Swap Bonds or Stocks, or Balance Trade! A Game of Implicit Environmental Policy

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Abstract

This paper develops a mechanism to correct production externalities between several parties, such as externalities motivating environmental policy between countries, using asset ownership. Efficiency can be obtained if each party retains less than the full share of their own gain from resource use materialized in gross product. The remainder of the product can be earned by other parties—in a reciprocal way. The resulting swaps can be enforced by using bonds or stocks or with balanced trade of shares, hindering free-riding. The best international climate policy thus may be swap contracts. (JEL classification numbers D62, H23, Q58)

Keywords: Free-rider behavior, Privately provided public good, Lindahl prices, Property rights, Climate agreement, General equilibrium.

1. Introduction

An efficient resource allocation requires the correction of large-scale externalities. For example, one immensely important problem are greenhouse gas (GHG) emissions from fossil fuel use and the release of carbon into the atmosphere through deforestation, which, with mounting evidence, alter the

climate. A policy framework, in short a mechanism, that controls the social costs of climate change at the global level would limit adverse impacts of climate change. Any convincing framework correcting the external effects of resource use between countries leading to the social costs needs to comprise strong economic incentives.

Identifying a framework with strong incentives is challenging, given that not all actions can be contracted upon, or all the way actions must be self-enforcing. The Paris Agreement 2015 of the United Nations’ Framework Conference on Climate Change (FCCC) has led to voluntary pledges and no binding targets for emissions by countries. That with such pledges, some countries ride free on other countries’ emissions reductions conforms to the outcomes of coalition formation among multiple countries if, defining a first viewpoint on climate policy, each country benefits from global emissions reductions (Barrett 2003, Finus 2001). With simultaneous moves on emissions, the best responses predict a Pareto nonoptimal allocation. In a second viewpoint, a coalition comprises nearly all countries majorly affected by climate quality so that other parties are largely unaffected by it (Harstad 2012a), implying leakage in the coalition’s policy. In a third viewpoint, some parties for some time benefit from emissions of other parties.

This paper provides a mechanism to achieve efficiency with externalities using verifiable and enforceable policies in the absence of a regulator, thus suiting environmental policy between countries or states, embracing all three viewpoints on climate policy.

1.1. Appraisal of swaps and structure of derivations

Classic solutions to the problem of implementing an efficient allocation under an externality from one party on the welfare of another party do not work. One, regulating by a global tax, traced to Pigou (1920), does not work as no institution prevails that could enforce taxes. By contrast, in this paper, by means of swaps, the parties use resources efficiently, because they antici-

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1 The Conference of the Parties of the FCCC has met annually since 1995 and still not found a climate agreement that fulfills their goal of limiting global warming to 2 degrees above preindustrial levels.

2 The framework can be interpreted with adaptation to climate change, for example, imposing cost of roof-repair or migration, and mitigation of climate change, imposing cost of changing production in given geographic regions. The language in the paper focuses on changing production.
ipate a tax (for an external cost such as environmental harm) implicit in a swap. Two, creating Lindahl markets, as studied by Arrow (1970), fails as no institution prevails that could enforce appropriate exchanges on markets. Swaps resolve this problem either because they can be used to structure further claims (swap bonds or stocks) or can balance out claims between parties (balanced trade). Three, that parties find their way to efficiency through negotiation with no further institution, associated with Coase (1960), cannot be argued as property rights to a public good such as climate quality do not exist. Swaps create property rights that shape incentives for efficient resource use.\(^3\) Four, a supranational institution that could enforce parties’ contributions to a public good such as country-level emissions does not exist. Swaps solve this problem, again, by contracting.

The nature of swaps. The framework in this paper prevents free-rider behavior and carbon leakage by countries’ forward trade in claims to gross product before countries set policies on trade of products. Efficiency can be attained when a party such as a country or state, that imposes an external cost onto another party, diverts some of its own gain from doing so. An amount diverted by some party is owned by another party. The resulting swaps are determined in bilateral contracts on a “swap market.” In the analysis, production economies are represented by parties containing at least one party that prefers a public good framing the externalities. Swaps developed from gross product, defined as the market value of final goods and intermediate externality-creating goods, make implicit policy by disentangling policy and public good provisions. If instead parties trade products only, then inefficiency results, as a party imposing an external cost onto, or harming, other parties, contributes to a public good such as environmental quality less than optimally.

Equilibrium, verification, and enforcement of swaps. The question arises how swaps can be formed. First I assume that parties can sign binding contracts about swaps, hence cannot renegotiate swaps. Contracts are thus enforceable with no restrictions. Then I put restrictions on contracts, and lastly assume that swap contracts are nonbinding, and observable, as parties can renegotiate the contracts. I adopt a simultaneous move structure both in contracting and taking actions influenced by the contracts.\(^4\)

\(^3\) Alchian (1965, p.817) defines property rights as ‘the rights of individuals to the use of resources’. This is the definition of property rights I use in this paper.

\(^4\) With sequential moves of public good provisions, an allocation can be efficient, for
I investigate how linked markets for permits to use resources can free swaps of side payments. This appears important when first transfers as side payments pose a challenge to verify swaps, and second transfers in general have a role for efficiency and existence of equilibrium. This role of transfers arises when the parties' tastes for the public good differ much and parties block allocations that do not Pareto improve. The parties block such allocations using one of the two equilibrium concepts.

I analyze how swaps developed in the contracting environment can be enforced. One, the claims in swaps can be enforced by paying them out as returns on bonds or stocks that are enforced. Governments issue bonds or stocks to private agents, and thus swap bonds or stocks. Two, in renegotiation-proof contracts, each one party and its swap trading partners pay out their swaps, or each two parties pay out their swap, in equal magnitudes. This uses nonnegative claims in swaps, expressing the lack of international lump-sum taxes. Renegotiation-proof swaps mean to balance trade, and because being self-enforcing, can be said to be agreed upon.

I first study an equilibrium in which, to change a given swap, the parties involved in the swap require the consent of other parties. This rule of consent prevents a no-contract equilibrium from prevailing, and guarantees that an equilibrium with efficiency exists. I then analyze an equilibrium with a protocol that parties use to propose and accept—unanimously approve—contracts, leading to both unanimous approval to create and change given swaps. In this case, no party blocks a Pareto improvement. This mirrors the notion that a coalition forms with unanimous approval by its members (Ray & Vohra 2001), yet contrasts the notion of an environmental agreement as a coalitional structure in the literature.5

Implications for environmental policy and relationship to the literature.—The current analysis of externality correction between parties generates broad insights for environmental policy and offers simplicity in the practical implementation of climate policy.

To attain efficiency with externalities between parties, (i) coalitions do not need to form, and (ii) a supra-party regulator can be absent. These two

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5 The Parties to the FCCC reached the Paris Agreement 2015 unanimously (New York Times 2015), though with no rule about unanimous decisions. Other policy spheres have adopted unanimity voting, one prominent example being the decision of member countries in the European Union on their multi-annual budget.
points are general. Coalitions do not need to form because of the rule of consent. This appears important for countries negotiating practices relevant to reducing waste plastic in oceans, sustaining biodiversity, or limiting climate change. A principal or regulator is not essential to attain efficiency with externalities between parties following from the enforcement of swaps, such as with balanced trade under no commitment to contracts. With no principal or regulator, efficiency can be attained through renegotiation-proof contracts among the parties.

(iii) Generally, externality-correcting swaps may include claims to gross product and state revenue based on Pigou externality pricing (see Section 4). To correct climate change externalities with no transfers, the exclusive use of claims to gross product in swaps appears sufficient, embraces both market and nonmarket economies, and reduces monitoring effort—to create and verify swaps with no transfers, already existing product accounts can be used, yet no new registry for emissions needs to be set up. To verify swaps with transfers, though, tradeable emissions permits, which require monitoring emissions, can be used.

(iv) Since its inception in 1992, the FCCC has seen incentive payments and transfers between Parties as elements of a climate agreement (both clean development mechanism in the Kyoto protocol and REDD+, and Article 11 in the Kyoto protocol). As the financial claims in swaps are budget-balanced, any forms of climate finance beside swaps are obsolete to reach the goal by the FCCC of preventing dangerous climate change.

(v) In the dynamic extension, various durations of an agreement on swaps lead to efficient emissions and investment. Acemoglu, Egorov & Sonin (2008)

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6 The FCCC still debates a rulebook for monitoring emissions pledges in current climate policy with no swaps (UN 2018).

7 See United Nations (1998) and “Warsaw Framework for REDD-plus,” Decision 9 to 15/CP19 in UNFCCC (2013). Figures for public climate finance range from 1.9 and 2.5 billion USD given as grants or concessional loans over 2013-2014 by climate funds run by multilateral development banks or bilateral development finance institutions beside United Nations agencies (UNFCCC 2016) to more than 100 billion USD over 2015-2016 on average per year given by public development and finance institutions, among which some are United-Nations related and some are owned privately or publicly in the recipient’s country, as revealed by the Climate Policy Initiative (2017). Monitoring and demonstration projects for REDD+ are being financed internationally. As public institutions lacked the mandate to verify and enforce emissions goals directly, “In 2010, UN Secretary General Ban Ki-Moon established a high-level advisory group and tasked it to find the best sources of climate finance.” (Fankhauser & Editors 2013).
argue that governments having been chosen more democratically can commit longer to policies. The irrelevance of the duration of swap contracts for efficiency thus implies that democratic versus other politics within each country do not matter for internalizing the externalities countries impose on each other with emissions.

(vi) Interpreting a polluting intermediate good as fuel, the second and third viewpoint on climate change have special implications. Coalition members and parties unaffected by global pollution each cease an equal share of their product in swaps, because the external costs they impose each are valued by equal sums of Lindahl prices. The parties use fuel efficiently, because they anticipate a tax (for harm) implicit in the swap. Equilibrium is also generally efficient if some party enjoys global pollution. The parties then use fuel efficiently, because they anticipate a tax (for harm or small joy) or a short sale (for large joy) in the swap. No further environmental regulation between parties is needed.

The main elements of a policy framework to correct externalities between parties in this paper—(a) shared ownership of gross product, (b) equilibrium (unanimous approval to change given swaps, and negotiation of bilateral or multilateral proposals), (c) verification of transfers with permits, and (d) enforcement with derived assets or balanced trade—contribute to different strands in the literature:

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<th>Literature/Elements</th>
<th>Welfare Economics</th>
<th>Mechanism design; Contracting and Linked markets for with externalities emissions permits</th>
<th>Climate contracts; Self-enforcing environmental policy; and Trade and environment</th>
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More detail on the relationship of the paper’s main findings to the literature can be found below.

Structure of the paper.—Section 2 sets up economies with a public good and swap contracts of gross product with the rules of a game using bilateral negotiation. Section 3 describes equilibrium properties when assuming that all parties weakly benefit from the public good and at least two parties benefit. Section 4 analyzes further structures of tastes and technology by parties, and presents swaps that are sourced more generally. Section 5 examines verification of swaps with transfers using linked permit markets, enforcement using bonds or stocks, self-enforcement, and voting on swaps for all parties as an alternative to bilateral negotiation. Section 6 presents extensions to the economic environment: heterogeneous impact of intermediate goods, green technology, deforestation, and dynamics. Section 7 relates my main findings to the literature. Section 8 provides conclusions. The appendices contain proofs of propositions and present further results on permits.

2. A game of implicit externality-correcting policy

Consider a one-period production economy with discretely many agents, called parties, forming the set $N$. If agents act collectively, then I write $M$ for the set of agents, or coalition, that does this, and $N^-=N\setminus M$ for the remainder set of agents, or nonmembers. Plausibly, the coalition contains some party marginally benefiting from a public good at an efficient allocation. The model fits a variety of public good problems. In presenting the model, I follow the topic of climate change, in which fossil fuel use creates an environmental externality, by making reference to the natural environment.

The economic environment.—A given party $i$ has preferences for consumption $x\in\mathbb{R}_+$ and a public good, say, environmental quality, $y\in\mathbb{R}$, represented by the utility $U_i = U_i(x, y)$, which strictly increases in $x$ and weakly in $y$, is twice-continuously differentiable, and strictly concave in $x$ and concave in $y$. These preferences can account for potentially heterogeneous impacts of pollution (for example, because of geography) while defining a global pollutant quantity below.

Each party can allocate one unit of labor between two sectors. For concreteness, the intermediate good fuel $r_i$ and labor $(1-n_i)$ produce consumption goods, and labor $n_i\in[0,1]$ produces fuel according to $y^Y_i = F^Y_i(r_i, 1-n_i)$ and $y^R_i = F^R_i(n_i)$ all $i\in N$. The production functions are increasing, con-
Parties can trade the consumption good and fuel,

\[ \sum_{i \in N} x_i = \sum_{i \in N} F^Y_i (r_i, 1 - n_i), \]

\[ \sum_i r_i = \sum_{i \in N} F^R_i (n_i). \]

Pollution by party \( i \) as measured by fuel reduces the public good below a number \( \bar{y} \in \mathbb{R} \) (with a pollution intensity of fuel equal to one),

\[ y = \bar{y} - \sum_{i \in N} r_i. \]

**Decisions by the parties.**—To allocate resources over the parties, I now introduce a three-stage game played simultaneously in each stage with trade in goods and claims to gross product. The parties decide in two stages, on swaps strategically at stage 1 and on labor, and intermediate and final goods, competitively at stage 2. At stage 3, the claims that define swaps pay out. Figure 1 depicts the timing of decisions and payout of claims.

1. **Swap-Setting Stage.**—All parties decide on swaps. Swaps lead to a payment from party \( i \) to party \( j \) denoted by \( v_{ij} \), which becomes swapped for \( v_{ji} \). With the price of fuel in terms of the consumption good \( p \in \mathbb{R}_+ \), gross product of party \( i \) used will be \( g_i(n_i, r_i, p) = F^Y_i + p F^R_i \). A swap between parties \( i \) and \( j \) stipulates a nonlinear tariff of their gross products involving (i) a payment proportional to gross product, \( \eta_{ij} g_i(\cdot) \) from party \( i \) to party \( j \), and based on gross product of party \( j \) in the opposite direction, and (ii) a side payment \( t_{ij} \in \mathbb{R}_+ \) from party \( i \) to party \( j \), or \( t_{ji} \) from party \( j \) to \( i \). Thus, a swap between the parties \( i \) and \( j \) can be stated as

\[ (q_{ij}, q_{ji}), \quad q_{ij} \equiv (\eta_{ij}, t_{ij}), \quad (i, j) \in N. \]

Payments are real, \( \eta_{ij} \in \mathbb{R} \) and \( t_{ij} \in \mathbb{R}_+ \), and occur between different agents, \( \eta_{ii} = 0 \) and \( t_{ii} = 0 \). Side payments are unidirectional, \( t_{ij} t_{ji} = 0 \) all \( (i, j) \in N \).

Using the tariff amount \( v_{ij} \equiv \eta_{ij} g_i(n_i, r_i, p) + t_{ij} \), it will be useful to define party \( j \)'s aggregate receipts less lump-sum expenditure on the swap market
(T_j) and the portion of their own gross product party i retains (\alpha_i),

\[ T_j = \sum_i v_{ij} - \sum_i t_{ji}, \]  
\[ \alpha_i = 1 - \sum_j \eta_{ij}. \]  

The parties can sign bilateral contracts on the “swap market.” Swaps are found, if and only if there exists no pair of parties \((i, j)\) at the given swaps such that both these parties are strictly better off by changing their swap with the consent of all other parties. One can think of negotiations between alternating pairs of parties until swaps are found. No swap trade by party \(i\), \(q_{ij} = (0, 0)\) and \(q_{ji} = (0, 0)\) all \(j \neq i\), can be understood as nonparticipation of party \(i\) in the swap market. Swaps thus allow a party to share their gains from pollution as materialized in gross product with other parties.

2. Production and Consumption Stage.—The decision-makers of the parties are price takers on world markets. In setting labor, fuel, and consumption
\((n_i, r_i, x_i) \in [0, 1] \times \mathbb{R}^2_+\), party \(i\) seeks to maximize utility

\[
U_i(x_i, y - r_i - \sum_{j \neq i} r_j)
\]

subject to the budget constraint

\[
x_i + pr_i = \alpha [F^y_i(r_i, 1 - n_i) + pF^R_i(n_i)] + T_i,
\]

taking as given the price \(p\) and swaps expressed by \((\alpha, T_i)\). Notice that the parties choose the contributions to the public good noncooperatively.\(^8\)

3. Swap Payout Stage.—The swaps pay out in terms of the consumption good.

By definition of swaps, all gross product is available for consumption and fuel acquisition. The swap market thus implies budget balance of the mechanism,

\[
\sum_{i \in \mathbb{N}} ([1 - \alpha_i] g_i + T_i) = 0.
\]

I can now define an equilibrium. The allocation to be decided on can be written \(q = \cup_{i,j} q_{ij} \setminus q_{ii}\) and \(v = \cup_i v_i\) for \(v \in \{n, r, x\}\). In an equilibrium of the game defined as an allocation and price system \((q, n, r, x, y)\) and \(p\), decisions are made as described, the markets for swaps and tradeable goods clear, and the public good \(y\) is incidentally produced.\(^9\)

2.1. Characterizing the best and no-contract outcomes

Equilibrium allocations will need to be compared to best outcomes defined as being Pareto optimal and the outcome with no contracting on swaps. This subsection thus compares the best and no-contract allocations.

