‘NEXT’ events
A cooperative game theoretic view to festivals

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Abstract

During a cultural festival, artists and theaters act as a cartel by agreeing on pricing decisions that maximize the groups’ profit as a whole. We model the problem of sharing the profit created by a festival among organizing theaters as a cooperative game. In such a game, the worth of a coalition is defined as the theaters’ profit from the optimal fixation of prices. We show that this class of games is convex and we axiomatically characterize the Shapley value (Shapley 1953) for this class of games. We also provide an axiomatic basis for another rule : the downstream incremental solution. Finally, we apply this model to the NEXT festival, for which we have collected data.We propose an approach to compute the games’ vector from the data and we compute the different solutions.

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1 Introduction

Music, theater or dance festivals are very usual ways to supply art to *amateurs*. A festival is an organized sequence of special events and performances, taking place in one or different spots in a short time. From an economic viewpoint, artists and theaters act as a cartel during the festival, i.e. they agree on decisions about prices, and performances’ type and quality, in order to maximize the groups’ profit as a whole. Interestingly, this behavior is not disciplined by authorities, and is even often encouraged by public

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subsidies.[1] These subsidizing practices may be justified by the fact that performing art festivals creates spillovers. Moreover, it could happen that artists and theaters taking part to a festival have heterogeneous characteristics. In this article, we investigate the problem of the distribution of a festival's profit, partly generated by spillovers, among heterogeneous theaters and artists.

We first model the attendance and theaters' behavior. The attendance of a festival can be seen as a flow partly going from one event to the next ones according to its positive and negative experiences within the festival[2]. Therefore, members of the cartel benefit from the spillover effect of the attendance's transfer to the next performances. Theaters cooperate by coordinating decisions, using spillovers as a mean to maximize the groups' profit as a whole.

To address the problem of sharing the profit created by the cooperation among theaters we introduce a class of cooperative games with transferable utility, or TU-games, defined on the set of theaters. In such games, the worth of a coalition is defined as the theaters' profit from the optimal fixation of prices. We show that these games are convex, which means that the Shapley value is in the core. Then we provide a characterization of the Shapley value on the class of festival games. [3]

Ginsburgh and Zang (2003) also apply cooperative games tools to cultural events by providing a cooperative solution to share the profit of a cultural pass among participant museums. Their solution is an elegant expression of the Shapley value that holds for the class of their museum pass games. Béal and Solal (2009) provide an axiomatic characterization of two solutions for the class of museum pass games: the one defined by Ginsburgh and Zang's, and the egalitarian sharing rule. The structure of our model is closely related to Ambec and Sprumont (2002) and Ambec and Ehlers (2008) who study fair distributions of welfare resulting from the optimal allocation of water among a set of riparian agents.

Actual implementations of cooperative game theoretic solutions are often impeded by practical difficulties. First, models are often highly stylized versions of real life situations. Heterogeneity among agents is the main justification for sophisticated sharing rules. However, this heterogeneity is often considered as given and easy to observe, which is not always the case. Second, although theoretical models typically consider an arbitrary number of players, the numerical complexity soon raises computational difficulties when many parts are involved. To convince the reader that our solution is indeed implementable, we provide a complete treatment of a real life case study.

Section 2 is devoted to the theoretical results. We set up the model, define a cooper-

[1]We are aware of a single case of legal investigation against a festival grounded on the US antitrust legislation. The investigation by Attorney General Lisa Madigan concerns exclusivity clauses which artists playing in the Chicago music festival Lollapalooza must sign. According to Hiller (2012), the number of venues in the cities covered by the clause significantly decreased.

[2]Levy-Garboua et al. (1996) show that due to the singularity of each experience, good or bad surprises experimented in the recent past have a strong impact on the demand for next performances.

[3]Mamoru Kaneko together with several co-authors studied in various papers a discrimination model in which a "festival game" is described. Our model bears little -if any- resemblance to Kaneko's works. We are grateful to Nicolas Gravel for mentioning this potential source of confusion.

ative festival game, and establish its convexity. In Section 3, we show that the balanced contributions property introduced by Myerson (1980) —together with efficiency— fully characterizes Shapley's sharing rule (Shapley 1953) for this class of games. We also provide an axiomatic basis for the downstream incremental solution (see vand den Brink*et al.* for details). In Section 4, we apply our model to the dance and theater NEXT festival for which we collected data. We estimate the demand functions for each performance and assess spillover effects as functions of explanatory variables. Finally, we use this estimation to compute a value sharing of the festival. An appendix Section contains some supplementary material about the NEXT festival.

2 A theoretical game for festivals

In this section, we set up a theoretical festival game. We then show that it admits a simple linear structure. More precisely we show that the payoff vector of any festival game is a linear combination of a limited number of "elementary" festival games. We establish that elementary festival games are convex. The convexity of any festival game follows. Finally we propose an axiomatic characterization of the Shapley value on the set of festival games.

2.1 Setup

We consider a set $N = \{1, \dots, n\}$, $n \in \mathbb{N}$, of theaters in which cultural performances may be organized. We assume that the program is already constrained so that theaters are indexed in the chronological order. A first performance is organized in theater 1 (and nowhere else), the next event takes place in theater 2, and so on. This hypothesis will be relaxed in section 2.5. We suppose that he issue of the best program was addressed in a first stage.

Before the i-th performance takes place, a queue of potential spectators forms. The size of this queue is $q_i > 0$. Then this potential demand "learns" the price p_i due to attend the performance. Notice here that the price representation is merely a simplification device. The model may be rewritten so that spectators are informed about the quality of the performance instead of the price.

The demand is assumed to be a strictly decreasing function of the price. The audience for the i-th performance is $S_i(p_i)q_i$. Moreover, we assume $S_i(p) = 1$ whenever $p \leq 0$ for all $i \in N$, in other words everybody in the queue attends the show if it is free or subsidized.[4] We assume that each show bears linear costs. The profit for show i is then given by $S_i(p_i)q_i(p_i - c_i)$.

After the i-th performance, a proportion $\rho_{i+1} \in [0, 1]$ of i's audience gathers to form the next queue together with some newcomers $e_{i+1} \geq 0$ hence

$$q_{i+1} = \rho_{i+1} S_i(p_i) q_i + e_{i+1}$$

[4] In this paper, S stands for "survival" since it should be interpreted as the proportion of the potential demand that "survives" the price's revelation.

The spillover effect increases with ρ_{i+1}. In accordance with the interpretation of newcomers in the rest of the festival, we shall consider that $q_0 = 0$, i.e. the attenders of the first performance are all "newcomers". This convention creates no extra difficulties in the remaining theoretical analysis.

Each theater is assumed to benefit from a quasi-linear utility function adding the profit and some side payment t_i. As usual, we assume that the side payments sum up to zero, so Pareto Efficiency among any set of players is obtained if the sum of the profits over this set is maximized. This maximization is assumed to rely only on the prices (this is the only variable for firms once the program of the festival is set). When they do not decide to coordinate their prices the theaters simply choose a monopoly pricing without taking downward spillovers into account. A set of theaters that coordinate their prices is called a coalition. The festival game is then defined as the cooperative game such that the value $v(T)$ of the coalition $T \subset N$ is the total profit of this coalition, with the convention $v(\emptyset) = 0$.

We assume that prices are credible commitments among the members of the coalition. This may be obtained by the "menu" device attached to the program of the festival or by formal conventions signed among festivals organizers. In the context of the applied exercise of Section 3 below, a formal convention exists since the NEXT festival is subsidized by the E.C. Interreg Program. According to this convention, the costs induced by any performance is cover up to 50% by the Interreg Program, provided the other members of the festival all agree to add the performance in the NEXT program.

Whether a collectively pricing rule may be harmful from a welfare viewpoint is beyond the scope of this paper.

It shall be stressed that the spectators are not informed of the suppliers agreements. Hence, whether a theater decides to join a coalition or not, the audience's reaction remains unchanged. In this respect, our model departs from Ambec and Sprumont [1]. In the river sharing model of Ambec and Sprumont, deviant players take all the remaining water. In our model, a deviant player may choose a prohibitive price but it is against its own interest. By playing in an "efficiently selfish way" a deviant player passes part of its audience to downward players, some of which may be members of the coalition. In this sense, the model is more closely related to Ambec and Elhers [2]. Ambec and Elhers consider a river sharing model in which deviation cannot outsets some bliss point. A major difference is that the core may be empty in the model considered by Ambec and Ehlers. As we claimed in the introduction, we shall show that any festival game is convex.

We end up this section by giving a formal definition of a festival game. To this end, let $\mathcal{S}$ be the set of differentiable functions from $\mathbb{R}$ to $\mathbb{R}$ such that $S(p) = 1$ whenever $p \leq 0$, $S'(0) = 0$ and $S''(p) < 0$ whenever $p \geq 0$.

Remark that for all $S \in \mathcal{S}$ there exist $p_S > 0$ such that $S(p_S) = 0$ and $S(p) < 0$ for all $p > p_S$.

