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Revisiting the "Missing Middle": Production and Corruption

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Abstract. This paper investigates empirically the relationship between firm size and production efficiency, and the relationship between firm size and the likelihood of paying bribes using firm-level data from Vietnam. Our analysis indicates that middle-sized firms' production efficiencies tend to be lower than small-sized or large-sized firms in most of the manufacturing industries, and as firm size increases, the likelihood of paying bribes also increases. Firm size distribution has been a particular concern of economists for almost a century, and more recently, there has been increasing interest in the firm size distribution of developing nations, particularly the missing middle phenomenon, which refers to the fact that the distribution of firm size in developing countries tends to be bimodal. Our empirical results shed light on the relationship between firm size, production and corruption, and provide insights into these relations.

Key Words: Bribe, Firm size distribution.
JEL Classification Numbers: D21, D22, L25.

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1 Introduction

Firm size distribution has been a particular concern of economists for almost a century. The missing middle refers to the fact that the distribution of firm size in developing countries tends to be bimodal. This paper studies this fact using firm-level data from Vietnam. We investigate empirically the relationship between firm size and production efficiency. Furthermore, we also study the relationship between firm size and the likelihood of paying bribes. We then present the following facts.

- Middle-sized firms' production efficiencies tend to be lower than those of small-sized or large-sized firms in most of the industries that we study here.

- As firm size increases, the likelihood of paying bribes also increases, according to our regression result.

The missing middle was first documented in Liedholm and Mead (1987), which presents evidence that most employment in developing countries is located in either small-sized or large-sized firm groups. Tybout (2000) also finds that the firm size distribution in developing countries is bimodal and argues that strong business regulation could be a reason behind the existence of too many small firms. The same evidence of bimodalism is presented in Little et al. (1987) for South Korea and Steel and Webster (1992) for Ghana in the 1980s.

Recently, Hsieh and Olken (2014) studied data from India, Indonesia and Mexico and presented the following results: (1) there are many small firms but not so many middle-sized or large-sized firms, and in this sense, there is no missing middle, (2) large firms rather than small firms seemingly incur the large fixed costs, (3) there is no evidence of discontinuities in regulatory obstacles across different firm sizes. Our results are consistent with the findings in Hsieh and Olken (2014) on these three points.

In response to the findings by Hsieh and Olken (2014), Tybout (2014a) (see also Tybout, 2014b) suggests that a better test of the missing middle is to ask whether the share of middle-sized firms, as opposed to small or large firms, is smaller than the share that one would observe in an undistorted economy and shows a concentration of small plants. Further, we also study whether the likelihood of paying bribes varies depending on the formality of firms. In the literature, some studies (e.g., Dutta et al., 2013; Mishra and Ray, 2013) suggest that small firms may refrain from becoming formal to hide themselves from the rent-seeking activities of civil servants. We also find that the formality is another important determinant in this aspect, which is consistent with the previous studies.

Our first contribution is to present empirical evidence showing that productivity is different across firm size and, in particular, small-sized firms are relatively more efficient at producing than middle-sized firms, using firm-level data. We also show that the difference varies depending on each sector. To the best of our knowledge, this is one of the first studies to connect the missing middle and production efficiencies across different firm sizes in each sector of a developing country.

Recently, there have been several studies on the relationship between firm size distribution and productivity in developed nations. Leung et al. (2008) investigate the US–Canada difference in the distribution of employment over firm size and confirm that a larger average size supports higher productivity at both the plant and firm level. Crosato et al. (2009) investigate the data from a micro-survey in Italy and, using nonparametric estimates, finds that firms in the concave part of the Zipf plot of Italian firm distribution overwhelmingly experiences increasing returns to scale, while firms in the linear part are mainly characterized by constant returns to scale. Diaz and Sanchez (2008) use a stochastic frontier model to investigate small-sized and medium-sized manufacturing enterprises in Spain and finds that these firms tend to be less efficient than their large-sized peers.

In contrast, the literature on firm size and productivity in developing countries is scarce, perhaps because of the unavailability of detailed data. Among a few studies, Johannes (2005) provides evidence on the differences in the evolution of firm size and productivity distribution across nine sub-Saharan African countries and developed countries including the United States. While in the United States, small firms normally have low productivity and survival rates, and incumbents who exit the market tend to experience a period of decreasing size and productivity, in Africa, the largest productive firms have the highest growth rate, and small firms rarely reach the top of the size and productivity distributions. Banerjee and Duflo (2005) provide surveys and analyze what causes lower total factor productivity (TFP) in poor countries (which they call "macro puzzles"). They mainly study access to technology or human capital for the reasons why small firms do not grow.

In this study, using firm-level data, we analyze productivity across different firm sizes and industries using the frontier approach and a nonparametric model, which is "best suit to describe industry and firm behavior" (see Mahadevan, 2004). As Kalirajan and Shand (1994) claim, the implicit assumption of the non-frontier method is that firms operate in the long-run equilibrium, and there is no technical inefficiency, whereas the frontier approach measures technical inefficiency of each firm as the radial distance from the observed production to the frontier. We present both efficiency scores, which measure the distance from the production frontier of each firm's production, and returns to scale, in each sector of the manufacturing industry. By so doing, we demonstrate that middle-sized firms have relatively low efficiency in production, although the scale at which minimum efficiency occurs and the level of this efficiency are different across industries. This insight adds to the literature by providing a more detailed analysis of the missing middle, as a macroeconomic phenomenon. Furthermore, we show that small firms have technologies with increasing returns to scale, which suggests that while they *may not want to grow*, they *should grow*.

To study whether there are any negative costs related to size, we analyze various aspects of corruption in the economy. Our second contribution is presenting empirical evidence that the likelihood of paying bribes is highly correlated to firm size. We also show that the location of firms affects the likelihood. We also analyze the relationship between the likelihood of bribes and firm age, which shows a negative relationship. This negative relationship could be thought of as being counterintuitive, although this may indicate that a firm tends to pay bribes at the start of business. Finally, our

analysis also indicates that the likelihood of paying bribes is different between formal and informal firms.

One may wonder why we study these two aspects of production and corruption, because they may sound unrelated. Our empirical analysis does not attempt to establish a link between the two; rather, we simply present evidence of potential productivity growth for small-sized firms in Vietnam and one possible explanation, which helps us to explain why they *do not grow*. Indeed, our findings suggest that bribes are positively correlated with size, and the likelihood of paying bribes is significantly different across different firm sizes. A striking fact about government corruption is that no matter how it is defined, it is higher in developing countries. According to the Transparency International Corruption Perception Index report[1], most developing countries are at the bottom of the corruption list (Pakistan, Indonesia, Tanzania, etc.), while developed economies such as New Zealand, Denmark and Sweden are in the top 20.[2]

In this study, we use two data sets from Vietnam. The data set that we use to measure productivity and efficiency is the Enterprise Census conducted by the General Statistics Office of Vietnam (GSO). The Enterprise Census contains information on all registered firms in Vietnam. The second data set, the Small and Medium Enterprises (SME) survey, which we use to analyze the relationship between bribes and firm size, provides comprehensive information on all forms of small-sized and medium-sized firms.[4]

Another feature of the SME survey is that it also contains formality information for each firm. We also analyze how the formality of firms is related to the likelihood of paying bribes. In the literature, using data from the World Business Environment Survey compiled by the World Bank for a large number of developing and developed countries, Dabla-Norris et al. (2008) study the link between firm size and informality. Our analysis also indicates that formality is another important determinant of the likelihood of paying bribes, and this finding is consistent with the precedent findings (see Dabla-Norris et al., 2008; Dutta et al., 2013; Mishra and Ray, 2013).

The rest of the paper is organized as follows. The second section describes our first data set and provides a summary of Vietnam's firm size distribution and average products. The third section describes our methodology for measuring productivity and efficiency, and then presents the analysis on production efficiency and estimation of returns to scale. The third section provides the empirical

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[1]http://www.transparency.org/cpi2013/results. Viewed on October 1, 2014.

[2]Evidence exists that corruption levels are significantly higher for developing and transitional countries (e.g., Nowak, 2001). Some studies (see He, 2000; CIEM, 2006)[3] suggest that the high level of corruption in these countries is perhaps due to (i) abuse of power by public officials, (ii) arbitrary decisions related to policies and administration, (iii) weak accountability of officials and government agencies, and (iv) weak state implementation and monitoring.

[4]The survey has been published in "Characteristics of doing business in Vietnam" conducted by the Central Institute for Economic Management (CIEM) in collaboration with the Vietnamese Institute of Labor Science and Social Affairs (ILSSA), the Department of Economics (DoE) of the University of Copenhagen, and UNU-WIDER together with the Royal Embassy of Denmark, every two years since 2005. A more detailed description of the process of collecting these data is provided in Rand and Tarp (2012).

analysis on bribes and firm size. The last section concludes.

2 Firm Size Distribution at a Glance in Vietnam

2.1 Census Date Description

In Vietnam, a census has been conducted annually since 2000. Enterprise, in the survey, is defined as "an economic unit that independently keeps a business account and acquires its own legal status." In this paper, we focus on the nine-year period from 2000 to 2008. We exclude the recent years (from 2009 to present) because the Vietnamese economy has suffered from the effects of the global financial crisis, and a 30% reduction of corporate income tax for qualifying entities was implemented in the fourth quarter of 2008 and all of 2009[5]. In addition, SME firms involved in labor-intensive production and processing activities will also benefit from the tax reduction under Decree 60/2011. We also exclude inconsistent data from our sample, such as observations that are recorded twice for the same firm in the same year, ones with negative or zero revenue values, or ones with an implausibly large number of employees. We include 32 industries that have at least 200 observations for each year, and the number of observations ranges from approximately 1,500 to 160,000 per industry.

2.2 Summary

Figure 1, Figure 2 and Figure 3 summarize the firm-size distribution for 2000, 2004 and 2008, and are the same as those produced by Hsieh and Olken (2014). Figure 1 shows the distributions of firm size measured by number of employees in each firm. Figure 2 shows the distributions of employment share by firm size; namely, showing what the percentage of firms having the number of employees that fall into each value of the x-axis are relative to the total number of firms. Figure 3 shows the distribution of ratios of production relative to capital, labor and the ones of revenue relative to raw material. To save space, we do not present the figures for the other years, which are quite similar to the ones presented here.

<div align="center">

Insert Figure 1

Insert Figure 2

Insert Figure 3

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To make the patterns more visible, in Figure 1 and Figure 2, we classify firm size into subintervals of 0 to 200 employees (column 1), 10 to 200 employees (column 2), 20 to 200 employees (column 3), 50 to 200 employees (column 4) and 200 to 3000 employees (column 5). These two histograms present the distribution of firm size in bins of 10 workers. In Figure 3, we take the maximums and the

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[5]See Circular No. 03/2009/TT-BTC published in January 2009 issued by Vietnam's Ministry of Finance.

minimums of $\log(\frac{\text{Value added}}{\text{Capital}})$ (the first row), $\log(\frac{\text{Value added}}{\text{Capital}})$ (the second row) and $\log(\frac{\text{Revenue}}{\text{Material}})$ (the third row), for the entire period of 2000 to 2008, and truncate the intervals into 50 bins.

The key observations in the figures include the following.

- From Figure 1, there are a large number of small firms compared with other sizes.

- From Figure 2, the distribution of employment in each column is more "flat" compared with the one in Figure 1.

