2008), Lozano et al (2008) and Rey-Maquieira et al (2009) study a small open economy specializing in tourism where the tourism package is a bundle of attributes provided by firms, the government and the natural environment.

Our model belongs to the tourism-led growth models literature developed by Hazari and Sgro (1995, 2004(ch.12)), Chao et al (2005), Nowak et al (2007) and Schubert and Brida (2011), among others. However, contrary to the mainstream, in our paper tourist arrivals are viewed as an endogenous process which depends on environmental quality. Several studies in micro-economic analysis lead to the belief that the ability of a country to attract tourists depends on its characteristics as a tourist destination (facilities, quality of hotels and services, environmental endowment and quality, cultural heritage, political stability, among others). Some of these elements, despite of being given, can be maintained (environmental quality) and can be improved through investment. Therefore, the saving/investment behavior of domestic agents affects the tourist attraction power of the economy and hence the long-term economic growth rate. Albaladejo and Martínez-García (2013) assume that tourists arrive at a rate that depends on the quality of the tourist services, which can be

¹Just to give an idea of the amount of money related to these taxes, some numbers about them. During the two years that eco-tax was in Baleares 84 millions euros were raised and they were used to conservation of touristic and natural places in Baleares. Since November 2012, the revenues in Catalonia has been 82 million euros. Among the different estimations considered by GLA Economics (2017), it appears that, applied to London, systems similar to the one in Paris (based on hotel star rating), to the one in Berlin (5% of the room rate per night) or the one in Barcelona (mainly four and five star hotels and a nominal rate for others) could generate £140, £240 or £77 million, that could be used to support the sector in various ways.
endogenously improved by the country. Albaladejo et al (2014) also propose that tourism arrivals and the quality of tourism services can be endogenously enhanced. Nevertheless, these works do not take into account the environment and the Tourist Area Life Cycle (TALC) by Butler (1980). In this paper the TALC hypothesis is at the core of our analysis and policy measures prevent the rate of tourism arrivals from declining.

The following section presents a TALC model in which the tourism carrying capacity depends on environmental quality. Tourism generates pollution that hurts environment but the economy can devote resources to abatement. Environmental investment will be the motor of growth for the carrying capacity. In the subsequent, third section, tourists’ preferences are modelled and the demand function for tourist services is obtained. Both the tourist services demand function and the flow of visitors arriving according to the TALC model are incorporated into an endogenous growth model in the fourth section. The host economy shares some of the elements of the seminal model by Smulders and Gradus (1996), which studies the conditions for sustained economic growth preserving the environmental quality if pollution is a by-product of production which can be reduced by abatement activities. Section five studies the optimal balanced growth path for our tourism based economy. Section six describes the market economy and discusses the effects of different taxes to finance abatement activities. Finally, the seventh section concludes.

2 Tourism carrying capacity

The TALC model (Butler, 1980) is one of the most accepted models in the tourism literature. Since 1980 many authors have tested the validity and usefulness of the TALC model in a variety of destinations (Lagiewski, 2006) and a considerable number of studies have found that the TALC model is well suited to describing the process of development of tourist destinations (Butler, 2009 and 2011). This process has six key phases: exploration, involvement, development, consolidation, stagnation and a post-stagnation stage when the destination has two options – decline or rejuvenation (Lundtorp and Wanhill, 2001; Butler, 2009, 2011; Cole, 2009; Butler, 2009 and 2011). Butler presented this life cycle as an S-shaped curve representing the arrivals of tourists until stagnation stage. The upper limit of the S-shaped curve is achieved as the level of the carrying capacity of the destination is reached. The implicit assumption in previous models is that the market equilibrium positions we observe do trace out the path of demand, meaning that supply is flexible enough to adjust to demand.

Lundtorp and Wanhill (2001, 2006) showed that the TALC model might be satisfactorily approximated assuming that the number of tourists arriving in the country, follows the pattern of a logistic growth model, that is,

\[ N_t = \frac{K}{1 + e^{-r(t-t_0)}} \]

We omit time subscripts in Eq. (1) and in the subsequent analysis whenever no ambiguity results.
\[ \dot{T} = \sigma T \left( 1 - \frac{T}{CC} \right) \]  

(1)

where \( \sigma \), assumed to be positive, is the intrinsic rate of tourism growth and \( CC \) is a measure of the carrying capacity in the destination. \( \dot{T} \) denotes the time derivative of variable \( T \).

