Mergers in a Mixed Oligopoly

Khanh Phan

University of Sydney,

School of Economics, Faculty of Arts and Social Sciences,

NSW, 2006, Australia.

E-mail: tpha3235@uni.sydney.edu.au

This paper examines potential mergers among a high cost public firm and two low cost private firms. Mergers can either involve two firms merging with a share of ownership or one firm purchasing the other through a lump-sum payment. It is shown that two private firms having the same costs do not merge, regardless of the merger type. However, a potential merger between a public and a private firm is possible. It is found that if the cost of the merged firm is the same as the public firm, neither the public nor the private firm want to merge. On the other hand, if the cost of the merged firm is the same as the private firm, the equilibrium outcome involves the public firm purchasing the private firm. If the marginal cost of the merged firm is between that of the public and private firms, the equilibrium outcome depends on both the cost difference between the public and private firm and the cost of the merged firm. When the cost difference between the public and the private firm is large, the public firm purchases the private firm. Conversely, when the public-private cost difference is small either type of merger can occur depending on the cost of the merged firm. Regardless, whenever the public firm chooses to merge welfare is increased.

Key words: oligopoly; market structure; public enterprises; public-private enterprises

JEL Classification: L13, L22, L32

1 Introduction

There is a large literature on mergers and their impacts. The seminal paper in the area is Salant, et al. (1983) who find that mergers are not profitable unless the market share of the merged firm is large. However, the fact that mergers are common requires further examination. Indeed, mergers are found to be profitable by Deneckere and Davidson (1985), Perry and Potter (1985) and Zhou (2008) if firms compete over price, or if the merged firm is able to access the combined productive capacity of both pre-merger firms, or if the merged firm experiences a lower cost in the presence of cost uncertainty.

Recently it has been observed that mergers occur in waves and the theoretical literature has been extended to account for this fact. Fauli-Oller (2000), Qiu and Zhou (2007) and Toxvaerd (2008) find that a merger wave is created by one merger inducing another merger where this process is triggered by a negative demand shock. Rodrigues (2001) examines how firms endogenously merge, and find that a firm's propensity to merge depends on the initial number of firms in the industry, expected competitive intensity, and the possibility of economising on fixed costs. Kamien and Zang (1990, 1993) and Granier (2008) examine the monopolisation process in mergers and show how firms' cost heterogeneity triggers mergers.

Mergers do not only happen among private firms. Mergers between private and public firms are also common. For example, in 2011, two private airlines, Aegean and Cronus, merged with a public airline Olympia and competed in a mixed oligopoly market (Artz, Heywood and McGinty, 2009), where mixed oligopoly refers to a market in which public and private firms compete. Similarly, in Canada, there have been extensive private mergers in mixed oligopoly markets such as telephone services, financial services, utilities and energy (Artz, Heywood and McGinty, 2009). In China, public-private mergers are so common that there exist legal firms that specialise in dealing with these merger transactions (Gelves and Heywood, 2013). Finally in the European auto-making industry (a mixed oligopoly market), there have been many mergers; Fiat - an Italian private automaker purchased another Italian private automaker Lancia in 1968, merged with Ferrari in 1969, and took over an Italian government-owned company, Alfa Romeo, in 1986; Volkswagen purchased SEAT - a Spanish publicly owned automobile manufacturer in 1986 (Barcena-Ruiz and Garzon, 2003); and the French firm Renault was privatized through merger with 44.2% of its shares owned by the French government (Barcena-Ruiz and Garzon, 2003).

A few studies that attempt to examine merging activities in a mixed oligopoly include Barcena-Ruiz and Garzon (2003), Kamaga and Nakamura (2007), Mendez-Naya (2008), Kamijo and Nakamura (2009),

Artz , Heywood and McGinty (2009), and Gelves and Heywood (2013).

One of the first studies to examine a merger between a public and a private firm is Barcena-Ruiz and Garzon (2003). They find that the decision to merge depends on the degree to which goods are substitutes and the percentage of the shares owned by the public shareholders in the merged firm. Following on from this study, Kamaga and Nakamura (2007) and Kamijo and Nakamura (2009) examine various ownership structures under a triopoly. By assuming that the merged firm adopts the lowest cost structure from the merger participants, the authors find that a merger between a public and a private firm is a potential equilibrium outcome. Mendez-Naya (2008) and Artz , Heywood and McGinty (2009) generalise the private-public merger analysis by allowing many private firms to exist in the market, and examine the profitability of mergers among private firms. They find that the presence of a welfare maximising public firm reduces the set of mergers that will be profitable. Recently, Gelves and Heywood (2013) examine a public-private merger in a Stackelberg game, with the public firm being an inefficient leader and all other private firms being efficient followers. Their results show that it is possible for public-private mergers to occur and these mergers often dominate a unilateral privatisation.

This paper follows Kamijo and Nakamura (2009), where there are three firms, one inefficient public firm and two efficient private firms, which compete in a Cournot Nash oligopoly. Kamijo and Nakamura (2009) assume that the merged firm fully obtains the private firm's production efficiency as does the rest of the literature (Kamaga and Nakamura (2007), Kamijo and Nakamura, 2009, Heywood and McGinty, 2011 and Gelves and Heywood, 2013). An innovation in this paper is to relax this assumption and allow partial efficiency adoption by the merged firm. Firms' decisions to merge will then depend on how efficient the merged firm becomes after merger. A further innovation is to allow the public firm to purchase a private firm with a lump-sum payment. This paper then examines both merger by purchase as well as merger through a share of ownership and the resultant merger equilibrium.