- From Figure 2, the employment share for very small firms increases over time (from 2000 to 2004 and from 2004 to 2008).

- From Figure 3, the distributions of average products do not look bimodal.

Similarly to the graphs presented in Hsieh and Olken (2014) for India, Indonesia, and Mexico for a similar period, our graphs show a large number of small firms.

In what follows, we conduct an empirical test, in order to give more detailed analyses on the firm size distribution. We first test the sample distribution against the normal distribution using the Shapiro–Wilk test, which examines the ratio of the best estimator of the variance and the usual corrected sum of squared estimator of the variance.(see Dufour et al., 1998) In the literature, there are over 40 normality tests (Dufour et al., 1998). The idea of a normality test is to match the data distribution function with the cumulative distribution function of the theoretical normal distribution. According to the results[6], we reject the null hypothesis of normality at the 1% significant level for all nine years.

We then use the Dip test to investigate the null hypothesis H_0 that the sample distribution has a unimodal density, against the alternative hypothesis H_1 that the sample distribution has more than one mode. A p-value is calculated by comparing the Dip statistics obtained from resamplings of the same size from a uniform distribution (see Hartigan and Hartigan, 1985). According to the results of the Dip test[7], the null hypothesis is rejected at the 5% significance level for all years. The Dip statistics are all smaller than 0.05, which indicates the presence of bimodality or multimodality.[8]

Finally, we conduct the bimodality coefficient test. The test examines the relationship between bimodality, skewness and kurtosis of the distribution. The bimodality coefficient (BC) statistics ranges from 0 to 1, with those exceeding 0.55 suggesting bimodality[9]. The BC's of all years are found in Table 1. As it shows, all BC statistics are higher than 0.55, which indicates the existence of bimodality.

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[6]The detailed result is presented in Table 13 of Appendix.

[7]The detailed result is presented in Table 14 of Appendix.

[8]The more detailed interpretation of this statistic is found at Freeman and Dale (2013). Finding out how many modes are present in the distributions is not our aim in this paper, and so we will not consider this issue further.

[9]For more details, see page 1258 in SAS Institute Inc. (2008). The bimodality coefficient is $\frac{m_0^2+1}{m_1+\frac{3(n-1)^2}{(n-2)(n-3)}}$ where m_0 is skewness and m_1 is the excess kurtosis. The BC test, however, is criticized because of its sensitivity to the skewness of the distribution. Thus, we also include the results of the Dip test, which is considered to be more robust (see Freeman and Dale, 2013).

Table 1: The Bimodality Coefficients

Year	'00	'01	'02	'03	'04	'05	'06	'07	'08
BC	0.69	0.72	0.71	0.71	0.63	0.69	0.71	0.62	0.56

3 Efficiency Scores and Estimation of Returns to Scale

In this section, we measure the technical efficiency of production, using the efficiency score, and returns to scale by firm size at the firm level using nonparametric methods, which impose no or very limited assumptions on the data.[10]. In particular, they provide the upper boundary estimates of the production set without imposing any restrictions on the parametric relationship between inputs and outputs. Specially because we do not know the functional form of each firm's production function and the parametric distribution of the error terms, these methods are appropriate in our case.

First, the efficiency scores measure how much a firm can reduce inputs while still obtaining the same level of outputs. Second, returns to scale measure how outputs change when all inputs change proportionally, compared with the optimal level in each industry. The operational scale of a firm may be small relative to the optimal scale, and in this case, production shows increasing returns to scale (IRS), suggesting that the firm should increase production. On the other hand, a firm's operational size may be too large compared with the optimal scale, and in this case, we say that the production shows decreasing returns to scale (DRS). If a firm's production shows either IRS or DRS, its efficiency might be improved by changing its scale of production.

We consider the situation where a firm uses capital, labor, and intermediate materials to produce goods, which are measured in monetary terms. Real revenue is used as the proxy for output. Three inputs are included in our estimation: intermediate inputs, labor and capital. All input values are adjusted to account for inflation to obtain a real value.

In our analysis, labor is measured by the total income of employees in a firm. This includes total wages and other employee labor-related costs such as social security, insurance and other benefits. The intermediate material includes costs such as fuel and the value of other materials. Capital is measured as assets to be used in production, rather than the recorded acquisition values.[11]

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[10]In recent years, many methods have been developed to estimate technical efficiency as well as returns to scale under the frontier approach. The production frontier can be estimated using parametric estimate (Deterministic Frontier Analysis or Stochastic Frontier Analysis) or non-parametric one (Data Envelopment Analysis or Free Disposal Hull). For more details, (see Mahadevan, 2004)

[11]At a firm level, prices and quantities may not be well measured, and revenue, instead of gross output or cost, is normally used to estimate RTS. In the literature, there are some arguments that the elasticities of labor and capital, in a revenue estimate, may be downward biased (Basu and Fernald, 1997). Klette and Griliches (1996) report that changes

Table 2 shows the sectors that we use in this section. These sector codes are based on the International Standard Industrial Classification (ISIC) codes. We choose these sectors because each sector based on the two-digit level ISIC code has more than 200 observations per year, so that we can ensure robustness of the nonparametric estimations that we report in this section.

3.1 Efficiency Score and Results

The efficiency score is defined by the distance between the observed quantity of inputs and outputs, and the quantity of inputs and outputs at the production possibility frontier, which is the best possible outcome for a firm in its cluster (industry). In a sense, the efficiency score measures technical efficiency in production.

We consider an industry where inputs $x \in \mathbb{R}_+^3$ are used to produce outputs $y \in \mathbb{R}_+$. Suppose that there are T periods and each period is denoted by $t = 1, ..., T$. Let N_t denote the number of firms in the data set at period t. At each period t, firm i's input and output combination is denoted by (x_{it}, y_{it}) with $x_{it} \in \mathbb{R}_+^3$ and $y_{it} \in \mathbb{R}_+$.

The production possibility set (hereafter PPS) at period t is:

$$\Psi^t = \{(x, y) : x \geq \sum_{i=1}^{N_t} \lambda^{it} x_{it}, y \leq \sum_{i=1}^{N_t} \lambda^{it} y_{it}, \text{ where } \sum_{i=1}^{n} \lambda^{it} = 1 \text{ and } \lambda^{it} \in \mathbb{R}_+ \forall i\}, \quad (1)$$

where $\lambda = \{\lambda^{it}\}$ is an intensity vector.

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in sector prices are substantially diversified and correlated with changes in labor and capital. However, according to Jacques and Jordi (2005), the introduction of individual output prices into the production function does not markedly affect the estimate of RTS. In addition, the estimation of a production function in terms of "physical quantities" is, in fact, meaningless, unless we confine the analysis to a very precisely defined industry where goods are so homogeneous that firm outputs can be well measured and compared across firms. Accordingly, we use this measure in the analysis.

Table 2: ISIC Codes and Industry Description

ISIC	Industry Description	ISIC	Industry Description
14	Other mining and quarrying	15	Food products and beverages
17	Textiles	18	Wearing apparel etc.
19	Tanning and dressing leather	20	Wood and wood products
21	Paper and paper products	22	Publishing, printing etc.
24	Chemical products etc.	25	Rubber and plastic products
26	Nonmetallic mineral products	28	Fabricated metal products
29	Machinery and equipment etc.	36	Furniture

Formally, the efficiency score for (x_{it}, y_{it}) is defined by:

$$E(x_{it}, y_{it}) = \inf_{\theta}\{\theta : (\theta x_{it}, y_{it}) \in \Psi^t\}. \tag{2}$$

The efficiency score[12] lies between 0 and 1, and represents the minimal proportional reduction of all inputs while maintaining the same output level within the PPS.

The PPS Ψ^t is assumed to satisfy the regularity conditions; namely, boundedness, closedness, no free-lunch[13], and free disposability[14]. We say that Ψ_t for each t exhibits *convexity* if for every $(x_1, y_1), (x_2, y_2) \in \Psi^t$, and any $\alpha \in [0, 1]$, $\alpha(x_1, y_1) + (1 - \alpha)(x_2, y_2) \in \Psi^t$ holds.

Efficiency scores depend on how the PPS is set. We use two different methods in this paper: *data envelopment analysis* (hereafter DEA), and *free disposal hull* (hereafter, FDH).[15]

1. The *DEA* method assumes that the PPS is convex;

2. The *FDH* method assumes that $\lambda^{it} \in \mathbb{N}$ and does not assume that the PPS is convex.

FDH is, to an extent, a more general version of DEA because it does not impose convexity and relies only on strong free disposability. In other words, if the true production possibility set is convex, both DEA and FDH are consistent. On the other hand, if the true PPS is nonconvex, DEA is a nonconsistent estimator, while FDH still preserves its consistency. However, the disadvantage of FDH is the low rate of convergence (because of the fewer assumptions that it requires), which makes FDH a less efficient estimator, compared with DEA. As a result, in this study, to check the robustness of our results, we present the results from both methods: FDH and DEA.

Figure 4 is a scatter-plot of efficiency scores in our sample industries for the entire nine-year period, and the curve shows the function representing the moving averages of the efficiency scores for every 0.1-interval of $log(size)$. The x-axis represents the logarithms of firms' sizes. To analyze the data further, we group the samples by number of employees into five groups, where groups 1 to 5 include firms with $1 - 10$, $11 - 50$, $51 - 100$, $101 - 200$, and $201-$ employees, respectively.[16] Figure 5 shows the average efficiency scores for each group in each sector. The x-axis represents firm-size groups. Each square dot (■) represents an FDH efficiency score for each group. Each round dot (●)

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[12]This is called the "input-oriented" efficiency score. There is another definition called the "output-oriented" efficiency score. In this analysis, we use the input-oriented efficiency score, because the results do not change substantially.

[13]A positive amount of production cannot occur without a positive amount of inputs.

[14]The increase in inputs must lead to increased or constant outputs, and a smaller output vector than a feasible vector is also feasible.

[15]A scaling parameter ϕ can be added to the constraints of the set Ψ^t so that Ψ^t can capture the general properties of production technologies.

[16]Throughout this paper, we use these cutoff sizes whenever we group the samples. This set of cutoff sizes ensures that we have enough firms in each group, and in a latter section, when we conduct ANOVA, we have a similar variability across groups (namely, the variance of paying bribes is not much different, as required by the assumptions for ANOVA. Whenever we use this grouping, we also try a different cutoff size, which does not yield substantially different results. As such, we present the results for this set of cutoffs).

represents a DEA efficiency score for each group. Figure 6 represents the ratios between DEA and FDH scores in each sample industry.

Insert Figure 4

Insert Figure 5

Insert Figure 6

Apart from Hsieh and Olken (2014), where average product of inputs of the economy is investigated, we examine the technical efficiency of firms at a sectorial level. Except in Sections 26 and 29, the relationship between firm size and efficiency score is U-shaped, which indicates that smaller firms and larger firms produce efficiently compared with middle-sized firms. Moreover, firm size at the bottom of the U-shape—namely, the lowest efficiency score—differs across groups. Our result shows that the efficiency of middle-sized firms is lowest across different size groups. In the literature, there has been an increasing interest in the relationship between firm sizes and their characteristics such as innovation and market structure (see Acs and Audretsch, 1987), growth and productivity (see Bentzen et al., 2011) or job creation (see Dalton et al., 2011). There are, however, only few empirical studies on the relationship between firm size and efficiency level, and particularly the one by using a developing country is rare. Also, in preceding works, large firms are found to be the most efficient (see Angelini and Generate, 2008; Leung et al., 2008). This study fills in the gap of the literature by analyzing firms in a developing country in which small firms dominate the economy and exhibit higher efficiency level, compared to their middle-sized firms. Moreover, the heterogeneity of firm size at the bottom of the U-shape across industries indicates the importance of sector level analysis in investigating the missing middle. Our analysis also contributes in this point.