Definition 1 *Let (N, v) be a transferable utility game. It belongs to the class of festival games, denoted by $\mathcal{F}$, if and only if there exists three sequences of positive real numbers*

$e' = (e_1, \ldots, e_n)'$, $c' = (c_1, \ldots, c_n)'$, and $\rho' = (\rho_2, \ldots, \rho_n)'$ and $S' = (S_1, \ldots, S_n)' \in \mathcal{S}^n_+$ such that for all $i \in N$ we have $S_i(c_i) > 0$ and for all $T \subset N$ we have

$$\begin{array}{lll} v(T) & = max & \sum_{i \in T}(p_i - c_i)d_i \\ wrt & p = (p_i)_{i \in T}\mathbb{R}^{Card(T)} & \\ & s.t. & d_i = (e_i + \rho_i d_{i-1})S_i(p_i) \\ & & d_0 = 0 \\ & & p_j = p^m_j = \arg\max_{p_j \in \mathbb{R}} \; S_j(p_j)(p_j - c_j), \; \forall j \notin T. \end{array} \quad (1)$$

When (N, v) is a transferable utility game such that for all $T \subset N$ the value $v(T)$ may be computed as in program (1) for some quadruplet (e, c, ρ, S) we shall say that (N, v) is *associated with* (e, c, ρ, S).

2.2 Convexity of festival games

As it is well known, convexity of a game entails several important property among which non emptiness of the core and stability of Shapley sharing rule.

We start our analysis with preliminary remarks and results about our class of games.

We first study the maximization of $S(p)(p-c)$ wrt to $p \in \mathbb{R}$ when $S \in \mathcal{S}$ and $S(c) > 0$. Notice that we shall consider the possibility $c < 0$ for reasons that will become clear as we proceed.

Clearly, by choosing $p = c$ we can secure $S(p)(p-c) = 0$. We also have $\frac{\partial}{\partial p}\left\{S(p)(p-c)\right\}_{p=c} = S(c) > 0$.

Hence as we assumed $S(c) > 0$, we may restrict our attention to the cases in which $p \in]c, p_S.]$

Now, if $p < 0$ then $S(p)(p-c) = p - c < -c = S(0)(0-c)$ so we may restrict our attention to the case $p \geq 0$. Moreover $S'(0) = 0$ and $S(0) = 1$ imply $\frac{\partial}{\partial p}\left\{S(p)(p-c)\right\}_{p=0} = 1 > 0$ so we now consider $p \in]\max\{c, 0\}, p_S]$. Over this interval we have

$$\frac{\partial^2}{\partial p_i^2}\left\{S_i(p_i)(p_i - c)\right\} = S''_i(p_i)(p_i - c) + 2S'_i(p_i) < 0$$

which implies the following result

Lemma 1 *For all $S \in \mathcal{S}$ and all real value c such that $S(c) > 0$ the program*

$$\max_{p \in \mathbb{R}} \; S(p)(p-c)$$

admits a unique solution.

For all $S \in \mathcal{S}$ and real value c such that $S(c) > 0$ define the function

$$p^m_S(c) = \arg\max_{p \in \mathbb{R}} \; S(p)(p-c).$$

Obviously if $c' < c$ then

$$S(p^m(c))(p^m_S(c) - c') > S(p^m_S(c))(p^m(c) - c)$$

which implies

Remark 1 *If $S \in \mathcal{S}$ and c is a real value such that $S(c) > 0$, the function $\pi^m_S(c) = \max_{p\in\mathbb{R}} S(p)(p-c)$ is a strictly decreasing function of c.*

Using lemma 1 the above remark implies

Remark 2 *If $S \in \mathcal{S}$ and c is a real value such that $S(c) > 0$, the function $p^m_S(c) = \arg\max_{p\in\mathbb{R}} S(p)(p-c)$ is an increasing function of c.*

We now study the pricing behavior inside a coalition containing at least two members. Consider the coalition $T = \{i_1 < i_2 < \ldots < i_T\}$. Fix the pricing rules of theaters 1 to $-1 + i_T$. It is important to stress that when considering theater i_T, this choice may not be optimal and that the behavior of theater inside and outside the coalition are assumed to be fixed.

According to definition 1, theater i_T must solve

$$\max_{p_{i_T}\in\mathbb{R}} S_{i_T}(p_{i_T})(p_{i_T} - c_{i_T})(e_{i_T} + \rho_{i_T} d_{-1+i_T})$$

where d_{-1+i_T} results from fixed choices of all upward theaters. Clearly, theater i_T must choose the monopoly price $p^m_{S_{i_T}}(c_{i_T})$ which uniquely defined according to lemma 1 and definition 1 since we assumed $S_{i_T}(c_{i_T}) > 0$. To save space, we shall write $p^m_{i_T}$ for $p^m_{S_{i_T}}(c_{i_T})$.

Now proceeding backward consider theater i_{T-1}. As in Stigler's famous argument on double marginalization, when spillover are effective it will not choose a monopoly pricing.

Indeed, fixing the pricing rules of all upward theaters up to index i_{T-1} excluded, this theater solves

$$\max_{p_{i_{T-1}}\in\mathbb{R}} S_{i_{T-1}}(p_{i_{T-1}})(p_{i_{T-1}} - c_{i_{T-1}})(e_{i_{T-1}} + \rho_{i_{T-1}} d_{-1+i_{T-1}}) + S_{i_T}(p^m_{i_T})(p^m_{i_T} - c_{i_T})(e_{i_T} + \rho_{i_T} d_{-1+i_T})$$

which is equivalent to

$$\max_{p_{i_{T-1}}\in\mathbb{R}} S_{i_{T-1}}(p_{i_{T-1}})(p_{i_{T-1}} - c_{i_{T-1}})(e_{i_{T-1}} + \rho_{i_{T-1}} d_{-1+i_{T-1}}) + S_{i_T}(p^m_{i_T})(p^m_{i_T} - c_{i_T})\rho_{i_T} d_{-1+i_T}$$

Now we have

$$d_{-1+i_T} = (e_{-1+i_T} + \rho_{-1+i_T} d_{-2+i_T}) S_{-1+i_T}(p_{-1+i_T})$$

If $-1 + i_T = i_{T-1}$ theater i_{T-1} must solve

$$\max_{p_{i_{T-1}}\in\mathbb{R}} S_{i_{T-1}}(p_{i_{T-1}})(p_{i_{T-1}} - c_{i_{T-1}} + S_{i_T}(p^m_{i_T})(p^m_{i_T} - c_{i_T})\rho_{i_T})(e_{i_{T-1}} + \rho_{i_{T-1}} d_{-1+i_{T-1}})$$

or

$$\max_{p_{i_{T-1}}\in\mathbb{R}} S_{i_{T-1}}(p_{i_{T-1}})(p_{i_{T-1}} - c_{i_{T-1}}(T))(e_{i_{T-1}} + \rho_{i_{T-1}} d_{-1+i_{T-1}})$$

with $c_{i_{T-1}}(T) = c_{i_{T-1}} - S_{i_T}(p^m_{i_T})(p^m_{i_T} - c_{i_T})\rho_{i_T}$.

Otherwise if $-1 + i_T > i_{T-1}$ we know from definition 1 that $S_{-1+i_T}(p_{-1+i_T}) = S_{-1+i_T}(p^m_{-1+i_T}) = S^m_{-1+i_T}$. So that theater i_{T-1} solves

$$\begin{aligned}\max_{p_{i_{T-1}}\in\mathbb{R}} \quad & S_{i_{T-1}}(p_{i_{T-1}})(p_{i_{T-1}} - c_{i_{T-1}})(e_{i_{T-1}} + \rho_{i_{T-1}} d_{-1+i_{T-1}}) \\ & + S_{i_T}(p^m_{i_T})(p^m_{i_T} - c_{i_T})\rho_{i_T}(e_{-1+i_T} + \rho_{-1+i_T} d_{-2+i_T}) S^m_{-1+i_T}\end{aligned}$$

which is equivalent to

$$\begin{aligned}\max_{p_{i_{T-1}}\in\mathbb{R}} \quad & S_{i_{T-1}}(p_{i_{T-1}})(p_{i_{T-1}} - c_{i_{T-1}})(e_{i_{T-1}} + \rho_{i_{T-1}} d_{-1+i_{T-1}}) \\ & + S_{i_T}(p^m_{i_T})(p^m_{i_T} - c_{i_T})\rho_{i_T}\rho_{-1+i_T} d_{-2+i_T} S^m_{-1+i_T}\end{aligned}$$

and proceeding upwards the objective of theater i_{T-1} is

$$\begin{aligned}\max_{p_{i_{T-1}}\in\mathbb{R}} \quad & S_{i_{T-1}}(p_{i_{T-1}})(p_{i_{T-1}} - c_{i_{T-1}})(e_{i_{T-1}} + \rho_{i_{T-1}} d_{-1+i_{T-1}}) \\ & + S_{i_T}(p^m_{i_T})(p^m_{i_T} - c_{i_T})\rho_{i_T}\left(\textstyle\prod_{1+i_{T-1}<j\le i_T} \rho_j S^m_j\right) d_{i_{T-1}}\end{aligned}$$

or, using $d_{i_{T-1}} = S_{i_{T-1}}(p_{i_{T-1}})(e_{i_{T-1}} + \rho_{i_{T-1}} d_{-1+i_{T-1}})$

$$\begin{aligned}\max_{p_{i_{T-1}}\in\mathbb{R}} \quad & S_{i_{T-1}}(p_{i_{T-1}})(p_{i_{T-1}} - c_{i_{T-1}})(e_{i_{T-1}} + \rho_{i_{T-1}} d_{-1+i_{T-1}}) \\ & + S_{i_T}(p^m_{i_T})(p^m_{i_T} - c_{i_T})\rho_{i_T}\left(\textstyle\prod_{1+i_{T-1}<j\le i_T} \rho_j S^m_j\right) S_{i_{T-1}}(p_{i_{T-1}})(e_{i_{T-1}} + \rho_{i_{T-1}} d_{-1+i_{T-1}})\end{aligned}$$

We may now observe that in any case, the objective of theater i_{T-1} may be written as

$$\max_{p_{i_{T-1}}\in\mathbb{R}} S_{i_{T-1}}(p_{i_{T-1}})(p_{i_{T-1}} - c_{i_{T-1}}(T))(e_{i_{T-1}} + \rho_{i_{T-1}} d_{-1+i_{T-1}})$$

where

$$c_{i_{T-1}}(T) = c_{i_{T-1}} - S_{i_T}(p^m_{i_T})(p^m_{i_T} - c_{i_T})\rho_{i_T} \prod_{1+i_{T-1}<j\le i_T} \rho_j S^m_j$$

with the convention $\prod_{i\in R} = 1$ whenever R is an empty set.