A Pareto optimal allocation \((n, q, r, x, y)\) can be said to be efficient. To characterize efficient and equilibrium allocations, preferences and production obey regularity conditions that ensure efficient consumption \((x_i > 0\) all \(i \in \mathbb{N}\)), and fuel use and employment in consumption goods production \((0 < r_i, n_i < 1, i \in \mathbb{N})\). Each party may or may not produce fuel, \(n_i \geq 0\), all \(i\), at an

\(^8\)A competitive environment for decisions not contracted upon agrees with considering that each party comprises many agents that demand and many agents that supply the associated goods and follows the literature on climate contracts (Harstad 2012b, Harstad 2016).

\(^9\)An equilibrium thus is a set of admissible and feasible actions, where admissible actions are defined as following the stage-wise moves subject to the mechanism of finding swaps, and feasible actions are defined as clearing markets.
efficient allocation. Partial derivatives are indicated by subscript.\textsuperscript{10} As fuel is used, labor is used to produce fuel in some party, \( \sum_i n_i > 0 \).

**Lemma 1.** An allocation is efficient if the resource constraints hold in the form (1), (2), and (3), and the following conditions are satisfied:

\[
W_i U_{x_i} = W_j U_{x_j}, \quad (i, j) \in \mathbb{N},
\]

\[
\sum_{j \in \mathbb{N}} (U_{x_j})^{-1} U_{y_j} = \mathcal{F}^Y_i - \frac{\mathcal{F}^Y_{1-nk}}{\mathcal{F}^R_{nk}}, \quad i \in \mathbb{N}, \quad n_k \in (0, 1),
\]

(7)

with weights \( W_i > 0 \) all \( i \in \mathbb{N} \) and partial derivatives indicated by subscript.

The best outcomes include efficient allocations under the Utilitarian welfare criterion of equal weights on utility and other weighting schemes to construct social welfare.\textsuperscript{11}

The first equation in (7) represents conditions governing the distributional concerns of private goods. The second equation balances the social marginal rate of substitution (involving the sum of parties’ ratio of marginal utility of public and private consumption, giving the efficient utility, or Pareto, frontier) and the marginal rate of transformation between the private and the public consumption good. The marginal rate of substitution of labor that appears in the marginal rate of transformation, \( \mathcal{F}^Y_{1-nk}/\mathcal{F}^R_{nk} \), appears to be positive because of some production of dirty fuel, given the Inada condition on fuel use.

The no-contracting outcome exhibits no trade in claims, \( \eta_{ij} = t_{ij} = 0 \) all \( (i, j) \in \mathbb{N} \). Typically in an efficient allocation more labor is allocated to the production of consumption and less labor is allocated to the production of fuel compared to no contracting on swaps. All parties are better off at some efficient allocation described by some welfare weights compared to the laissez-faire (which can be seen from the proof of Pareto optimality).

\textsuperscript{10}To guarantee positive consumption, \( x_i > 0 \), at an optimum, marginal utility of consumption \( U_{x_i} \) approaches infinity as consumption tends to zero, \( \lim_{x \to 0} U_{x_i} = \infty \). Resources are used for producing consumption goods, \( 1 - n_i > 0 \) all \( i \), by assuming the Inada condition that the marginal product of the second input in \( \mathcal{F}^Y_i \) approaches infinity as the input amount tends to zero, \( \lim_{n_i \to -1} \partial \mathcal{F}^Y_i / \partial (1 - n_i) = \infty \). Resources are needed to produce fuel, \( \mathcal{F}^R_i(0) = 0 \). Each party uses fuel, by assuming that the marginal product of the first input in \( \mathcal{F}^Y_i \) approaches infinity as the input amount tends to zero, \( \lim_{r_i \to -0} \partial \mathcal{F}^Y_i / \partial r_i = \infty \).

\textsuperscript{11}Kreps (1990, p. 161) shows that every efficient allocation maximizes the weighted sum of utilities of agents.
Lemma 1 holds if parties have different productivity expressed by heterogeneous technology. For example, relevant to the climate problem, fossil fuel may be extracted from deposits, or transported, or processed at costs that differ over geographical space. Thus the model can reflect that some countries produce fossil fuel, and some do not, but all countries use fossil fuel. With or without swaps, a party that does not produce fuel can import fuel for their own use and pay for it by exporting the consumption good.

3. Contracting on swaps

This section qualitatively characterizes equilibrium, shows multiplicity of equilibrium allocations, and considers domestic policy to correct externalities between parties by affecting decisions of private agents.

The consumption decision, and the market for fuel.—Utility-maximizing parties allocate consumption $x$ and fuel use $r$ so that

$$F^Y_{ri} - [(1 - \alpha_i)F^Y_{ri} + (\mathcal{U}_{xi})^{-1}\mathcal{U}_{yi}] = p. \quad (8)$$

Equation (8) reveals two effects undermining the price for the externality-creating good, or fuel, below the marginal product of fuel use. The second term in brackets describes that party $i$ views their own contribution to the public good. The first term in brackets describes that party $i$ diverts some of their own production to other parties. This effect, sharing the private benefit from pollution, can induce to pollute less than with no policy.

Production and the market for labor.—From the choice of labor $n$, the fuel price in equilibrium equals the marginal rate of transformation of labor,

$$p = \frac{F^Y_{1-n_j}}{F^R_{n_j}}, \quad n_j \in (0, 1). \quad (9)$$

To interpret what follows, denote the relative desire for the public good, or harm of pollution, by $\gamma_i$ and the ratio of the marginal rate of transformation of labor between two uses and the social marginal rate of substitution by $\varepsilon$,

$$\gamma_i \equiv W_i\mathcal{U}_{yi}/\sum_{j \in \mathbb{N}} W_j\mathcal{U}_{yj}, \quad i \in \mathbb{N},$$

$$\varepsilon \equiv W_i\mathcal{U}_{xi}\frac{F^Y_{1-n_k}}{\sum_{j \in \mathbb{N}} W_j\mathcal{U}_{yj}}, \quad i \in \mathbb{N}, \quad n_k \in (0, 1).$$

\footnote{Switzerland, for example, does not produce fossil fuel.}
The social marginal rate of substitution is given by \( \Delta \equiv \sum_{j \in N} W_j u_{ij} / W_i u_{xi} \).

To set the stage, I will focus on the first viewpoint on international climate policy by assuming that all parties are weakly harmed by pollution and at least two are harmed \((u_{yi} \geq 0, \text{all } i \in N, \text{and } u_{yi} > 0, \text{at least two } i \in N)\) at any efficient allocation in the remainder of this section.

### 3.1. Equilibrium correction of externalities

I can derive efficiency and universal participation by parties in contracting given verifiability of swaps with side payments and binding contracts with no renegotiation, which will be relaxed in Section 5.

The decisions on swaps.—The conditions for social and private optimality (7) and (8) can be aligned when every party consumes a positive amount \((x_i > 0 \text{ all } i)\).

**Lemma 2.** Efficient fuel use results only if

\[
\alpha_i = \frac{\gamma_i}{1 + \varepsilon} + \frac{\varepsilon}{1 + \varepsilon}, \quad i \in N.
\]  

Each party retains a higher portion of output \(\alpha_i\) the greater their own relative harm from pollution \(\gamma_i\) (the Pareto-weighted marginal effect relative to the total weighted marginal effect of pollution) adjusted by the marginal rate of transformation in fuel production in \(\varepsilon\). The more important the own harm from pollution for overall welfare, the less the framework needs to adjust gains from polluting, as each party takes into account the effect of own emissions on own harm

\[
\delta_i \equiv u_{yi}/u_{xi}.
\]

Allocations with \(\alpha_i < 1\) require party \(i\) to participate in issuing claims in swaps. Given all parties are weakly harmed by pollution and no coalition has formed comprising all parties harmed \((u_{yi} \geq 0, \text{all } i \in N, \text{and } u_{yi} > 0, \text{at least two } i \in N)\), \(\alpha_i < 1, \text{ all } i \in N\), as any party’s fuel use affects some other party. Thus, to attain efficiency, every party needs to participate in contracting on swaps.

Swap contracts utilize that efficiency can attained by reducing the own gain from polluting. A party cedes ownership of some share of own gross product and also gains ownership of a share in some other party’s gross product. Thus,
each of two parties can mutually swap claims to gross product, or parties can swap claims through overall changes of ownership of gross product (see below for structures of swaps and how asymmetric tastes require interpreting swaps).

In general, I can derive the following result.

**Proposition 1 (i).** Equilibria with the market for swaps exist and are efficient with no collective action, $M = \emptyset$.

Contracting yields efficiency. If the contracts on swaps allocated resources in an inefficient way, then two parties could gain from a different contract and obtain the consent of other parties for it. First, with swaps the parties can contract to the Pareto frontier, forward-looking to their choices of fuel. The parties anticipate how their decision on swaps at stage 1 affects their decision on fuel use at stage 2, and in turn how their decision on fuel use at stage 2 influences the public good. Second, parties comply to swaps as any change to given swaps needs to be unanimously approved. These two points imply that no inefficient allocation results, and show that efficiency results—the set of equilibria that satisfy the social optimality conditions (7) contains the set of equilibria with the properties (8) and (9) which characterize individual parties’ choices. Then the efficient asset ownership (10) characterizes equilibrium.

The efficiency result shows that all parties participate in the swap market in equilibrium, $q_{ij} \neq (0,0)$ all $i$ some $j$, given that at least two parties are harmed by pollution, $U_{yi} > 0$, at least two $i \in N$. All parties participate in the swap market, or contract on swaps, because any excluded party could reap gains by joining the market.

Swaps shape the incentives to provide a public good such as pollution abatement for environmental policy. With claims to gross product, swaps make implicit environmental policy. To create property rights where property rights on a public good are missing in the first place because of multi-sided externalities and the nonexcludability from the benefits of a public good, swaps use unanimity in decision-making of parties. Thus, the efficiency result in an environment otherwise bound to free-rider behavior uses no cooperation in choosing emissions.

**Lindal prices and allocation.**—To relate swaps to Lindahl markets, and to better understand the role of bilateral contracts for enforcement of payments between parties in Section 5.3, I will now show that swaps which price the
external cost of a party’s fuel use account for the sum of Lindahl prices. A supra-party government, if it were to exist, would set the Lindahl price \( \gamma_j \) to be paid from party \( i \) to party \( j \) for each unit of fuel used by party \( i \), or the Lindahl price on party \( i \)'s gross product

\[
\tau_{ij} = \frac{\gamma_j}{1 + \varepsilon}.
\]

Any two parties “pay” the same price implicit in swaps to another party, because anyone’s fuel use has the same effect on the public good. In equilibrium, party \( i \in N \) thus relinquishes a portion of gross product equal to the sum of Lindahl prices, \( 1 - \alpha_i = \sum_{j \neq i} \tau_{ij} \). This can be derived by using (10) and the identity \( \gamma_i + \sum_{j \neq i} \gamma_j = 1 \).

With no side payments, swaps can yield the Lindahl allocation, that is, the allocation when ownership shares and rebates in all budget constraints are evaluated at Lindahl prices and there are no further transfers between agents. This can be seen from the budget constraint of a party (6). With no side payments and two parties (\( N = 2 \)), the outcome is automatically a Lindahl allocation.

Domestic externality pricing (Section 3.3) and general sourcing of swaps (Section 4.3) use the internality and externality of fuel use by party \( i \), which can be exhibited together,

\[
\sum_j W_j U_{yj}/W_i U_{xi} = \frac{U_{yi}/U_{xi}}{\alpha_i} + \sum_{j \neq i} \tau_{ij}(1 + \varepsilon) \Delta, \\
\gamma_i = \frac{\delta_i}{\Delta}.
\]

The social cost of fuel use, \( \Delta \), is the same for all parties as their weighted marginal utility of consumption is equalized, \( W_i U_{xi} = W_j U_{xj} \) all \( (i, j) \in N \), at an efficient allocation.

3.2. Equilibrium allocations

To continue, I will now further characterize equilibrium allocations.

Allocative indeterminancy.—A continuum of efficient allocations with positive consumption for any party can be implemented, because side payments are continuous. Agreement on a set of welfare weights would pin down a
consumption distribution. Depending on the tastes for consumption and environmental quality, the efficient and thus equilibrium level of environmental quality may be unique. It appears unique if tastes satisfy independence of the distribution of consumption and the efficient level of environmental quality. See Bergstrom & Cornes (1983) for a definition. There are thus multiple equilibrium allocations.

Pareto improvement or nonimprovement.—Some equilibria have allocations that Pareto improve, that is, yield weakly higher utility for all parties and strictly higher utility for some party. However, in equilibrium some party may receive lower utility than with no swaps, because the first ‘proposal’ negotiated, or to be changed, is undetermined by the mechanism. Those equilibria can be ruled out by refinement by eliminating the agenda to make the first ‘proposal’ Pareto improving, or with making proposals and voter on them (Section 5.5). Notice that, in the first viewpoint on climate policy adopted in this section, the Lindahl allocation may not Pareto improve when parties have sufficiently heterogeneous tastes for emissions reductions.

3.3. Decision-making by private agents and domestic policy

To better follow the analyses of general sourcing of swaps in Section 4.3 and verification of swaps in Section 5.1, I will now provide one possible decentralization of the economy in each party. This also shows that private agents in any party have to pay a uniform price for emissions, given emissions are created in proportion to fuel use.

In each party’s jurisdiction, private agents fail to internalize the effect of fuel use on their own society, in our case, on utility, if there is such an effect ($U_{yi} \neq 0$). To internalize the effect, each party regulates fuel use domestically, for the sake of illustration, using a fuel tax. To internalize the transboundary externalities, each party levies a tax on gross product.

Each party is populated by a continuum of identical households and firms each with measure 1. Underlying the budget constraint of a party (6), households receive factor payments and profits from domestic production, hence the wage $w_i$ and profits $\pi_i$ in party $i$’s economy. A representative household of party $i$ consumes

$$x_i = w_i + \pi_i + \tau_i,$$

taking as given the wage, profits, and the lump sum $\tau_i$. The lump sum, when being positive, means a transfer from the government of the party. Firms
allocate labor, produce consumption goods, and produce and use fuel. A representative firm in party $i$ chooses labor and fuel $(n_i, r_i) \in [0, 1] \times \mathbb{R}_+$ to seek to maximize profits

$$\pi_i = (1 - \tau_i^y) g_i(n_i, r_i, p) - w_i - (p + \tau_i^R) r_i$$

(12)

taking as given the prices $(p, w_i)$ and tax rates $(\tau_i^y, \tau_i^R)$. The government of each party sets taxes in stage 1 or 2 before households and firms make decisions. The budget constraint of the government of party $i$ reads

$$\tau_i^y g_i + \tau_i^R r_i + \left( \sum_j v_{ji} - \sum_j v_{ij} \right) = \tau_i.$$ 

(13)

The government of a party administers swaps so that households receive the domestic tax revenue plus the payments made to the party less the payments made by the party on the swap market. Notice that the budget constraint of each household (11), the profit of each firm (12), and the budget constraint of the government (13), in a given party imply the budget constraint of the party (6).

Profit-maximizing firms set labor and fuel use $(n_i, r_i)$ so that

$$\mathcal{F}_i^y - \left( \tau_i^y \mathcal{F}_i^y + \tau_i^R \right) = p.$$ 

(14)

To align the party’s and the firms’ necessary equilibrium conditions (8) and (14) with generating exactly as much revenue from taxing gross product as required for swaps of gross product, the government in party $i$ sets domestic policy $\tau_i^y = 1 - \alpha_i$ and $\tau_i^R = \delta_i$.\footnote{Using the tax rates and evaluating swaps at efficient ownership of gross product, the budget constraint of the government of party $i$ reads

$$\delta_i r_i + \left( \sum_j v_{ji} - \sum_j t_{ij} \right) = \tau_i.$$ 

Characterizing the degree of homogeneity in tastes or technology necessary for nonnegative transfers from government to private agents and nonnegative firm profit is beyond the scope of this paper.}
use equal to \( \Delta \) as \( x_i^Y - \Delta = p \). Hence emissions have a uniform price equal to the social cost of emissions for private agents in any party’s economy.