It should be worth mentioning that the ratios between the DEA scores and the FDH scores are less than one for most of the firms, which indicates that there is nonconvexity in the production technologies. The curves in Figure 6 indicate that in some industries such as Sector 26, these curves are also U-shaped, and the degree of nonconvex technology may be associated with size, although these observations are not consistent across industries.[17]

3.2 Returns to Scale and Results

In this section, we measure returns to scale at the firm-level in each industry. Figure 6 in the previous section indicates the existence of some nonconvexities in production technologies.[18] Thus, to estimate the RTS, we use an FDH estimate, which does not require the assumption of convex technology.

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[17]The results are similar for each year in each industry.

[18]Farrell (1957) emphasizes indivisibilities and economies of scale as important sources of nonconvexity and emphasizes that convexity can only be justified in terms of time divisibility (ignoring not only any setup times but also indivisibilities, increasing RTS, positive or negative production externalities, etc. that each can lead to nonconvexity). For some recent theoretical or empirical studies, see Brown (1991) or (Ramey, 1991).

The FDH RTS identification algorithm that we adopt here is developed by Soleimani-Damaneh and Reshadi (2007). Suppose that we are given a data set of n observations whose generic element is denoted by (x_i, y_i). Let N denote the index set of firms in the data set. The index set for inputs is given by S.

Algorithm.

Step 1. *For each $j \in N$, denote by D_j, a set of observations (x_i, y_i). Compute λ^{ij} for all $i, j \in N$ by $\lambda^{ij} = \frac{y_i}{y_j}$ where $y_i, y_j \in D_j$.*

Step 2. *Compute θ^{ij} for all $i, j \in N$ by $\theta^{ij} = \max_{s \in S}\{\frac{x_{is} \cdot \lambda^{ij}}{x_{js}} : (x_i, y_i) \in D_j\}$.*

Step 3. *Compute θ_j and A_j for all $j \in N$ by $\theta_j = \min_{i \in N} \theta^{ij}$ and $A_j = \{i \in N : \theta^{ij} = \theta_j\}$.*

Step 4. *Compute λ_j^+ and λ_j^- for all $j \in N$ by $\lambda_j^+ = \max_{i \in A_j} \lambda^{ij}$ and $\lambda_j^- = \min_{i \in A_j} \lambda^{ij}$.*

Step 1 computes the ratio of outputs between each firm i and j. Step 2 computes the ratio of inputs rescaled by the ratio of outputs between each firm i and j. Noting $\frac{y_i}{x_i} = \theta^{ij}\frac{y_j}{x_j}$, the ratio θ^{ij} represents the productivity of firm j relative to firm i. Step 3 computes the minimum of such ratios for firm j. The set A_j is a set of firms which are more productive than firm j. Finally, Step 4 studies the output ratios for firm j and firm i, compared with which firm j's production is the least productive, and then obtains the maximal ratio λ_j^+ and the minimum ratio λ_j^-.

An interpretation of these ratios is that when $\lambda_j^+ < 1$, firm j can produce more outputs for a given level of inputs than other firms, while if $\lambda_j^- > 1$, firm j should decrease production because firm j does not obtain enough outputs relative to inputs, compared with other firms.

We say that production by firm j for $j \in N$ *exhibits IRS*, if $\lambda_j^+ < 1$; *exhibits DRS*, if $\lambda_j^- > 1$; otherwise *exhibits CRS*, at (x_j, y_j), respectively. This algorithm determines whether or not production at each observation exhibits RTS by calculating the ratios between inputs and outputs.

Figure 7 presents the bar graphs for the percentages of firms whose production is IRS (dark color), CRS (white) and DRS (light color) in each group of firm sizes. According to the categorization defined above, we calculate how many firms exhibit IRS, DRS, and CRS production and then divide these numbers by the total number of firms in each group, so that we can obtain the percentages of firms in each category of each group. We conduct this procedure for all industries.

Insert Figure 7

Figure 7 shows that in most of the industries, the proportion of firms with IRS is the highest in Group 1. In Sectors 28 and 29, Group 1's firms with IRS production do not have the highest percentage compared with Group 2, although compared with Group 5, they still have the highest percentage. This indicates that the small firm category includes the most firms that could improve their efficiency by expanding their production scale, compared with other firm categories, in most of

the industries. This might sound counter-intuitive, because we would expect that larger-sized firms may tend to benefit from economies of scale. Together with the estimation of efficiency scores in the previous section, our results show that small firms have some potential to expand their production in most of the manufacturing industries that we consider in this analysis. In contrast, we observe that the firm size distributions are rather stable in Vietnam; for example, as shown in Figure 1, and as shown in the transition matrix 3, there are not many micro-sized or small-sized firms who shift to a larger-scale category.

Our results show that the proportions of firms with CRS productions are small, compared to those with DRS and IRS productions across all the sectors we study. In the literature, CRS is one of the common assumptions on productions, whereas our results cast a doubt on the validity of this assumption when studying the missing middle phenomenon.

4   Bribes and Firm Size

4.1   Date Description

The data set that we use to analyze the relationship between bribes and firm size is the Small and Medium Manufacturing Enterprise (SME) survey conducted by the Central Institute for Economic Management (CIEM) in collaboration with the Vietnamese Institute of Labor Science and Social Affairs (ILSSA), the Department of Economics (DoE) of the University of Copenhagen, and UNU-WIDER together with the Royal Embassy of Denmark in Vietnam. We exclude joint ventures from the sample because of the high degree of governmental and foreign involvement in ownership structures. Unlike the first data set, this survey only deals with nonstate manufacturing enterprises. It excludes all state-owned firms as well as firms in other sectors such as the service sector. Third, firms in the survey include both formally registered ones (enterprises with a business registration license and/or a tax code) and informal households. All informal firms included in the survey operate alongside the officially registered enterprises. These informal household establishments (firms without a business registration license or a tax code and not registered with District authorities) are included in the surveys based on the on-site identification process. The inclusion of unregistered firms is another important contribution of the survey.

Our initial sample contains 9567 observations, which include all firms in the SME survey covering four years: 2005, 2007, 2009 and 2011. We exclude all firms for which we do not have information on firm size, wage and total assets. We also discard firms for which we do not have information on firm age. The final sample consists of 8421 nonstate manufacturing firms in 10 provinces of Vietnam[19]. Employees in our analysis include both regular full-time and casual ones.

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[19]The 10 provinces of Vietnam to be surveyed are: Hochiminh City (HCMC), Ha Noi, Hai Phong, Long An, Ha Tay, Quang Nam, Phu Tho, Nghe An, Khanh Hoa and Lam Dong. These selected provinces cover about 30% of the total manufacturing enterprises in Vietnam.

Table 3: Employment Transition Matrix (2005–2011)

'05 \ '07	Micro	Small	Medium
Micro	1,280 (93.2%)	94 (6.8%)	0 (0%)
Small	169 (26.5%)	433 (67.9%)	36 (5.6%)
Medium	6 (3.8%)	52 (32.9%)	100 (63.3%)

'07 \ '09	Micro	Small	Medium
Micro	1259 (92.8%)	95 (7%)	3 (0.2%)
Small	135 (25%)	377 (69.7%)	29 (5.4%)
Medium	3 (2.3%)	37 (28.9%)	88 (68.8%)

'09 \ '11	Micro	Small	Medium
Micro	1,255 (93.4%)	86 (6.4%)	3 (0.2%)
Small	131 (25.%)	362 (69.3%)	29 (5.6%)
Medium	2 (1.5%)	41 (30.8%)	90 (67.7%)

Table 4: The Number of Firms Bribing/Not Bribing in Each Group

	group 1	group 2	group 3	group 4	group 5
Not Bribe	3184	1449	171	91	54
Bribe	1383	1605	304	186	60

4.2 Snapshots

By using the SME survey, we provide Table 3, which is a matrix of employment transitions that shows what percentage of firms shift from one group to another.[20] The definition of each group is such that a micro group consists of firms with one to nine employees, a small group consists of firms with 10 to 49 employees and a medium group consists of firms with 50 to 300 employees.[21] It appears that not many firms shift from the Micro category to the Small or Medium category, or from the Small category to the Medium category. Most firms remain in the same category. Another feature is that there are not many firms who shift down from Medium to Small. This matrix indicates that in these years, not many firms shift up from Micro to another category, and on the other hand, very few firms shift down from Medium to Micro.

Furthermore, in the survey, each firm is asked whether they had paid bribes in the surveyed year. Table 4 shows the number of firms who bribe or do not bribe in each group. It is clear that the proportion of firms paying bribes is much higher in the last three groups, compared to the first or the second group.

Here, we present a set of figures from Figure 8 and Figure 9 that are based on a classification by a relative ranking.[22] We rank firms in ascending orders of size, and partition them.[23] In Figure 8, each

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[20]For a data description, refer to Section 4.1.

[21]This is the definition of small and medium enterprises given in Vietnamese law (Decree no. 90/2001/CP-ND on "Supporting for Development of Small and Medium Enterprises").

[22]Figures based on a classification by the absolute number of employees are also available upon request. The figures show that the likelihood of paying bribes increases with size based on a relative ranking.

[23]If there are some firms of the same size, we use a computer program to permutate them randomly. We repeat this

group contains 20 firms, and in Figure 9, each group represents 10 firms. Each dot shows how many firms paid bribes in each group. For example, the first dot shows the proportion of the smallest 20 firms who pay bribes, out of these 20 firms in this group.

Moreover, in Figure 9, we separate the firms into two groups, depending on whether or not each firm has a tax code. We call those with tax codes *formal* firms and the rest *informal* firms. In Figure 9, we select firms having fewer than 200 employees. Informal firms tend to be smaller, and for a good comparison, we select 200 firms instead of 500, unlike Figure 8.

<div style="text-align:center">

Insert Figure 8

Insert Figure 9

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From Figure 8 and Figure 9, it appears that larger-sized firms are more likely to pay bribes, and formal firms are more likely to pay bribes than informal firms in each size group, particularly in 2005 and 2007.

4.3 Logit Regression on Bribes

As a preliminary investigation, we conduct an analysis of variance (ANOVA) on the categorized samples as in Table 4. Let Y_{ij} denote the portion of firms bribing out of the total number of firms in each group $i \in \{1, \cdot, 5\}$ for the year $j \in \{05, 07, 09, 11\}$. Then, the ANOVA model is represented by:

$$Y_{ij} = \mu + \alpha_j + \epsilon_{ij},$$

where μ is the overall average, α_j is the treatment effect for group j, and the error terms, ϵ_{ij}, are assumed to be independent and normally distributed with mean 0 and standard deviation σ.

Next, we conduct a logit regression on bribes. Assets include both short-term and long-term assets of the firm. Tax code (denoted by TAX) is a dummy that represents the formality of firms. This variable has the value 1 if a firm acquires both a business registration certificate (BRC) and a tax code, and 0 otherwise.[24] The 10 dummies for firms' location are represented by PRO_j for each district j. Finally, TYP_k represents the five firm-type dummies for each k. The 10 provinces include Hà Nội, Phú Thọ, Hồ Chí Minh City, Hà Tây, Hải Phòng, Nghệ An, Quảng Nam, Khánh Hòa, Lâm

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procedure several times to see whether the feature of increasing likelihood of bribes as size increases still holds.