As we assumed in definition 1 that $\rho_i \ge 0$ and $S_{i_{T-1}}(c_{i_{T-1}}) > 0$ we have $S_{i_{T-1}}(c_{i_{T-1}}(T)) > 0$ and we then deduce the optimal pricing rule for theater i_{T-1} is $p^m_{i_{T-1}}(c_{i_{T-1}}(T))$ with transparent notations.

Observe the objective functions of theaters i_{T-1} and i_T are now similar, we may proceed further upward using the same approach to derive the optimal pricing rule for all theaters in coalition T. We then derive the following result:

Lemma 2 *Let $T = \{i_1 < i_2 < \ldots < i_T\} \subset N$. The pricing rule $p(T) \in R^{Card(T)}$ given by the following recursive formula*

$$\begin{aligned} & c_{i_T}(T) = c_{i_T} & \\ & p_{i_j}(T) = \arg\max_{p\in\mathbb{R}} S_{i_j}(p)(p - c_{i_j}(T)) & 1 \le j \le T \\ & c_{i_{j-1}}(T) = c_{i_{j-1}} - S_{i_j}(p_{i_j}(T))(p_{i_j}(T) - c_{i_j}(T))\rho_{i_j} \textstyle\prod_{1+i_{j-1}<k\le i_j} \rho_k S^m_k & 2 \le j \le T \end{aligned}$$

maximizes the payoff of the coalition T.

The above lemma says that cooperation among theaters inside a coalition amounts to choose monopoly prices for 'fitted' costs. As we assumed that $\rho_i > 0$ for all i fitted costs are lower that 'actual' ones. It should be stressed that fitted costs depends on characteristics of theaters outside the coalition.

This lemma has an important consequence. It appears that the optimal pricing rule of any coalition is independent of e. This implies that v depends on e through $d_1, \ldots, d_n$ only. But according to $d_i = (e_i + \rho_i d_{i-1})S_i(p_i)$ these are linear functions of e.

Now remark that for fixed prices, the value of any coalition is a linear combination of the vector e. Hence we have

Lemma 3 *Let $u_1, \ldots, u_n$ be the canonical basis of R^n. Let $(N, v) \in \mathcal{F}$ be associated with (e, c, ρ, S) then for any $T \subset N$ we have*

$$v(T) = \sum_{i=1}^{N} e_i v_i(T)$$

where v_i is associated with (u_i, c, ρ, S).

Next, we investigate the impact of a theater leaves a game. More precisely we now show that the remaining members act as in a new festival game.

This is trivial if the last theater leaves. If $n > 2$, consider $T \subset N$, $1 < i < n$ and $i \notin T$ we have $p_i = p_i^m$ and $d_i = (e_i + \rho_i d_{i-1})S_i^m$ where $S_i^m = S_i(p_i^m)$.

We may then write:

$$\begin{aligned}
&d_1 = e_1 S_1(p_1) \\
&d_2 = (e_2 + \rho_2 d_1)S_2(p_2) \ldots \\
&d_{i-1} = (e_{i-1} + \rho_{i-1} d_{i-2})S_{i-1}(p_{i-1}) \\
&d_{i+1} = (e_{i+1} + \rho_{i+1} e_i S_i^m + \rho_{i+1}\rho_i S_i^m d_{i-1})S_{i+1}(p_{i+1}) \\
&d_{i+2} = (e_{i+2} + \rho_{i+2} d_{i+1})S_{i+2}(p_{i+2}) \ldots \\
&d_n = (e_n + \rho_n d_{n-1})S_n(p_n)
\end{aligned}$$

Finally if 1 leaves coalition T we have

$$\begin{aligned}
&d_1 = e_1 S_1^m \\
&d_2 = (e_2 + \rho_2 e_1 S_1^m)S_2(p_2) \\
&d_3 = (e_3 + \rho_3 d_2)S_3(p_3) \ldots \\
&d_n = (e_n + \rho_n d_{n-1})S_n(p_n)
\end{aligned}$$

Thus in any case, if theater i leaves, the remaining players enter a new festival game in which e_{i+1} and ρ_{i+1} are redefined. More formally, we established the following lemma.

Lemma 4 *For any $(N, v) \in \mathcal{F}$ where $n > 1$ and $i \in N$ we define the game $(v_{-i}, N\backslash\{i\})$ with characteristic function $v_{-i}(T) = v(T)$ whenever $i \notin T$. Then $(v_{-i}, N\backslash\{i\}) \in \mathcal{F}$.*

Moreover if (N,v) is associated with the sequences e, c, ρ, D then $(v_{-i}, N\backslash\{i\})$ is associated with the sequences

$$\begin{array}{l} e_{-i} = e_1, \ldots, e_{i-1}, e_{i+1} + \rho_{i+1} e_i S_i^m, e_{i+2}, \ldots e_n \\ c_{-i} = c_1, \ldots, c_{i-1}, c_{i+1}, \ldots, c_n \\ \rho_{-i} = \rho_1, \ldots, \rho_{i-1}, \rho_i \rho_{i+1} S_i^m, \rho_{i+2}, \ldots \rho_n \\ S_{-i} = S_1, \ldots, S_{i-1}, S_{i+1}, \ldots, S_n \end{array}$$

This asserts that any sub-game of a festival game is also a festival game. Notice in particular that e_{-i} is an affine transformation of e so the result of lemma 3 remains.

We may now prove that festival games are convex.

Proposition 1 *If (N,v) is a festival game then (N,v) is convex.*

Proof: Using Lemma 3, we may considered the case $e = u_n$.

The case $n = 2$ is easy since convexity amounts to show $v(\{1,2\}) \geq v(\{1\}) + v(\{2\})$. Now if 1 and 2 choose to keep their monopoly prices the total payoff of the grand coalition is exactly $v(\{1\}) + v(\{2\})$.

Now assume the property is established for all festival games of size smaller or equal to $n-1$ and consider the case of game of size n. We have to show that $R \subset T$ and $i \notin T$ imply

$$v(T \cup \{i\}) - v(T) \geq V(R \cup \{i\}) - v(R).$$

But as showed in lemma 4, if a theater leaves a festival, the remaining players are involved in a new festival with one player less. Hence, by induction we only need to establish that the previous inequality holds in the case $T \cup \{i\} = N$ and $R = N\backslash\{i,j\}$. We then need to show

$$v(N) - v(N\backslash\{i\}) \geq v(N\backslash\{j\}) - v(N\backslash\{i,j\}).$$

Notice that this inequality is equivalent to

$$v(N) \geq v(N\backslash\{i\}) + v(N\backslash\{j\}) - v(N\backslash\{i,j\}),$$

hence, by symmetry we may consider the case $j > i$.

First assume $i = 1$. Using lemma 4, we have

$$v(N) - v(N\backslash\{1\}) = -v(N\backslash\{1\}) + \max_{p_1} \left\{ S_1(p_1)(p_1 - c_1) + \frac{S_1(p_1)}{S_1^m} v(N\backslash\{1\}) \right\}$$

where $S_1^m = S_1(p_1^m)$. Similarly, we get (for $j > 1$)

$$v(N\backslash\{j\}) - v(N\backslash\{1,j\}) = -v(N\backslash\{1,j\}) + \max_{p_1} \left\{ S_1(p_1)(p_1 - c_1) + \frac{S_1(p_1)}{S_1^m} v(N\backslash\{1,j\}) \right\}$$

Consider the following function

$$f_1(x) = -x + \max_{p_1} \left\{ S_1(p_1)(p_1 - c_1) + \frac{S_1(p_1)}{S_1^m} x \right\}$$

for positive values of x.

The envelope theorem asserts that $f_1'(x) = \frac{S_1(p_1(x))}{S_1^m} - 1$ where

$$p_1(x) = \arg\max_{p_1} \left\{ S_1(p_1)(p_1 - c_1) + \frac{S_1(p_1)}{S_1^m} x \right\} = \arg\max_{p_1} \left\{ S_1(p_1) \left(p_1 - \left(c_1 - \frac{x}{S_1^m} \right) \right) \right\}$$

As x is a positive value, using Remark 2 and $S'() < 0$ we get $f_1'(x) > 0$.