The findings show that a merger between two private firms having the same marginal costs is not an equilibrium. However, an equilibrium involving a public firm merging with a private firm may be possible. It is found that if the cost of the merged firm is the same as the public firm, neither the public nor the private firm want to merge. On the other hand, if the cost of the merged firm is the same as the private firm, the equilibrium outcome involves the public firm purchasing the private firm. If the marginal cost of the merged firm is between that of the public and private firms, the equilibrium outcome depends on both the cost difference between the public and private firm and the cost of the merged firm. When the cost difference between the public and the private firm is large, the public firm purchases the private firm.

Conversely, when the public-private cost difference is small either type of merger can occur depending on the cost of the merged firm.

In the next section, the main model and assumptions are developed. A detailed merger analysis and merger equilibria are then derived in Section 3. In Section 4, the paper concludes. The appendix is included at the end to show some proofs, followed by the list of preferences.

2 Main model

In this section, the main model is set up and and all the assumptions are stated. There are three firms - one public and two private in the market. Each of the private firms has a constant marginal cost of c. The public firm is less efficient with a constant marginal cost of $c + \epsilon$, where $\epsilon > 0$.[1]

It is assumed that there is no entry and no fixed costs. All firms produce homogeneous products and face a general market demand $p = a - bQ$ where Q is the total quantity demanded in the market, b is the slope of the demand function and $a > c > 0$ to guarantee non-negative firm outputs in equilibrium.

Let q_0 be the quantity produced by the public firm, firm 0, and q_i be the output of the private firms where $i = \{1, 2\}$. Also, denote π_i as firm i's profit. Each private firm maximises profit, and the public firm maximises welfare (W)- sum of consumer surplus and all firms' profits (Matsumura (1998), Fjell and Pal, 1996; Barcena-Ruiz and Garzon, 2003; Artz, et al., 2009; Heywood and McGinty, 2011; Gelves and Heywood, 2013). It is reasonable to assume that the public firm only targets welfare because the firm is one of the instruments used by public authorities to correct market failures and distortions (DeFraja and Delbono, 1990). In addition, in the real world, many public firms are financed by government tax revenues and a rationale for their existence is to improve social welfare (Kamaga and Nakamura, 2007). While some papers assume public firms pursue different objectives, maximisation of social welfare appears to be the most common public firm's target.[2] Specifically, the objective function for each private firm and for the public firm are, respectively,

$$\pi_i = (a - bQ - c)q_i, \quad i = \{1, 2\}; \tag{1}$$

$$W = \frac{1}{2}(Q)^2 + \pi_0 + \pi_i. \tag{2}$$

[1] If the public firm faces the same linear cost as the private firm then it simply produces the competitive quantity (DeFraja and Delbono, 1990). This a trivial case and is not of interest.

[2] Merrill and Schneider (1966) assume the public firm's objective is to maximise total amount of commodity produced, which can be seen as a proxy for employment. Coloma (2006) considers the public firm as an entity that tries to maximise its managers' utility, assuming that managers' utility is an increasing function of the firm's expenditures on production factors and inputs.

3 Pre-merger equilibrium

In this section, the pre-merger equilibrium is examined. All firms are assumed to compete in a Cournot Nash game. Let the superscript P denote the pre-merger equilibrium. In equilibrium, the quantities produced by each of the private firms, the public firm's quantity and total market output are respectively given by

$$\begin{aligned} q_0^P &= \frac{(2-b)(a-c)-3b\epsilon}{b(2b-1)}; \\ q_i^P &= \frac{(b-1)(a-c)+b\epsilon}{b(2b-1)}, \quad i=\{1,2\}; \\ Q^P &= \frac{a-c-\epsilon}{2b-1}. \end{aligned}$$

The market price, profit for each of the private firms, the public firm's profit and social welfare prior to merger are respectively given by

$$\begin{aligned} p^P &= \frac{b(a+c+\epsilon)-a}{2b-1}; \\ \pi_i^P &= \frac{1}{b}\left[\frac{(b-1)(a-c)+b\epsilon}{2b-1}\right]^2, \quad i=\{1,2\}; \\ \pi_0^P &= \frac{(b-1)(a-c-\epsilon)\left[(2-b)(a-c)-3b\epsilon\right]}{b(2b-1)^2}; \\ W^P &= \frac{1}{2}(Q^P)^2+\pi_0^P+\pi_1^P+\pi_2^P. \end{aligned}$$

In the pre-merger equilibrium each firms' outputs and profits as well as social welfare depend on the demand intercept (a), the marginal costs of firms (c and ϵ), and the slope of the demand function (b). However, these expressions are cumbersome; therefore, to simplify the analysis, it is assumed that $b=1$.

With $b=1$, the pre-merger equilibrium firms' outputs, market price, profits and social welfare are respectively given by

$$q_0^P = a-c-3\epsilon; \quad q_i^P = \epsilon, \quad i=\{1,2\}; \tag{3}$$

$$p^P = c+\epsilon; \tag{4}$$

$$\pi_i^P = \epsilon^2, i=\{1,2\}; \quad \pi_0^P = 0; \tag{5}$$

$$W^P = \frac{1}{2}(a-c-\epsilon)^2+2\epsilon^2. \tag{6}$$

From (3), to ensure positive production by all firms in the market, it is assumed that $a - c - 3\epsilon > 0$. In the pre-merger equilibrium, each private firm produces ϵ and the market price is equal to the marginal cost of the public firm. This implies that the public firm produces a quantity such that price equals its marginal cost in order to maximise total welfare. Meanwhile, private firms produce positive outputs as they are more efficient than the public firm and their production depends on how much more efficient they are compared to the public firm, that is, on the size of ϵ. If the public firm is just as efficient as the private firm ($\epsilon = 0$), the private firms produce nothing and the public firm supplies the competitive quantity (DeFraja and Delbono, 1990).