[24]There are two ways to define a formal firm in the survey: (i) a firm that holds a BRC, and (ii) a firm that holds both a BRC and a tax code. A firm can have a BRC without a tax code, but it is not possible to have a tax code without a BRC, because the tax authorities require a BRC before issuing a tax code. The two definitions differ in the level of commitment to formalization and thus the extent to which firms are visible to civil servants. Further investigation using the transition matrix of formal incidence using the first definition provides some contradictions. For example, about 10.7% of firms obtained BRCs in 2007, while about 13.1% of registered firms lost their BRCs in that year. This may indicate some misreporting. Therefore, we use the second definition in the analysis.

Đồng, and Long An. The firms' dummies include five types, which are a household business, a limited liability company (Ltd hereafter), a private enterprise, a joint stock, and a cooperative.[25]

Unregistered firms usually have restricted access to credit, infrastructure, public services and markets. Such firms, therefore, may try to become formal to overcome such limitations. On the other hand, remaining informal enables firms to escape the heavy burdens of regulations such as taxes or labor-related requirements. It is, therefore, of interest to see which force is dominant in the Vietnamese case.

Let Y_{ij}^k denote whether firm i of type k in district j pays bribes. If the firm pays bribes, Y_{ij}^k equals one, or zero otherwise. Then, we consider the following formulation:

$$Y_{ij}^k = \beta_0 + \beta_1 \cdot \ln(size_i) + \beta_2 \cdot \ln(age_i) + \beta_3 \cdot \ln(asset_i) + \beta_4 \cdot TAX_i + \beta_5^j \cdot PRO_j + \beta_6^k \cdot TYP_k + \epsilon_{ij}^k.$$

To estimate the logit regression, we use the dummy of household business from the set of firm type dummies and the dummy of Ho Chi Minh City from the set of province dummies for normalization.

Finally, we estimate two sets of regressions in order to check the robustness of our regression results. The first one is a fixed-effects regression that uses each firm's ID, because there might be some reverse causality between bribes and firm size. The second one takes differences in formality into account using a mixed-effect regression, and we assume a fixed effect over the formality, and random effects over age and location.[26]

4.4 Empirical Findings on Bribes

Table 5 shows that the p-value from the ANOVA is 0.0039, and we reject the null hypothesis that the proportion of firms paying bribes comes from the same distribution across different groups at the 5 % significance level. Our regression results from the logit regression are summarized in Table 7, and Table 9 shows the odds ratio and the 95 % confidence interval. Moreover, the results from the fixed-effect and mixed-effect regressions are presented in Table 8. The odds ratios computed from the results of these regressions are also presented in Table 9.

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[25]A household business is a business that has not registered as an enterprise under Vietnam's Enterprise Law. Not all businesses are required to register as an enterprise. Thus, many businesses operate as a household business, both informally (i.e.,without a license) and formally (i.e., with a license). An Ltd is established by member capital contribution to the company. As an Ltd is a legal entity separate from the owner(s), the owner's liability for the firm's debts and obligations is limited to his capital contribution (Socialist Republic of Vietnam, 2005). A private enterprise is a firm owned by an individual, who is its legal representative. The owner has total discretion in making business decisions and is liable for its operations because he owns all firm assets. Each individual can only establish one private enterprise (Socialist Republic of Vietnam, 2005). A joint stock company is a company whose capital is divided into shares, and the liability of each shareholder is limited to the par value of the shares held by him/her (Socialist Republic of Vietnam, 2005). A cooperative is a collective economic organization that is founded by individuals, households or legal entities who have common demands and benefits, and who volunteer to contribute capital and labor (Socialist Republic of Vietnam, 2003).

[26]We estimated various combinations of fixed and random effects over these variables and used Akaike Information Criteria (AIC) to choose this particular combination.

Table 5: ANOVA Table

Source	SS	d.f.	MS	F	Pr(> F)
Columns	0.34	4	0.086	6.14	0.0039
Error	0.21	15	0.014		
Total	0.55	19			

Table 9 indicates that the likelihood of bribing increases by 1.44 times per unit increment of size. Observing a negative effect of age may appear to be counterintuitive because the longer the firm has operated, the greater the firm's visibility is likely to be, which may lead to an increased likelihood of paying bribes. Our interpretation of this result is that a firm is perhaps more likely to need to pay bribes at the start of business. Comparing the odds ratio from the ordinal OLS and the one from the fixed-effect regression, we see that age has a negative effect in the ordinal OLS, whereas age has a positive effect in the fixed-effect regression. We can conclude that if a firm pays bribes at the start of business, then the likelihood of paying bribes also increases over time.

Table 9 also indicates that firms' location is important to the likelihood of paying bribes. Relative to being located in Ho Chi Minh City, being in Province 9 has the highest likelihood of paying bribes. Province 7 has the second highest likelihood of paying bribes. Firm type also has an interesting effect on the likelihood of paying bribes. Ltd and Private firms have higher likelihoods than cooperative or joint stock firms.

Finally, Figure 9 shows that the formality of firms is also quite an important determinant of the likelihood of paying bribes. It is well known that informal sector plays an important role in many developing economies. According to Schneider and Enste (2002), the informal sector generates from 10 to 20% of the aggregate output in developed countries, and more than 30% for developing countries, with some reaching more than 50%. Our analysis indicates that corruption could be one possible explanation for the dominance of the informal sector in a developing country.

It is also known that corruption is one of the most significant barriers to economic growth. Many studies have provided an evidence that corruption reduces human capital, discourages investment, leads to a mis-allocation of resources, and slows down economic development. However, few studies have analyzed possible causes of corruption and most of them use cross-country data on corruption perception(Méndez and Sepúlveda, 2006; Mo, 2001), rather than firm level data in one nation. Our analysis provides insights on the dynamics of bribery at the firm level.

Table 6: Logit Regression: Some Statistics

		Model likelihood ratio test		Discrimination indexes		Rank discrim. indexes	
# Observations	7696	LR χ^2	1296.86	R^2	0.209	[6]C	0.732
# 0	4457	d.f.	17	[2]g	1.054	[7]Dxy	0.465
# 1	3239	Pr($> \chi^2$)	< 0.0001	[3]gr	2.870	[8]gamma	0.466
[1]max |deriv|	3.00E-12			[4]gp	0.226	[9]tau-a	0.226
				[5]Brier	0.205		

[1] This is the maximum absolute value of the first derivative of the log likelihood function.

[2] The value g is Gini's mean difference of $X_i\hat{\beta}$.

[3] The value gr is the exponential of g.

[4] In logit regressions, dependent variables (Ys in this case) can be transformed into a probability estimate, and the value gp is Gini's mean difference of these probabilities (those for paying the bribes in this case).

[5] The Brier score is $\frac{1}{N}\sum_{t=1}^{n}(f_t - o_t)^2$, where f_t is the probability that was forecast, o_t is the actual outcome of the event at instance t (0 if it does not happen and 1 if it does happen), and n is the number of forecasts.

[6] C denotes the c-index, and a c-index of 0.5 denotes random splitting, whereas a c-index of 1 denotes perfect prediction.

[7] Dxy represents Somers' Dxy rank correlation between the predicted probabilities and the observed responses. Between Dxy and the c-index, $Dxy = 2(c - 0.5)$ holds. A Dxy of 0 occurs when the model's predictions are random, and when it equals 1, the model is perfectly discriminating. The definition is taken from Somers (1962).

[8] Gamma is Goodman Kruskal's γ, which measures the similarity of the orderings of the data when ranked by each of the quantities. The values range from -1.0 (no association) to 1.0 (perfect association). The definition is taken from Kruskal (1958).

[9] Tau-a is Kendall's τ_a rank correlations between predicted probabilities and observed responses. The definition is taken from Kendall (1938).

Note: A detailed description of these indexes is taken from Harrell (2014) and Harrell (2001).

Table 7: Logit Regression: Results

	β's	S.E.	Wald Z	Pr($> |Z|$)
(Intercept)	-2.99	0.27	-11.21	<0.0001
$\ln(size)$	0.36	0.03	11.53	<0.0001
$\ln(age)$	-0.19	0.04	-5.38	<0.0001
$\ln(asset)$	0.11	0.02	5.38	<0.0001
Tax code	0.52	0.07	7.65	<0.0001
Province 1	0.85	0.08	10.31	<0.0001
Province 2	0.70	0.11	6.28	<0.0001
Province 3	-0.27	0.09	-2.83	0.0046
Province 4	0.69	0.10	7.30	<0.0001
Province 5	0.41	0.10	4.23	<0.0001
Province 6	-0.11	0.12	-0.88	0.3787
Province 7	1.15	0.13	9.00	<0.0001
Province 8	0.84	0.14	5.89	<0.0001
Province 9	1.23	0.13	9.67	<0.0001
Ltd	0.36	0.10	3.66	0.0002
Cooperative	-0.05	0.10	-0.53	0.5964
Private	0.38	0.10	3.70	0.0002
Joint stock	-0.34	0.06	-5.60	<0.0001

Note: Because our study is about corruption, which may be a delicate issue, we do not disclose each province's name here.

5   Conclusion

In this paper, we have empirically studied the relationship between firm size and production efficiency, as well as the relationship between firm size and the likelihood of paying bribes using Vietnamese data. We have presented results indicating the following.

- A middle-sized firm tends to produce less efficiently relative to small-sized or large-sized firms.

- The likelihood of paying bribes is very likely correlated with firm size.

Needless to say, one might question whether our findings from a transition economy—Vietnam—provide a sufficient basis for generalization, particularly because much of the missing middle literature concerns Africa.

Several interesting future research paths follow from our analysis. A next step is to conduct a similar analysis with data from developed countries on production efficiency and to compare the results for Vietnam. If we still observe lower productivity for middle-sized firms, we could then consider the cause. If we only observe this U-shaped pattern in developing countries, we could examine the reasons that this pattern exists in developing countries but not in developed countries.

Finally, it would be very interesting and important to study the causes of these differences in productivity across different firm sizes in developing countries. OECD Publishing (2013) reports two main reasons among OECD countries for the differences in productivity across different firm sizes: (i) firm size matters for productivity, and (ii) structural differences in the industrial composition of economies impact on the relative performance of large and small firms across countries. First, in most countries, there is evidence of increasing returns to scale. Larger firms are on average more productive than smaller ones, and this generally holds for all industries. Second, in countries with large industrial sectors and relatively low income per capita, large firms are, on average, 2–3 times as productive as smaller firms. However, in countries with large services sectors and relatively high income per capita, small firms are often more productive than large firms. It would be interesting to investigate whether these factors are also important in developing nations.