Now we conclude, using $v(N\backslash\{1\}) \geq v(N\backslash\{1,j\})$,

$$f_1(v(N\backslash\{1\})) = v(N) - v(N\backslash\{1\}) \geq v(N\backslash\{j\}) - v(N\backslash\{1,j\}) = f_1(v(N\backslash\{1,j\}))$$

A similar argument may be used if $n \geq 3$ and $i > 1$. More precisely, let us write $v(N) - v(N\backslash\{i\})$ as

$$\begin{array}{lll} \max_{p_i} \{ & \max_{p_1,p_2,\ldots p_{i-1}} \{ & S_1(p_1)(p_1 - c_1) + \rho_2 S_1(p_1) S_2(p_2)(p_2 - c_2) + \ldots \\ & & \ldots + \rho_2 \times \ldots \times \rho_{i-1} S_1(p_1) \times \ldots \times S_{i-1}(p_{i-1})(p_{i-1} - c_{i-1}) \\ & & +\rho_2 \times \ldots \times \rho_{i-1}\rho_i S_1(p_1) \times \ldots \times S_{i-1}(p_{i-1}) \times S(p_i)(p_i - c_i) \\ & & +\frac{S_i(p_i)}{S_i^m} \frac{S_1(p_1)\times\ldots\times S_{i-1}(p_{i-1})}{S_1^m\times\ldots\times S_{i-1}^m} v(N\backslash\{1,\ldots,i\}) \\ & & \} \\ & \} & \end{array}$$

$$\begin{array}{ll} -\max_{p_1,p_2,\ldots p_{i-1}} \{ & S_1(p_1)(p_1 - c_1) + \rho_2 S_1(p_1) S_2(p_2)(p_2 - c_2) + \ldots \\ & \ldots + \rho_2 \times \ldots \times \rho_{i-1} S_1(p_1) \times \ldots \times S_{i-1}(p_{i-1})(p_{i-1} - c_{i-1}) \\ & +\frac{S_1(p_1)\times\ldots\times S_{i-1}(p_{i-1})}{S_1^m\times\ldots\times S_{i-1}^m} v(N\backslash\{1,\ldots,i\}) \\ & \} \end{array}$$

Accordingly, as we assumed $j > i$ we may write $v(N\backslash\{j\}) - v(N\backslash\{i,j\})$ as

$$\begin{array}{lll} max_{p_i} \{ & \max_{p_1,p_2,\ldots p_{i-1}} \{ & S_1(p_1)(p_1 - c_1) + \rho_2 S_1(p_1) S_2(p_2)(p_2 - c_2) + \ldots \\ & & \ldots + \rho_2 \times \ldots \times \rho_{i-1} S_1(p_1) \times \ldots \times S_{i-1}(p_{i-1})(p_{i-1} - c_{i-1}) \\ & & +\rho_2 \times \ldots \times \rho_{i-1}\rho_i S_1(p_1) \times \ldots \times S_{i-1}(p_{i-1}) \times S(p_i)(p_i - c_i) \\ & & +\frac{S_i(p_i)}{S_i^m} \frac{S_1(p_1)\times\ldots\times S_{i-1}(p_{i-1})}{S_1^m\times\ldots\times S_{i-1}^m} v(N\backslash\{1,\ldots,i,j\}) \\ & & \} \\ & \} & \end{array}$$

$$\begin{array}{ll} -\max_{p_1,p_2,\ldots p_{i-1}} \{ & S_1(p_1)(p_1 - c_1) + \rho_2 S_1(p_1) S_2(p_2)(p_2 - c_2) + \ldots \\ & \ldots + \rho_2 \times \ldots \times \rho_{i-1} S_1(p_1) \times \ldots \times S_{i-1}(p_{i-1})(p_{i-1} - c_{i-1}) \\ & +\frac{S_1(p_1)\times\ldots\times S_{i-1}(p_{i-1})}{S_1^m\times\ldots\times S_{i-1}^m} v(N\backslash\{1,\ldots,i,j\}) \\ & \} \end{array}$$

Following the previous argument let us define

$$
\begin{aligned}
g_i(p_i,x) = \max_{p_1,p_2,\dots p_{i-1}} \{ \quad & S_1(p_1)(p_1-c_1)+\rho_2 S_1(p_1)S_2(p_2)(p_2-c_2)+\dots \\
& \dots+\rho_2\times\dots\times\rho_{i-1}S_1(p_1)\times\dots\times S_{i-1}(p_{i-1})(p_{i-1}-c_{i-1}) \\
& +\rho_2\times\dots\times\rho_{i-1}\rho_i S_1(p_1)\times\dots\times S_{i-1}(p_{i-1})\times S(p_i)(p_i-c_i) \\
& +\frac{S_i(p_i)}{S_i^m}\frac{S_1(p_1)\times\dots\times S_{i-1}(p_{i-1})}{S_1^m\times\dots\times S_{i-1}^m}x \\
& \} \\
h_i(x) = g_i(p_i^m,x) = \max_{p_1,p_2,\dots p_{i-1}} \{ \quad & S_1(p_1)(p_1-c_1)+\rho_2 S_1(p_1)S_2(p_2)(p_2-c_2)+\dots \\
& \dots+\rho_2\times\dots\times\rho_{i-1}S_1(p_1)\times\dots\times S_{i-1}(p_{i-1})(p_{i-1}-c_{i-1}) \\
& +\frac{S_1(p_1)\times\dots\times S_{i-1}(p_{i-1})}{S_1^m\times\dots\times S_{i-1}^m}x \\
& \} \\
f_i(x) = \max_{p_i}\{g_i(p_i,x)\} - h_i(x) &
\end{aligned}
$$

We shall prove that $f_i' > 0$ so the conclusion follows as in the case $i = 1$. The envelope theorem applies, but one must be cautious for the sequences $p_1(x), \dots, p_{i-1}(x)$ for which the maximum of the function defining g_i and h_i are not the same. To this end, define for all given value of x the function $p_i(x)$ as the value of p_i for which the function defining g_i achieves its maximum and $p_1(p_i(x), x), \dots, p_{i-1}(p_i(x), x)$ the remaining terms of this sequence whereas $p_1(x), \dots, p_{i-1}(x)$ correspond to the sequence of prices associated with the definition of h_i. We then have

$$
f_i'(x) \quad = \frac{S_i(p_i(x))}{S_i^m}\frac{S_1(p_1(p_i(x),x))\times\dots\times S_{i-1}(p_{i-1}(p_i(x),x))}{S_1^m\times\dots\times S_{i-1}^m} - \frac{S_1(p_1(x))\times\dots\times S_{i-1}(p_{i-1}(x))}{S_1^m\times\dots\times S_{i-1}^m}
$$

Now using again $x > 0$ and Remark 2 we have $S_k(p_k(x)) < S_k(p_k(p_i(x), x))$ for all $k \in \{1, i-1\}$ and $\frac{S_i(p_i(x))}{S_i^m} > 1$ hence $f_i' > 0$. ■

3 Value sharing in festival games

The property 1 is a strong statement in favor of the cooperative approach. In particular, it is well known that the core of any convex game is non-empty. More precisely, for any ordering π define $s_i(\pi) = v(\{\pi(1), \dots \pi(i)) - v(\{\pi(1), \dots \pi(i-1))$ for all $i > 1$ and $s_1(\pi) = v(\pi(1))$. then it is known that when π ranges over all possibles values, the sharing rule $(s_1(\pi), \dots, s_n(\pi))$ ranges over all vertices of the core. As a uniform average over all theses imputations, Shapley sharing rule cannot be blocked. We consider two rules, namely the Shapley and downstream incremental solutions (see van den Brink *et al.* (2011)).

3.1 Shapley solution

We will now provide a characterization of the Shapley value on the set of festival games. To this end, we closely follow Myerson (1980). The Myerson approach to Shapley value in graphs rests on an equalization of rewards and threats among coalition participants. Together with efficiency this property characterizes the Shapley rule of sharing on the set of games with transferable utility (or TU-games). We now show that these properties also lead to the Shapley value for the class of festival games.

Proposition 2 *Let* $(N, v) \in \mathcal{F}$ *and* $\psi(v) = (\psi_1(v), \ldots, \psi_n(v)) \in R^n$ *be a solution concept such that*

$$\begin{array}{l} \psi_i(v) - \psi_i(v_{-j}) = \psi_j(v) - \psi_j(v_{-i}) \, \forall \, i, j \in N \times N \\ \sum_{i \in N} \psi_i(v) = v(N) \end{array}$$

Then ψ *coincides with the Shapley value.*

Proof : First consider the case $n = 2$. The two conditions on ψ_1 and ψ_2 write as

$$\begin{array}{l} \psi_1(v) - v(\{1\}) = \psi_2(v) - v(\{2\}) \\ \psi_1(v) + \psi_2(v) = v(\{1, 2\}) \end{array}$$

Hence $\psi(v)$ is uniquely defined and the result holds for $n = 2$. Now consider ψ coincides with the Shapley, value for all $(N, v) \in \mathcal{F}$, with $2 \leq m < n$ and consider $(N, v) \in \mathcal{F}$. Using the recurrence hypothesis and the fact that removing one player leaves us with a festival game of size $n - 1$, we have $\psi_i(v_{-j}) = \phi_i(v_{-j})$ for all i, j (where $\phi()$ stands for the Shapley value). Now the linear system

$$\begin{array}{l} \psi_i(v) - \phi_i(v_{-j}) = \psi_j(v) - \phi_j(v_{-i}) \, \forall \, i, j \in N \times N \\ \sum_{i \in N} \psi_i(v) = v(N) \end{array}$$

admits a single solution hence ψ coincides with the Shapley value on $\mathcal{F}$. ■

3.2 Shapley solutions in the multi events case

Up to now we assumed that each theater organizes a single event. The Shapley solution may easily be extended to the multi-event case. More formally, consider k theaters organize a festival and denote N the set of events (again organized in chronological order). Let $T_1, \ldots, T_k$ be the partition of N such that T_1 collects the indexes in N corresponding to the events organized by theater 1, and so on. Myerson's characterization of Shapley's rule is commonly justified by balanced contributions arguments. More precisely the requirement

$$\psi_i(v) - \psi_i(v_{-j}) = \psi_j(v) - \psi_j(v_{-i})$$

amounts to balance the loss caused to i when j deviates with that caused to j when i deviates. However, when one theater organizes several events, it does not make much sense to balance the losses caused by deviations of i, j events if both events are organized by the same theater. If decision units are the theaters then we shall consider that deviations from the grand coalition should concern theater and not events. We thus need a variation of Myerson's characterization of Shapley's rule to cover this case.