4 Merger analysis

In this section, both private and private-public mergers are analysed. As mentioned in the Introduction section, mergers can either involve both merger participants agreeing upon a specific value of share of ownership or one firm purchasing another through a lump-sum payment. In general, both types of mergers are similar in the sense that one firm makes a payment to another firm either in the form of a share payment or as a lump-sum. If the merger occurs between two private firms, both types of mergers are practically the same in the sense that the latter is just the former with one party holding a 100% share of ownership. However, if the merger occurs between a private and a public firm, due to different objectives that each merger participant pursues, the former type of merger gives each party the right to make decisions that favour their interest. Thus when merger participants decide on each party's share of ownership, they take into account the fact that this will affect the output decisions of the merged firm as well as the outsiders. This is the strategic effect of a merger. In the case where a merger involves a lump-sum payment, there is no strategic effect.

4.1 A private merger

The first merger considered is a merger between two private firms. As is well known, mergers between private firms are unprofitable, given there are no cost advantages involved. Salant et al. (1983) show that as long as there exists at least one excluded rival, no merger between two firms with the same marginal costs will be profitable in equilibrium under Cournot competition.

The question in this section is whether, in the presence of a public firm, it is profitable for two private firms to merge.

Let the superscript M denote the equilibrium in which the two private firms merge. Let α^M be the ownership share of private firm 1 and also the weight attached to firm 1's objective in the merged firm's objective function.[3] In the case of one firm purchasing another through a transfer payment, α^M can either be 0 or 1 depending on whether it is firm 1 or 2 that purchases the other firm, and the purchasee gets a fixed lump-sum payment T^M. As both merger participants are private firms and they both aim to maximise profit, the merged entity's objective is to maximise the merged firm's profit. This is denoted $\pi_{1,2}^M$.[4]

Subsequent to the merger, two firms operate in the market - one public and one private merged firm. The public firm maximises social welfare $W^M = \frac{1}{2}(Q^M)^2 + \pi_0^M + \pi_{1,2}^M$ and the private merged firm maximises its profit $\pi_{1,2}^M$. In a Cournot equilibrium, market price is equal to $p^M = c + \epsilon$ and the merged firm's profit is equal to $\pi_{1,2}^M = \epsilon^2$.

In the case of an ownership share, the firms' shareholders will only want to merge if their shares of profit in the merged firm subsequent to merger, $\alpha^M \pi_{1,2}^M$ and $(1-\alpha)\pi_{1,2}^M$, are at least equal to the profits they make in the pre-merger equilibrium. As can be seen from the post-merger equilibrium, the profit of the merged firm is the same as each firm made pre-merger, that is $\pi_{1,2}^M = \epsilon^2 = \pi_i^P$. Therefore, there is no α^M that can increase the profit of one private firm without reducing that of the other. Thus, at least one firm has an incentive not to merge.

In the case of purchasing through a lump-sum payment, the purchaser will get full control of the firm and obtain a profit $\pi_{1,2}^M$, while the purchasee receives a transfer payment. The firms' owners will only want to merge if their payoffs after merger, that is $\pi_{1,2}^M - T^M$ for the purchaser and T^M for the purchasee, are at least equal to their pre-merger profits. Again, there is no T^M that increases one firm's profit without reducing that of the other relative to the pre-merger equilibrium. Therefore, merger does not occur. This is summarised in the following Proposition

PROPOSITION 1 *If three firms operate in a market, in which there is one inefficient public firm and two efficient private firms, it is not profitable for the two private firms to merge.*

The intuition is clear. As in Salant, et al. (1983), when two private firms merge, the merged firm produces less output than the sum of the output of the two firms pre-merger. In response, the welfare-maximising public firm expands output, as the merger between the two private firms restricts output below

[3]Firm 1 and 2 are symmetric, therefore choosing α^M to be the ownership share of firm 1 or firm 2 does not change the results.

[4]The merged firm's objective function is the weighted average of the merger participants' objective functions. In this case, it is $\alpha^M \pi_{1,2}^M + (1-\alpha^M)\pi_{1,2}^M = \pi_{1,2}^M$ for all $\alpha \in [0,1]$.

the welfare-maximising level.

Kamaga and Nakamura (2007) and Artz, Heywood and McGinty (2009) use a convex cost structure for firms, and also find that the presence of a welfare maximising public firm reduces the set of mergers that will be profitable.

4.2 A public-private merger

In the previous section, a merger between two private firms was examined and the conclusion was that a profitable private merger is not possible. In this section, a merger between a public and a private firm is analysed.

Let the superscript N denote the public-private-merger equilibrium. Without loss of generality assume the merger is between the public firm 0 and the private firm 1. The merged firm's marginal cost is a weighted average of its pre-merger constituents' costs - $c_{0,1}^N = (1-\theta)(c+\epsilon) + \theta c$, where $\theta \in [0,1]$ is the weight associated with the private firm's cost in the merged firm's cost function. Effectively, θ measures the efficiency of the merged firm. The larger the value of θ, the closer the merged firm's marginal cost is to that of the private firm. If the merged firm's marginal cost is the lowest among that of its pre-merger constituents as is used in the literature (Kamaga and Nakamura, 2007, Kamijo and Nakamura, 2009, Heywood and McGinty, 2011, Gelves and Heywood, 2013), it is shown later in the paper that the public firm always chooses to purchase the private firm and pay the private firm a lump-sum payment. Thus, allowing the merged firm's marginal cost to be $c_{0,1}^N \in [c, c+\epsilon]$ is necessary for a more general examination of merger.