There are many definitions of carrying capacity in the context of tourism. In its most traditional sense, it is understood as the maximum number of tourists or tourist uses that can be accommodated within a specific geographical destination (O’Reilly, 1986). This capacity has been identified in terms of limits of environmental, social, economic or physical factors (Butler, 1980; Saveriades, 2000; Cole, 2009; Diedrich and García-Buades, 2009). The environmental factor has been widely used in tourism studies owing to concern over the negative impacts of tourism. Recently, Lozano et al (2008), using an environmental growth model for an economy specializing in tourism, show that its evolution does not contradict the evolution derived by the TALC model. Albaladejo and Martínez-García (2017) prove that a multi-logistic growth model fixes better the evolution of a mature touristic destination:

![Figure 2: Passengers to Bornholm (Albaladejo and Martínez-García, 2018).](image)

Although the micro-foundations of the life-cycle predicted by TALC model are demand-oriented (Plog, 1974), appropriate policy measures may not only sustain tourism flows over time but may also rejuvenate resorts, initiating a new life cycle (Papatheodorou, 2004). Environmental quality, the goods and services offered by the destination (accommodation, transport, shopping, attractions, events) and the type of consumption of the tourists can be the responsible factors for increasing /decreasing the maximum number of tourists that could be accommodated by a destination.
The higher environmental quality and the wider and more varied number of services and attractions the more number of tourists can be received simultaneously in the same geographical space and they can enjoy several goods and services simultaneously. However, the higher per capita tourism consumption (longer stays, higher demand in restaurants, for example) the fewer the number of tourists that can be attended. In this paper, the tourism carrying capacity is considered as an increasing function of the environmental quality and tourism services supply, and decreasing function of the per capita tourism consumption. For simplicity, we consider the carrying capacity as the following function

\[ CC = \frac{Y E}{c_T}. \]  

(2)

where \( c_T \) is per capita tourism consumption and E the environmental quality.

According to the idea of Smulders and Gradus (1996), we interpret that the environmental quality is directly and negatively related to the level of tourist pollution. So,

\[ E = P^{-1}. \]

Furthermore, as in Cerina (2007) and Lozano et al. (2008), we consider that tourism activity has damaging effects on the environment but, as Marsiglio (2015) propose, the tourism economy may undertake abatement activities to neutralize the negative effects of the tourists. Given these ideas we assume that tourist pollution, \( P \), depends positively on tourism consumption, \( c_T T \), and negatively on the abatement activities, \( A \), according to the following function

\[ P = \frac{c_T T}{A}. \]

Traditionally, tourism carrying capacity has been considered as a rigid and static value. However, as several authors argue (Saveriades, 2000; Cole, 2011), it can evolve with time due to changes in economic or environmental conditions, as we have assumed in (2).

Note that equation (1) follows the logistic growth pattern (sinusoidal shape) only if the carrying capacity (\( CC \)) is a fixed point. Conversely, if \( CC \) grows, as we propose in this paper, the number of tourists could be continuously increasing.

### 3 The behavior of tourists

As understood by Eugenio-Martín (2003), tourists face a multi-stage decision problem in which the decisions about the destination and budget are taken at different stages. Following this idea, we shall start by assuming that the decision on the destination has somehow been taken previously. Once this decision has been taken and our country is the chosen destination, tourists must decide the budget for the vacation, denoted by variable \( b \), which will somehow determine their tourism consumption.
As it is proved in Albaladejo and Martínez-García (2015), tourists’ international preferences can be represented by a utility function displaying a constant elasticity of substitution (CES), that is

\[
U(c_T, c_T^d) = \left[ \xi (c_T^{-\delta} + (1 - \xi) (c_T^d)^{-\delta}) \right] ^{-1/\delta}
\] (3)

where \(0 < \xi < 1\), \(\delta > -1\), \(c_T\) is consumption of tourism services and \(c_T^d\) is domestic consumption (in their origin country).