More precisely, we establish the following property

Proposition 3 *Let $T \subset N$. Define $v_{|T}(R) = v(R)$ if $R \subset T$ and $v_{|T}(R) = 0$ otherwise.*

Let $N, v \in \mathcal{F}$ and $T_1, \ldots, T_k$ be the partition of N There exists a single vector $\psi' = (\psi_1, \ldots, \psi_k)'$ such that

$$\psi_{k_1}(v) - \psi(v_{|N\backslash\{T_{k_2}\}}) = \psi_{k_2}(v) - \psi(v_{|N\backslash\{T_{k_1}\}}) \, \forall \, k_1, k_2 \in \{1, \ldots, k\} \times \{1, \ldots, k\}$$
$$\sum_{l=1}^{k} \psi_l(v) = v(N)$$

Proof: The case $k = 1$ is trivial since it leads to $\psi_1 = v(N)$. Now let $T \subset N$. By repeated uses for all $i \in T$ of the fact that removing one player leaves us with a (sub) festival game, we establish that $(v_{|N\backslash\{T\}}, N\backslash\{T\}) \in \mathcal{F}$ (with straightforward notation).

Consider the case $k = 2$. Since $v_{|N\backslash\{T_1\}}$ and $v_{|N\backslash\{T_2\}}$ both belong to $\mathcal{F}$ we get

$$\psi_1(v) - \psi_1(v_{|N\backslash\{T_2\}}) = \psi_2(v) - \psi_2(v_{|N\backslash\{T_1\}})$$
$$\psi_1(v) + \psi_2(v) = v(N)$$

which is equivalent to

$$\psi_1(v) - v(T_2) = \psi_2(v) - v(T_1)$$
$$\psi_1(v) + \psi_2(v) = v(N)$$

and this system admits a unique solution. Finally assume the property has been established up to $k - 1$ theaters and proceed as in the characterization proof.■

This proposition tells us that we can safely apply the Shapley sharing rule among the theaters in the case of a multi-events program. Notice however that the above solution implicitly assume that once a theater leaves the coalition, it charges the monopoly price for all shows it organizes. This is not too harsh an assumption if the spillover between the shows organized in the same theater are small (for instance because it is the very same event or if different events are chronologically far from each other). But if the spillover among different events organized by the same theater are large, then when it deviates, these must be taken into account. In the applied part of the part of the paper, we compute these spillover effect and show they are quite small.

3.3 Downstream incremental solution

As the full computation of the game is needed for the derivation of the Shapley sharing rule, this solution suffers from two drawbacks. First, the computational burden increases exponentially fast. Second, as argued by van den Brink *et al.* the cooperative game may be viewed as an unnecessarily clumsy 'detour' towards the main goal: outcome sharing. In particular, the axioms proposed to justify the Shapley solution stem from cooperative game theory and are not directly related to the parameters of the supply/demand functions. As the decision makers typically base their behaviors on these parameters and not on game theoretic considerations the axioms should rely entirely on the former and not on the latter.[5]

For reasons that will become clearer in the newt section, a 'parametric' set of axioms for the Shapley solution seems difficult to find. But at least another solution may

[5]We thank Philippe Solal for raising this point.

be parametrically justified, namely the downstream incremental solution. The idea of incremental solutions is to take into account the linear structure of the game. In the downstream incremental version, the first theater receives its stand-alone value $v(\{1\})$, whereas the second one receives $v(\{1,2\})-v(\{1\})$ and so on until the last theater receives $v(N) - v(N\backslash\{n\})$.

Obviously, this solution satisfy efficiency. Also, since festival games are convex, it belongs to the core. In particular it fulfills the individual rationality criterion. The downstream incremental solution is independent from downstream characteristics of the game. In particular, for all $i < n$ the share distributed to theater i in the downstream incremental solution is independent from ρ_{i+1}. It is also independent from downstream inflows, as the Shapley solution.

We now propose a set of axioms for the downstream incremental solution.

Axiom 1 *Let $(N, v) \in \mathcal{F}$. A solution concept $\psi(v) = (\psi_1(v), \ldots, \psi_n(v)) \in R^n$ satisfies the efficiency property if and only if*

$$v(N) = \sum_{i \in N}^{n} \psi_i(v)$$

This first axiom is usual and already fulfilled by the Shapley sharing rule. Notice it imposes to be able to observe and/or compute $v(N)$.

Axiom 2 *Let $(N, v) \in \mathcal{F}$. A solution concept $\psi(v) = (\psi_1(v), \ldots, \psi_n(v)) \in R^n$ satisfies the no contribution property if and only if*

$$\psi_i(v) = 0$$

whenever $d_i = 0$ for all $(p_1, \ldots, p_n) \in \mathbb{R}^n$.

In a rather trivial way, this axiom tells us that if a theater cannot expect to have any audience it shall receive zero payment.

Axiom 3 *Let $(N, v), (N, v^\star) \in \mathcal{F} \times \mathcal{F}$. Let (e, c, ρ, S) (resp. $(e^\star, c^\star, \rho^\star, S^\star)$) be associated with (N, v) (resp. $(N, v^\star)$). A solution concept $\psi(v) = (\psi_1(v), \ldots, \psi_n(v)) \in R^n$ satisfies the downstream independence property if and only if*

$$\left(\forall j \leq i \; e_j = e_j^\star \text{ and } \rho_j = \rho_j^\star\right) \Rightarrow \psi_i(v) = \psi_i(v^\star)$$

This last axiom asserts that the share received by theater i does not depend on the characteristics of the downstream part of the festival. In particular, value sharing that fulfills downstream independence are robust to some "manipulations" by downstream agents which may pretend that the spillover effect is lower than its actual value.

We are now in a position to assert the following

Proposition 4 *A value sharing is equal to the downstream incremental solution if and only if it satisfies efficiency, no contribution and downstream independence properties.*

Proof : The proof follows the lines by van den Brink *et al.* and we provide it for completeness. It is trivial that the downstream incremental solution satisfies the three above axioms. For sufficiency, consider (N, v) associated with (e, c, ρ, S) and let us start with theater 1. Consider the game $(N, v_1^\star)$ associated with

$$\begin{array}{l}(e^\star)' = (e_1, 0, \ldots, 0)' \\ c^\star = c \\ (\rho^\star)' = (0, 0, \ldots, 0)' \\ S^\star = S\end{array}$$

By the no contribution property we have $\psi_i = 0$ for all $i \geq 2$ and by efficiency we get $\psi_1 = v(\{1\})$. Now consider the property is fulfilled up to theater $i-1 < n-1$. Consider the game $(N, v_i^\star)$ associated with

$$\begin{array}{l}(e^\star)' = (e_1, e_2, \ldots, e_{i-1}, e_i, 0, \ldots, 0)' \\ c^\star = c \\ (\rho^\star)' = (\rho_2, \rho_3, \ldots, \rho_{i-2}, \rho_i, 0, \ldots, 0)' \\ S^\star = S\end{array}$$

By induction and the no contribution property we have $\psi_j = 0$ for all $j > i$. Now by induction and efficiency we get $\psi_i = v(N\backslash\{i+1, \ldots, n\}) - v(N\backslash\{i, \ldots, n\})$. Now for theater n by induction and efficiency we get $\psi_n = v(N) - v(N\backslash\{n\})$. ■

The three axioms are logically independent. If all players receive nothing, this solution satisfied axioms 2 and 3 but not axiom 1. If we give $v(N)$ to the last player, this rule satisfies axioms 1 and 3 but not axiom 2. Finally, the Shapley rule satisfies axioms 1. As for axiom 2, remark the no-contribution property applies to player i if and only if $\max_{j \in N, j \leq i}\{e_j\} = 0$. Now in this case, it is clear that the surpluses generated by player i are all zero, so Shapley rule also fulfills axiom 2. Consider the case $N = \{1, 2\}$, player 1 gets $v(\{1\}) + \frac{1}{2}(v(N) - v(\{2\}))$ if we adopt the Shapley rule. The quantity $v(\{1\})$ does not depend on e_2. But, as a consequence of Lemma 3, $\frac{1}{2}(v(N) - v(\{2\}))$ is a linear function of e_2 and it is trivial to verify that this quantity strictly increases with e_2 whenever $\rho_2 > 0$. so the Shapley rule does not verify axiom 3.

3.4 Comparison of solutions

We end up this section by a comparison of the different solutions. Consider the case $N = \{1, 2, 3\}$, $\alpha_i = \sqrt{8}$, $c_i = 1$ for $i \in N$ $\rho_i = 1$ for $i \in \{2, 3\}$, and $e' = (1, 0, 0)'$. In this case we have

$$\begin{aligned} v(\{i\}) &= 2^{-i} \\ v(\{1,2\}) &= \frac{97^{3/2}-287}{864} \\ v(\{1,3\}) &= \frac{131\sqrt{393}-1143}{2304} \\ v(\{2,3\}) &= \frac{v(\{1,2\})}{2} \\ v(N) &= \frac{(8-p^2(N))(p(N)-c(N))}{8} \end{aligned}$$

with $p(N) = \frac{c(N)+\sqrt{24+c^2(N)}}{3}$ and $c(N) = \frac{1151-97^{3/2}}{864}$.