Both merger by share of ownership and merger through a lump-sum payment are examined, and the conditions under which one type is preferred to the other are analysed in order to find the merger equilibria.

4.2.1 Share of ownership

The public firm 0 and the private firm 1 merge through both parties agreeing upon an ownership share. Let α be the ownership share of the private firm and also the weight attached to the private owner's objective in the merged firm's objective function. The public owner wants to maximise welfare while the private owner is only concerned about profit. The objective function of the merged entity is therefore a weighted average of welfare and profit $(1-\alpha)W^N + \alpha\pi_{0,1}^N$, where $\pi_{0,1}^N$ is profit of the merged firm and $W^N = \frac{1}{2}(Q^N)^2 + (1-\alpha)\pi_{0,1}^N + \alpha\pi_{0,1}^N + \pi_2^N$ is welfare. This objective function has been widely used in the literature including Barcena-Ruiz and Garzon (2003), Artz, Heywood and McGinty (2009), Kamijo and

Nakamura (2009) and Gelves and Heywood (2013).

Subsequent to merger, there remains one private outsider - firm 2, and one mixed merged firm - firm $(0,1)$ in the market. Firm 2's objective is to maximise its profit π_2^N. These two firms compete in a Cournot duopoly. As a result, the firm quantities, the market quantity and the market price are as follows:

$$q_2^N = \frac{\alpha(a-c)+(1-\theta)\epsilon}{1+2\alpha}; \tag{7}$$

$$q_{0,1}^N = \frac{a-c-2(1-\theta)\epsilon}{1+2\alpha}; \tag{8}$$

$$Q^N = \frac{(\alpha+1)(a-c)-(1-\theta)\epsilon}{1+2\alpha}; \tag{9}$$

$$p^N = \frac{\alpha a+(1+\alpha)c+(1-\theta)\epsilon}{1+2\alpha}. \tag{10}$$

Firm profits and welfare are:

$$\pi_{0,1}^N = (p-c)q_{0,1} = \frac{\alpha[(a-c)-2(1-\theta)\epsilon]^2}{(1+2\alpha)^2}; \tag{11}$$

$$\pi_2^N = (p-c)q_2 = \frac{[\alpha(a-c)+(1-\theta)\epsilon]^2}{(1+2\alpha)^2}; \tag{12}$$

$$W^N = \frac{[(1+\alpha)^2+2\alpha+2\alpha^2](a-c)^2-2(1+3\alpha)(a-c)(1-\theta)\epsilon+(3+8\alpha)(1-\theta)^2\epsilon^2}{2(1+2\alpha)^2}. \tag{13}$$

From (7) and (10), firm 2's output and the market price are both positive. The positive production condition in section 3 guarantees that the merged firm's output in (8) is also positive. It is clear that the merged firm's output decreases in α and it can be shown that the outsider's output increases in α. In addition, the merged firm's profit is positively related to the private share of ownership, α, for any $\alpha \leq 1/2$ and negatively related to α for $\alpha > 1/2$. In order to explain the results here, there are two effects, which are the profit maximum effect and the strategic effect of the outsider, that have to be considered. The first implies that an increase in α, which gives more weight to profit in the merged firm's objective function, induces the firm to produce less so that the market price and the merged firm's profit are higher. The second however implies an increase in α which induces a lower production by the merged firm strategically encourages the outsider, firm 2, to produce more. As a result, the merged firm's profit is decreased. It is found that the profit maximum effect dominates when α is small and the strategic effect of the outsider dominates when α is large.

Having determined the post-merger equilibrium, conditions under which the merger occurs are now

examined. In order for the private firm to merge, its share of the merged firm's profit subsequent to merger must be at least as large as its pre-merger profit, while the public firm will only merge if post-merger welfare is at least as large as its pre-merger level.

Firstly, the merger condition for the private firm is considered. As mentioned above, the private firm will only merge if $\alpha\pi_{0,1}^N \geq \pi_1^P$, where α is the private ownership share and also the private share in the merged firm's profit and π_1^P is the pre-merger profit of the private firm. Using (5) and (11) above, it can be shown that $\alpha\pi_{0,1}^N \geq \pi_1^P \Leftrightarrow \alpha \geq \frac{\epsilon}{a-c-2\epsilon(2-\theta)}$. Let $\frac{\epsilon}{a-c-2\epsilon(2-\theta)} = A1$. It is shown in the Appendix that $0 < A1 < 1$. The condition for the **private firm** to accept the merger is summarised in the following Lemma:

Lemma 1 *The private firm will merge with the public firm by agreeing upon a private share of ownership, α, if and only if $\alpha \geq A1$.*

It is clear that $A1$ decreases in θ and increases in c and ϵ.

This implies that if θ is large, the private shareholders only requires a small share of ownership, α, to accept the merger. θ represents the merged firm's production efficiency or how close the merged firm's marginal cost is to the private cost. Therefore, when θ is high, the merged firm's marginal cost is low and the firm's profit is large. With a large profit of the merged firm, private shareholders only need a small share of it to be as well off as in the pre-merger equilibrium and accept the merger.

In contrast, when the production inefficiency of the public firm ϵ or marginal cost of production c or both are high, the private shareholders require a large ownership share in the merged firm to accept the merger. The reason is that when ϵ or c or both are high, the merged firm's marginal cost is high and the merged firm's profit is low. The private firm therefore will only merge if it receives a large share of this profit.