Table 8: Fixed-Effect and Mixed-Effect Regressions

	Fixed-effect	Mixed-effect
(Intercept)	0.21	-2.88***
	(0.25)	(0.29)
Tax code	0.05*	0.54***
	(0.02)	(0.07)
$\ln(size)$	0.07***	0.37***
	(0.02)	(0.03)
$\ln(age)$	0.01	–
	(0.01)	–
$\ln(asset)$	0.01	0.10***
	(0.01)	(0.02)
Ltd	0.14***	0.34***
	(0.03)	(0.10)
Cooperative	-0.00	-0.06
	(0.03)	(0.10)
Private	0.07	0.39***
	(0.03)	(0.10)
Joint stock	-0.07***	-0.36***
	(0.01)	(0.06)
Residual standard error	0.4466	
d.f.	4294	
Multiple R^2	0.54	
Adjusted R^2	0.18	
F-statistic	1.50	
p-value	<0.0001	
AIC		9265.45
BIC		9334.93
Log likelihood		-4622.72
Num. groups: age		62
Num. groups: Firm ID		10
Variance: age.(Intercept)		0.02
Variance: Firm ID.(Intercept)		0.23
Variance: Residual		1.00
# Observations	7696	

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table 9: Odds Ratio (OR) and 95 % Confidence Interval

	OR	2.50%	97.50%	OR (FE)[1]	OR (ME)[2]
(Intercept)	0.05	0.03	0.08	1.24	0.06
$\ln(size)$	1.44	1.35	1.53	1.07	1.44
$\ln(age)$	0.82	0.77	0.88	1.01	-
$\ln(asset)$	1.11	1.07	1.16	1.01	1.11
Tax code	1.68	1.47	1.93	1.05	1.71
Ltd	1.43	1.18	1.74	1.15	1.41
Cooperative	0.95	0.78	1.15	1.00	0.94
Private	1.46	1.20	1.78	1.07	1.48
Joint stock	0.71	0.63	0.80	0.93	0.70
Province 1	2.35	2.00	2.76		
Province 2	2.01	1.62	2.50		
Province 3	0.77	0.64	0.92		
Province 4	2.00	1.66	2.41		
Province 5	1.51	1.25	1.83		
Province 6	0.90	0.70	1.14		
Province 7	3.15	2.46	4.05		
Province 8	2.31	1.75	3.05		
Province 9	3.42	2.67	4.39		

[1] FE stands for fixed effect;

[2] ME stands for mixed effect.

List of Tables

Table 10: Summary of Enterprise Census Data Statistics (Firm Size)

Sector	# Observations	Size			Capital			Material		
		Mean	Median	St. Dev.	Mean	Median	St. Dev.	Mean	Median	St. Dev.
Sector 14	10200	70.39	25	201.44	16183.77	1330.00	70968.11	937.32	169.50	4958.17
Sector 15	32006	103.60	13	309.88	22026.78	1213.00	142222	5408.13	262.00	33063.95
Sector 17	6637	202.61	42	541.41	40813.60	3052.00	265189.39	9241.37	768.00	43373.63
Sector 18	10700	342.11	72	720.55	15284.91	2342.50	46559.26	3027.77	421.00	13524.88
Sector 19	3541	1135.40	211	3341.28	63691.13	7163.00	556305.05	9674.50	843.00	42378.52
Sector 20	12270	68.06	20	168.40	5226.62	995.00	23149.73	1808.58	322.50	8281.56
Sector 21	7363	75.70	30	187.99	16018.26	2750.00	77713.21	4087.83	607.00	23673.22
Sector 22	8896	40.47	10	95.67	7709.19	852.00	42009.32	2029.04	209.00	12778.10
Sector 24	7597	96.35	23	262.72	32341.05	3149.00	168550.07	8674.42	924.00	28441.16
Sector 25	9811	87.53	26	207.97	18970.10	2988.00	67558.16	4669.97	895.00	14612.27
Sector 26	12798	133.68	41	288.33	37335.68	1916.00	260515.38	6395.12	444.00	36957.37
Sector 28	16814	52.89	15	141.30	11259.46	1176.50	47465.02	3699.33	492.00	16725.93
Sector 29	4492	90.28	25	203.45	19246.81	1899.00	95030.86	5647.87	821.00	17728.35
Sector 36	11665	150.28	26	405.82	14114.21	1571.00	77846.93	4317.39	550.00	16302.37

[1] Size is measured by the number of employees.

[2] Capital and Material are measured in home currency (not inflation adjusted).

[3] St. Dev. is standard deviation.

Table 11: Summary of Enterprise Census Data Statistics (Number of Observations)

Year	Total	Sec 14	Sec 15	Sec 17	Sec 18	Sec 19	Sec 20	Sec 21	Sec 22	Sec 24	Sec 25	Sec 26	Sec 28	Sec 29	Sec 36
2000	8004	488	2418	362	498	236	908	244	244	397	445	499	558	220	487
2001	10107	596	1744	420	609	265	741	666	666	500	554	1672	742	294	638
2002	12386	786	2963	544	769	311	944	627	533	620	756	1218	1115	361	839
2003	14280	870	3147	603	939	341	1263	642	648	709	862	1287	1415	449	1105
2004	17663	1016	3674	744	1277	433	1315	865	967	876	1095	1528	1949	543	1381
2005	21203	1043	5018	904	1410	490	1508	1015	1184	1018	1344	1652	2332	633	1652
2006	18388	901	3815	824	1301	399	1611	842	1398	922	1178	1365	2000	480	1352
2007	26675	1217	4952	1206	1958	538	2131	1263	1785	1340	1877	1919	3502	836	2151
2008	26084	3283	4275	1030	1939	528	1849	1199	1471	1215	1700	1658	3201	676	2060

Note: Each number shows the number of observations in each sector of each industry.

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Table 12: Summary of Data Statistics (Number of Employees)

	2005		2007		2009		2011	
	Formal	Informal	Formal	Informal	Formal	Informal	Formal	Informal
Min.	1	2	1	2	2	1	2	1
Max.	1929	90	1300	120	2561	65	496	321
Median	15	5	11	5	11	5	8	8
Mean	37.72	8.13	30.57	7.92	27.01	8.07	20.02	21.62
St. Dev.	87.89	8.54	70.13	10.40	78.23	9.56	37.11	40.10
# Observations	1083	494	1530	597	1568	446	1364	614
Total	1577		2127		2014		1978	

[1] All entries are measured in terms of the number of employees, except for the number of observations.

[2] St. Dev. is standard deviation.

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Table 13: Shapiro–Wilk Test for Normality

Year	# Observation	W^1	Z	$\Pr(> |Z|)$
2000	23842	0.08	25.91	<0.0001
2001	38038	0.07	26.33	<0.0001
2002	47505	0.07	26.91	<0.0001
2003	55634	0.07	27.31	<0.0001
2004	72083	0.13	27.76	<0.0001
2005	89799	0.06	28.53	<0.0001
2006	101695	0.05	28.84	<0.0001
2007	128251	0.09	29.22	<0.0001
2008	169155	0.06	29.91	<0.0001

[1] The test statistic W is given by $W = \frac{\left(\sum_{i=1}^{n} a_i x_{(i)}\right)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$, where $x_{(i)}$ are the ordered sample values, and a_i are constants generated from the means, variances, and covariances of the order statistics of a sample of size n from a normal distribution, and $\bar{x}$ is the mean of the samples x_is.

[2] In general, the statistic is positive and less than or equal to one; being close to one indicates normality.

Note: A more detailed description about the test is found at Shapiro and Wilk (1965).

Table 14: Dip Test for Unimodality

Year	# Observation	Dip^1	$Pr(>|Z|)^2$
2000	32842	0.038	<0.0001
2001	38038	0.031	<0.0001
2002	47505	0.033	<0.0001
2003	55634	0.029	<0.0001
2004	72083	0.031	<0.0001
2005	89799	0.035	<0.0001
2006	101695	0.049	<0.0001
2007	128251	0.046	<0.0001
2008	169155	0.044	<0.0001

[1] The Dip statistics measures the departure from unimodality, by computing the maximum distance between the sample distribution and the best fitting unimodal distribution function.

[2] The p-value is calculated by comparing the Dip statistics obtained from the data, and the one from random resamplings.