As in this example the game is strictly monotonic, we have

$$\min\{v(N) - v(\{2,3\}), v(\{1,2\}) - v(\{2\}), v(\{1,3\}) - v(\{3\})\} > v(\{1\})$$

hence we get

$$\phi_1 = \frac{1}{3}v(\{1\}) + \frac{1}{3}\left(v(N) - v(\{2,3\})\right) + \frac{1}{6}\left(v(\{1,2\}) - v(\{2\})\right) + \frac{1}{6}\left(v(\{1,3\}) - v(\{3\})\right) > \frac{1}{3}v(\{1\}) + \frac{1}{3}v(\{1\}) + \frac{1}{6}v(\{1\}) + \frac{1}{6}v(\{1\}) = v(\{1\})$$

and the first player gets strictly more with Shapley value than in the downstream incremental solution. Numerically, the difference in our example is about 0.0231. A similar comparison holds in the opposite direction for player 3

$$\phi_3 = \frac{1}{3}v(\{3\}) + \frac{1}{3}\left(v(N) - v(\{1,2\})\right) + \frac{1}{6}\left(v(\{1,3\}) - v(\{1\})\right) + \frac{1}{6}\left(v(\{2,3\}) - v(\{2\})\right) < v(N) - v(\{1,2\})$$

Indeed, strict convexity implies

$$v(N) - v(\{1,2\}) > \max\{v(\{3\}), v(\{1,3\}) - v(\{1\}, v(\{2,3\}) - v(\{2\})\}$$

Comparison for player 2 is ambiguous. In our example, this player is better treated by the Shapley rule. But this no longer the case if, for instance, the spillover effect is low enough. [6]

The above comparison also highlights the differences between festival and river sharing games. Indeed, in the river sharing game introduced by Ambec and Sprumont (2002), downstream incremental and Shapley rules are known to coincide. In our model, the difference comes from two effects. First, when a theater leaves the festival it is not its own interest to leave the downstream part of the festival without audience. In this regard, the festival game is more closely related to Ambec and Ehlers (2008). But there is another difference. When it joins a coalition the pricing efforts of the downstream members of the coalition are more efficient. This is not the case in river sharing games,

[6] This is the case, when —other things equal— $\rho_i = 0.1$ for $i \in \{2, 3\}$.

as the utility of water for a given player does not depend on the efforts made by its upstream fellows.

The above argument also shows that "parametric" justifications of the Shapley solution is quite difficult. When applied to festival games, the Shapley rule takes into account all the externalities in a complex way. It is also difficult to provide parametric justifications for the incremental upstream solution. In particular, this solution is not independent from upstream characteristics since the market shares of downstream theaters depend on upstream inflows and spillover effects.

To end up this comparison, notice that from a computational viewpoint, the downstream incremental solution is also much simpler to compute if the festival is very large, since we only need to know n coordinates of v. On the other hand, it is difficult to generalize it to the multi events if the same decision unit is both in a downstream and in an upstream position with respect to another one.

Computational as well as theoretical considerations then lead us to prefer the downstream incremental solution when the festival consists in a large number of separate decisions units each providing a single show (as it is typically the case in music festivals, for instance).

4 Implementation for the NEXT festival

This section is devoted to the presentation of a concrete example. The aim of this section is to convince the reader that the computation of shared values is indeed feasible in a real life example. The main difficulty is that the vector game is of course not observed. Indeed, if we do trust the cooperative approach, then the grand coalition must form and no other behavior is observable. We then need to combine an econometric model and extra observations to recover the missing part of the game. It must be stressed that in this paper, econometrics do not aim at testing our model or the validity of cooperative approach. Rather we seek for counterfactuals to compute the payoffs if such-and-such coalition would have formed. Also we do not claim that the statistical avenue we propose is the only (nor the best) one. Other routes may be followed that lead to other estimates of the shares.

We first present the available data set, then we propose an applied version of the theoretical model. Finally we discuss the estimations.

4.1 Data

We collected data related to a dance and theater festival that took place in the north of France and southern part of Belgium between 02/18/2012 and 03/03/2012. The festival is organized on a yearly basis. The theaters involved change slightly from one edition to the other. For the 2012 edition 9 theaters (5 French, 4 Belgian) were involved. The theaters display some heterogeneity : the largest total capacity is 800 and the smallest is 70, one theater is entirely new whereas the oldest theater is more than one century old, etc. Each of these theaters have their own program during the remaining part of the year, some of them are also involved in other festivals. One common feature is the "avant-garde" nature of the works presented. The program presents a selection of contemporaneous, ambitious, young and well-established performers, dancers and directors (the agenda is provided in the appendix section). The audience of this festival mainly consists in aware amateurs, professional and semi-professional artists, so the spillover effects during the festival are likely large enough for the above model to be useful. Spillover effects are a major issue for the organizers as the report for public subsides explicitly mentions audience mobility as a major objective of the festival. Organizers of this festival particularly promoted trans-borders mobility. To this end, Dutch subtitles are displayed for many French speaking performances (and vice-versa) and a shuttle service has been organized.

For each performance, we observe the audience and the total capacity of the theater. We used the presentation of the performance provided by the festival's program to collect information about the nature of the performance (dance, theater or other type of performance). To measure relative notoriety we counted Google citations of each performances. [7] We also observed characteristics that are probably linked to the cost such as the number of artists and the duration of the show.

Of course, the program provides a full description of the prices. The full price is charged for the first ticket only. A 2 € discount is offered for all other tickets. Finally less than 26 years old people benefits from a special tariff (7 €) for all performances.

We also have extra information about mobility. Surveys have been collected before and during most of the performances. [8] Each survey consists in 44 questions (an English translation of the complete survey is displayed in appendix). Questions 34 to 42 explicitly measures the mobility of the audience. We use the answers to 476 surveys to compute the following Markov transition matrix.

Table 1

[7] We searched for exact citations of the artist(s) and/or the show. The mention "NEXT" has been deleted for the search and we also restrict to dates before November 2012.

[8] Three performances out of 22 could not have been covered. One of them, -"On the Concept of the Face, regarding the Son of God" by Romeo Castellucci- experienced violent demonstrations from extreme Christian activists. As the survey have been conducted by students we prefer to cancel the interviews for this performance. Notice the demonstrations did not discourage audience, since both representations of this performance were sold out weeks before the performance. For two other performances, we ran out of survey conductors.

dest. / org.	Budascoop	Espace Pasolini	La Rose des vents	Maison de la culture de Tournai	Maison Folie de Wazemmes	Phenix	Schouwburg	Théâtre du Nord	Transfo
Budascoop	0.18	0.06	0.11	0.14	0.09	0.09	0.15	0.10	0.09
Espace Pasolini	0.08	0.17	0.14	0.10	0.12	0.17	0.06	0.12	0.04
La Rose des vents	0.04	0.04	0.22	0.13	0.19	0.13	0.05	0.19	0.01
Maison de la culture de Tournai	0.05	0.06	0.17	0.23	0.14	0.11	0.09	0.13	0.02
Maison Folie de Wazemmes	0.10	0.06	0.16	0.10	0.22	0.08	0.08	0.15	0.04
Phenix	0.07	0.10	0.16	0.11	0.12	0.20	0.04	0.16	0.01
Schouwburg	0.17	0.06	0.11	0.11	0.09	0.10	0.20	0.10	0.06
Théâtre du Nord	0.02	0.05	0.19	0.14	0.21	0.13	0.03	0.22	0.02
Transfo	0.15	0.06	0.12	0.13	0.08	0.08	0.16	0.10	0.13

Table 1 above should be red as follows. The probability that a spectator interviewed at 'Espace Pasolini' will attend the next performance at 'la Rose des Vents' is about 14%. Special buses where provided by the organizers of the festival in order to enhance this "natural" mobility and to promote public transport. For two couples of performances packages were proposed which include transportation from one theater to the other in order to attend both performances on the same day. In a similar fashion, we also collected the travel times between places to the other.

4.2 Inference and estimation results

For our cooperative approach to be implemented, we need for each event : the survival function, the cost, a measure of the queues q_i a decompositions between newcomers and spillover effects $q_i = e_i + r_i$ and a dispatching of r_i among ancestors of event i. These issues raise different problems that we addressed with specific inference techniques.

4.2.1 Survival functions

The estimation strategy cannot rest upon the hypothesis that actual prices have been chosen as to maximize the total welfare. Indeed, only three (full) prices are observed (20 €,14 €, 9 €). Moreover one performance (Syndrome Collective) is free. As this part of the paper is mainly descriptive, we did not take into account the full variety of prices, and used full prices as a proxy in the demand equation.

We first have to choose $\mathcal{S}$. Recall we need $S(0) = 1; S'(0) = 0$ and $S" < 0$. Bearing in mind the relatively modest data set, we choose one of the simplest functional form, namely $S(p) = 1 - (p/\alpha)^2$ with $\alpha \geq p \geq 0$.

Our estimation strategy rests on 'robust' assumptions about the choices of full prices. First, the monopoly price must be larger than the observed price since even if the theater leaves the coalition, it is against its own interest to set a price larger than the monopoly price. The monopoly price for a linear cost c is

$$\frac{1}{3}\left(c + \sqrt{3\alpha^2 + c^2}\right)$$

Now when theater i joins the coalition, it chooses a price as if it was in a monopoly position with a smaller linear cost (Cf equation (**??**) above). We assume this effort cannot exceed the case in which the "corrected" value for c is negative, for otherwise it would be optimal to charge a negative price. Hence we have for all theater i

$$\alpha_i \geq p_i \geq \alpha_i/\sqrt{3} \Leftrightarrow p_i \geq \alpha_i \geq \sqrt{3}p_i$$

As the prices are observed, we may calibrate α_i by interval regression using a specification such as $\alpha_i = \beta' X_i + \epsilon_i$ where X_i are some covariates and ϵ_i is and iid sample in the $\mathcal{N}(0, \sigma^2)$ distribution. At this stage, the free event has not been used since, for this event we have $S_i = 1$ by assumption.