Using a combination of gross product and fuel tax to separately address the external and internal cost here provides a gradual transition to Section 5.1 on the verification of swaps with transfers.

4. Further results

In this section, I relax several assumptions on tastes and technology. Efficiency is shown to be robust with respect to other viewpoints on public good provision (some collective action; and a taste structure in which some or all parties not members in a coalition are marginally unaffected by pollution or parties have asymmetric tastes for a public good) in Section 4.1 and 4.2, claims on government revenue from explicit externality pricing in Section 4.3, and a general nonexternality-creating intermediate good in Section 4.4. The analysis of the taste structure builds on the analysis of collective action.

In Section 4.1, all parties are harmed or unaffected by pollution \( (U_i \geq 0, \text{all } i \in N) \).

4.1. Collective action

That efficiency requires no collective action makes coalition formation unnecessary for reach efficient public good provisions. As we shall see shortly, with some collective action efficiency can be obtained as well.

Section 3 has viewed no collective action. Let us now assume that a coalition \( M \) with at least two members, \( |M| \geq 2 \), has formed prior to choosing swaps. The coalition includes at least one member harmed. Nonmembers may be harmed or unaffected by fuel used. The typical setup prone to leakage in the private provision of a public good arises if all nonmembers are unaffected. For example, in the problem of an externality of fossil fuel used onto the climate, \( M \) is the climate coalition. With only trade in products, the decisions by nonmembers that are marginally unaffected by the public good at an optimum \( (U_i = 0, i \in N^-) \) imply leakage defined as increased fuel use by them to unilateral efforts to reduce fuel use by the coalition. Here, however, nonmembers have access to the swap market.

The coalition chooses consumption \( x_i \), labor input \( n_i \), and fuel use \( r_i \), \( i \in M \), so as to maximize the weighted sum of utilities of their members,
\[ \sum_{i \in M} W_i U_i(x_i, y), \] subject to individual budget constraints, so that
\[
(W_i U_i)^{-1} \left[ \sum_{j \in M} W_j U_{ij} \right] = \alpha_i \frac{\mathcal{Y}_{ri} - p}{\gamma_i}, \quad i \in M, \tag{15}
\]
with the fuel price given as before (9). Nonmembers individually set consumption \( x_i \), labor input \( n_i \), and fuel use \( r_i \), \( i \in N^- \), as before, yielding (8).

Conditions (8) and (15) differ such that the coalition views contributions to the public good by all the coalition members. Then efficiency requires that
\[
\alpha_i = \frac{\sum_{j \in M} \gamma_j + \varepsilon}{1 + \varepsilon} \text{ if } i \in M, \quad \alpha_i = \frac{\gamma_i + \varepsilon}{1 + \varepsilon} \text{ if } i \in N^-.
\]
Nonmembers retain the portion of gross product equal to the individual relative harm as before (using \( \gamma_i \)). Coalition members thus, according to the first equity principle of swaps, retain the same portion of gross product following the sum of the coalition members’ relative marginal effect of the public good on utility (using \( \alpha_i = \alpha_j \), \((i, j) \in M \)). The effects are adjusted by the marginal rate of transformation between the two sectors using a common factor such as labor.

**Proposition 1 (ii).** Equilibria with the market for swaps exist and are efficient with some collective action, \( M \neq \emptyset \).

Proposition 1(ii) holds by the same token as Proposition 1(i). That the swap market helps to achieve efficiency with collective action (Proposition 1(ii)) is unsurprising, because the swap market leads to efficiency with no collective action (Proposition 1(i)).

Importantly, Propositions 1(i)-(ii) hold when some party \( i \) is unaffected by the public good \( y \) at an efficient allocation \( (U_{yi} = 0) \). Efficiency thus requires, expressing the second equity principle of swaps, each unharmed nonmember to retain the same portion of gross product (using \( \gamma_i = 0 \)).

**Addressing leakage.**—In one case when some collective action occurs and some party \( i \) is unaffected by the public good \( y \) at an efficient allocation \( (U_{yi} = 0) \), all nonmembers are unharmed by pollution. Then the coalition does not create an externality, and hence does not share their gross product \( (\sum_{j \in M} \gamma_j = 1) \). This case characterizes the viewpoint often taken to analyze...
carbon leakage, the second viewpoint on climate policy. The swap market thus addresses leakage.

In the second viewpoint on climate policy, the Lindahl allocation does not Pareto improve, because, in this allocation, the coalition does not compensate nonmembers for paying implicit taxes that internalize the externality nonmembers impose on the coalition. The coalition does not impose an externality on nonmembers, and hence pays no implicit Lindahl taxes to attain the Lindahl allocation.

4.2. Asymmetric tastes for environmental quality

This subsection analyzes the correction of externalities using swaps with parties that benefit from fuel use (so that $U_j$ decreases and is convex with respect to $y$ for some party $j$), at least locally, that is, for some range of the public good $y$. For example, a warmer climate could defrost soil and thus enlarge the area of arable land or reduce the cost to extract fuel. Overall, reductions in fuel use contribute to a public good ($\sum W_i U_{yi} > 0$). This means that tastes for environmental quality are asymmetric as fuel use harms some party (so that utility $U_i$ increases and is concave with respect to $y$ for some party $i$), which pertains to the third viewpoint on climate policy defined above.

An equilibrium with swaps is efficient as long as each nonmember in the climate coalition retains or sells short own gross product, $\gamma_i + \varepsilon \neq 0$, $i \in N^-$. The case beside this condition directly follows from Lemma 2. Party $i$ that incurs a positive effect of global pollution equal to the marginal rate of transformation of labor at an efficient allocation ($-\frac{U_{yi}}{U_{xi}} = p$, equivalent to $\gamma_i = -\varepsilon < 0$) retains none of their own gross product in swaps, $\alpha_i = 0$. Thus, as fuel can be produced, a party enjoying fuel used does not need to be in the coalition to guarantee efficiency except in a knife-edge case. Swaps thus generally address the asymmetric free-rider problem posed by tastes for and against a public good.

That a party benefits from global pollution can yield the following property of swaps which cannot emerge with weakly harmful pollution for every party analyzed in Section 3 and and 4.1. At an efficient allocation, a party $i$ benefiting from fuel used may sell gross product short with respect to other parties, $\alpha_i < 0$, or equivalently, $\gamma_i + \varepsilon < 0$. Such a short sale means that gross product of some other party $j$ as a whole becomes implicitly subsidized.
in swaps \((1 - \alpha_j < 0, i \neq j)\). Eventual subsidies of gross product because of asymmetric environmental effects can be inverted to yield positive claims in the opposite direction.

Notice that in domestic policy the government of party \(i\) subsidizes fuel when \(U_{yi} < 0\), and thus \(\delta_i < 0\).

### 4.3. Claims on government revenue from explicit externality pricing

In presenting a party’s problem, I have focused on swaps of gross product to embrace both market and nonmarket economies. A government centrally planning allocations for a society may need to write contracts on gross product. This subsection generalizes swaps with a mixture of claims on gross product and government revenue from externality pricing such as emissions taxes or sale of emissions permits with decentralized decision-making.

To show that various combinations of gross product tax and fuel tax can price a party’s social cost of reducing a public good, define the variable component in the payment from party \(i\) to \(j\) in a swap as \(\chi_{ij} \equiv \eta_{ij}^y g_i + \eta_{ij}^R r_i\). A swap between the parties \(i\) and \(j\) can be stated with

\[
q_{ij} \equiv (\eta_{ij}^y, \eta_{ij}^R, t_{ij}), \quad (i, j) \in N.
\]

Taxation of fuel use newly creates a rent. The ownership shares specific to gross product and fuel use \(\eta_{ij}^y\) and \(\eta_{ij}^R\) then determine the portion of gross product and the portion of the rent of fuel use that are retained by party \(i\) as

\[
\alpha_i^y \equiv 1 - \sum_j \eta_{ij}^y, \quad \alpha_i^R \equiv 1 - \sum_j \eta_{ij}^R, \quad i \in N.
\]

Party \(i\) now chooses \((n_i, r_i, x_i)\) so to maximize utility viewing the budget constraint

\[
x_i + (p + 1 - \alpha_i^R)r_i = \alpha_i^y [F_i^y(r_i, 1 - n_i) + pF_i^R(n_i)] + T_i.
\]

**Contracting on swaps.**—Let us restrict attention to swaps that price the externality, as before, though regulatory revenue exists from pricing the internality and externality of a party’s fuel use. This assumption is made to accord with the intention to use swaps—to address free-rider behavior resulting from an uncorrected externality. The exogenously given parameter \(\kappa\) indicates the weight in claims on gross product, which was previously taken
to be one. For simplicity, this parameter is equal for all parties. Then party $i$ retains the following portions of gross product and fuel rent.

**Lemma 3. Efficiency with claims on emissions tax or permit sale revenue requires that**

$$
\alpha^Y_i = \frac{(1 + \varepsilon) - \kappa(1 - \gamma_i)}{(1 + \varepsilon)}, \quad \alpha^R_i = 1 - (1 - \kappa)(1 - \gamma_i)\Delta, \quad i \in N.
$$

**Decision-making by public and private agents.**—The budget constraint of the government of party $i$ is the same as before, namely (13). Profit-maximizing firms of party $i$ set labor and fuel use $(n_i, r_i)$ so that (14) holds. One can interpret the gross price of fuel $(p + \tau^R_i)$ such that firms in party $i$ buy fuel at price $p$ on the fuel market, and at price $\tau^R_i$ from the domestic government and other parties’ governments together.

The government in party $i$ then sets domestic policy $\tau^Y_i = 1 - \alpha^Y_i$ and $\tau^R_i = \delta_i + (1 - \alpha^R_i)$. Two special cases are exclusive use of gross product taxes ($\kappa = 0$) with $\tau^Y_i = (1 - \gamma_i)/(1 + \varepsilon)$ and $\tau^R_i = 0$ analyzed above, and exclusive use of fuel taxes or domestic emissions permits ($\kappa = 1$) with $\tau^Y_i = 0$ and $\tau^R_i = \Delta$. For any tax mix ($0 \leq \kappa \leq 1$), emissions have a uniform price for private agents in any party’s economy. Notice that $\delta_i \leq \tau^R_i \leq \Delta$. One can now interpret the gross price of fuel $(p + \tau^R_i)$ such that firms in party $i$ buy fuel at price $p$ on the fuel market, at price $\delta_i$ from the domestic government, and at price $(1 - \alpha^R_i)$ from other parties’ governments.

**4.4. General nonexternality-creating intermediate good**

Using general labor for consumption goods specializes the production of the second intermediary good. At the same time, the nonexternality-generating intermediate good becomes nontradeable between parties, which simplifies notation. Suppose, more generally, consumption goods use the intermediary goods $r$ and $s$. The second intermediary good uses labor $(1 - n)$, and can be traded, at price $\phi$, between parties so that $\sum_{i \in N} s_i = \sum_{i \in N} F^S_i (1 - n_i)$.

Party $i$ decides on $(n_i, r_i, s_i, x_i) \in \mathbb{R}_+^3$ so as to maximize utility

$$
\mathcal{U}_i(x_i, y - r_i - \sum_{j \neq i} r_j)
$$

subject to the budget constraint

$$
x_i + pr_i = \alpha_i[F^Y_i (r_i, s_i) + pF^R_i (n_i) + \phi(F^S_i (1 - n_i) - s_i)] + T_i, \quad (16)
$$
taking as given the prices \((p, \phi)\) and swaps expressed by \((\alpha_i, T_i)\).

The price for the externality-creating intermediate good then can be stated as

\[
p = F^Y_{si}(F^S_{1-ni}/F^R_{ni}).
\]

Equilibrium can be straightforwardly defined as in Section 2 and \(\varepsilon\) be generalized with \(F^Y_{sj}F^S_{1-nj}\) in place of \(F^Y_{1-nj}\).

From the above, it is clear that the characterization of shared ownership of gross product given by Lemma 2 does not change when the second intermediary good can or cannot be traded between parties.

5. Limited contracting, enforcement, and Pareto improvement

To strengthen swaps as a framework to correct externalities between parties, this section presents institutions and features of swaps that help to verify and enforce swaps. This section first discusses verification of swaps with limited knowledge about side payments (Section 5.1), followed by a presentation of two approaches to enforce swaps—enforcement of derived assets (Section 5.2) and self-enforcement with balanced trade (Section 5.3 and 5.4), and a mechanism (in game form) that ensures a Pareto improvement (Section 5.5). The self-enforcement is viewed with the lack of side payments (weak verification) and presence of merely nonnegative payments, or implicit taxes (weak enforcement), in swaps.

5.1. Weak verification: Distributing emissions permits

Swaps with no side payments \((t_{ij} = 0, \text{ all } (i, j) \in N)\) can be verified given the tax on gross product implicit in swaps can be viewed as a domestic tax, as used in Section 3.3. The parties can learn the tax basis in party \(i\)'s jurisdictional region by comparing the tax rate \((1 - \alpha_i)\) and the sum of swap payments \((1 - \alpha_i)g_i\) to actual gross product party \(i\) reports, for example, publishes in national accounts, \(g_i\). Hence, swaps with no side payments can be verified. I say that swaps can be weakly verified when swaps with side payments cannot be verified. Side payments are one form of transfers.

Transfers support Pareto improvements when tastes for environmental quality are somewhat heterogeneous, which motivates to achieve transfers
Transfers aside from swaps.—Distributing permits to use the externality-creating good allows to verify swaps when swaps with side payments cannot be verified and redistribution through transfers is desirable.

I assume that taxes (or subsidies) are used to price the externalities, so permits are used to price the internality. To regulate fuel use chosen by firms, firms decide on permit holdings, in turn demanding that:

(i) Household ownership of firms is well-defined. To replace side payments in swaps, households in different parties need to have different initial permit wealth. As firms are endowed with permits, in targeting household permit wealth, regulators need to backtrack firm ownership among households from different parties. For simplicity, I assume that this can be achieved as households earn profit only from domestic firms.

(ii) Permits are tradeable. Firms in one party need to gain access to permits initially held by firms in another party, assuming households wholly own domestic firms. This access implies that permits need to be tradeable between the regions defined by the parties. I will characterize the allowance ratio of permits in each region required for efficiency.

A setup of linked permit markets with allowance ratios.—To continue, denote by $u(i) \in \mathbb{R}$ the number of usage rights, or permits, held by firms in party $i$. To each firm in region $i$ permits limit fuel use. With the allowance ratio $\rho_i$ giving the number of usage rights allowing to one unit of fuel use,

$$\rho_i r_i \leq u(i). \quad (17)$$

Let $\bar{u}(i) \in \mathbb{R}$ be the quantity of permits initially distributed to firms in party $i$. To implement an allocation which binds fuel use by $\rho_i r_i = u(i)$, $i \in \mathbb{N}$, the regulators of the parties which are part of the governments choose the permits $(\bar{u}(1), \bar{u}(2), \ldots, \bar{u}(N))$ to balance demand and supply of permits,

$$\sum_j u(j) = \sum_j \bar{u}(j). \quad (18)$$

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14In the second viewpoint on climate policy, with no transfers some parties become worse off than with no swaps, an undesirable feature. If one limited the first ‘proposal’ on the agenda during negotiation to be Pareto improving or used proposals of swaps for all parties (see Section 5.5), transfers would be needed for an equilibrium with swaps to exist. Any Pareto improving allocation can be implemented for a suitable choice of transfers that can be continuously varied to move from a first ‘proposal’ to be changed in negotiation.
Each firm can sequester permits with the regulator of any party—the number of permits \( \rho_i \) for a unit of fuel. To avoid double sequestering, some monitoring will be required.

**The decision problems and the government budget constraint.** While permits do not alter the household problem, it may be noted that if permits have a positive price, households receive wealth from initially freely distributed permits (distributed to households through profits) and use it to buy more costly goods compared to no regulation.

To limit the use of lump-sum payments of households for parties making transfers, a party’s regulator can sell permits. Conferring to party \( i \), regulators sell to domestic firms the portion of permits these firms hold initially \((1 - \varphi_i)\) at the price \( \phi^\text{permit} \in \mathbb{R}_+ \). The remainder portion is freely distributed.

Each firm in party \( i \) chooses the amounts of labor \( n_i \), fuel \( r_i \), and permits \( u(i) \), so to maximize profits

\[
\pi_i = (1 - \tau_i^y)g_i(n_i, r_i, p) - pr_i - \phi^\text{permit}[u(i) - \varphi_i \bar{u}(i)] - w_i,
\]

subject to the limiting of fuel use \((17)\), taking as given the prices of fuel, labor, and permits \((p, w_i, \phi^\text{permit})\) and the allowance ratio and tax rate \((\rho_i, \tau_i^y)\).