Secondly, the condition for the public firm to merge is examined. As mentioned above, the public firm accepts the merger if and only if post-merger welfare is at least as large as pre-merger welfare. Using (6) and (13) above, it can be shown that $W^N \geq W^P \Leftrightarrow A3 \leq \alpha \leq A2$, where

$$A2 = \frac{2\epsilon^2(2\theta^2-4\theta-3)+\epsilon(1+3\theta)(a-c)+[a-c-2\epsilon(1-\theta)]\sqrt{\epsilon[2(a-c)\theta+\epsilon(4\theta^2-8\theta-1)]}}{(a-c-4\epsilon)^2+4\epsilon^2}, \quad \textit{and}$$

$$A3 = \frac{2\epsilon^2(2\theta^2-4\theta-3)+\epsilon(1+3\theta)(a-c)-[a-c-2\epsilon(1-\theta)]\sqrt{\epsilon[2(a-c)\theta+\epsilon(4\theta^2-8\theta-1)]}}{(a-c-4\epsilon)^2+4\epsilon^2}.$$

Let $\frac{\sqrt{(a-c-4\epsilon)^2+4\epsilon^2}-(a-c-4\epsilon)}{4\epsilon} = E1$. The condition for the **public firm** to accept the merger is

summarised in the following Lemma:

Lemma 2 *Assume $\theta > E1$, then $A2$ and $A3$ are real with $A2 > A3$. The public firm will accept the merger if and only if the private share of ownership $A3 \leq \alpha \leq A2$.*

The condition $\theta > E1$ guarantees that the expression inside the square root term in $A2$ and $A3$ are positive. Given $A2, A3$ are real, and they only differ by the sign between the first and second term in their expressions, it is clear that $A2 > A3$. The case where $A2 = A3$ is eliminated because it results in a decrease in welfare under the merger relative to the pre-merger equilibrium. The intuition behind this Lemma is elaborated later in that of Proposition 2.

In Lemmas 1 and 2 above, conditions were derived so that a merger between the public and private firm was profitable from the perspective of the private firm (Lemma 1) and welfare increasing from the perspective of the public firm (Lemma 2). In the Appendix it is shown that $A3 < A1$, so for a merger to be accepted by both the private and the public firm, it must be that $A3 < A1 \leq \alpha \leq A2$.

It is clear that as α is a share, it must be that $\alpha \in [0, 1]$. It was stated right before the Lemma 1 that $0 < A1$. Also, it can be shown that $A2 < 1$ if $\theta < E3$ and $A2 = 1$ if $\theta \geq E3$, where $E3 = \frac{3\sqrt{3(a-c)^2-22(a-c)\epsilon+55\epsilon^2}-4(a-c)+11\epsilon}{11\epsilon}$. Therefore, the condition that $\alpha \in [0, 1]$ is satisfied. Now, for α to lie between $A1$ and $A2$ as stated above, it must be that $A2 \geq A1$. This requires $\theta \geq E2$, where $E2 = \frac{\sqrt{(a-c-4\epsilon)^2+3\epsilon^2}-(a-c-4\epsilon)}{3\epsilon}$. It is shown in the Appendix that $E2 > E1$, so the condition for the existence of an α that is accepted by both the public and private firm ensures that the condition in Lemma 2 is satisfied. It is also shown that $0 < E2 < 1$ (in Appendix) so the condition that $\theta \in [0, 1]$ is satisfied. Therefore, the conditions for a profitable and welfare increasing merger between the public and the private firm is summarised in the following Proposition

PROPOSITION 2 *Both the public firm and the private firm accept the merger by agreeing upon a private share of ownership, α, if and only if the weight associated with the private cost in the merged firm's cost function θ and the private share of ownership α satisfy the following conditions:*

$$\theta \geq E2 \text{ and } \alpha \in [A1, A2]$$

The Proposition shows conditions on the cost parameter of the merged firm, θ, and the private ownership share,α, for which a merger between a public and a private firm is desirable for both.

The condition on θ indicates that the merged firm must become sufficiently efficient after merger in order for the merger to occur. It is shown in the Appendix that $E2$ increases with ϵ, implying that the more inefficient the public firm is relative to the private firm, the more efficient the merged firm is required to become for a merger to occur. The intuition is that when the public firm is largely inefficient, the merged firm's marginal cost, which is the weighted average of that of the public and the private firms, is large and its profit is small. From Lemma 1, the private shareholders require a large share of ownership, α, to accept the merger. However, if α is too large, the public shareholders will have no incentive to merge as specified in Lemma 2. Thus, a higher θ that lowers the merged firm's marginal cost and increases the firm's profit is required so that a moderate α can be accepted by both merger parties.

The condition on α indicates that a private share of ownership, α, must be sufficiently large for the private shareholders to accept the merger, but must not be too large to discourage the public firm from merging. It was stated after Lemma 1 that $A1$ decreases in θ and it can be shown that $A2$ increases in θ. Thus, the range of α over which a merger is desirable for both the public and the private firms is larger the larger is θ. The intuition is that when θ is large, the merged firm's marginal cost is low and its profit is high. The private shareholders therefore accept the merger with a small private share of ownership and the public shareholders are willing to give more ownership share to the private firm. If $\theta \geq E3$, this range can include complete privatisation ($\alpha = 1$) as with $\theta \geq E3$, $A2 = 1$.

Consider the following example. Let $a - c = 1$ and $\epsilon = 0.1$. In this case, $E2 = 0.08$ and $E3 = 0.53$. Now, for a merger between a public and a private firm to occur, it must be that $\theta \geq E2 = 0.08$. However, the range of α over which the merger occurs depends on the relationship between θ and $E3$. If $\theta < E3$, for example $\theta = 0.1$, it is found that $A1 = 0.16 \leq \alpha \leq A2 = 0.26$; while if $\theta \geq E3$, for example $\theta = 0.6$, it is found that $A1 = 0.14 \leq \alpha \leq 1 = A2$.