A representative tourist must solve the following static optimization problem

\[
\max_{c_T, c_T^d} U(c_T, c_T^d)
\]

s.t. \(p_T(1 + \tau_T) c_T = b\) \hspace{1cm} (5)

\(p_T^d c_T^d = R - b\) \hspace{1cm} (6)

where \(R\) is the tourist’s annual income, \(b\) is the budget for tourism, \(p_T^d\) is the price of domestic consumption goods and \(\tau_T\) is a tax affecting tourists activities imposed by the host country. As in Gago et al. (2009), we have introduced a specific tourism tax, \(\tau_T\), that finances projects aimed at improving the quality of tourist activities and preserving the environment.

Solving (4)-(6), the relative demand as a function of the prices can be obtained, that is,

\[
c_T = \left( \frac{1 - \xi}{\xi} \right)^{-\beta} p^{-\beta} c_T^d, \hspace{1cm} (7)
\]

where \(\beta = 1/(1 + \delta)\) is the elasticity of substitution and \(p\) is the ratio of prices \(p_T(1 + \tau_T)/p_T^d\).

Taking into account (5) and (6), a relation among the terms of trade, \(p\), and the marginal propensity to tourism consumption \(t = b/R\) can be obtained. That is

\[
p^{1-\beta} = \left( \frac{1 - \xi}{\xi} \right)^{\beta} \frac{t}{1 - t}, \hspace{1cm} (8)
\]

being \(t\) a parameter which usually depends on the idiosyncrasy of the visitors. If \(\beta\) is lower than 1, this is a positive relation: If the tourist agrees to pay a higher price it would be since she has a higher marginal propensity to tourism consumption.

We have started this section taken from granted that the decision of visiting our country had been taken previously, and our country was the chosen destination. But, who are those tourists who had decided to come to our country?, where are they from?. As it is proved in Albaladejo and Martínez-García (2015), tourists visiting our country are those from countries with a marginal propensity to tourism consumption \(t\) satisfying equation (8). Increments in the terms of trade \(p\) could determine tourists origin countries.
4 The host economy

We model an economy that produces a composite good that, for the sake of simplicity, is used either as tourism services and for domestic consumption and investment. Production of this good requires domestic and foreign capital. Foreign capital is a necessary input for production and it is imported in exchange of tourism services. Hence, it must be imported by trading domestic production with nonresidents, that is, tourists. Additionally, tourism deteriorates environmental quality, which has a negative impact on consumers welfare.

a) Firms

Final output production is described by a Cobb-Douglas function with constant returns to scale and it can be written as

$$Y = BK^\alpha X^{1-\alpha}, \quad 0 < \alpha < 1, \quad B > 0$$

(9)

where $K$ is domestic capital and $X$ is an intermediate input produced with imported foreign capital $K_f$. Domestic capital should be interpreted as a broad measure of knowledge, technology, physical and human capital. Foreign capital can be used either for abatement activities $A_T$ or as intermediate input, that is

$$K_f = X + A_T.$$ 

Since final-goods production is constant returns to scale, without loss we can consider a single price-taking firm when solving for the competitive outcome. When the price of output is normalized to unity in every period, profit maximization is given by

$$\max (1 - \tau_Y) Y - rK - p_X X$$

where $r$ is the net rate of return to households that own domestic capital and $p_X$ is the price paid for intermediate input. Note that, perfect competition and free-entry condition in the intermediate good sector implies that the price paid for one unit of $X$ equals the price of foreign capital $1/p$, if a one-to-one production function is assumed. We have introduced a value added tax, $\tau_Y$, in order to discuss policy measures. The tax $\tau_Y$ makes firms internalize the negative impact of tourism on welfare.