Note: A more detailed description of the test is found at Hartigan and Hartigan (1985).
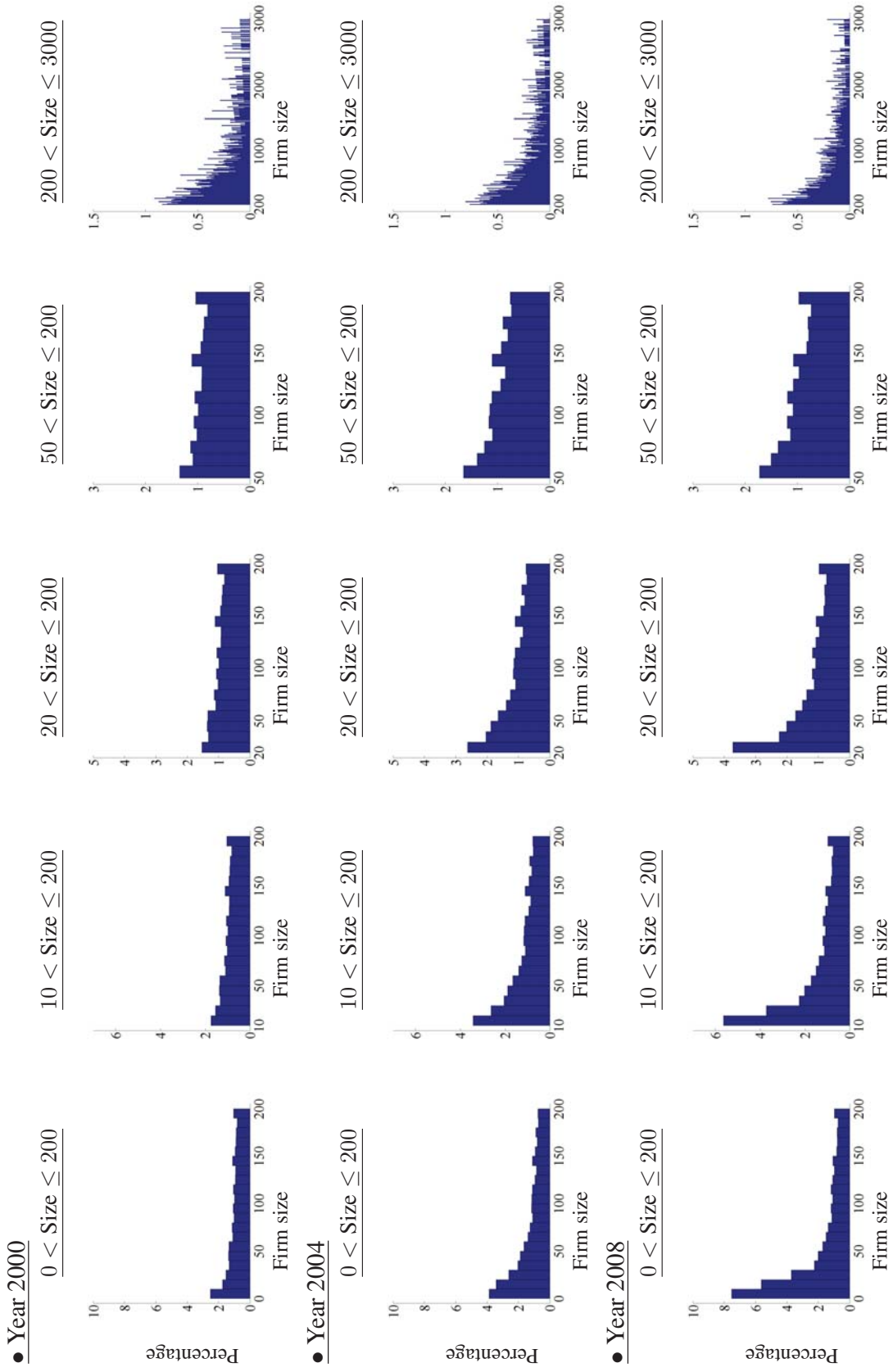
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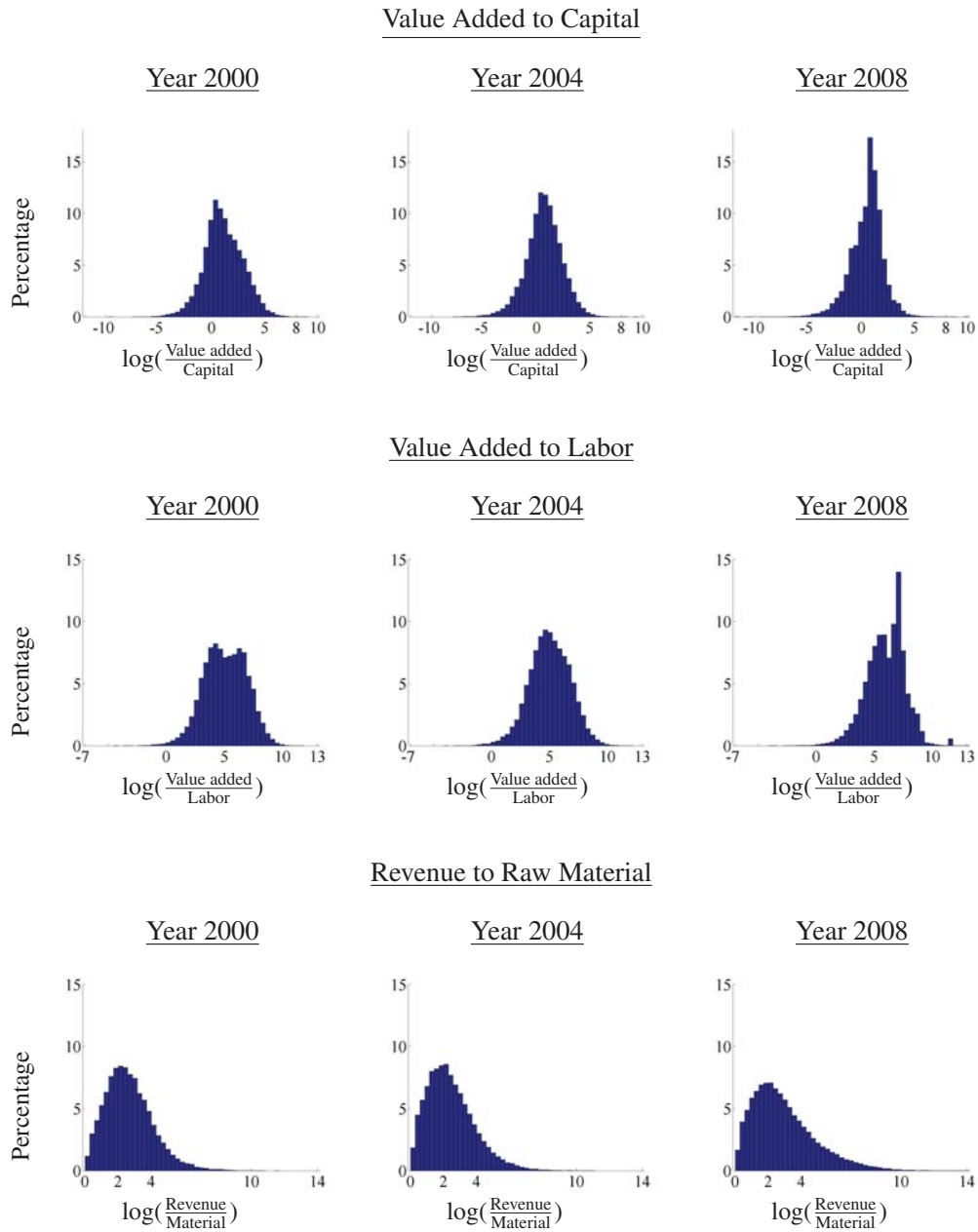Figure 1: Distribution of Firm Size Measured by Number of Employees
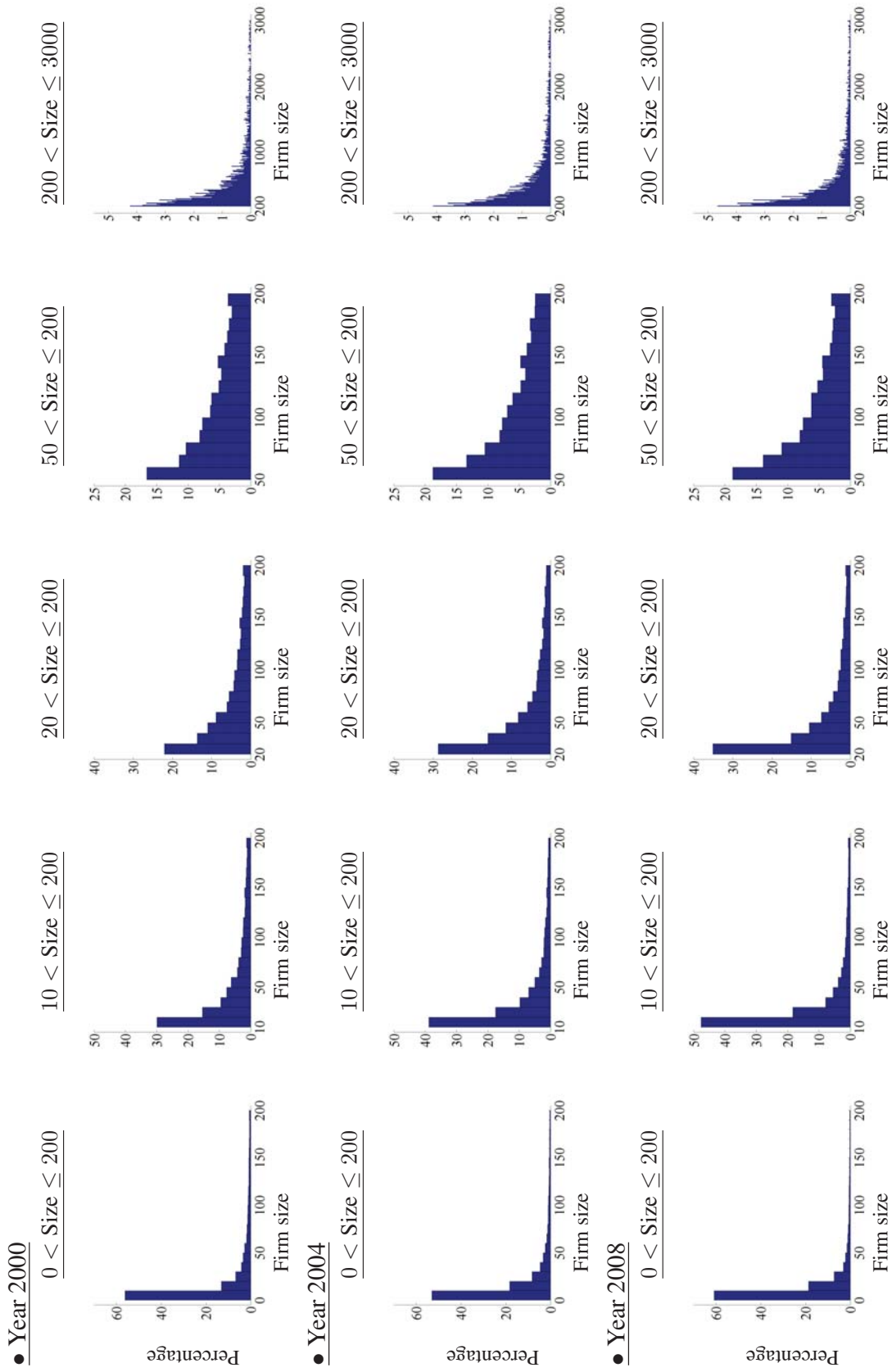
Note: The bin size is 10 employees. Each bin contains the upper bound and not the lower bound.

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Figure 2: Distribution of Employment Share by Firm Size

Note: The bin size is 10 employees. Each bin contains the upper bound and not the lower bound.