4.2.2 Lump-sum payment

In this section, the option of the public firm, firm 0, purchasing the private firm, firm 1, with a lump-sum payment is examined.

It is assumed that the public firm can give the private firm a lump-sum payment that makes the private firm agree to the purchase. It is also assumed that the public firm need not have a positive profit to make a payment to the private firm. It is not unrealistic to assume that the public firm, which aims for welfare maximisation, can get funding from the government to finance its transactions if those transactions increase welfare. While Artz, et al. (2009) assume that the merged firm's post merger profit

must be enough to pay the private firm for the firm to accept the purchase, this assumption is relaxed in this paper. The purpose of doing this is to examine the public firm's merger decision in equilibrium in the most general circumstance.

Now, the public firm makes a proposal to purchase the private firm and pays the private firm an amount that leaves the private firm just as well off as it was in the pre-merger equilibrium. Given that the private firm is only concerned about profit, the public firm gives the private firm a payment that is equal to the private firm's pre-merger profit, and the private firm accepts this proposal. The public firm on the other hand will only propose to buy the private firm if the purchase increases welfare.

Denote T^N as the lump-sum payment made by the public firm to purchase the private firm. The lump-sum payment is equal to the private firm's pre-merger profit, therefore $T^N = \epsilon^2$. The public firm after purchasing the private firm acts as a pure welfare maximiser with objective function $\hat{W^N} = 1/2(Q^N)^2 + (\pi_0^N - T^N) + T^N + \pi_2^N$, where π_0^N is the public firm's post-merger profit, and π_2^N is firm 2's post-merger profit. Note that the public merged firm's marginal cost is still a weighted average of the public and the private firms' marginal costs, that is $c_0^N = \theta c + (1-\theta)(c+\epsilon)$.

Calculation reveals post-merger welfare to be $\hat{W^N} = \frac{(a-c)^2 - 2(a-c)(1-\theta)\epsilon + 3(1-\theta)^2\epsilon^2}{2}$. In order for the public firm to carry out the purchase, post-merger welfare must be at least as large as pre-merger welfare, that is, $\hat{W^N} \geq W^P$. Let $E4 = \frac{\sqrt{(a-c-3\epsilon)^2 + 6\epsilon^2} - (a-c-3\epsilon)}{3\epsilon}$. In the Appendix, it is shown $0 < E4 < 1$. A condition for the purchase by the public firm to increase welfare is stated in the following Proposition:

PROPOSITION 3 *Both the public and private firms agree to the public firm purchasing the private firm with a lump-sum payment if and only if the weight attached to the private cost in the merged firm's cost function, θ, satisfies the condition $\theta \geq E4$.*

Proposition 3 shows that a public firm only purchases a private firm if the marginal cost of the public merged firm is close to that of the pre-merger private firm or in other words, the merged firm becomes sufficiently efficient after purchase. The merged firm maximises welfare and so tends to expand output. For this to be welfare increasing the production cost of this extra output must be sufficiently low.

In the Appendix it is shown that $E4$ is positively related to the cost difference between the public and the private firms, ϵ. Therefore, the more inefficient the public firm is relative to the private firm prior to the purchase, the more efficient the merged firm must become after the purchase in order for the purchase to occur.

4.2.3 Merger decisions

Having analysed two types of possible mergers between a public and a private firm, this section analyses the merger decisions and derives the merger equilibrium.

From Proposition 1, a merger between a public and a private firm occurs if the merged firm's marginal cost is sufficiently close to that of the private firm, that is $\theta \geq E2$; and both firms agree on a private share of ownership, α, belonging to the range $[A1, A2]$. This range can include a complete privatisation ($\alpha = 1$) if $\theta \geq E3$, but a full nationalisation ($\alpha = 0$) is not possible because holding no share in the merged firm entitles private shareholders to no profit and they will not accept such a merger. Nevertheless, nationalisation (full public ownership) is possible if the public firm gives the private firm a lump-sum payment that makes the private firm as well off as it was in the pre-merger equilibrium. From Proposition 2, the public firm only chooses to completely purchase the private firm if $\theta \geq E4$.

Now, if it is possible for the public firm to choose either to completely purchase the private firm or to merge with the private firm with a specific share of ownership, the public firm's decision will depend on which option results in higher welfare. It can be shown that welfare is higher with complete purchase if and only if $\alpha \geq A4$, where $A4 = \frac{2\epsilon(1-\theta)}{a-c-6\epsilon(1-\theta)}$ (in the Appendix). Therefore, the public firm chooses to purchase the private firm if $\alpha \geq A4$, and to merge with a specific share of ownership otherwise. The equality, $\alpha = A4$, indicates that the public firm is indifferent between the two options. It is also shown in the Appendix that $0 < A4 < 1$ so the condition that $\alpha \in [0, 1]$ is satisfied.

As for the private firm, it always prefers to merge with a share of ownership ($\alpha \geq A1$) if both types of merger are possible. This follows because the least the private firm gets from the merger with a share of ownership is equal to what it gets under the purchase.[5]

In general, which type of merger that is optimal depends on the parameters of the model. The merger equilibria are now examined under various parameter assumptions.