The government budget constraint states that the receipts from the sale of permits \( R_i = \phi^\text{permit}(1 - \varphi_i)\bar{u}(i) \) replace the fuel tax revenue that generated government revenue in \((13)\),

\[
R_i + \tau_i^y g_i + \left( \sum_j v_{ji} - \sum_j v_{ij} \right) = \tau_i.
\]

**The resource constraints.** The resource constraints \((1)\) and \((2)\) distribute consumption goods and fuel, while the resource constraint \((18)\) equalizes demand for and supply of permits.

**Weak verification of swaps.**—To weakly verify swaps, it will now be shown that an efficient allocation which can be decentralized with side payments in swaps can also be decentralized with: (i) a set of allowance ratios \( \rho_1, \rho_2, \ldots, \rho_N \), (ii) earmarking the tax on gross product for claims in swaps, \( \tau_i^y g_i(\cdot) = \sum_j v_{ij}, \) \( i \in N, \) and (iii) no side payments in swaps, \( t_{ij} = 0, (i, j) \in N. \) Then side payments in swaps can be replicated by permit wealth.

**Proposition 2.** There exists an initial distribution of emissions permits \((\bar{u}(1), \bar{u}(2), \ldots, \bar{u}(N))\) that nullifies side payments in swaps \((t_{ij} = 0, (i, j) \in N)\).
In pursuit of task (i), the choices of fuel and permits \( r_i > 0 \) and \( u(i) \) need to balance their marginal benefit and cost, \((1 - \tau Y_i) F_r Y - p = \rho_i v_i \) and \( v_i - \phi_{\text{permit}} = 0 \), letting \( v_i \) be the shadow price of the constraint (17) limiting fuel use. Therefore, the marginal value of an additional emissions permit equals the price of a permit, \( v_i = \phi_{\text{permit}} \). Assuming that swaps price the externality, as before, the number of permits allowing to the use of one unit of fuel (allowance ratio) for party \( i \) thus amounts to

\[
\rho_i = \delta_i / \phi_{\text{permit}}, \quad \phi_{\text{permit}} > 0.
\]

Permits sold in advance make no difference for the allowance ratio, because permits sold only shift wealth between profit and government transfers. Households receive profits that are reduced by the value of sold permits and obtain transfers that are raised through the value. To achieve (ii), the parties set the gross product tax rate \( \tau Y_i = (1 - \gamma_i)/(1 + \varepsilon) \). The task (iii) can then be accomplished by picking the initial permits

\[
\bar{u}(i) = \left( \delta_i r_i - \left[ \sum_j t_{ij} - \sum_j t_{ji} \right] \right) / \phi_{\text{permit}}, \quad i \in N.
\]

With earmarking of taxes or subsidies on gross product for payments in swaps, the tax or subsidy rate can be verified by observations on output. Permits thus can help to verify swaps when direct transfers in the form of side payments in swaps cannot be verified.

Two design features of tradeable permits that replace side payments in swaps emerge.

*Net transfers determine initial permits.* The initial emissions permits that nullify side payments in swaps appear unique up to given net transfers in swaps that they replace \((\sum_j t_{ij} - \sum_j t_{ji}) \) for party \( i \) and the price of permits \((\phi_{\text{permit}} > 0, \text{which can be normalized})\). This is revealed in the construction of an according initial distribution of permits in the proof of Proposition 2, being reproduced above.

*A pre-emptive right to buy permits may be used.* Some transfers may require issuing negative permits. For example, a party that produces nearly all fuel, whose utility does not change with fuel use directly, and that receives utility as high as with no swaps, needs to hold initially more permits than it will use. In turn, another party has to receive negative permits initially.
Equivalent to issuing negative permits, a regulator can distribute a nonnegative number of permits to domestic firms and has the pre-emptive right to buy permits from these firms, that is, before the firms can sequester permits.

**Generalizations.**—Claims in swaps can contain emissions tax revenue, or equivalently, fuel tax revenue. See the Appendix B for a generalization in this direction.

Claims can be generalized so that swaps price at most, rather than exactly, the external costs. Let swaps of gross product price the portion \( c \in (0, 1] \) of the externalities. The allowance ratio for party \( i \) then becomes \( \rho_i = (c\delta_i + (1 - c)\Delta)/\phi^\text{permit} \), provided a price for permits \( \phi^\text{permit} > 0 \) which can be normalized. The parties then set the gross product tax rate \( \tau^y_i = c(1 - \gamma_i)/(1 + \varepsilon) \).

Thus, an initial distribution of permits can be used to replicate side payments in swaps.

**Corollary 1.** *Equilibrium swaps can be weakly verified.*

Weak verification of swaps follows as it has been shown that permits to the use of externality-creating goods can achieve transfers. In detail, linking markets for permits transfers wealth so that swaps feature only variable payments (with gross product or fuel use). Then swaps can be verified. A lack of side payments in swaps then does not pose a threat to efficiency when an efficient equilibrium requires a Pareto improvement.

Next I will describe enforcement. A decision to pursue either one of the two ways of enforcement presented when both are available must be motivated outside the model.

**5.2. Enforcement I: Assets**

Returns on assets such as bonds or stocks can be enforced if citizens have the right to receive these returns. The parties thus have an incentive to establish their right to the claims in swaps if citizens can claim returns derived from swaps. This shows that swaps can be enforced, as the claims in swaps can be used to create further claims which can be enforced.

To internalize the externality of a party’s fuel use on other parties, the party’s government raises funds through domestic taxes on gross product, or, with considerably asymmetric tastes for the common good, may subsidize gross product using funds received from swaps, see Section 4.2.
funds are promised to parties. Now, these funds are restructured in the form of claims promised to households and directly promised to households as a return on an asset in the two regimes of bundled and mixed claims. Then enforcing asset returns induces all parties to set up an institution that enforces swaps. I will now define asset returns.

Positive and negative swaps and asset returns.—With asymmetric tastes for the common good, subsidies to gross product can arise, as Section 4.2 shows, leading to negative swaps. Adopting the view that a holder of an asset cannot be forced into making a payment on the asset, then the direction of payment becomes reversed. To add returns, denote positive claims in swaps with a plus superscript, and denote negative claims with a minus superscript and revert their direction of payment.\(^{15}\) Thus, the payments from party \(j\) to party \(i\) in swaps are given by \((v_{ji}^+ + v_{ij}^-)\).

**Bonds**

Governments issue bonds to households; the government of party \(j\) issues to a household of party \(i\) the bonds denoted by \(b_j(i)\). Outstanding bonds held by households of party \(i\) are

\[
B_i \equiv \sum_j b_j(i).
\]

As parties, through households and government, have claims on each other, bonds, or more precisely, their interests, are swapped.

**Bundled claims.**—Governments promise payments they receive from other governments on the swap market by issuing bonds to households. Define the sum of payments party \(i\) makes as asset returns by \(O_i \equiv \sum_j (v_{ji}^+ + v_{ij}^-)\). Assuming that the composition of funds with payments from different parties can be verified, payments from party \(i\) to households in the form of bond returns are given by \(O_i = (r - 1) \sum_j b_i(j)\) using the gross interest rate on bonds \(r\). The budget constraint of a representative household in party \(i\) reads

\[
x_i = w_i + \pi_i + (r - 1)B_i + \tau_i.
\]

The budget constraint of the government of party \(i\) expands relative to (20) to reflect that the claims from swaps are no longer paid out to the governments

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\(^{15}\)I define the positively valued functions \(f^+ = f\) if \(f > 0\) and 0 else, \(f^- = -f\) if \(f < 0\) and 0 else. For the sake of generality, I include side payments \((v^+ = \eta^+ g + t\) and \(v^- = \eta^- g + t\)). However, one can think of claims that vary with gross product only as explained in Section 5.1 \((v^+ = \eta^+ g\) and \(v^- = \eta^- g\)).
for free use but are designated for payment to households,

\[ R_i + \tau^Y_i \gamma_i + \left( \sum_j (v_{ji}^+ + v_{ij}^-) \right) = \tau_i + \left( \sum_j (v_{ij}^+ + v_{ji}^-) \right) + O_i. \]  (22)

Bond supplies here determine a common gross interest rate. Interestingly, the distribution of bonds \((b_j(1), b_j(2), \ldots, b_j(N)), \ j \in N\), may be picked with no change in household consumption relative to an equilibrium with no bonds. For example, households receive interest payment only on bonds funded by claims which are promised to the domestic government, using

\[ b_i(j) = 0 \text{ if } i \neq j. \]

This example of bundled claims makes sense if bonds issued to households at home may be enforced better than those issued to households abroad, which can be argued in line with current law as governments bail out firms majorly owned by domestic households and default on foreign debt. Verification and enforcement can thus be complementary, which is desirable.

**Mixed claims.**—Each government of a given party obliged to make payments in swaps directly issues bonds to foreign households, and the government of a party the households belong to guarantees these payments. Governments make those guarantees frequently in foreign trade.

To focus on enforcement, abstract from redistribution effects by assets. Thus let payments from party \(i\) to households of party \(j\) in the form of bond returns be denoted by \(o_i(j) \equiv v_{ij}^+ + v_{ji}^-\) so that \(o_j(i) = (r - 1)b_j(i)\). Households of party \(i\) receive from foreign governments \((r - 1)B_i\). The budget constraint of these households remains, albeit with a different meaning of bonds, (21). The budget constraint of the government of party \(i\) shrinks compared to (22) to account for the fact that households of party \(i\) receive the claims from swaps targeted to this party,

\[ R_i + \tau^Y_i \gamma_i = \tau_i + \left( \sum_j (v_{ij}^+ + v_{ji}^-) \right). \]  (23)

Bond supplies here determine a common gross interest rate, and the distribution of bonds \((b_j(1), b_j(2), \ldots, b_j(N)), \ j \in N\), automatically avoids redistributive effects as governments issue bonds only to households of parties that otherwise receive claims in swaps,

\[ b_i(j) = 0 \text{ if } i = j. \]
Verification and enforcement can thus be complementary as desired.

**Interpretation of swaps that fund interests on bonds.**—With bundled or mixed claims, suppose at least two parties issue positive claims ($\sum_j v_{ij} > 0$ at least two $i \in N$). From the perspective of parties as decentralized economies, then interest payments, funded by taxes levied abroad, on bonds newly issued to households at home, are paid with interest, funded by domestic tax payments, on bonds newly issued to households abroad. Thus, interests on bonds are swapped. Generally, interests on bonds are swapped, including the possibility that interest payments from swap claims paid and received are in the same direction for some parties.

**Stocks**

Governments issue securities, or stocks, to households. The number of stocks with claims on party $j$ each household in party $i$ wishes to hold and the number of stocks issued to the household are denoted by $s_j(i)$ and $z_j(i)$. The total number of stocks outstanding and issued to each household of party $i$ are

$$S_i \equiv \sum_j s_j(i),$$

$$Z_i \equiv \sum_j z_j(i).$$

As parties, through households and government, have claims on each other, stocks, or more precisely, their dividend payments, are swapped.

**Bundled claims.**—Governments promise payments they receive from other governments on the swap market by issuing stocks to households. Assuming that the composition of funds with payments from different parties can be verified, payments from party $i$ to households in the form of stock returns are given by $O_i = \lambda \sum_j z_i(j)$ with the dividend $\lambda$. Stocks have the price $\phi^\text{stock} \in \mathbb{R}_+$. The budget constraint of a representative household in party $i$ then reads

$$x_i + \phi^\text{stock} S_i = w_i + \pi_i + (\phi^\text{stock} + \lambda) Z_i + \tau_i. \tag{24}$$

Unsurprisingly, the budget constraint of the government of party $i$ can be stated as (22). Each household of party $i$ wishes to hold a finite number of stocks $s_j(i)$ only if the price of stocks vanishes, $\phi^\text{stock} = 0$, explaining why one can abstract from stock prices being specific to the governments of parties which issue stocks. Thus, equilibrium here entails no actual trade in stocks, $s_j(i) = z_j(i)$. 

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The initial distribution of securities, or stocks, \((z_j(1), z_j(2), \ldots, z_j(N))\), \(j \in N\), may be picked with no change in household consumption relative to an equilibrium with no stocks. For example, the dividend equals claims promised to a party divided by the number of outstanding stocks in the same party, \(O_i = \lambda Z_i\), using

\[ z_i(j) = 0 \text{ if } i \neq j, \]

The expression for claims \(O_i = \lambda Z_i\) gives the dividend of stocks. This example of bundled claims appears sensible as stocks issued to households at home may be enforced better than those issued to households abroad, given the current law enforcement as discussed with respect to bonds above. Verification and enforcement can thus be complementary as desired.

Mixed claims. — Analogously to bonds, each government of a given party obliged to make payments in swaps directly issues stocks to foreign households, and the respective foreign governments guarantee these payments. Guaranteeing payments appears in line with guarantees frequently made in foreign trade.

To focus on enforcement, abstract from redistribution effects by stocks. Thus let payments from party \(j\) to households of party \(i\) in the form of dividends on stocks be denoted by \(a_j(i) = \lambda z_j(i)\). With the number of stocks outstanding held by households of party \(i\) and the number of stocks issued to them \(S_i\) and \(Z_i\), the budget constraint of the government and of a representative household in party \(i\) are (23) and (24). Households of party \(i\) do not trade stocks, \(s_j(i) = z_j(i)\), and receive from foreign governments \(\lambda Z_i\).

Stock supplies here determine a common dividend. Interestingly, the distribution of stocks \((z_j(1), z_j(2), \ldots, z_j(N))\), \(j \in N\), automatically avoids redistributive effects as, analogously to bonds,

\[ z_i(j) = 0 \text{ if } i = j. \]

Verification and enforcement can thus be complementary as desired.

Interpretation of swaps that fund dividends on stocks. — With bundled or mixed claims, consider claims in swaps such that at least two parties issue positive claims \((\sum_j v_{ij} > 0 \text{ at least two } i \in N)\). From the perspective of decentralized economies, then dividend payments, funded by taxes levied abroad, on stocks newly issued to households at home, are swapped with dividends, funded by domestic tax payments, on stocks newly issued to households abroad. Thus, dividends on stocks are swapped. Generally, dividends
on stocks are swapped, including the possibility that dividend payments from swap claims paid and received are in the same direction for some parties.

5.3. Enforcement II: Balanced trade

This subsection derives balanced trade of claims in swaps. Balanced trade defined in two forms as swap payments of equal magnitude between each one party and all its trading partners in swaps and each two parties contracting on swaps are desirable as they are not renegotiated, hence can be said to be agreed upon by the parties involved in trading claims, or self-enforced. I can thus assume here that the parties can renegotiate swaps. Equilibrium can then literally mean that at some time each party makes promises, for example, at a conference, and later honors the promises they have made.

Briefly, a swap of a given party would not be renegotiated after external effects have been imposed, or fuel has been used, with the consent of all its counterparties (only the counterparty in this swap) if swaps were balanced (the swap was balanced), as then a party gained from renegotiation only if the parties that need to (party that needs to) approve would lose. Balanced trade in swaps \( \sum_j(v_{ij} - v_{ji}) = 0, \text{ all } i \in N \) appears to be strongly renegotiation-proof, and bilaterally balanced trade in swaps \( (v_{ij} - v_{ji}) = 0, \text{ all } (i, j) \in N \) appears robust to a weak form of renegotiation, see Section 5.4.

Let us sharpen swaps further by requiring weak verification and enforcement. As lump-sum taxes (negative rebates) may not be enforced, I use the notion of weak enforcement to express that swaps do not use lump-sum taxes, \( T_i \geq 0, i \in N \). Balanced trade then requires, for all parties or a division of the set of all parties into parties and a coalition of parties in the collection \( H \), the following.

Condition 1. (Nonnegative swaps) \( \delta_i \leq \Delta, \text{ all } i \in H, \text{ strict at least two } i \in H \).

This condition means that at least two sets of parties become implicitly taxed for reducing the public good and none becomes implicitly subsidized for it. This holds, for example, if all parties or parties and a coalition of parties do not like fuel use, and at least two like reductions of fuel use \( \delta_i > 0 \text{ at least two } i \in H \), or a coalition likes reductions of fuel use and all nonmembers, at least two in number, are unaffected by fuel use \( \delta_i = 0 \text{ all but one } i \in H \).\[16\]

\[16\] Another reason to study swaps with no implicit subsidies to reducing the public good
Implications of balanced trade for climate policy.—Clearly, in the absence of lump-sum taxes, optimal trade of claims in swaps can be balanced only if, for each party, the external cost imposed on some parties outweighs the external benefit imposed on other parties. This means, if necessary, a modification of the third viewpoint on climate policy. For example, a climate coalition composed of $(N - n)$ parties needs to absorb a party that likes fuel use, if the remaining parties are unaffected by fuel use, $1 < n < N - 2$. With no lump-sum taxes, optimal trade of claims in swaps can be balanced only if at least two parties impose negative externalities, hence tax reducing the public good to attain an optimum ($\alpha_i < 1$ at least two $i \in N$). Thus to enforce swaps by balanced trade, the second viewpoint of climate policy qualifies. Optimal trade of claims in swaps can be balanced if all parties dislike fuel use and there is no coalition, and hence efficiency in the absence of lump-sum taxes does not require a coalition to form, in the first viewpoint on climate policy.