Assuming perfect competition, instantaneous profit maximization leads to the following demand functions for inputs:

$$\alpha (1 - \tau_Y) \frac{Y}{K} = r$$

(10)

$$(1 - \alpha) (1 - \tau_Y) \frac{Y}{X} = p_X = 1/p.$$ 

(11)

b) Households

Households own financial assets $a$ in the form of ownership claims on domestic capital. Assets deliver a rate of return $r$ and households use the capital
income that they do not consume to accumulate more capital. The households’ welfare depends not only on per capita consumption of material goods but negatively on tourist pollution, \( P \), that is
\[
W = \int_0^\infty U(c, P)e^{-\rho t} dt
\] (12)

where
\[
U(c, P) = \begin{cases} \frac{(c^{-\theta})^{1-\theta} - 1}{1-\theta} & \text{for } \theta \neq 1 \\ \ln c - \phi \ln P & \text{for } \theta = 1 \end{cases}
\]

Per capita consumption is denoted by \( c \). By abstracting from population growth and normalizing population size to one, it equals aggregate consumption \( c = C \). The positive parameter \( \theta \) is the inverse of the elasticity of intertemporal substitution. The higher \( \theta \), the more the consumers prefer a uniform, smooth consumption path. For \( \theta = 1 \) a logarithmic utility function is considered. A higher \( \phi > 0 \) reflects a higher concern for the quality of the environment. The parameter \( \rho > 0 \) is the rate of time preference.

Therefore, domestic household decides upon per capita consumption, \( c \), in order to maximize overall utility (12), taken the tourist pollution \( P \) as given. Optimality conditions for an interior solution leads to the usual Ramsey rule which drives the dynamics of per capita consumption.\(^3\)

\[
g_c \equiv \frac{\dot{c}}{c} = \frac{1}{\theta} (r - \rho) = \frac{1}{\theta} \left( \alpha (1 - \tau_Y) \frac{Y}{K} - \rho \right)
\] (13)

c) Government

Tourism generates pollution and abatement is necessary in order to improve environmental quality and the touristic appeal, as it is stated in (1). Government taxes firms, at a rate \( \tau_Y \), and tourists, \( \tau_T \), in order to finance abatement activities, \( A \), to regenerate the environment. That is
\[
A = A_Y + \frac{A_T}{p}
\] (14)
where \( A_Y = \tau_Y Y \) and \( A_T = p\tau_T c_T T \) is the abatement directly related to the tourism tax. We assume that tourists are identical and that each tourist consumes the same quantity of tourism services \( c_T \).

5 Market equilibrium

We focus on an equilibrium solution where international trade is balanced. Our model represents an open economy which imports foreign capital \( K_f \) in exchange of tourism services, that is

\(^3\)Here, and henceforth, \( g_x \) denotes the growth rate of variable \( x \).
The market equilibrium is given by equations (1), (7), (10), (11), (13), (14) and (15), taking as given the terms of trade according to equation (8).

Note that from (7), (11) and (15) the terms of trade are given by

\[ p = \frac{\xi}{(1 - \xi)} \left( \frac{1 - \tau_T}{(1 - \alpha)(1 - \tau_Y)} \right)^{1/\beta} \left( \frac{c_T^d T}{Y} \right)^{1/\beta} \]

which is time varying according to the equation

\[ \frac{\dot{p}}{p} = \frac{1}{\beta} (g_{\text{world}} + g_T - g) \]

where \( g_{\text{world}} \equiv c_T^d / c_T^d \) is the growth rate of the foreign economy, \( g = \dot{Y} / Y \) is the growth rate of the host economy and \( g_T \) the growth rate of the number of tourists.

Defining abatement, input and tourism expenditure shares as \( s_A = A / Y \), \( s_X = X / pY \) and \( s_T = c_T T / Y \), from (11), (14) and (15), tourism expenditure, input and abatement shares are constant and dependent on taxes

\[ s_X = (1 - \alpha)(1 - \tau_Y) \]

\[ s_T = \frac{s_X}{(1 - \tau_T)} = \frac{(1 - \alpha)(1 - \tau_Y)}{(1 - \tau_T)} \]

\[ s_A = \tau_Y + \tau_T s_T = \tau_Y + \tau_T \frac{(1 - \alpha)(1 - \tau_Y)}{(1 - \tau_T)} \]

Together with \( s_A, s_X \) and \( s_T \), the terms of trade \( p \), consumption share \( s_c = C / Y \) and the ratio \( Y / K \) remain constant in a balanced growth path. Therefore, \( C, A, c_T T, K, X \) and \( Y \) grow at the same rate \( g \). Note that

\[ \frac{Y}{K} = B \left( \frac{X}{K} \right)^{1-\alpha} = B \left[ ps_X \frac{Y}{K} \right]^{1-\alpha} \Rightarrow \frac{Y}{K} = B^{1/\alpha} (ps_X)^{(1-\alpha)/\alpha} = y(ps_X). \]

The following proposition proves the existence of a balanced growth path.