Survival parameter α_i (QML estimation)

	Estimate	Std. Err	z	p-value
intercept	-25.775	0.548	-47.036	0.000
theater	1.8203	0.250	7.274	0.000
log(capacity)	9.266	0.074	124.424	0.000
log(notoriety)	0.791	0.066	12.034	0.000
concurrence	0.449	0.019	23.342	0.000
conc. $\times$ star	-0.107	0.007	-15.348	0.000
σ	0.329	0.006	51.488	0.000

The sensibility to prices is lower for theater shows than for dance or performance, it is affected by the size of the the theater, the notoriety of the performer, the number of shows on the same day (captured by 'concurrence'). Notice this last effect displays a counter intuitive sign. Remark also the effect of concurrence is mitigated when the performer is a 'star'. [9]

4.2.2 Costs

Clearly, since the monopoly price is a given function of α and c we may recover the cost from the previous estimates of α and some knowledge about the monopoly prices.

To this end, we used again a robust bracketing argument. We know that the observed price cannot be larger than the monopoly price. Now, the effort when joining a coalition is maximized when the theater assumes that the audiences of downward shows result only of spillovers from its own audience. Moreover we know in this case that the effect on the linear cost is exactly equal to the per capita sum of profit of downward firms. Hence the difference between $p_i^m - p_i$ cannot be larger than

$$\frac{1}{3} \sum_{j>i} \pi_j / d_j$$

[9] The 'star' dummy variable equals one if the number of links found by Google exceeds 1.000 (otherwise it is zero). The threshold has been so chosen to reflect the clear distinction between the number of links.

We do not observe directly this profit (and it is unlikely that the theaters could do so before the end of festival) but it cannot be larger than the sum of observed (and committed) observed prices. [10] We then get the following brackets

$$p_i + \frac{1}{3}\sum_{j>i} p_j \geq p_i^m \geq p_i$$

We used this bracket to perform an estimation of the monopoly price assuming again a linear gaussian specification for the logarithm of the monopoly price. Using the forecast values of p_i^m and α_i we may recover the costs c.

We finally obtained the following maximum likelihood estimates (we do not report the estimations results for the monopoly prices, as they are not directly related to our purpose). The following table provides some summary descriptive statistics related to costs (markup is computed as observed full price minus cost).

Summary statistics for costs and markups

	Mean	Std. Err	Min.	Max
cost	6.970	2.787	3.746	13.223
markup	7.259	2.863	2.860	12.511

The markups are slightly larger than expected, but one should recall that the full price is charged only for the first event. Also remark the order of magnitude of the costs is consistent with an average zero markup for youngsters. Finally recall the Interreg program allows to subsidize costs (up to half of their value).

4.2.3 Queues

As previously mentioned, we then used the estimate for α_i as well as the actual prices to compute an estimated version of the queue q_i. At this stage we face another issue since 13 events appeared to be sold out. For these events the computation of the queue as the ratio of the observed attendance over the survival rate $1 - (p_i/\alpha_i)^2$ is a right-censored version of the actual queue. To overcome this problem, we specified a Tobit model to estimate the a corrected version of the queue.

The 'sold out' outcome may result from very different factors. Of course, notoriety is expected to play a major role, but the capacity of the theater, the presence of competing cultural events within or outside the festival may also be important. Unfortunately the sample is rather small and we have no information about competing events outside the festival.

We performed a Principal Component Analysis on the available exogenous variables and then used the factors as explanatory variables in the Tobit equation. Significant

[10] A minor detail must be mentioned at this stage. For many days, several shows takes place at the same time. We treated downward shows as a whole, meaning that we compute we replace p_j for $j > i$ in the above equation by the average prices observed for each date to come. Details are available upon request.

factors are 1,3,6 and 7. We dot not report the Tobit estimation because PCA factors are difficult to interpret directly, but the following table provides the most significant correlations between the corrected queue estimates and the explanatory variables.

explanatory variable	correlation with q_i
capacity	0.878
star	0.796
staff	0.422
dance	0.421

It appears, as expected that queues are the biggest in front of larger theaters and for 'stars'. The two last positive correlations ('dance' and 'staff') are less expected. The 'staff' variable is a measure of the number of actors/dancers/performers/musicians on stage. Together with the fact that the event involve dancers, it appears to be positively correlated with the estimations for queue.

4.2.4 Spillovers

Now, we need a strategy to decompose the corrected queue between newcomers and spillovers effects. The theoretical model tell us that whatever the coalition, prices should be not correlated with newcomers. We may then use prices as instruments to decompose $q_i = e_i + r_i$.

The best model with came up with (in terms of significance of coefficients and relevance of instruments) is a two stages least squares with capacity and factor 5 of the PCA as (possibly endogenous) explanatory variables together with actual and three previous prices as instruments. We are able to recover the spillovers up to some constant (see below how we did calibrate the intercept parameter).

4.2.5 Dispatching

Finally for each show i we must decompose r_i over all possible previous shows. Unfortunately, we do not have direct access to this information in the data. We assumed:

- spillover from more than one day are negligible;
- nobody attends the same show twice in a row; [11]
- the transition probabilities are as in Table 1.

An example may be in order to explain the full process. Consider for instance the seventh performance that took place on november the 20-th at "Budascoop". This was the last performance of the play "Frustrating Picture Book for Adults" by N. Penino and the audience was 58 people.

[11] Notice this assumption does not strictly preclude attending twice the same show, since one can attend another show between two different performances of the same play.

First, the estimation of the queue tells us that $q_7 = 89$ people were initially "interested" by the show. The decomposition of the queue tells us that $r_7 = 24$ —out of 89— people have attended a show in the NEXT festival the day before. On november the 19-th four plays were displayed : "Kiss and Cry" at "Maison de la Culture de Tournai", "Cinquanta urlanti, quaranta ruggenti, sessanta stridenti" at Schouwburg, "Eden Central" at "Budascoop" and the first performance of "Frustrating Picture Book for Adults" also at "Budascoop". The spillover r_7 has then be dispatched as follows. The probability for someone arriving at "Budascoop" form "Maison de la Culture de Tournai" conditionnally on the fact that he/she either come from "Maison de la Culture de Tournai", "Budascoop" or "Schouwburg" may be computed form table 1 and it is almost 13%. So the estimated spillover from "Kiss and Cry" on 19-th to "Frustrating Picture Book for Adults" on the 20-th is $r_7 \times 0.1299 \simeq 3$ people.

A final detail is in order. Recall we are only able to estimate spillover (and consequently, the newcomers) up to some constant. This constant may be bounded from above, as the number of newcomers cannot be negative. Also, we assumed that there is no "word-of-mouth-effect". This last assumption means that the total amount of spillover that may be attributed to a given performance cannot exceed the total audience of this performance.[12] Also the constant may be bounded from below as spillover must be positive. To assess the sensitivity of our computations to this parameter, we computed the Shapley values using three different magnitudes (high, middle, low) but the differences are negligible. The table below gives some summary statistics about the spillover effects.

Spillover : Estimated number of spectator received and sent

	received	sent
min	0	0
Q_1	6.80	4.29
Q_2	9.59	7.45
Q_3	15.98	15.31
max	37.74	45.18

The model allows to compute the largest contributors and recipients from the spillover effects. The events organized at the 'Transfo' and 'Maison folie de Wazemmes' (which are 'new' places and whose performances were organized near the end of the festival) seem to benefit the most from the spillovers, whereas most providers are related to stars and –by construction– the first show.

4.3 Value sharing

We now have all the necessary inputs to compute the game. Notice that value sharing of the festival will be computed according to the Shapley rule since we are in a multi events

[12]Using notations of the theoretical part, this amount to say that $0 < \rho_i \leq 1$. Notice that this does not hamper our theoretical results, but this is a reasonable assumption in our setting and it does gives us a rather sharp bound on the constant.

case. Notice this exercise is quite time-consuming since we allow for many transitions. However, we consider that the decisions are taken by the 9 theaters and we apply results of subsection 2.3 above.

For each coalition we computed the optimal pricing rule (considering the outside member charge a monopoly pricing rule for each event). We derived the attendances (controlled for the capacity constraints) and the payoffs.[13]

We have checked the convexity of the game. Recall this is not a consequence of our theoretical result, since capacity constraints are binding. It turns out that our game is not convex, but the violation of the inequality $v(S \cup T) - v(S) - v(T) + v(S \cap T) \geq 0$ are extremely small as the minimum of $\frac{v(S \cup T) - v(S) - v(T) + v(S \cap T)}{v(S \cup T)}$ over all possible coalition S, T is -2.05×10^{-5}.

The following table provides a comparison of the the relative shares obtained by the solution proposed in Section 3 and three 'rules of thumb' considering attendances, costs and payoffs. We used attendances (referred to as 'Att.' in the table below) times full prices as a proxy for unobservable payoffs. Also, recall that we are in a multi event setting. Hence the downstream incremental solution is not directly applicable. We propose to compute the downstream incremental solution as if the different days were separate decision units. When several theaters organized a show on the same day, we have to split the value between these theaters. We compare two solutions. The first (reffered to as 'per capita' in the table below) is an equal sharing between theaters. The second (reffered to as 'per Att.' in the table below) is proportional to observed attendances.