Figure 3: Distribution of Average Products

Value Added to Capital

| Year 2000 | Year 2004 | Year 2008 |

Value Added to Labor

| Year 2000 | Year 2004 | Year 2008 |

Revenue to Raw Material

| Year 2000 | Year 2004 | Year 2008 |

Note: We take the maximums and the minimums of $\log(\frac{\text{Value added}}{\text{Capital}})$ (the first row), $\log(\frac{\text{Value added}}{\text{Capital}})$ (the second row) and $\log(\frac{\text{Revenue}}{\text{Material}})$ (the third row), respectively and truncate the intervals into 50 bins. Each bin contains the upper bound and not the lower bound.
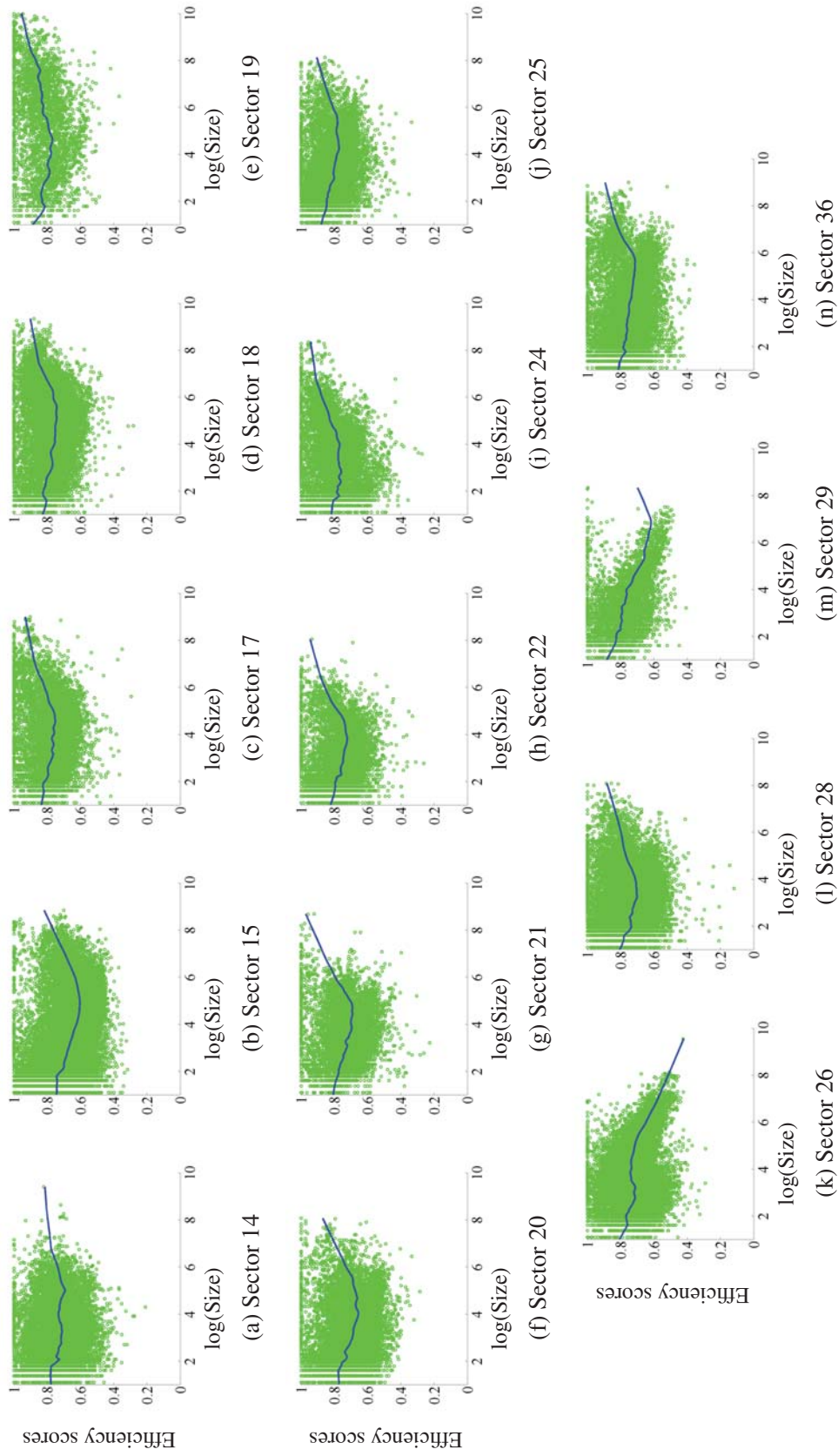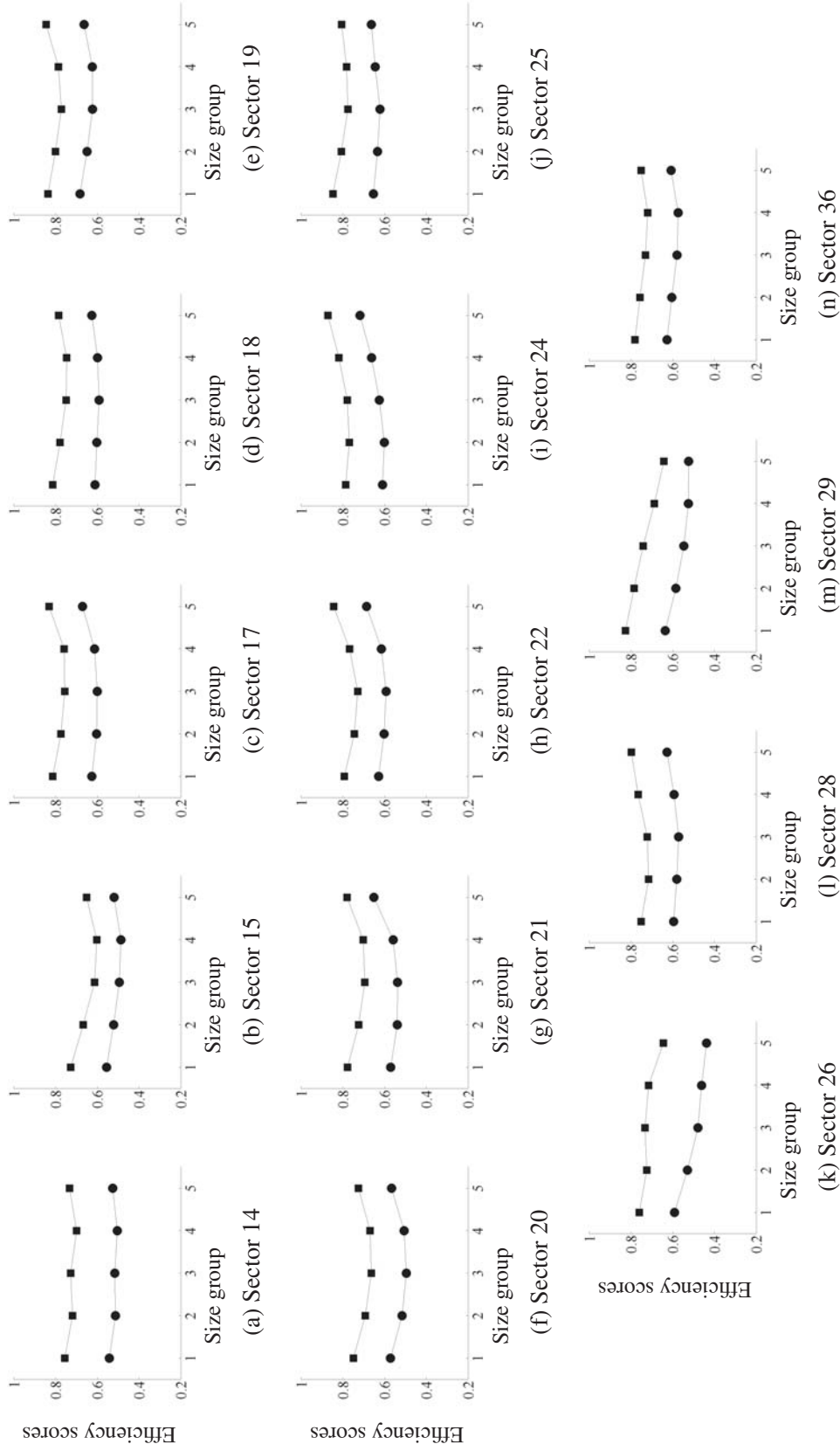
Figure 4: Efficiency Scores in Each Industry

(a) Sector 14
(b) Sector 15
(c) Sector 17
(d) Sector 18
(e) Sector 19
(f) Sector 20
(g) Sector 21
(h) Sector 22
(i) Sector 24
(j) Sector 25
(k) Sector 26
(l) Sector 28
(m) Sector 29
(n) Sector 36

Figure 5: Averaged Efficiency Scores in Each Groups

Note: The x-axis represents firms' size groups. Each square dot (■) represents an FDH efficiency score for each group. Each round dot (●) represents a DEA efficiency score for each group.