For simplicity, albeit with slight abuse of language, I will call each member of the set $H$ a party in what follows in this subsection.

Balanced trade.—Denote by $H$ the number of parties in $H$. With two parties, $H = 2$, weak verification, weak enforcement, and balanced trade require that these parties issue the same values $((1 - \alpha_1)g_1 = (1 - \alpha_2)g_2 > 0)$. To characterize swaps involving more than two parties, suppose the triangle inequality that, out of three values, the sum of any two be no less than the remainder value.

**Proposition 3.** Under Condition 1, with three or more parties, $H \geq 3$,

(i) Equilibrium swaps can be weakly verified, weakly enforced, and balanced if the triangle inequality holds for the three largest values issued by parties.

(ii) Some equilibrium swaps that can be weakly verified, weakly enforced, and balanced, are bilaterally balanced, $v_{ij} = v_{ji}$, all $(i, j) \in N$.

Efficiency therefore results with swaps under restrictions yielded by weak verification, weak enforcement, and the ability to renegotiate swaps, given the mild assumption that the three largest values of claims issued by parties are of similar size. Notice that $H \geq 3$ captures the second viewpoint on climate

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is that with no side payments as required for weak verification, balanced swaps can be trivially found with short sale of gross product with respect to another party ($\eta_{ij} < 0$, some $(i, j) \in N$), which means that some party $j$ subsidizes gross product of party $i$. 

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policy with minimum two trading parties. As an immediate implication, an international environmental agreement arises as a set of renegotiation-proof swap contracts.

To illustrate how balanced swaps can be characterized, consider an example with five parties \((N = 5)\) where \(N = H\). In this example, payments in swaps form the directed network with nodes designating parties and edges representing claims Figure 2 shows. The parties are listed in descending order of the value they relinquish to other parties \((d_1 \geq \ldots \geq d_5)\), where \(d_i = \sum_j v_{ij}\). Assume that all parties prefer the public good. Then one set of renegotiation-proof, weakly verified, and weakly enforced swap contracts involves party 1 to obtain claims on every other party, and issues claims to party 2 and 3 \((b = d_2\) and \(a = d_1 - d_2)\). Party 2 bilaterally trades with party 1 (receives from and sends to \(b\) and \(a + b - c\), where \(c = d_3\)) and makes payments to party 3 \((c - a)\). Party 3 bilaterally trades with party 1 (receives from and sends to \(a\) and \((c - f)\), where \(f = d_4\)), makes payments to party 4 \((f)\), and receives payments from party 2. The remaining parties’ claims follow a pattern that applies generally, with five or more parties. Party 4 makes

Fig 2. Balanced trade of swaps: Example with five parties \((N = 5)\)
payments to the first party and the own successor, *party 1* and 5 (*f* − *g* and *g*, where *g* = *d*₅), and receives payments from the own predecessor, *party 3*. Finally, the last one, *party 5*, transfers claims on own gross product to *party 1* (*g*) and receives a portion of its predecessor’s, *party 4*’s, gross product in equal magnitude. The triangle inequality on the top three values ensures that *party 3* has a positive claim to *party 2*’s gross product. The sorting of the values guarantees that the remaining claims are positive. Another example of renegotiation-proof, weakly verified, and weakly enforced swap contracts can be derived from reversing the directions of payments in this example.

By way of examples proving Proposition 3, swaps with different ownership structures can implement the same allocation.

**Corollary 2.** In equilibrium swaps, ownership of positive claims on some party *i* can be diversified (*η*ᵢⱼ > 0 at least two *j*) or concentrated (*η*ᵢⱼ > 0 at most one *j*).

In the example shown in Figure 2, ownership of claims to gross product of *parties 1* to 4 appears diversified, while ownership of claims to *party 5* appears concentrated.

### 5.4 Renegotiation

The purpose of this subsection is to show that swaps which are balanced are robust to renegotiation.

Suppose that parties can only make nonbinding contracts, because they can renegotiate the terms of swaps. A party can renge on their promise to meet the terms of a swap contract with no punishment.¹⁷ Consider an adaptation of the model in Section 3 requiring that, at stage 3, thus after fuel use has occurred, before payout of swaps, no party deviates from swaps agreed upon at stage 1. Out of equilibrium a party can deviate from a swap. Notice that party *i* makes a surplus from a swap (*v*ᵢⱼ < *v*ᵢⱼ) only if the counterparty *j* makes a loss from the swap (*v*ᵢⱼ > *v*ᵢⱼ).

Equilibria with swaps are said to be weakly renegotiation-proof if, to renegotiate a swap, a party involved in the swap requires the consent of all parties contracting in swaps with this party. A renegotiation leads to no gain or loss

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¹⁷ This contrasts the notion of renegotiation-proofness in Rubinstein and Wolinsky (1992) where two parties find it mutually beneficial to renegotiate after more information has arrived.
on all swaps taken together. Thus, incentives to renegotiate swaps are absent if the payments in swaps between any given party and all other parties are zero on net. Any party making a loss from the payout of swaps would renege on their promise to make payments. A weakly renegotiation-proof equilibrium then involves only balanced swaps, that is, with zero net payment between any given party and all other parties, because these swaps are not renegotiated.

Equilibria with swaps are said to be strongly renegotiation-proof if, to renegotiate a swap, a party involved in the swap requires the consent of the counterparty in the given swap. Though in the latter concept less parties need to give consent, the number of swaps makes the concept more restricted. A renegotiation leads to no gain or loss on any swap. Incentives to renegotiate are absent if all swaps pay in and out at equal magnitude. Any party making a loss from the payout of a swap would renege on their promise to make payments. A strongly renegotiation-proof equilibrium then involves only bilaterally balanced swaps, that is, with zero net payment between any two parties, because these swaps are not renegotiated.

5.5. Equilibrium concept

In Section 3, some party can obtain lower utility in equilibrium than with no swaps as the first ‘proposal’ negotiated is undetermined. Despite the Pareto optimality of resource use, one may ask if swaps cannot do worse for all parties using other rules of the game. The mechanism that follows ensures this, thus Pareto improves. Notice that Pareto improving allocations can be implemented, because continuous transfers (side payments or permits for resource use) are available, which is a version of the second welfare theorem.

Equilibrium uses a protocol on events in two substages in the first stage of the game. In substage 1, all parties submit proposals about swaps between any two parties and can submit an arbitrary number of proposals, or make no proposal. In substage 2, the parties accept proposals. Parties vote either “approve” or “not approve” on every proposal made. Any proposal that will be approved by all parties will be said to be accepted. When less than $N$ proposals become accepted, the outcome will be the same as without swaps. If at least $N$ proposals are accepted, one of them will be randomly chosen to form binding contracts. With no further refinement, I let break equivalence in favor of an efficient outcome. Below, I discuss a refinement not needing
this preference.

The two main insights from the protocol investigated here can now be stated.

**Proposition 4.** With no collective action, \( M = \emptyset \), and voting, equilibria with swaps exist and are characterized by (i) efficiency, and (ii) that every party’s utility weakly improves, compared to no swaps.

Equilibrium allocations are characterized by efficiency and determined as the special allocations in which \((N - 1)\) parties receive their outside option value. The assumptions in the protocol leading to these insights are now being discussed.

The unanimity in accepting proposals prevents inefficiency as parties are eager to make proposals which lead to efficiency to increase their own payoff. Thus swaps exploit gains from trade as in the basic equilibrium concept in Section 3. As in equilibrium defined with approving changes to given swaps in Section 3, if swaps contracted in stage 1 allocated resources in a different way, then two parties could gain from a different swap and obtain the consent of other parties for it. Here, in equilibrium with approving proposed swaps, any party would block contracts that make themselves worse off than in the outside option which arises with no swap contracts.

A minimum acceptance clause prevents that a party does not approve any proposal another party prefers. The clause implies that of \( N \) accepted proposals one becomes implemented. With no minimum acceptance clause, \( N \) equilibria exist with certain outcome as equivalence is broken in favor of efficiency. The minimum requirement would become obsolete to define an equilibrium, if a party was fixed or selected randomly as the single proposer.\(^{18}\)

Multiple proposers and proposals are not necessary for efficiency but desirable. Allowing for multiple proposers improves expected utility for all but one party compared to a fixed single proposer as each of the \( N \) proposals made favors increasing a different party’s utility relative to free-riding with no swaps. There are thus also multiple equilibrium allocations compared to a fixed single proposer. Expected utility would be the same for all parties with a randomly selected proposer as with the mechanism used, if to determine a set of contracts that form, one attaches the same probabilities

\(^{18}\) With no minimum requirement, if exactly one proposal becomes accepted, this one forms binding contracts, and if multiple proposals are accepted, one of them will be randomly chosen to form binding contracts.
to the selection of the proposer as to accepted proposals. Hence, multiple proposers improve utility in expectation, or overcome any practical difficulty that exists with selecting a proposer at random and that does not prevail with randomly choosing an accepted proposal, for example, from fairness considerations. Allowing to make multiple proposals, while not being required for efficiency compared to the alternative that each party makes at most one proposal (approve-or-leave-it offer), appears attractive as it enables the use of a device for coordination on one proposal. Such a device can focus on a range of welfare changes for all parties relative to the outcome with no swaps as described next.

A further refinement yields strictly improved utility for all parties and does not require to break equivalence in favor of an efficient outcome. One can guarantee each party utility above the outside option value arising with no swap contracts ($U_i$). To do so, restrict the voting process to deliver acceptance or binding contracts only if the outcome yields utility a certain percentage above the outside option value for all parties ($U_i \geq (1 + \epsilon)\bar{U}_i$, $i = 1, 2, \ldots, N$). Provided $\epsilon > 0$ is not too large, this constraint can be accommodated, and will be binding for $(N - 1)$ parties in equilibrium as it yields reservation utility above the outside option value for all parties. This refinement to yield strictly improved utility for all parties appears noncredible and thus needs commitment.

The mechanism just described includes approving proposed swaps to ensure a Pareto improvement. With this mechanism, enforcement with secondary claims, or with balanced trade in primary claims, as with the simple mechanism leaving free the first ‘proposal’ to be negotiated, can be used.

**6. Extensions of the economic environment**

In this section, I analyze extensions of the model to show that the efficiency result appears robust with respect to other modeling choices of technology (heterogeneous impact of intermediate goods on public good; green technology; deforestation) and dynamics (with labor input, and with investment goods). The extension with deforestation builds on the extension with heterogeneous impact of intermediate goods.
6.1. Heterogeneous impact of intermediate goods on public good

In Section 3 the intermediate good fuel reduces the public good by the same number of units. Different, Holland & Yates (2015) analyzed nonuniformly mixed emissions of nitrogen oxides. The main purpose here for deviating from uniformly mixed emissions is to represent average fuel combinations that may vary by location relevant to simple models of climate change. Considering an emissions intensity differing from one also reinforces the understanding of (10) and serves as a preliminary step to analyze swaps with multiple sources influencing a public good.

Let $b_i$ be the number of mixed emissions units per unit of fuel use by party $i$. Then fuel use by party $i$ has the marginal impact on party $j$ given by the coefficient $b_i$ multiplied by the marginal utility of the public good $U_{ij}$. The linear transformation frontier between fuel and the public good can be stated as

$$ y = \bar{y} - \sum_i b_i r_i. $$

**Uniform impact of private good on public good.** To reinforce the understanding of (10), let the impact of the private good fuel onto the public good $b_i = b$ be identical over parties. Then naturally efficiency requires to retain the portion of gross product

$$ \alpha_i = \frac{b \gamma_i}{b + \varepsilon} + \frac{\varepsilon}{b + \varepsilon}. $$

The relative valuation of the impact of party $i$'s fuel use on the public good ($b \gamma_i$) compares to the unit impact ($b$) skewed with the aggregate valuation of the public good per unit of fuel ($\varepsilon$). The aggregate valuation of the public good per unit of fuel appears common to all parties because fuel can be traded between all parties. The portion of gross product retained $\alpha_i$ can thus be interpreted as the individual relative valuation of the public good in terms of fuel as indicated in Section 3.

Having uniform impacts with $b \neq 1$ serves as a preliminary step to analyze swaps with multiple sources that have effects on a public good such as fuel use and deforestation.

**Nonuniform impact of private good on public good.** To account for different fuel mixes in different locations, the impact of the private good in question onto the public good $b_i$ (with $b_i \neq b_j, i \neq j$) contributes a source of
heterogeneity for the portion of gross product retained

\[ \alpha_i = \frac{b_i \gamma_i}{b_i + \varepsilon} + \frac{\varepsilon}{b_i + \varepsilon}. \]

The relative valuation of the impact of party \( i \)'s fuel use on the public good \((b_i \gamma_i)\) and the unit impact \((b_i)\) now differ between parties with same relative marginal utility of the public good \((\gamma_i)\). The aggregate valuation of the public good per unit of fuel \((\varepsilon)\) appears the same for all parties because fuel can be traded. The interpretation of \( \alpha_i \) as the individual relative valuation of the public good in terms of fuel continues to hold.

### 6.2. Green technology

This subsection extends the model to production using intermediate goods such as energy derived from fossil fuel, harming the environment, and intermediate goods, for example renewable energy, which do not harm the environment. To keep the model simple, each one dirty and clean intermediate good are produced and consumed by each party.

Consumption goods can be produced using clean energy \( m_i^c \), dirty energy \( m_i^d \), and labor \((1 - n_i)\) according to \( y_i^Y = \mathcal{F}_i^Y(m_i^c, m_i^d, 1 - n_i) \). Energy in turn uses labor, \( m_i^d = \mathcal{M}_j^d(n_j) \), \( j \in \{c, d\} \). One unit of dirty energy requires \( \psi > 0 \) units of fuel, \( r_i = \psi m_i^d \). All labor is used in production, \((1 - n_i) + n_i^c + n_i^d + n_i^R = 1\). Equation (3) gives the environmental quality \( y \). Notice that I adopt a formulation of the climate problem which can express fossil and renewable energy at various substitution strengths.

Efficiency of consumption \( x_i \), labor used in dirty energy production \( n_i^d \), fuel use \( r_i \), and labor used to produce fuel \( n_i^R \) requires that (with party \( j \) producing fuel)

\[ \gamma_i \left[ \left( \frac{\mathcal{F}_{di}^Y \mathcal{M}_{ni}^d - \mathcal{F}_{1-ni}^Y}{\psi \mathcal{M}_{ni}^d} \right) - \frac{\mathcal{F}_{1-nj}^Y}{\mathcal{F}_{nj}^R} \right] = \delta_i, \quad i = 1, 2, \ldots, N. \]

In stage 2, party \( i \) chooses consumption \( x_i \), labor \( n_i^c, n_i^d, n_i^R \), and fuel use \( r_i \) subject to the budget constraint

\[ x_i + p r_i = \alpha_i [\mathcal{F}_i^Y(\mathcal{M}_i^c(n_i^c), \mathcal{M}_i^d(n_i^d), 1 - n_i) + p \mathcal{F}_i^R(n_i^R)] + T_i. \]
The choices of the parties of consumption, labor used to produce dirty goods and fuel, and fuel use deliver
\[ \alpha_i \left( \frac{\mathcal{F}_{d1}^Y \mathcal{M}_{ni} - \mathcal{F}_{1-ni}^Y}{\psi \mathcal{M}_{ni}^d} \right) - p = \delta_i, \quad i = 1, 2, \ldots, N, \]
and the fuel price (9). Both efficient planning and the parties’ decision-making imply, through the choice of labor used in clean production
\[ F_{ci}^Y \mathcal{M}_{ni}^c \leq \mathcal{F}_{1-ni}^Y, \quad \text{with equality if } n_i^c > 0. \]

Eliminating the term in parentheses and rearranging implies the portion of gross product retained \( \alpha_i \) given in (10), which yields the tax implicit in swaps, as before.

Efficiency may require to produce not all intermediate goods depending on the substitutability of the inputs and the tastes for the public good. Notice that swaps can implement such an efficient allocation, hence one with no fuel use.\(^{19}\)

### 6.3. Deforestation

This subsection extends the model to two sources of emissions to analyze emissions from the use of fossil fuels and deforestation through the use of old-growth forest for wood or conversion into arable land. Alternatively, one can consider the use of different fossil fuels.