**Proposition 1** There exists a balanced growth path for the market economy with a positive long-run growth rate for the economy \( g \) if and only if \( \tau_T > 0 \) or \( \tau_Y > 0 \).

**Proof.** See the Appendix. \( \blacksquare \)

Then, the long run growth rate in the market economy is given by

\[ g^* = g_{\text{world}} + \sigma \left\{ \frac{(1 - \alpha)(1 - \tau_Y)^2}{1 - \tau_T \tau_Y (1 - \tau_T) + (1 - \alpha) \tau_T (1 - \tau_Y)} \right\} \]

Moreover, the balanced growth path is saddle path stable, as it is also proved in the Appendix.
In a pure market economy with $\tau_Y = \tau_T = 0$, the spending ratio on abatement, $s_A$, is zero. The market economy ignores the effect of tourism congestion on consumers’ utility. In this case the government cannot afford abatement activities and the tourism carrying capacity will decrease. Once tourism occupation reaches the carrying capacity, the number of visitors will not grow and stagnation and start to decline. Tourism/environmental policy may be aimed at moving the economy from an unsustainable towards a sustainable growth path.

5.1 The effects of taxation on the long-run growth rate and the terms of trade

On the following figures we have represented the households desired growth rate for the economy (blue line), given by the Ramsey rule (13), together with the growth rate of tourism expenditure (red line). At the equilibrium both lines intersect.

We can distinguish between two cases. The first case is the one with a small $\tau_Y$. If this is the case, the introduction of a new touristic tax $\tau_T > 0$ will reduce congestion ($T/CC$) and increase the tourism expenditure growth rate. At the equilibrium the terms of trade, the long run growth rate and household welfare are higher.

Figure 3: The effect of introducing a touristic tax on the long-run growth rate

If $\tau_Y$ is high, congestion increases with the introduction of a tourists tax. The effects on the terms of trade, long run growth and welfare are negative, as they are illustrated in the following figure.

Figure 4: The effect of introducing a touristic tax on the market long-run growth rate
6 Optimal taxes

To obtain the balanced optimal growth, the social planner seeks to maximize the welfare (12) by the choice of abatement (A), consumption (C) and intermediate input (X) subject to production function (9), the inflow of intermediate goods (15), tourism evolution (1) and the economic constraint given by the equation

\[ \dot{K} = Y - C - AY - cT \]  

where the foreign capital that tourists bring, \( \frac{K_f}{\tau_T} \), and the consumed tourism services, \( cT \), are in equilibrium according to equation (15). The solution of the social planner problem implies the following result

**Proposition 2** If \( \theta \geq 1 \), a value added tax \( \tau_Y > 0 \) is not sufficient to align the optimal and market balanced growth paths, a specific tourism tax \( \tau_T > 0 \) is necessary for the market economy reaches the social optimum.

**Proof.** See the Appendix.

7 Concluding remarks

In this paper a dynamic model of economic growth, tourism and environment is studied. The model considers tourists in the economy as optimizing agents, interacting with domestic consumers. The number of tourists arriving in the country follows the Tourism Are Life-cycle model with a tourism carrying capacity dependent on environmental quality. Tourism hurts environment and abatement activities have to be carried out in order to keep the economy growing. A social planner determines the optimal levels for the long-run growth rate, abatement share and tourism congestion.