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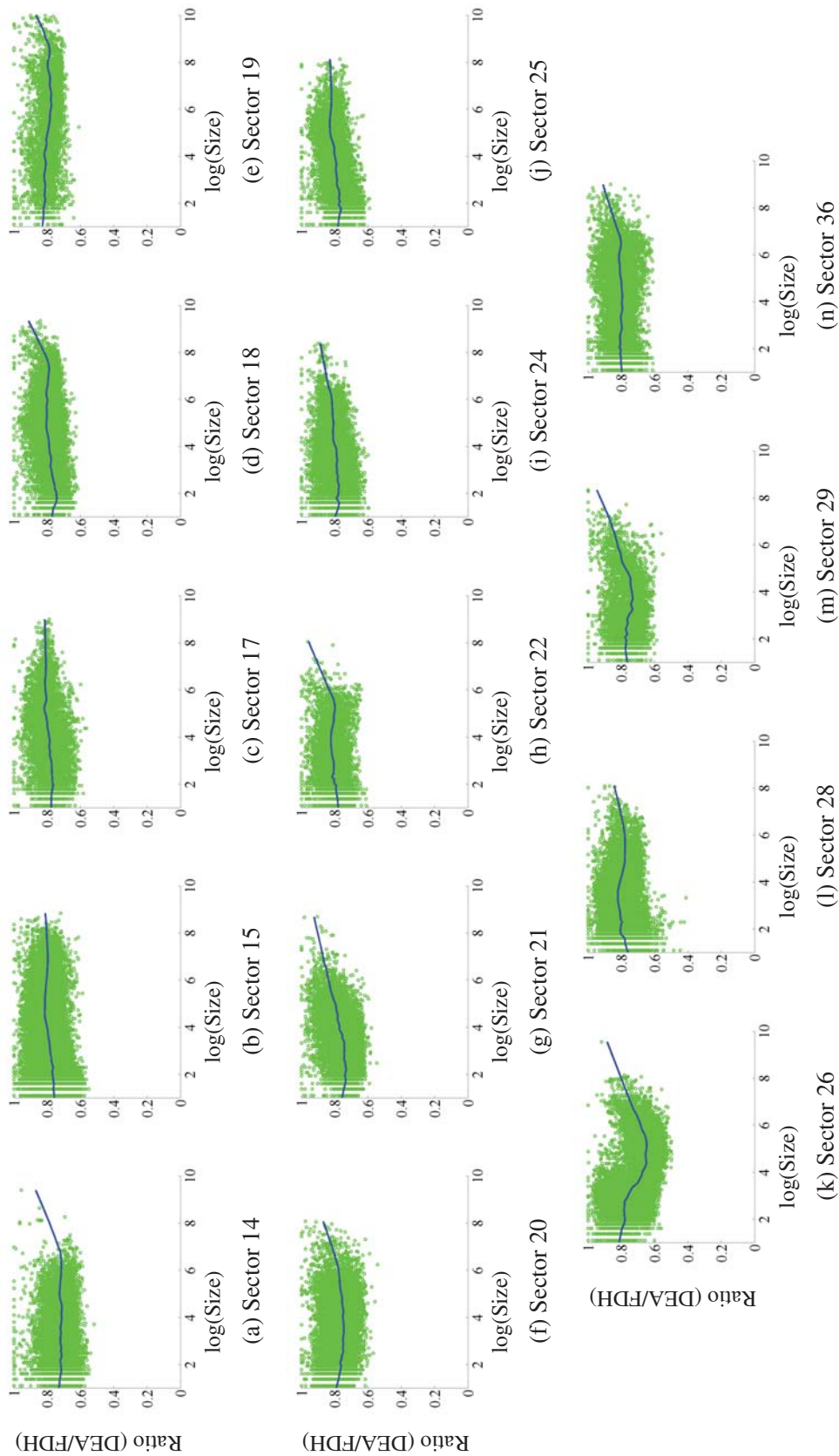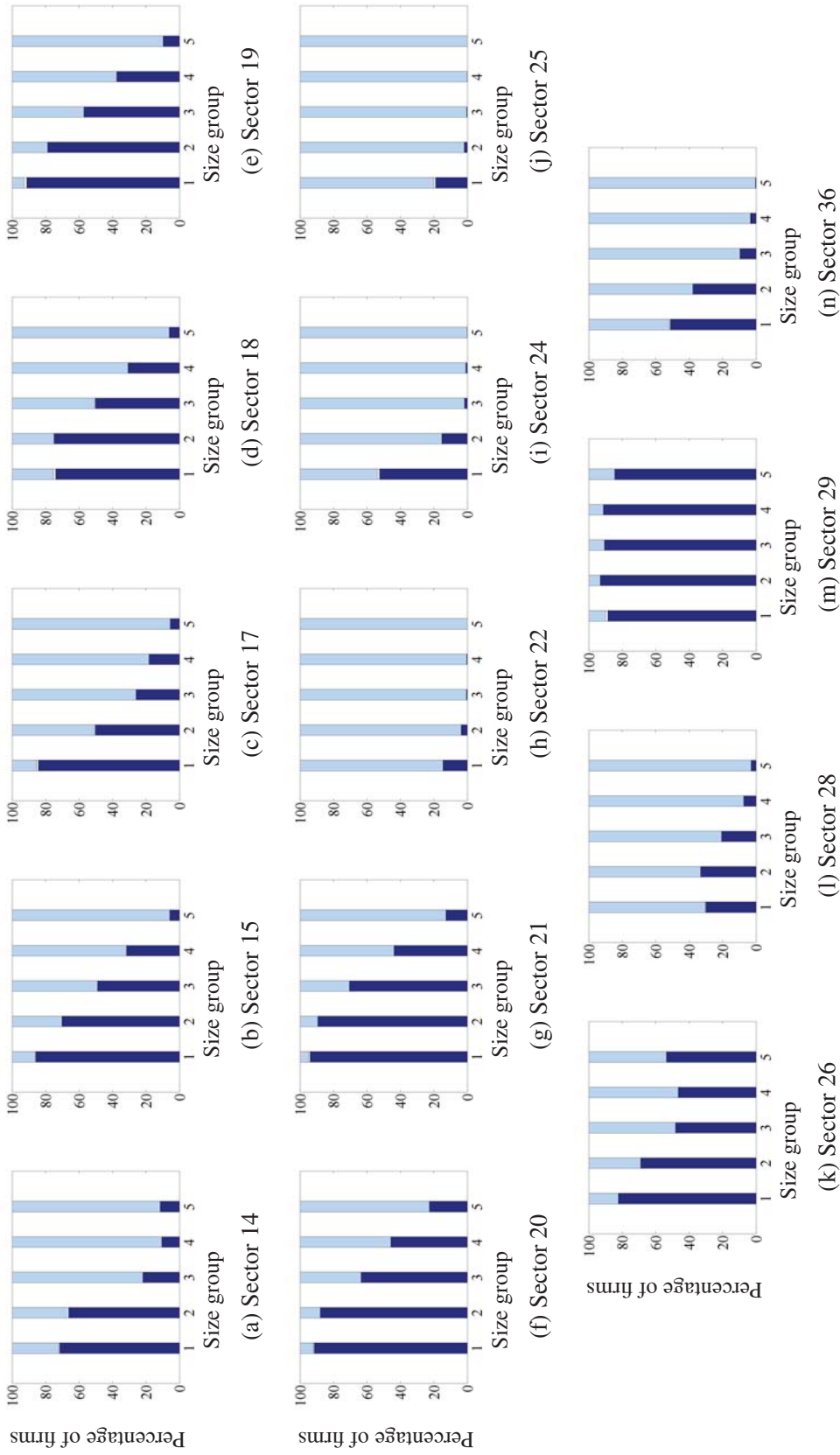Figure 6: Ratio of the Two Efficiency Scores in Each Industry
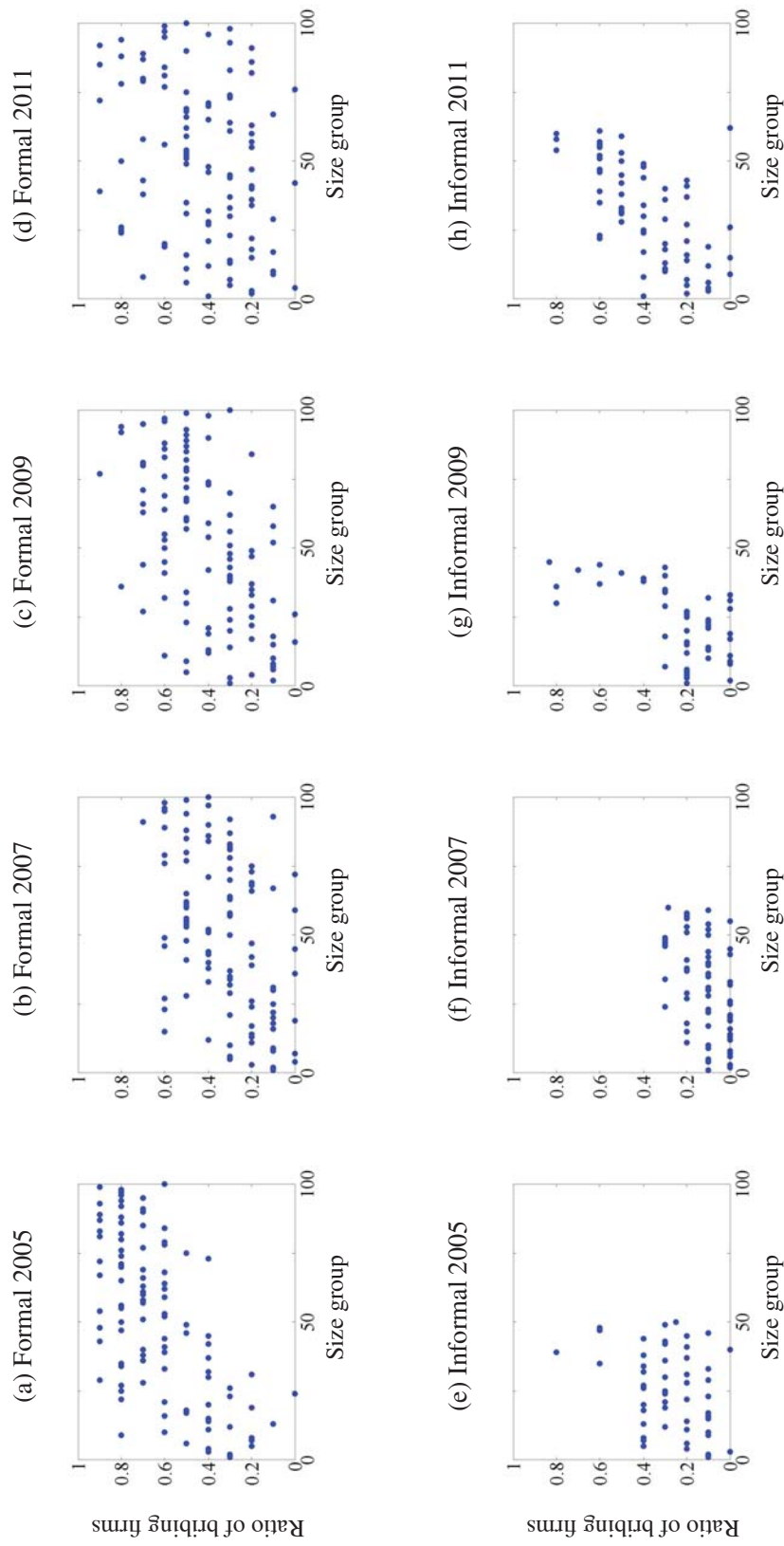
Figure 7: Returns to Scale in Each Industry

(a) Sector 14   (b) Sector 15   (c) Sector 17   (d) Sector 18   (e) Sector 19

(f) Sector 20   (g) Sector 21   (h) Sector 22   (i) Sector 24   (j) Sector 25

(k) Sector 26   (l) Sector 28   (m) Sector 29   (n) Sector 36

Note: A thin color represents a portion of firms for DRS, white color represents a portion of firms for CRS, and a thick color reporesents a portion of firms for IRS. The x-axis represents each group, namely i is Group i for each $i = 1, \cdots, 5$.

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Figure 8: Ratio of Bribing Firms for Every 20 Firms (Total)

Year 2005

Year 2007

Year 2009

Year 2011
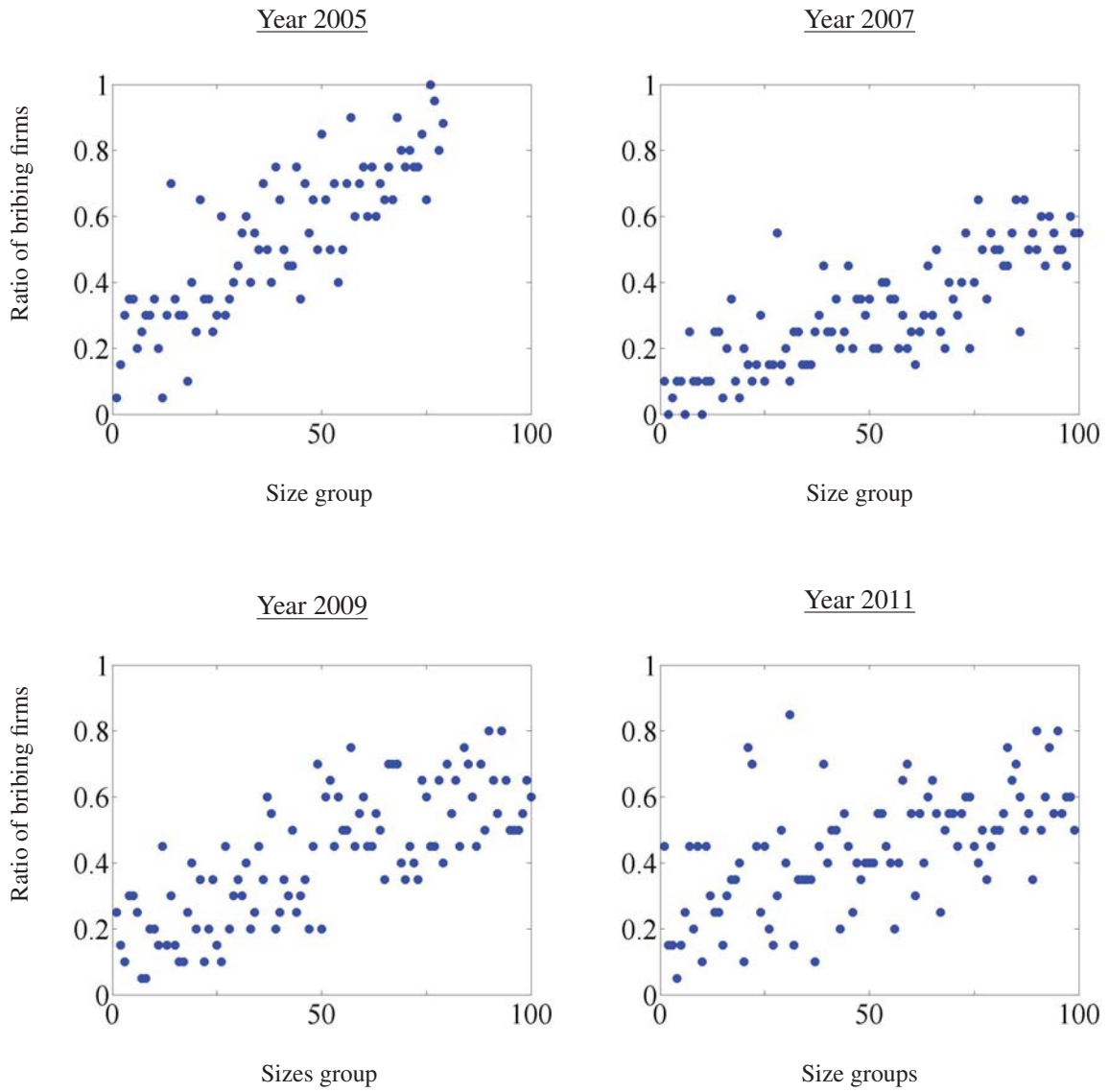
Note: The x-axis represents firms' size groups. The n's group is the n-th largest group for $n = 1, \cdots, 100$.

Figure 9: Ratio of Bribing Firms for Every 10 Firms (Formal and Informal)

(a) Formal 2005

(b) Formal 2007

(c) Formal 2009

(d) Formal 2011

(e) Informal 2005

(f) Informal 2007

(g) Informal 2009

(h) Informal 2011

Note: The first row is the figures of formal firms who have tax codes. The second raw is the figures of informal firms who do not have tax codes. The x-axis represents firms' size groups. The n's group is the n-th smallest group for $n = 1, \cdots, 100$. Each dot represents a ratio of firms paying bribes compared to the total number of firms in each group.

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