Consumption goods can be produced using energy \( m_i^1 \), wood products \( m_i^2 \), and labor \( (1 - n_i) \) according to \( y_i^Y = \mathcal{F}_i^Y(m_i^1, m_i^2, 1 - n_i) \). I abstract from the use of labor in energy and wood production. One unit of energy requires \( \psi^1 > 0 \) units of fuel, and of wood products requires \( \psi^2 > 0 \) units of forest, \( r_i^j = \psi_j m_i^j, j \in \{1, 2\} \). Fuel and forest can be produced using labor \( n_i^R \) and \( n_i^O \) according to \( y_i^R = \mathcal{F}_i^R(n_i^R) \) and \( y_i^O = \mathcal{F}_i^O(n_i^O) \). All labor is used in production, \((1 - n_i) + n_i^R + n_i^O = 1\). Fuel and forest use diminish the environmental quality \( y \). With the intensities of \( b^1 > 0, b^2 > 0, \)
\[ y = \bar{y} - b^1 \sum_i r_i^1 - b^2 \sum_i r_i^2. \]

\(^{19}\)The fuel price vanishes, \( p = 0, F_{ni}^R \rightarrow \infty \) as \( n_i^R \rightarrow 0 \). Then \( \alpha_i[(F_{di}^Y \mathcal{M}_{ni} - \mathcal{F}_{1-ni}^Y)/(\psi \mathcal{M}_{ni}^d)] - \mathcal{F}_{1-ni}^Y/F_{ni}^R \leq \delta_i \) implies that \( \alpha_i \leq (\gamma_i + \varepsilon)/(1 + \varepsilon) \).
Fuel and forest can be traded between parties,

\[ \sum_i r_i^1 = \sum_i y_i^R, \]

\[ \sum_i r_i^2 = \sum_i y_i^O. \]

Efficiency of consumption \( x_i \), labor used in fuel and forest production \( n_i^R \) and \( n_i^O \), and fuel and forest use \( r_i^1 \) and \( r_i^O \) requires that (with party \( j \) and \( k \) producing fuel and forest)

\[ \gamma_i \left[ \left( \frac{F_{1i}^Y - F_{1-ni}^Y}{\psi_1} \right) - \frac{F_{1-j}^Y}{F_{nj}^R} \right] = b^1 \delta_i, \quad i = 1, 2, \ldots, N, \]

\[ \gamma_i \left[ \left( \frac{F_{2i}^Y - F_{1-ni}^Y}{\psi_2} \right) - \frac{F_{1-k}^Y}{F_{nk}^O} \right] = b^2 \delta_i, \quad i = 1, 2, \ldots, N. \]

Fuel and forest trade at the prices \( p^1 \) and \( p^2 \), and are subsidized at the rates \((1 - \sigma^1)\) and \((1 - \sigma^2)\) (or taxed at the negative of these rates). In stage 2, a price-taking party \( i \) chooses consumption \( x_i \), labor \((n_i^R, n_i^O)\), and natural resource use \((r_i^1, r_i^2)\) viewing the budget constraint

\[ x_i + \sigma^1 p^1 r_i^1 + \sigma^2 p^2 r_i^2 = \alpha_i [F_i^Y(m_i^1, m_i^2, 1 - n_i) + p^1 F_i^R(n_i^R) + p^2 F_i^O(n_i^O)] + T_i. \]

The choices of the parties of consumption, labor, and resources deliver

\[ \alpha_i \left( \frac{F_{1i}^Y - F_{1-ni}^Y}{\psi_1} \right) - \sigma^1 p^1 = b^1 \delta_i, \quad (25) \]

\[ \alpha_i \left( \frac{F_{2i}^Y - F_{1-ni}^Y}{\psi_2} \right) - \sigma^2 p^2 = b^2 \delta_i, \quad (26) \]

and the analogues to equation (9) giving the fuel and forest price

\[ p^1 = \frac{F_{1-ni}^Y}{F_{ni}^R}, \]

\[ p^2 = \frac{F_{1-nk}^Y}{F_{nk}^O}. \]
Eliminating the terms in parentheses in (25) and (26) and rearranging implies two equations which can be solved for combinations of the implicit tax on gross product \((1 - \alpha_i)\), fuel \((\sigma_i^1 - 1)\), and forest \((\sigma_i^2 - 1)\). Let us use the degree of freedom obtained from introducing the two instruments on fuel and forests to fully offset taxes with subsidies, \((1 - \sigma^1)p^1r^1 + (1 - \sigma^2)p^2r^2 = 0\), omitting a subscript for party. Resulting is the portion of gross product retained after swaps and complementary policies on fuel and forests are accounted for

\[
\alpha_i = \frac{E_i\gamma_i}{E_i + (\varepsilon^1r_i^1 + \varepsilon^2r_i^2)} + \frac{(\varepsilon^1r_i^1 + \varepsilon^2r_i^2)}{E_i + (\varepsilon^1r_i^1 + \varepsilon^2r_i^2)}, \quad i \in N.
\]

With a single source or multiple sources of emissions, swaps implicitly tax gross product using the same principle. The emissions \(E_i = (b^1r_i^1 + b^2r_i^2)\) generalize the emissions \(br_i\); and the weighted sum of resource use \((\varepsilon^1r_i^1 + \varepsilon^2r_i^2)\) generalizes the product \(\varepsilon r_i\), when premultiplying both the numerator and denominator in (10) by the amount of fuel \(r_i\).

6.4. Dynamics

In this subsection, I consider environmental quality and capital stocks that evolve over time. The number of periods, \(T > 1\), can be finite or infinite \((T < \infty\) or \(T = \infty\)). Each party discounts utility between successive periods with the time discount factor \(\beta \in (0, 1)\).

To formalize, a value function for each party gives the present discounted value of utility depending on all stocks predetermined before current decisions are taken. The value function contains time as an argument, and thus refers to time as a state, which can summarize technological change.

Environmental stock.—Consider the evolution of the global public good, or environmental stock,

\[
1 - y' = \Psi(r_i + \sum_{j \neq i} r_j, 1 - y),
\]

where prime denotes next period. As fuel use and the current stock reduce and amend the future stock, the partial differentials of \(\Psi\) with respect to its first and second argument are given by \(\Psi_1 > 0\) and \(\Psi_2 > 0\). The environmental stock \(y\) is the only payoff-relevant stock which amounts to a public good or bad to a specific party according to how the stock enters the utility
function of each party that likes or dislikes reduced fuel use. Time flows discretely, as $t' = t + 1$ starting with some $t$, $t \leq T$.

Decisions appear as follows. In each period, there are three stages. In the first stage, parties contract on swaps $((\xi_{ij}, t_{ij}), \forall (i, j) \in \mathbb{N})$ implying the portion of gross product retained and incoming payments from swaps $((\alpha_i, T_i), i = 1, 2, ..., N)$. In the second stage, each party $i$ solves the problem

$$V_i(y, t) = \max_{n_i, r_i, x_i} \{U_i(x_i, y) + \beta V_i(y', t')\}$$

subject to the budget constraint (6) and the transition of environmental quality (27) taking as given the fuel price $p$, the share of own gross product retained $\alpha_i$, net transfers $T_i$, and the current environmental stock $y$. In the third stage, swaps pay out. The parties thus look forward to the evolution of environmental quality (27).

Environmental and reproducible stock.—To obtain technology stocks, I will now introduce investment in dirty and clean technologies. The stock of environmental quality $y$ evolves according to (27). Denote by $R$ the vector of all parties’ reproducible stocks $R = \{R_{d1}, R_{d2}, ..., R_{dN}, R_{c1}, R_{c2}, ..., R_{cN}\}$. Here $R_{di}$ and $R_{ci}$ represent the stock of reproducible capital in dirty and clean technology $d$ and $c$ of party $i$. For simplicity, reproducible capital fully depreciates within one period; and dirty energy uses fuel and capital, and clean energy uses capital only.

Decisions in each period occur as follows. In the first stage, parties choose swaps $((\xi_{ij}, t_{ij}), \forall (i, j) \in \mathbb{N})$ determining the portion of gross product retained and incoming net payments from swaps $((\alpha_i, T_i), i = 1, 2, ..., N)$. In the second stage, each party $i$ solves the problem

$$V_i(R, y, t) = \max_{R_{di}, R_{ci}, x_i} \{U_i(x_i, y) + \beta V_i(R', y', t')\}$$

subject to the budget constraint

$$x_i + p r_i = \alpha_i \left[ F_i^d(M_{di}(R_{di}, r_i), M_{ci}(R_{ci}), 1 - n_i) - R_{di}' - R_{ci}' + p F_i^R(n_i) \right] + T_i$$

and the transition equation (27), taking the fuel price $p$, the share of gross product retained $\alpha_i$, net transfers $T_i$, and the stocks $(R, y)$ as given. In the third stage, swaps pay out.\(^{20}\)

\(^{20}\)With environmental stock, or with environmental and reproducible stock, the value
Analysis.—As in the one-period economies, swaps lead to efficiency. Using the equilibrium concept that no two parties can improve their welfare with the consent of other parties as in Section 3 and 4, swaps exploit gains from trade. Using the equilibrium concept that parties make and accept proposals as in Section 5.5, swaps also exploit gains, and in addition, determine the special equilibrium allocations in which \((N-1)\) parties receive their outside option value.

The outside option value for each party as indicated by no swaps depends on the current stock in a Markov perfect equilibrium. In equilibrium, stocks take efficient levels. In particular, with investment in clean and dirty technology, there appears no hold-up problem, because emissions and investment cannot be contracted. Swaps can be contracted which gives incentives to efficiently choose emissions and investment.

I will now characterize the tax on gross product implicit in the swaps. To account for the dynamic provision of the public good, define the analogues to \(\gamma_i\) and \(\varepsilon\) at date \(t\) as

\[
\gamma_i^* \equiv W_i \mathcal{V}_{yi}(\cdot,t')/\sum_{j \in N} W_j \mathcal{V}_{yj}(\cdot,t'), \quad i \in N,
\]

\[
\varepsilon^* \equiv W_i \mathcal{U}_{xi}/\mathcal{F}_{1-nk}^{\gamma} \psi_1 \sum_{j \in N} W_j \mathcal{V}_{yj}(\cdot,t'), \quad i \in N, \quad n_k \in (0,1).
\]

The conditions for social and private optimality can now be aligned when every party consumes a positive amount \((x_i > 0 \text{ all } i)\) with the portion of gross product retained through swap contracts \(\alpha_i = (\gamma_i^* + \varepsilon^*)/(1 + \varepsilon^*)\). The tax rate implicit in swaps equals \((1 - \alpha_i^*)\).

Different durations of agreements on swaps beside a one-period agreement investigated above can arise, provided the conditions in Section 5.3 hold, because the swap contracts would not be renegotiated. There can be an Arrow-Debreu (AD) agreement contracting swaps for the whole time horizon in the first stage of the initial period. There can be a sequence of agreements on swaps enduring more than one period as a medium case between a sequence of one-period agreements and an AD agreement.

\text{Function evaluated at the payoff-relevant state given at the date } t \text{ can be stated as } \mathcal{V}(\cdot,t) = \sum_{t'=1}^T \beta^{t-t'} \mathcal{U}(x_{tr},y_{tr}). \text{ Equilibrium can be straightforwardly defined similar as in Section 2 with decisions now occurring in a sequence of periods.}
With equilibrium as defined in Section 2, the game has multiple equilibria, all of which are weakly renegotiation-proof, as defined by Farrell & Maskin (1989). Adopting the voting architecture from Section 5.5 in each round of determining swaps, the game appears stochastic, and as such, equilibrium is strongly renegotiation-proof in the sense of Farrell & Maskin (1989).

7. Relationship to the literature

This section describes the paper’s contributions to the literatures on trading with externalities and international environmental policy.

7.1. Trading with externalities

Contributions of the paper to welfare economics, linked markets for emissions permits, mechanism design, and contracting with externalities are discussed.

i. This paper lays down a form of pricing of external cost using a two-sector model with general tastes and technology. As is well-known from literature on welfare economics, Pigou or Lindahl pricing can correct externalities (see Meade (1952) or Lindahl (1919) and Arrow (1970)).

   However, overlooked, in a production economy, product taxes account for the missing markets of external effects under the reasonable assumption that a party views the effect of their own resource use on a public good. From the perspective of a party as a whole economy providing a public good, for example a country or state in the important class of public good problems framing environmental externalities, the gross product taxes that achieve efficiency can be implicit when being interpreted as shares of gross product ceased to other parties. The reciprocal nature of such claims lends to swaps.

   To internalize public good externalities, Pigovian taxes or resource permits, such as in related proposals for climate policy described below, in the literature raise the cost of polluting (precisely, in proposals for climate policy, the direct cost of own emissions or opportunity cost of nonparticipation in cooperation by a party). Contrasting this intuition, this paper proposes to reduce the benefit from polluting with implicit taxes on output measures. For the sake of generality, including the second viewpoint on climate policy

   \[21\text{A translation of Lindahl's relevant fourth Chapter into the English language entitled “Just Taxation - A Positive Solution” appeared in Musgrave & Peacock (1958).}\]
that there are parties unaffected by a public good, the paper uses one particular simple price system when externality-creating goods are tradeable, which can be interpreted as an implicit tax on gross product formed by the value of final goods, the value of externality-generating intermediate goods, and the trade surplus of between-parties tradeable nonexternality-generating intermediate goods. One reason for emphasizing swaps of output in the form of gross product that output may be easier to verify than emissions important with environmental externalities given no transfers between parties. An extension considers raising the cost of polluting by including environmental tax or permit revenue in swaps.

ii. This paper shows that permits covering resource use proportional to public good provisions can transfer wealth. Showing this form of transfers alternative to side payments, conceptually and by using an allowance ratio, contributes to literature on linked markets for emissions permits.

In the literature, markets for emissions permits have a role for parties acting alone or noncooperatively. Parties that act alone can benefit from linking permits markets, because the issue of permits by one region strategically alters the price of abatement also to the other region (Holtsmark & Midttømme 2015) or in the presence of heterogeneous cost (Flachsland, Marschinksi & Edenhofer 2009). Copeland & Taylor (1995) show that tradability of pollution permits issued by noncooperative regions can prevent global pollution from increasing when the regions open up for trade. See also Chichilnisky (1994) and Pethig (1976) for the desirability of permit trade between noncooperative regions. Copeland & Taylor (2005), however, find that permit trade can make noncooperative parties worse off because of trade in consumption goods. In other analyses, such as Harstad (2016), wealth-transferring emissions permits or side payments do not affect the outcome of parties negotiating emissions, as the parties are identical. McGinty (2007) distributes emissions permits among coalition members to achieve transfers, but neglects the effect fuel use has on own welfare in choosing emissions.

This paper establishes a novel role of linking permit markets: transferring wealth to substitute for side payments so to better verify the correction of externalities when parties are not identical. This takes into account the effect fuel use has on own welfare in choosing emissions. To attain efficiency, the ratio of permits tradeable on markets between parties must account for heterogeneous tastes for emissions reductions.

iii. This paper develops a contracting environment, or mechanism, for
public good provision among several parties in a general class of economies. The rules of the game that are key for attaining efficiency are shared ownership with payments in the form of swaps, unanimous approval by parties to change given swaps in a first stage, and choices of public good provisions in a second stage. To unfold the unanimity rule in negotiation, equilibrium either uses the assumption that the parties bilaterally negotiate or propose swaps for all parties.

Given the lack of a regulator, I develop institutions to enforce contracts when parties lack commitment to contracts by enforceable assets derived from claims or by self-enforced trade in claims. The development of institutions, or commitment devices, that help to achieve efficiency with a public good with no commitment are novel relative to the literatures on mechanism design and contracting with externalities.

The rules of the game that are key for attaining efficiency, and derived enforcement in the environment, advance contracting with implicit policy under externalities proposed in the form of refunding and contracting on an input using a deposit market. In refunding schemes (Gerber & Wichardt 2009, Gersbach & Winkler 2011), parties pay a fee and receive payments contingent on own actions. This paper shows that to derive enforcement of such payments, one may specify the recipients of the payments. For example, the payments can be enforced by further promising them in a form that can be enforced, or balancing trade between parties that becomes self-enforced. Swaps specify the recipients of payments by construction. In a market for fuel deposits with multiple parties harmed, in the first viewpoint on climate policy, efficiency can be attained if parties can commit to not use fuel deposits they have bought (Section IV.D in Harstad (2012a)). If they cannot commit, coordination after fuel deposits have been traded has a role for efficiency. This paper shows that to prevent the need of commitment to not use fuel, or coordination, to reach efficiency, one may set incentives for fuel use with swaps that pay out after production, and accomplish this with self-enforcement.