Comparison between value sharing rules (Relative shares in % of total revenue)

Theaters	Shapley	Downstream incremental		Rules of thumb		
		(per capita)	(per Att.)	Att.	Att. × full price	Costs
Budascoop	6.31	17.91	7.58	8.70	6.12	5.07
Espace Pasolini	2.42	3.36	2.27	2.05	1.27	1.61
La rose des vents	21.05	13.82	16.46	21.12	23.14	18.76
Maison de la culture de Tournai	21.46	17.37	30.36	20.27	23.82	25.96
Maison folie de Wazemmes	2.51	6.18	19.67	2.82	1.49	1.57
Phenix	6.78	7.36	5.47	7.13	5.87	5.81
Schouwburg	11.64	12.02	11.91	12.71	10.41	14.20
Théâtre du Nord	23.52	15.28	20.73	20.25	23.81	23.95
Transfo	4.27	6.67	3.32	4.92	4.05	3.05
Belongs to the core	yes	no	no	no	no	no

[13]Notice that the capacity constraint may cause a non convexity in the game. We checked that our applied game is convex despite this generalization, but it is quite easy to cook up theoretical examples in which convexity breaks down in presence of severe capacity constraints.

First, it is seen that the two versions of the downstream incremental solution differs very much form the other distribution rules. Moreover, these two versions differs quite sharply from one to another. This is tell us that although axiom 3 implies sharply different sharing, it leaves quite a large place to bargain in the multi event case. In one word, the downstream incremental solution, does not seem to be very appropriate in our setting.

Now comparing Shapley with various versions of 'rules of thumb', we see that despite similar orders of magnitude, the Shapley value does not exactly correspond to a mere redistribution of each participant's payoffs nor attendances nor production costs. It should be mentioned that using our proxy for payoffs a one point difference worth approximatively 1.800 €. Though modest, the differences between the sharing rules are not negligible.

As a stability check, we looked whether theses rules belong to the core of the game. If capacity constraints are never binding, the game is convex so the Shapley sharing rule belongs to the core. But stringent capacity constraints may cause the Shapley sharing rule to be outside the core and may even induce emptyness of the core. We found that the Shapley rule is inside the core, but none of the rule of thumb rules is.

It is not very meaningful to check whether the two versions of the downstream incremental solution belong to the core, since decision units are different. We nevertheless can check that this in not the case.

It could be claimed that the Shapley solution is not suited since the optimal pricing among theater differs from the actual pricing. Also, it takes into account possible deviations from the grand coalition, which also lead to different pricing rules. Now, if we consider the grand coalition pricing we see that the sum of the deviations from the monopoly pricing amounts to 3.44 €, which is close to the minimum cost. Taking efficiently spillover into account allows to cover the cost of one show. Incidentally we recall that one of the show was indeed a free event. As another point of comparison, recall the full price must be paid for the first event only whereas all the remaining one benefit from a 2 € cut. Also recall the Interreg program cover half the costs, the net effort corresponding to the cut amounts to 1 euro by shows. All these comparison tells us that the pricing rules used to derive the Shapley solution are in reasonable accordance with actual pricing choices.

4.4 Conclusion

We present a cooperative approach to the microeconomic study of cultural festival. The incentive to cooperate rests on spillover effects created by attenders moving from one performance to the other. The theoretical game is shown to be convex, and we show that Myerson's axiomatic characterizes Shapley value in the set of festival games. We also provide a generalization of Myerson approach adapted to the case in which some theaters may organize several events. Finally we propose another set of axioms which may be justified from a 'parametric' viewpoint. This new set leads to the downstream incremental solution and we discuss the sources of difference between the two possible solutions. We apply our model to the real life case of the NEXT festival organized on a yearly basis in the North of France and Southern part of Belgium. A structural estimation strategy is used to derive sharing rules among the places. The downstream incremental rule does not seem very appropriate since we deal with a multi event case. We show that Shapley rule does not exactly coincide with various versions of rules of thumb. Moreover we checked that Shapley is the only stable rules according to the core concept.

5 Appendix

5.1 Program

performance or artist name	dates (from 18/11. to 3/12)	Place	type	price (full, in €)
M. A. Demey /J. van Dormael	18,19,20	Tournai	Dance	20
T. Castellucci /D. Dell	19	Schowburg	Dance	9
M. Depauw	19	Budascoop	Theater	14
N. Penino	19,20,22,23	Budascoop	Theater	9
B. Lachambre	22	Phenix	Dance	14
C. De Smedt	21, 22	Espace Pasolini	Dance	9
D. Veronese	22 to 26	Théâtre du Nord	Theater	20
F. Jaâbi /J. Baccar	23	Phenix	Theater	14
O. Dubois	23,24	Rose des Vents	Dance	14
A.C. Vandalem	24,25	Tournai	Theater	20
Gob Squad	25,27	Budascoop	Theater	14
I. Van Hove	26,27	Schouwburg	Theater	20
N. Lucas /H. Heisig	28	Espace Pasolini	Dance	9
L. Rodrigues	29	Phenix	Dance	14
R. Castellucci	29 30	Rose des Vents	Theater	20
C. Loemij /M. Lorimer	30	Schouwburg	Dance	9
E. Joris	1 to 3	Wazemmes	Theater	9
Berlin	30 to 3	Transfo	Theater	14
T. Castellucci /D. Dell	1,2	Budascoop	Dance	9
W. Vandekeybus	2,3	Rose des Vents	Dance	20
O. Normand /Y. Barelli	2	Espace Pasolini	Dance	14
Syndrome Collective	2	Schouwburg	Theater	0 (free event)

5.2 Survey Questions

1. Place where the survey has been conducted (9 items)
2. Where in this place (3 items : queue, bar, other)
3. Gender (2 items : Female, Male)
4. Current occupation (14 items)

5. "Where are you coming from" (free answer)
6. ZIP code (free answer)
7. Transportation device (5 items: pers. veh., someone else's veh., public trans., Next bus, other)
8. If previous "other" describe (free answer)
9. Age (14 items : 5 years intervals from less than 15 to more than 74)
10. What kind of performance do think you are about to attend (2 items yes/no per category : classic, experimental, international masterpiece, new generation, some artist I know)
11. How many cultural performance do you attend on a yearly basis (4 item once a year, 2 to 4, 5 to 12, more than 12 per category : general, dance, theater, classic, experimental)
12. Do you see a difference between Next and the usual program (2 items yes/no)
13. If previous "yes" describe (free answer)
14. Do you attend other festivals (2 items yes/no)
15. If previous "yes" which one(s) (free answer)
16. Do you go to other theaters (2 items yes/no)
17. If previous "yes" which one(s) (open question)
18. Which other Next performances do you intend to /have you already see(n) (open question)
19. Do you have a personal practice of theater/dance (3 items : yes as amateur, yes as a professional, no)
20. How do you came to known NEXT's festival (6 items)
21. How do you came to known NEXT's program (8 items)
22. If previous "other" describe (free answer)
23. What type of tariff do you have (4 items unit price, subscription, free, other)
24. If previous "other" describe (free answer)
25. In case you know it what price did you pay (free answer)
26. Do you known the following theaters (4 items : by name, already went to, will soon go to, subscriber per theater : 9 possible)

27. Have you already answer this survey (2 items yes/no)
28. If previous "yes" at which performance (free answer)

5.3 Map

Figure 1: Map of the Festival

(1:Budascoop, 2:Espace Pasolini, 3:Rose des Vents, 4:Maison de la culture de Tournai, 5:Maison Folie de Wazemmes, 6:Phenix, 7:Schouwburg, 8:Théâtre du Nord, 9:Transfo)

Distance and travelling times (by car) between major relevant cities

	Tournai	Kortrijk	Valenciennes
Lille	26 kms ; 26 min.	35.2 kms ; 30 min.	44.5 kms ; 38 min
Tournai	.	35.5 kms ; 38 min.	58.4 kms ; 40 min
Kortrijk	.	.	76 kms ; 49 min

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References

[1] Ambec, S., Sprumont, Y., 2002. Sharing a river, *Journal of Economic Theory*, 107, 453-462.

[2] Ambec, S., Ehlers, L., 2008. Sharing a river among satiable agents *Games and Economic Behavior* 64 (1), 35-50.

[3] Béal S., Solal P., 2010. Allocation rules for museum pass programs *Revue Economique*, 61 (6), 1099-1109.

[4] van den Brink R., Estévez-Fernández A., van der Laan G. and Moes N., 2011. Independence Axioms for Water Allocation, *Tinbergen Institute Discussion Paper*, TI 2011-128/1.

[5] Ginsburgh, V., Zang, I., 2001. Sharing the Income of a Museum Pass Program, *Museum Management and Curatorship*, 19 (4), 371-383

[6] Myerson, R.B., 1980. Conference structures and fair allocation rules *International Journal of Game Theory*, 9, 169-182.

[7] Hiller S., 2012. Exclusive Dealing and its effects : the Impact of Large Music Festivals on local Venues, University of Colorado, mimeo.

[8] Levy-Garboua, L., Montmarquette C., 1996.A Microeconometric Study of Theater Demand, *Journal of cultural economics*, 20, 25-50.

[9] Shapley. L.S., 1953. A Value for n-person Games *in* Contributions to the Theory of Games, volume II, by H.W. Kuhn and A.W. Tucker, editors. *Annals of Mathematical Studies* 28, 307-317.