In a pure market economy growth is unsustainable. Producers ignore the effects of production on tourism congestion and its effect on consumers’ utility. Equally, tourists don’t internalize their effect on domestic consumers utility. In this case government cannot afford abatement activities. As a consequence, the tourism carrying capacity cannot grow, it remains fixed. It is only dependent on the natural amenities and, since no regenerative or abatement activities are carried out, tourism growth will stop, once tourism occupation reaches the fixed carrying capacity. Consequently, foreign capital inflows stop growing and the decreasing returns on domestic capital drive the economy to the stagnation.

Tourism/environmental policy moves the economy from the unsustainable outcome to the optimal long-run growth. Taxing firms revenues and tourist consumption would move the market economy to the social optimum. Nevertheless, a tourism tax to finance abatement has two counteracting effects. Abatement increases the tourism carrying capacity which has a positive impact on long-run growth rate. However, since an increase in the tourism tax is understood by tourists as an increase in the price, they would reduce their demand of tourism services, which harms growth. We reach to the conclusion that there
exists a trade-off between the two taxes: lower value added tax would require a higher specific tourism tax, which affects visitors and reduce our market share. Nevertheless, it would imply a higher growth rate of the host economy.

References


8 Appendix

Proof of Proposition 1: Taking into account the asset temporal evolution and (10) and (11), it is obtained that aggregate capital evolves according to

\[ \dot{K} = (1 - \tau Y)Y - C \]

The consumption share \( s_c = C/Y \), the ratio \( Y/K = y(ps_X) \) evolve according to the following equations

\[
\begin{align*}
\frac{\dot{Y}}{Y} &= \alpha \frac{\dot{K}}{K} + (1 - \alpha) \left( \frac{\dot{p}}{p} + \frac{\dot{c}_p}{c_p} + \frac{\dot{T}}{T} \right) \\
&= \alpha \left( (1 - \tau Y) - s_c \right) y(ps_X) + (1 - \alpha) \left[ (1 - \beta) \frac{\dot{p}}{p} + g_{world} + \sigma \left( 1 - \frac{s^2_c}{s_A} \right) \right]
\end{align*}
\]
That is
\[
\frac{\dot{Y}}{\dot{Y}} = \frac{\alpha \beta}{1 - \alpha + \beta \alpha} [(1 - \tau_Y) - s_c] y(p s X) + \frac{1 - \alpha}{1 - \alpha + \beta \alpha} \left[ g_{world} + \sigma \left( 1 - \frac{s_c^2}{s_A} \right) \right]
\]

Then,
\[
\frac{\dot{s}_c}{s_c} = \frac{\dot{C}}{C} \frac{\dot{Y}}{Y} = \frac{1}{\alpha} \left( (1 - \tau_Y) y(p s X) - \rho \right)
\]

\[
= \left\{ \frac{\alpha \beta}{1 - \alpha + \beta \alpha} [(1 - \tau_Y) - s_c] y(p s X) + \frac{1 - \alpha}{1 - \alpha + \beta \alpha} \left[ g_{world} + \sigma \left( 1 - \frac{s_c^2}{s_A} \right) \right] \right\}
\]

\[
\frac{\dot{p}}{p} = -\frac{\alpha}{1 - \alpha + \beta \alpha} [(1 - \tau_Y) - s_c] y(p s X) + \frac{\alpha}{1 - \alpha + \beta \alpha} \left[ g_{world} + \sigma \left( 1 - \frac{s_c^2}{s_A} \right) \right]
\]

Equalizing these equations to zero we obtain the values
\[
s_c^* = (1 - \tau_Y) - (y^*)^{-1} \left\{ g_{world} + \sigma \left[ 1 - \frac{(1 - \alpha)(1 - \tau_Y)^2}{\tau_Y + \tau T (1 - \alpha)(1 - \tau_Y)} \right] \right\}
\]

\[
p^* = \frac{B^{-1/(\alpha - 1)}}{(1 - \alpha)(1 - \tau_Y)} (y^*)^{\alpha/(1 - \alpha)}
\]

with \( y^* \) given by
\[
y^* = \left( \frac{Y}{K} \right)^* = \frac{\theta \left\{ g_{world} + \sigma \left[ 1 - \frac{(1 - \alpha)(1 - \tau_Y)^2}{\tau_Y + \tau T (1 - \alpha)(1 - \tau_Y)} \right] \right\}}{\alpha(1 - \tau_Y)}
\]