Mechanisms to correct externalities including the provisions of pure public goods have been proposed by Varian (1994) and Danziger & Schnytzer (1991), using Lindahl price announcements when a regulator that possesses incomplete information about technology can enforce payments, and agents are fully informed. Duggan & Roberts (2002) provide a variant with explicit pollution and payments to or from a regulator. In contrast, in this paper,
payments aggregate Lindahl prices, allowing to implement allocations beside
the Lindahl allocation. This allocation is not renegotiation-proof—as it does
not balance out claims—when parties’ tastes for the public good are strongly
heterogeneous. In these papers and the current analysis, incentives to com-
ply and participate do not trade off, as parties are fully informed. While
there is a trade off in other mechanisms with incomplete information among
parties, for example, in Martimort & Sand-Zantman (2016) on pollution, the
equilibrium concept differs.

A large literature studies features of commitment to payments from several
agents to a principal, such as between retailers and a manufacturer in De-
quiedt & Martimort (2015), or reversely, from a principal to several agents,
such as between an environmental protection agency and resource owners in
Harstad & Mideksa (2017), when there are externalities among the agents.
Segal (1999), unifying applications and emphasizing the direction of exter-
nalities between agents, shows that efficiency can be attained if every agents’
payoff in the outside option is independent on the other agents’ trades. Many
applications of contracting with externalities contain a positive payment be-
tween a principal and a given agent in an obvious direction.

In contracting between agents with multi-sided externalities, I show that
unanimity in approving changes to given trades or approving trades yields
efficiency. This condition allows agents to block an outcome worse than
their outside option such that the blockade is independent on every other
agent’s trades. This condition implies that in equilibrium swaps, ownership
of claims can be diversified or concentrated. In addition, between equilibria
yielding the same allocation, I show a trade flow indeterminancy. In one
example, payments that achieve efficiency can go from the party that imposes
the highest external costs on all others or from all others to that party, by
transposing payment flows in a given payment structure.

Similarly to contracting on a polluting input in implicit policy under exter-
nalities proposed in the literature (Böhm 1993, Hoel 1994, Harstad 2012a),
the forward trade in claims in swaps utilizes that enforced property rights to
public good provision are missing. Property rights that shape the incentives
for the provision may be established first so to control the social costs (and
benefits) from reduced provisions to a public good.

The unanimity in approving contracts avoids the inefficient no-contract
outcome. This helps to better understand what is required for Coase’s (1960)
conjecture to hold that parties find an efficient outcome given well-defined
property rights and zero transaction costs. In Ellingsen & Paltseva (2016), with two parties efficiency results, because, in line with this paper, then unanimity prevails, but with three or more parties may fail. This paper shows that, with three or more parties, efficiency can be obtained in contracting with externalities using unanimity in approving contracts—with or with no commitment to not renegotiate. Thus, rules of unanimity in negotiation support the creation of property rights in the form of claims in swaps. This reveals that existing property rights, thought to be an assumption in Coase (1960), merely determine the outside option of each party—new ones may be created so to internalize the externalities. This agrees with Demsetz (1967), according to whom agents set up property rights when net benefits from internalizing externalities between these agents are positive. This paper goes beyond this argument by showing rules of unanimity in approving contracts that ameliorate the setup of property rights—the consent needed to change given swaps or the unanimity needed to accept swaps at once for all parties. The current analysis thus shows that the lack of unanimity can explain that property rights are absent. At the same time, this paper expresses the view that limits to verification and enforcement appear key for absent property rights.\footnote{Coase’s conjecture is known as the Coase theorem. It was noted, e.g. Arrow (1971), that public good externalities are associated with the lack of property rights, however leaving open possible reasons for the absence of property rights.}

7.2. International environmental policy

This paper provides a framework to limit free-rider behavior and leakage in environmental policy between countries or states advancing literature in three ways.

One, a contracting environment—a set of contracts consisting of claims in swaps, unanimity in finding swaps, and negotiation giving proposed swaps—with commitment, or external enforcement of contracts, and with no commitment, or derived enforcement of contracts (using bonds or stocks or balanced trade of claims in swaps), forms a novel framework to achieve efficiency relative to the literature on climate contracts using refunding state revenue from externality pricing in Gerber & Wichardt (2009) and Gersbach & Winkler (2011, 2012), contracting on a polluting input in Harstad (2012a), or contracting on emissions in Harstad (2012b).
Two, the current proposal, with the concepts of claims in swaps, unanimity in finding swaps, negotiation, and balanced trade of claims, carries a novel way of thinking about an international environmental agreement as a set of renegotiation-proof swap contracts relative to the literature on self-enforcing environmental policy insisting on an agreement in the form of coalition membership. This literature makes diverse proposals for climate policy: external trade sanctions with coalitions, for example, in Hagen & Schneider (2017), Nordhaus (2015), and Barrett (1997), exclusion of parties which are not members in a coalition from the benefits of cooperation on technology adoption or development, for example, in Carraro & Siniscalco (1995), Barrett (2006), and Hoel & de Zeeuw (2010), transfers among coalition members or between a coalition and outsiders, for example, in Carraro & Siniscalco (1993), Barrett (2001), and Carraro, Eyckmans & Finus (2006), retaliation against deviations to efficient actions, for example, in Chander & Tulkens (1995), Heitzig, Lessmann & Zou (2011), Kratzsch, Sieg & Stegemann (2012), and Harstad, Lancia & Russo (2017), and commitment to duration of a coalition in Battaglini & Harstad (2016). The current proposal provides a way to design an international treaty which at the same time induces parties to comply to emissions reductions and to participate, which in the literature trade off because of the lack of instruments for finding and enforcing an agreement. In the literature, with no proposed strategy added to internal and external stability for a coalition, efficiency is limited to specific heterogeneous tastes and technology (Finus & McGinty 2017), or a small number of parties (Barrett 1994, Diamantoudi & Sartzetakis 2006, Rubio & Ulph 2007). In this paper, efficiency can be generally attained. In addition, this paper shows that the combination of a commitment device of climate policy with chosen emissions, as in refunding, self-enforcing agreements in the literature, or swaps drawn on government revenue from explicit externality pricing, restricts attention to economies with decentralized decision-making and relies on the verification of emissions.

Three, this paper provides a flexible trade-theoretical general equilibrium framework for analyzing the limiting of free-rider behavior in environmental policy between countries or states, studied in the literature using stylized partial equilibrium frameworks. Other general equilibrium analysis of trade and the environment assumes a mode of cooperation, see, for example, Chichilnisky (1994) and Copeland & Taylor (2005). Variations and extensions such as with heterogeneous parties’ tastes and technology, deforestation,
and dynamics, and the use of general functional forms for production and utility underline the generality of the approach.

8. Conclusion

This paper has illustrated that shared ownership of a party’s own gain from polluting can prevent free-rider behavior and leakage in the provision of environmental quality, which is a public good. The shared ownership creates incentives to efficiently provide the public good, and occurs, because of the reciprocal nature of shared ownership, with swaps. In the model, parties that can represent countries or states, swap gross product. Importantly, the parties can commit to claims in the form of swaps as swap contracts can be enforced either through incentives from enforcing secondary claims or through renegotiation-proof swaps with balanced trade in the primary claims.

The analysis shows that, viewing claims as property rights, those rights need to be created first so to internalize transboundary environmental externalities. Consequently, the analysis carries a novel way of thinking about an international environmental agreement as a set of renegotiation-proof swap contracts. Swaps can form a major pillar of a post-Paris agreement on international climate policy with verification and enforcement, different than pledges for emissions reductions or sources of public climate finance the United Nations’ FCCC presently considers as parts of an agreement to reach their goal of preventing dangerous climate change. As swaps generally hinder free-rider behavior in the presence of environmental externalities, they can also enhance negotiations on other important environmental problems, for example, dumped waste plastic in oceans, biodiversity loss through agricultural practices, and the depletion of the ozone layer.

Appendix A: Proofs

Proof of Lemma 1
Denote the shadow price of the constraint (1) and (2) by $\lambda^Y$ and $\lambda^R$. Differentiate Lagranges’ function $\sum_j W_j U_j(x_j, \bar{y} - \sum_i r_i) + \lambda^Y (\sum_i F_i^Y(n_i, r_i) - \sum_i x_i) + \lambda^R (\sum_i F_i^R(1 - n_i) - \sum_i r_i)$ with respect to $x_i$, $r_i$, and $n_i$. A social welfare-maximizing allocation $x_i > 0$, $r_i > 0$, and $0 \leq n_i < 1$ satisfies, all
\[ i \in N, \]
\[ W_i U_{xi} = \lambda^Y, \]
\[ \lambda^Y F_{ri}^Y = \lambda^R + \sum_{j} W_j U_{yj}, \]
\[ \lambda^R F_{ni}^R \leq \lambda^Y F_{1-ni}^Y \quad \text{with equality if } n_i \in (0, 1). \]

These conditions can be combined to yield (7). The second equation in (7), strictly speaking, has utilized the first equation as the second equation reads

\[ (W_i U_{xi})^{-1} \sum_{j \in N} W_j U_{yij} = F_{ri}^Y - \frac{F_{nk}^Y}{F_{1-nk}^R}, \quad i \in N, \quad n_k \in (0, 1). \quad \text{QED} \]

**Proof of Lemma 2**

It is useful to first prove the following result.

**Result A.1.** In equilibrium, individual decisions \( x_i > 0, r_i > 0, \) and \( 0 \leq n_i < 1, \) yield (8) and (9).

Proof. Denote the shadow price of the budget constraint (6) by \( \mu_i. \) By differentiation of Lagrange’s function \( U_i(x_i, 1 - \sum_j r_j) + \mu_i(\alpha_i g_i(n_i, r_i, p) + T_i - p r_i - x_i) \) with respect to \( x_i, r_i, \) and \( n_i, \)

\[ U_{xi} = \mu_i, \]
\[ \mu_i(\alpha_i F_{ri}^Y - p) = U_{yji} \]
\[ p F_{ni}^R \leq F_{1-ni}^Y \quad \text{with equality if } n_i \in (0, 1). \]

Combining the first two equations yields (8). The remainder equation can be written as (9). QED

The necessary equilibrium conditions (8) and (9) derived from Result A.1 and the necessary social optimality conditions (7) given in Lemma 1 are aligned only if (10) holds. QED

**Proof of Proposition 1**

Part i: It is useful to first prove the following result.

**Result A.2.** Party \( j \) has an incentive to change swaps to Pareto improve if and only if, some \( i \in N, \alpha_i \neq (\gamma_i + \varepsilon)/(1 + \varepsilon). \)
Proof. Consider swaps involving some party $j$ and fix all other swaps so that output markets clear. Hold constant utility of any other party $i$, that is, $i \neq j$. Party $j$ can share gains from trade, so if party $j$ can be made better off, so can be two parties.

“only if.” Differentiate party $i$’s utility $U_i(x_i,y)$ using the choice of fuel (8), the choice of labor (9), and the provision of the public good ($\sum_i dr_i = \text{d}y$),

$$dU_i = U_{xi} \left[ (\mathcal{F}_i^Y + p\mathcal{F}_i^R) d\alpha_i + (U_{yi}/U_{xi}) dr_i \right] - r_i dp + \alpha_i \mathcal{F}_i^R dp + dT_i - U_{yi} \sum_i dr_i.$$  \hspace{1cm} (A1)

The terms with $d\alpha_i$, $dp$, and $dT_i$ vanish when aggregating. Swaps in the aggregate must be balanced, $\sum_i [(1 - \alpha_i)(\mathcal{F}_i^Y + p\mathcal{F}_i^R) - T_i] = 0$, so that

$$\sum_i \left[ (\mathcal{F}_i^Y + p\mathcal{F}_i^R) d\alpha_i + \alpha_i \mathcal{F}_i^R dp + dT_i \right] = \sum_i \left[ (1 - \alpha_i) \mathcal{F}_i^Y dr_i + \mathcal{F}_i^R dp \right]$$  \hspace{1cm} (A2)

using the choice of labor (9). Multiply both sides of (A1) with $W_i > 0$. Combining (A1) and (A2), and using clearing in the fuel market ($\sum_i \mathcal{F}_i^R = \sum_i r_i$) then yields the change in the sum of utility

$$\sum_i (dU_i/U_{xi}) = \sum_i \left[ (1 - \alpha_i) \mathcal{F}_i^Y + (U_{yi}/U_{xi}) \right] dr_i - \sum_i W_i (U_{yi}/W_i U_{xi}) \sum_j dr_j.$$  \hspace{1cm} (A3)

Suppose that $dU_i = 0$ all $i \neq j$. In equilibrium, the term in brackets equals the net benefit of fuel use ($\mathcal{F}_i^V - p$) from (8). Evaluating with constant $\mathcal{F}_i^V$ and $W_i U_{xi}$, the right side equals zero if condition (10) holds, restated here for convenience as

$$\alpha_i = (\gamma_i + \varepsilon)/(1 + \varepsilon) \quad \text{all } i \in N.$$  

Thus, party $j$ has an incentive to change swaps only if the converse of (10) holds. Given decisions in the second stage of the game, only a change in swaps can yield a change in fuel use.
“if.” The right side of equation (A3) exceeds zero for $dr_j \neq 0$, so there are gains from trade if

$$\alpha_i \neq (\gamma_i + \varepsilon)/(1 + \varepsilon) \quad \text{some } i \in N.$$ 

Swaps exploit these gains with or without side payments as $dT_i = 0$ all $i \in N$ could be used. QED

The proof proceeds in two steps.

Step 1: An equilibrium with efficiency exists if the allocations in some equilibrium of the game are socially optimal. By backward induction, this requires:

(a) No single party has an incentive to deviate from efficient actions in the second stage, and

(b) No two parties have incentives to switch to another contract among themselves in the first stage with the consent of all other parties.

The necessary equilibrium conditions (8) and (9) and the equation (10) given in Result A.1 together imply the social optimality conditions (7) derived from Lemma 1 for $1/\mu_i = W_i/\lambda^y$ and $p = \lambda^R/\lambda^y$, because $\alpha_i \neq 0$. That $\alpha_i \neq 0$ can be seen from $\gamma_i \geq 0$ and $\varepsilon > 0$. The conditions (1), (2), (3), and (7) are sufficient for a Pareto, or social, optimum implied by the concavity assumptions on production and utility. Clearly, (a) is satisfied, as at existing swaps implying efficiency, promises of payouts are honoured thanks to assuming binding contracts.

Suppose that some parties $i$ and $j$ seek to change their swap to improve on their welfare. If other parties give their consent only if an allocation weakly improves their own welfare beyond what would be obtained with current swaps, then the two parties $i$ and $j$ do not change their swap so to yield lower welfare to some other party. Now (b) holds using Result A.2 “only if” as no party has an incentive to deviate in the first stage given decisions in the second stage implying efficiency.

An efficient equilibrium thus exists. The equilibrium concept does not specify which efficient allocation will be selected.

Step 2: That no equilibrium with an inefficient allocation exists can be seen as follows. If an equilibrium can be Pareto improved through reallocating fuel use among parties or changing aggregate fuel use, as Lemma A.2 “if” indicates, then there are gains to have from changing swaps.

Part ii: Now the coalition $M$ and other parties can swap gross product. For example, $M$ can swap gross product with $i$ to keep utility $U_i$ constant.
and increase M’s utility. Otherwise M has not maximized their utility. The desired result then follows along the lines of the proof of Proposition 1(i).