PROPOSITION 4 *If the cost difference between the public and private firms is relatively small, in particular* $\epsilon < 4/17(a - c)$*, then*

(i) for $0 \leq \theta < E2$*, the equilibrium involves no merger between the public and private firms nor any purchase of the private firm by the public firm,*

(ii) for $E2 \leq \theta \leq E4$*, the equilibrium involves a merger between the public and private firms with a*

[5]The private firm gets at least its pre-merger profit when $\alpha = A1$ when it merges with the public firm through a share of ownership. For any $\alpha > A1$, the private firm gets a higher payoff. With the purchase, the maximum payoff the private firm gets is its pre-merger profit.

share of ownership, α, in the range $\alpha \in [A1, A2]$,

(iii) for $E4 < \theta \leq 1$,

(a) if $E4 < \theta < 1/2$ so $A1 < A4 < A2$, the equilibrium involves a merger between the public and private firms with a share of ownership in the range $\alpha \in [A1, A4]$,

(b) if $1/2 \leq \theta \leq 1$ so $A4 \leq A1 < A2$, the equilibrium involves the public firm purchasing the private firm with a lump-sum payment.

Proposition 4 (i) is clear from Propositions 2 and 3. It was stated in Proposition 2 that a merger between the public and private firms with a share of ownership is not possible if $\theta < E2$. As $E2 < E4$ given the condition $\epsilon < 4/17(a - c)$, this makes $\theta < E4$ and a purchase of the private firm by the public firm is impossible following Proposition 3. The intuition is that if the merged firm does not become sufficiently efficient after merger/purchase, the merged firm's profit and social welfare are low. This discourages both firms from merging. Kamaga and Nakamura (2007) using a convex cost structure also find that without any productivity-improving effect with a merger, the mixed oligopoly is a stable market structure.

In Proposition 4 (ii), firms agree to merge with a share of ownership in the range $\alpha \in [A1, A2]$, but the public firm does not want to purchase the private firm as θ satisfies the condition in Proposition 2, but does not satisfy the condition in Proposition 3. The merged firm's production efficiency is low after merger, and therefore having private shareholders holding an ownership share in the merged firm commits the firm to not producing a large output with a low efficiency. The efficient outsider strategically reacts to this small production of the merged firm by producing more, and welfare increases.

In Proposition 4 (iii), the merged firm now becomes sufficiently efficient after merger/purchase and the conditions on θ in both Proposition 2 and 3 are satisfied. Both merger with a share of ownership and purchase of the private firm by the public firm are possible.

If the merged firm does not become significantly efficient after merger/purchase (Proposition 4 (iii) (a)), the public firm chooses to merge with the private firm with a share of ownership in the range $\alpha \in [A1, A4]$, as having private shareholders holding an ownership share in the merged firm commits the firm to not producing a large output with a low efficiency. However, the private shareholders cannot ask for a higher ownership share ($\alpha > A4$) as with such a large private share of ownership, the merger results in a smaller increase in welfare than the purchase or even results in a decrease in welfare relative to the pre-merger level. As a result, the public firm chooses to purchase the private firm. As mentioned above, the private firm always prefers to merge with a share of ownership, so by mutually agreeing on an α in the range $[A1, A4]$, both firms are better off with a merger.

If the merged firm becomes significantly efficient after merger/purchase (Proposition 4 (iii) (b)), the public firm purchases the private firm through a lump-sum payment. The public firm then acts as a pure welfare maximiser and largely expands its output to maximise welfare. It is obvious that if the merged firm becomes as efficient as the pre-merger private firm ($\theta = 1$), the public firm purchases the private firm, produce its output at a competitive level and welfare is maximied.

PROPOSITION 5 *If the cost difference between the public and private firms is relatively large, in particular* $\epsilon \geq 4/17(a-c)$*, then*
(i) for $0 \leq \theta < E4$*, the equilibrium involves no merger between the public and private firms nor any purchase of the private firm by the public firm,*
(ii) for $E4 \leq \theta \leq 1$*, the equilibrium involves the public firm purchasing the private firm with a lump-sum payment.*

Proposition 5 (i) makes the same argument as that in Proposition 4 (i). It is clear that neither the condition on θ in Propositions 2 nor that in Proposition 3 is satisfied as $\theta < E4$ and $E4 \leq E2$ by the condition $\epsilon \geq 4/17(a-c)$.

In Proposition 5 (ii), the merged firm now becomes sufficiently efficient ($\theta \geq E4$). Given the condition $\epsilon \geq 4/17(a-c)$ in this Proposition, it is found that $E4 \geq 1/2$. With such a large production efficiency obtained by the merged firm after merger ($\theta \geq 1/2$), the public firm chooses to purchase the private firm through a lump-sum payment following the argument in Proposition 4 (iii) (b).

Both Propositions 4 and 5 established that the merged firm must become sufficiently efficient for a merger/purchase to occur. However, this condition on the merged firm's production efficiency is looser when the cost difference between the public and private firms is relatively small than when it is large. In particular, from Proposition 4, it is possible for a merger to occur when $\theta < 1/2$ while from Proposition 5, it is required that $\theta \geq E4 \geq 1/2$ for firms to have incentives to merge/purchase. It can be seen that if the cost difference between the public and private firms is small, the merged firm's marginal cost is small. As a result, the merged firm's profit and welfare can both increase after the merger without the need for a large efficiency gain for the merged firm after merger.

Also, when it is possible for the public firm to either merge with the private with a share of ownership or purchase the private firm through a lump-sum payment, the public firm chooses to merge with a share of ownership when $\theta < 0.5$ and to purchase the private firm when $\theta \geq 0.5$. Therefore, in Proposition 4, it can be seen that an equilibrium involving a merger between the public and private firms through

a share of ownership is possible as the merged firm does not need to become significantly efficient after merger for the merger to occur. In contrast, as established in Proposition 5, the merged firm is required to be significantly efficient after merger for merger/purchase to occur as the public firm is largely inefficient pre-merger. Given that the merged firm becomes such an efficient firm after merger, it is optimal for the firm to purchase the private firm, expand it output and act as a pure welfare maximiser in equilibrium.