**Proof of the saddle path stability:** The Jacobian matrix evaluated at the steady state is
\[
J^* = \begin{pmatrix}
\frac{\alpha \beta}{1 - \alpha + \beta \alpha} s_c^* y^* & \frac{\alpha \beta}{1 - \alpha + \beta \alpha} [(1 - \tau_Y) - s_c^*] \frac{\partial p}{\partial p} - \sigma s_c^* \frac{1 - \alpha}{1 - \alpha + \beta \alpha} \frac{\partial p}{\partial p} \\
p^* \frac{\beta}{1 - \alpha + \beta \alpha} y^* & -p^* \frac{\beta}{1 - \alpha + \beta \alpha} [(1 - \tau_Y) - s_c^*] \frac{\partial y}{\partial p} + \sigma p^* \frac{1 - \alpha}{1 - \alpha + \beta \alpha} \frac{\partial p}{\partial p} \end{pmatrix}
\]

Since
\[
\left| \begin{array}{cc}
\frac{\alpha \beta}{1 - \alpha + \beta \alpha} s_c^* y^* & \frac{\alpha \beta}{1 - \alpha + \beta \alpha} [(1 - \tau_Y) - s_c^*] \frac{\partial p}{\partial p} - \sigma s_c^* \frac{1 - \alpha}{1 - \alpha + \beta \alpha} \frac{\partial p}{\partial p} \\
p^* \frac{\beta}{1 - \alpha + \beta \alpha} y^* & -p^* \frac{\beta}{1 - \alpha + \beta \alpha} [(1 - \tau_Y) - s_c^*] \frac{\partial y}{\partial p} + \sigma p^* \frac{1 - \alpha}{1 - \alpha + \beta \alpha} \frac{\partial p}{\partial p} \end{array} \right| < 0
\]
one of the eigenvalues is negative, which gives the saddle path stability.

**The social planner problem and optimal taxes:** The static first order condition for the social planner problem is
\[
\frac{s_X}{1 - \alpha} = 1 + s_A - \phi s_c
\]
while the dynamic first order conditions are

\[
g = \frac{\dot{c}}{c} = \frac{1}{\theta} \left( \frac{\alpha}{1 - \alpha} B^{1/\alpha} p^{(1-\alpha)/\alpha} s_X^{1/\alpha} - \rho \right) \tag{24}
\]

\[
g_{\text{world}} + \frac{\dot{T}}{T} = \frac{\sigma s_T^2}{s_A} \left( \frac{2s_A - \phi s_c}{s_A - \phi s_c} \right) + \frac{\alpha}{1 - \alpha} B^{1/\alpha} p^{(1-\alpha)/\alpha} s_X^{1/\alpha} \tag{25}
\]

while the growth rate for the number of tourists and capital are

\[
\frac{\dot{T}}{T} = \sigma \left( 1 - \frac{s_T^2}{s_A} \right) \tag{26}
\]

\[
\frac{\dot{K}}{K} = (1 - s_c - s_A - s_X) B^{1/\alpha} p^{(1-\alpha)/\alpha} s_X^{1/\alpha} \tag{27}
\]

Taking into account (23), (24) and (25) we obtain

\[
g = \frac{\sigma}{\theta - 1} \frac{s_T^2}{s_A} \left( \frac{(1 - \alpha) s_A}{(1 - \alpha) - s_X} - 1 \right) - \frac{\rho}{\theta - 1} \tag{28}
\]

From (26) we have that

\[
g = g_{\text{world}} + \sigma \left( 1 - \frac{s_T^2}{s_A} \right) \tag{29}
\]

At the optimal solution both curves (28) and (29) intersect. For the market solution to reach the optimum, expressions (17)-(19) should allow the intersection of (28) and (29). Nevertheless, it can be easily proved that if \( \tau_Y = 0 \) then

\[
\frac{(1 - \alpha) s_A}{(1 - \alpha) - s_X} - 1 = 0
\]

Which means that (28) doesn’t intersect (29) if \( \theta \geq 1 \).