QED

Proof of Lemma 3

To restrict attention to swaps which price the externality, deduct the internality from the regulatory revenue in expressing the sum of the variable components in swaps,

\[ \sum_j x_{ij} = \tau_i^y g_i + (\tau_i^r - \delta) r_i, \]

and define the fuel tax as a weighted average between the internality and the sum of internality and externality imposed by fuel use of party \( i \), \( \tau_i^r \equiv \kappa \delta_i + (1 - \kappa) \Delta. \)

The social optimality condition

\[ F_{ri} - (F_{1-nj}^y / F_{nij}^r) = \Delta \quad \text{all} \ j \in N \ \text{with} \ n_j \in (0,1), \]

and the necessary equilibrium conditions (9) and (14) are aligned only if

\[ \Delta - \tau_i^r = (p + \Delta) \tau_i^y. \]

This identifies the sum of \( \eta_{ij}^y \) and \( \eta_{ij}^r \) over \( j \neq i \) as desired. QED

Proof of Proposition 2

Use the firm profit and government budget constraint (19) and (20), and the household budget constraint (11), to yield party \( i \)’s balance of payments, where the value of net exports of goods equals the net outflow of funds,

\[ (F_i^y - x_i) + (p F_i^r - pr_i) = \sum_j (x_{ij} - x_{ji}) + \sum_j (t_{ij} - t_{ji}) + A \phi_{\text{permit}}(u(i) - \bar{u}(i)). \]

Here the variable part of swaps has been denoted by \( \chi_{ij} \equiv \eta_{ij} g_i \). Consider two regimes:

- With some side payments in swaps and equal initial and equilibrium holdings of permits for all parties, \( u(i) = \bar{u}(i) \), \( i \in N \), implying that \( B = 0 \).
- With no side payments in swaps, \( t_{ij} = 0 \), \( (i,j) \in N \), implying that \( A = 0 \), and unequal initial and equilibrium holdings of permits for some parties.
The regimes have equivalent allocations if the budget constraints of all parties coincide, and first-order conditions of the decentralized economies satisfy those of the social welfare maximization. The regime with side payments in swaps yields the budget constraint of a party (6). Using the equilibrium price of permits, the budget constraints coincide if

\[ \sum_j t_{ij} - \sum_j t_{ji} = \phi_{\text{permit}}(u(i) - \bar{u}(i)), \quad \phi_{\text{permit}} > 0. \]

The first-order conditions of the household and firm problems with respect to consumption, labor, fuel, and permits, satisfy the first-order conditions of the social welfare maximization with respect to consumption, labor, and fuel.

The equilibrium permit holdings \( u(i) = \rho_i r_i \) then imply the initial permit holdings

\[ \bar{u}(i) = \frac{\delta_i r_i - \left( \sum_j t_{ij} - \sum_j t_{ji} \right)}{\phi_{\text{permit}}}, \quad \phi_{\text{permit}} > 0. \]

Notice that summing both sides over \( i \) implies the market clearing condition of permits, (18). This follows by using the emissions permits in (17) at equality and the zero sum of net transfers \( \sum_i (\sum_j t_{ij} - \sum_j t_{ji}) = 0 \). QED

**Proof of Proposition 3**

Two structurally different examples will be shown. With no side payments, \( t_{ij} = 0 \), all \( i \in \mathbb{N} \), party \( i \) issues to party \( j \) the claim \( v_{ij} = \chi_{ij} = \eta_{ij} g_i \).

Part i: Let \( d_i = \sum_j v_{ij} \) be the value issued by party \( i \). Proving the result amounts to showing balanced swaps with no side payments and with implicit taxes on gross product. Balanced swaps with no side payments can be ascertained by showing a solution \((\chi_{i1}, \ldots, \chi_{iN})\), \( i \in \mathbb{N} \), to the system of equations

\[ \sum_j \chi_{ij} = d_i, \quad i \in \mathbb{N}, \quad \sum_j \chi_{ji} = d_j, \quad j \in \mathbb{N}, \]

giving equal payments from party \( i \) and to party \( i \). These equations, of which one can be omitted by Walras’ law, express equality of row and column sums in the matrix representing \( \chi_{ij} \) all \((i, j) \in H\), with all entries on the main diagonal being equal to zero \((\chi_{ii} = 0)\). Division of the trade \( \chi_{ij} \) by gross product \( g_i \) then gives numbers that represent the share of a party’s gross product relinquished to another party \((\eta_{ij})\). Implicit taxes require nonnegative shares. Hence all nonzero entries must be positive in

\[ \chi = (\chi_{ij}). \]
Alternative 1. To analyze several viewpoints on climate policy, sort the parties in ascending order of their values issued, $d_1 \leq d_2 \leq \ldots \leq d_H$. Notice that two values, say $d_i > 0$ and $d_j > 0$, $i \neq j$, can be the same.

Some hollow matrices $\chi$ with element $\chi_{ij}$ in row $i$ and column $j$ indicating payment flowing from party $i$ to $j$ show the result:

$$
\chi = \begin{pmatrix}
0 & 0 & d_1 \\
0 & d_1 + d_2 - d_3 & 0 \\
0 & d_3 - d_2 & d_2 \\
\end{pmatrix}, \quad H = 3,
$$

$$
\chi = \begin{pmatrix}
0 & \cdots & 0 & 0 & d_1 \\
0 & \cdots & 0 & 0 & d_2 - d_1 \\
\vdots & \ddots & \vdots & \vdots & \vdots \\
0 & \cdots & d_{H-3} & 0 & 0 & d_{H-2} - d_{H-3} \\
0 & \cdots & 0 & d_{H-2} + d_{H-1} - d_H & 0 & d_H - d_{H-2} \\
0 & \cdots & 0 & d_H - d_{H-1} & d_{H-1} & 0
\end{pmatrix},
$$

$H \geq 4$.

The row and column sums pertaining to the same index are equal as required for balanced swaps with no side payments. The element in row $(H-1)$, column $(H-2)$, appears nonnegative, because of the assumed triangle inequality among the three ordered top values, $d_{H-2} + d_{H-1} \geq d_H$. The element in row $H$, column $(H-2)$, appears nonnegative, $d_H - d_{H-1} \geq 0$. The $H$th column has nonnegative entries, $d_1 \geq 0$, $d_2 - d_1 \geq 0$, $\ldots$, $d_{H-2} - d_{H-3} \geq 0$, and $d_H - d_{H-2} \geq 0$.

Alternative 2. The bilaterally balanced swaps constructed in part (ii) establish the result.

Part ii: With three parties issuing positive values, $H = 3$, the result can be shown directly. For $H \geq 4$, proving the result under conditions more general than for the triangle inequality on the top three values helps to show the result. It will be useful to define the sum of successors on a sequence of values issued,

$$
D_j \equiv \sum_{k=j+1}^{H} d_k, \quad 1 \leq j \leq H - 1.
$$

Lemma A.1. Given $H \geq 4$, there are bilaterally balanced swaps that can be weakly verified and enforced if (a) $d_i = D_i$ and $d_j < D_j$, some $i = \ldots$
Proof. The conditions (a)-(c) are related in two ways. If \( d_i = D_i \) for two \( i \in H \) and \( d_j \leq D_j \), all \( j < i \), then one can check the strict inequality for the lowest index \( i \) for what needs to proven as indicated by condition (a). This way, (b) extends (a) for \( i = 1 \). Notice that (c) naturally extends (a) and (b) for \( i = (H - 1) \). The point of conditions (a)-(c) is that they include allocations with \( d_i = D_i \) and \( d_j > D_j \), \( i < j \), violating \( d_i \leq D_i \), all \( i = 1, 2, \ldots, H - 2 \).

First examine (a)-(c), and then use (d). Let the sum of values issued be \( D \equiv \sum d_i \).

Consider the following algorithm for constructing swaps among parties. Starting with \( i = 1 \) and \( \xi_1 = d_1/(D - d_1) \), let party \( i \) exchange the amount \( d_j \xi_i \) with all parties \( j > i \), where by recursion

\[
\xi_i = \frac{d_i}{D - \sum_{k=1}^i d_k} \left( 1 - \sum_{k=1}^{i-1} \xi_k \right), \quad i = 2, \ldots, H - 1.
\]  

(A5)

Repeat until \( i = H - 1 \). Let the sum of values issued up to the party of the current index be \( K_i \equiv \sum_{k=1}^i \xi_k \). Following this, a bilaterally balanced swap between parties \((H - 1)\) and \( H \) requires that

\[
(1 - K_{H-2})d_{H-1} = (1 - K_{H-2})d_H.
\]  

(A6)

Suppose that (a) or (b) hold. Then \( K_{H-2} = 1 \), precisely what is required to obtain a bilaterally balanced swap between the parties \((H - 1)\) and \( H \), condition (A6). Notice that \( K_{H-k} = 1 \) implies that \( d_{i-k} = D_{i-k} \) or \( K_{H-(k+1)} \), \( k = 2, \ldots, H - 2 \). Suppose that (c) holds. Clearly, this satisfies (A6). What remains to be shown by induction is that \( \chi_{ij} \geq 0 \), which in turn requires that \( K_{i-1} \leq 1 \) from (A5). Notice that \( K_{i-1} = K_{i-2} + \xi_{i-1} \), so that \( K_{i-1} \leq 1 \) if \( K_{i-2} \in [0, 1] \) and \( d_{i-1} \leq D_{i-1} \), \( i \geq 3 \). Now, \( K_1 = \xi_1 \leq 1 \) by construction and \( d_2 \leq D_2 \) by assumption, that is, for \( i = 3 \). Furthermore, \( d_{i-1} \leq D_{i-1} \) by assumption, \( i > 3 \). By induction, the desired result thus follows.

To use (d), run the above algorithm until but \((H - 3)\). Then let party \((H - 1)\) and \( H \) exchange \((d_{H-1} + d_H - d_{H-2})/2\), party \((H - 2)\) and \( H \) trade \((d_{H-2} + d_H - d_{H-1})/2\), and party \((H - 2)\) and \((H - 1)\) swap \((d_{H-2} + d_{H-1} -
The swaps then imply implicit taxes with nonnegativity given by the triangle inequality for \((d_H - 2, d_H - 1, d_H)\). QED

I can now continue with the proof of part (ii).

For \(H = 3\), using the swaps as described in the proof of Lemma A.1 (d) yields the desired result. Then party 2 and 3 exchange \(\chi_{-1} = (d_2 + d_3 - d_1)/2\), party 1 and 3 trade \(\chi_{-2} = (d_1 + d_3 - d_2)/2\), and party 1 and 2 swap \(\chi_{-3} = (d_1 + d_2 - d_3)/2\),

\[
\chi = \begin{pmatrix}
0 & \chi_{-3} & \chi_{-2} \\
\chi_{-3} & 0 & \chi_{-1} \\
\chi_{-2} & \chi_{-1} & 0
\end{pmatrix}.
\]

Implicit taxes are achieved as the claims are nonnegative from the triangle inequality for \((d_1, d_2, d_3)\). Notice that this includes the second viewpoint on climate policy when there are two parties unaffected by fuel use, \(d_1 = 0\), \(d_2 > 0\), and \(d_3 > 0\). The triangle inequality then requires that \(d_2 = d_3\).

For \(H \geq 4\), sort the values in ascending order. This ensures that \(d_i \leq D_i, \ i = 1, \ldots, H - 1\). The triangle inequality for the three top values then implies that Lemma A.1 (a)-(d) can be applied. Notice that one of (a)-(c) may be be invoked depending on the sorting of the values and the values themselves. QED

**Proof of Proposition 4**

Part i: Equilibria exist and are efficient if

(a) At least one proposal of swaps made leads to efficiency,

(b) Some proposal will be made, and

(c) Only a proposal that leads to efficiency will be unanimously approved.

These conditions will be shown to hold in three steps.

Step 1: Assume some proposal will be made, and confirm in Step 2. If no proposal leads to efficiency, there are gains from trade from changing any proposal which become exhausted by a Pareto improving proposal that will be unanimously approved, or accepted. Suppose one proposal will be accepted, and confirm in Step 3. Then a Pareto improving proposal will be accepted. Hence, no swaps leading to inefficiency are proposed. Then (a) follows, because all proposals lead to efficiency.

Step 2: A party can appropriate all gains from trade suggesting that extreme proposals could be made and accepted, with swaps entitling to consumption of one party up to the feasible upper bound. There are, however, lower limits to consumption as any party can decide to not approve a given

\[d_H)/2.\]
proposal. A proposal yielding lower utility for sure than in the outside option for some party would not be approved by this party. Thus, a party’s preferred proposal prioritizes own consumption and keeps utility of all other parties at their outside option value. This means that every accepted proposal for sure features the utility of the outside option or more for every party.

Thus, that everyone knows that proposals that Pareto improve will be made, and only such proposals will be accepted, shows (b).

Step 3: Each party becomes strictly better off in the Pareto optimal proposal that keeps all other parties at their outside option value, and thus strictly prefers to make this proposal relative to make a Pareto nonoptimal proposal that keeps all parties at their outside option value or make no proposal. Given all parties approve one party’s preferred proposal, this party has an incentive to not approve any other proposal. This can be prevented by a minimum number of approved proposals. With breaking equivalence in favor of efficiency, each party approves every party’s preferred proposal. Each party only approves these proposals by Step 2. Then (c) follows.

That proposals that are made will become accepted given the minimum acceptance clause confirms the conjecture made in Step 1 that one proposal will be accepted. (That one proposal becomes accepted by all parties given no minimum acceptance clause can confirm the conjecture given no such clause.)

Part ii: All proposals made are Pareto optimal, which was used to derive (a), every accepted proposal features the utility of the outside option or more for every party, which was used to prove (b), and proposals made will become accepted, which was used to show (c). QED

Appendix B: Emissions permits

This section combines the use of tradeable permits for transfers between parties with generalized sourcing of claims in swaps.

Claims can now be funded with revenue from a tax on the externality-creating good fuel such as an emissions tax (or a sale of domestic emissions permits). The remainder portion $\kappa \in [0, 1]$ becomes funded with gross product, which can be be taxed in an economy with decentralized decision-making. Taxes on fuel use and tradeable permits account for the portion $\epsilon_i \in [0, 1]$ and $(1 - \epsilon_i)$ of the internality. Denote party $i$’s tax rate on fuel use and gross product by $\tau_i^R$ and $\tau_i^Y$. To express the variable components of payments in
swaps, thus deduct the amount of the internality accounted for by the fuel tax, \( \sum_j \chi_{ij} = \tau_i^Y g_i + (\tau_i^R - \epsilon_i \delta_i) r_i \).

Each firm in party \( i \) chooses the amounts of labor \( n_i \), fuel \( r_i \), and permits \( u(i) \), so to maximize profits

\[
\pi_i = (1 - \tau_i^Y) g_i(n_i, r_i, p) - (p + \tau_i^R) r_i - \phi_{\text{permit}}[u(i) - \varphi_i \bar{u}(i)] - w_i.
\]

subject to limited fuel use (17), taking as given the prices of fuel, labor, and permits \( p, w_i, \phi_{\text{permit}} \) and the tax rates \( \tau_i, \tau_i^Y \).

The choices of fuel and permits \( r_i > 0 \) and \( u(i) \) need to balance their marginal benefit and cost, \((1 - \tau_i^Y) F_{ri}^Y - p = \tau_i^R + \rho_i v_i \), and \( v_i = \phi_{\text{permit}} = 0 \), letting \( v_i \) be the shadow price of the constraint (17) limiting fuel use.

The allowance ratio for party \( i \) thus amounts to

\[
\rho_i = (1 - \epsilon_i) \delta_i / \phi_{\text{permit}}, \quad \phi_{\text{permit}} > 0.
\]

This expression generalizes the allowance ratio derived under no fuel taxes in the main text. Fuel taxes may price some portion of the internality \( \delta_i \).

This gives the initial permits

\[
\bar{u}(i) = \left( (1 - \epsilon_i) \delta_i r_i - \left[ \sum_j t_{ij} - \sum_j t_{ji} \right] \right) / \phi_{\text{permit}}.
\]

**Relation to Section 4.3 and 5.1.**— The gross product tax rate \( \tau_i^Y = \kappa(1 - \gamma_i)/(1 + \varepsilon) \) the government in party \( i \) sets remains the same as with no tradeable permits in Section 4.3. The tax rate on fuel \( \tau_i^R = \kappa \delta_i + (1 - \kappa) \Delta - (1 - \epsilon_i) \delta_i \) adjusts to the use of taxes to potentially price some portion of the internality. If fuel taxes fully price the internality, \( \epsilon_i = 1 \) all \( i \in N \), the optimal fuel tax rate appears as in Section 4.3. If fuel taxes price no portion of the internality, \( \epsilon_i = 0 \) all \( i \in N \), the allowance ratio appears as in Section 5.1.

**Permit holdings.**—Given relative sources of swaps as indicated by a constant \( \kappa \), the minimum tax rate on fuel for a party \( i \) preferring the public good \( (\delta_i > 0) \), and the maximum tax rate on fuel for a party disliking the public good \( (\delta_i < 0) \) arises for \( \epsilon_i = 1 \). Notice that firms do not hold permits in equilibrium, \( u(i) = 0 \) all \( i \in N \), if permits merely transfer wealth, \( \epsilon_i = 1 \) all \( i \in N \), and thus \( \rho_i = 0 \) all \( i \in N \). To transfer wealth, then governments issue positive permits to firms in some party and negative permits to firms
in some other party. To a party \(i\) unaffected by the public good \((\delta_i = 0)\), the exogenously given share \(\epsilon_i\) does not matter for permits initially distributed or held in equilibrium, as in Section 5.1.

Thus, to implement an optimum when emissions permits transfer wealth using allowance ratios, a fuel tax (or other domestic pricing of fuel) can be used regardless of whether all or a subset of parties are affected by the public good.

References