5 Conclusion

The paper has examined the possibility of a merger between two private firms in the presence of a public firm and shown that a merger between two private firms having the same marginal costs is not profitable.

This paper then examined a potential merger between a public and a private firm and found that different outcomes result depending on the parameters of the model. It is shown that if the merged firm becomes as efficient as the pre-merged private firms, the equilibrium involves the public firm purchasing the private firm with a lump-sum payment. Conversely, if the merged firm has the same marginal cost as the public firm, no merger occurs and the equilibrium is a mixed triopoly.

If the merged firm's marginal cost is between that of the public and private firms, the equilibrium outcome depends on both the cost difference between the public and private firms and the marginal cost of the merged firm. The paper has employed θ as the weight attached to the private cost in the merged firm's cost function to examine different equilibria when θ varies from 0 to 1. It is found that if the cost difference between the public and private firms, ϵ, is relatively small, θ need not to be large in order for a merger/purchase to occur. In this case, both merger with a share of ownership and purchase through a lump-sum payment are possible depending the values of θ. In particular, if θ is relatively large ($\theta \geq 1/2$), it is optimal for the public firm to purchase the private firm with a lump-sum payment. Conversely, if θ is relatively small, it is optimal for both firms to merge with a share of ownership in equilibrium.

On the other hand, if ϵ is relatively large, θ must be sufficiently large ($\theta \geq 1/2$) in order for a merger/purchase between the public and private firms to occur. With such a large θ, it is found that the public firm optimally chooses to purchase the private firm in equilibrium.

Furthermore, the public firm maximises welfare, so regardless of whether the public firm purchases the private firm or merge with the private firm with a share of ownership, welfare increases. Otherwise the public firm will not agree to the purchase or merger.

APPENDIX

1. **Proof of** $0 < A1 < 1$

$$A1 = \frac{\epsilon}{a - c - 2\epsilon(2 - \theta)}$$

Assume $A1 > 0 \Leftrightarrow [a - c - 2\epsilon(2 - \theta)] > 0 \Leftrightarrow \theta > \frac{4\epsilon - a + c}{2\epsilon}$.

It can be proved that $E1 > \frac{4\epsilon - a + c}{2\epsilon} \Leftrightarrow \sqrt{(a - c - 4\epsilon)^2 + 4\epsilon^2} > -(a - c - 4\epsilon)$ which is true for all values of a, c and ϵ.

Also, $\theta > E1$ as conditioned in Lemma 2. By transition, $\theta > \frac{4\epsilon - a + c}{2\epsilon}$. Therefore, $A1 > 0$.

Assume that $A1 < 1 \Leftrightarrow \theta > \frac{5\epsilon - (a - c)}{2\epsilon}$.

It can be proved that $E2 > \frac{5\epsilon - (a - c)}{2\epsilon} \Leftrightarrow 2\sqrt{(a - c - 4\epsilon)^2 + 3\epsilon^2} - 2(a - c - 4\epsilon) > 15\epsilon - 3(a - c) \Leftrightarrow (a - c - 3\epsilon)^2 > 0$, which is true for all values of a, c and ϵ.

Also, $\theta \geq E2$ as conditioned in Proposition 2. By transition, $\theta > \frac{5\epsilon - (a - c)}{2\epsilon}$. Therefore, $A1 < 1$.

2. **Proof of** $A1 > A3$

This has not been proved algebraically but has been tested with many examples, and the results show that the statement is true.

3. **Proof of** $E2 > E1$

Assume that

$$\begin{aligned}
E2 > E1 &\Leftrightarrow \frac{\sqrt{(a - c - 4\epsilon)^2 + 3\epsilon^2} - (a - c - 4\epsilon)}{3\epsilon} > \frac{\sqrt{(a - c - 4\epsilon)^2 + 4\epsilon^2} - (a - c - 4\epsilon)}{4\epsilon} \\
&\Leftrightarrow 4\sqrt{(a - c - 4\epsilon)^2 + 3\epsilon^2} > 3\sqrt{(a - c - 4\epsilon)^2 + 4\epsilon^2} + (a - c - 4\epsilon) \\
&\Leftrightarrow (a - c - 4\epsilon)^2 + 2\epsilon^2 > (a - c - 4\epsilon)\sqrt{(a - c - 4\epsilon)^2 + 4\epsilon^2} \\
&\Leftrightarrow (a - c - 4\epsilon)^4 + 4\epsilon^4 + 4\epsilon^2(a - c - \epsilon)^2 > (a - c - 4\epsilon)^2[(a - c - 4\epsilon)^2 + 4\epsilon^2] \\
&\Leftrightarrow 4\epsilon^4 > 0
\end{aligned}$$

which is true for all value of ϵ.

4. **Proof of** $0 < E2 < 1$

$E1 = \frac{\sqrt{(a - c - 4\epsilon)^2 + 4\epsilon^2} - (a - c - 4\epsilon)}{4\epsilon}$. It is easy to see that $E1 > 0$.

As shown in Appendix 3, $E2 > E1$. Thus, $E2 > 0$.

Assume that $E2 < 1 \Leftrightarrow \sqrt{3\epsilon^2 + (a-c-4\epsilon)^2} - (a-c-4\epsilon) < 3\epsilon \Leftrightarrow 3\epsilon^2 + (a-c-4\epsilon)^2 < (a-c-\epsilon)^2 \Leftrightarrow a - c > 3\epsilon$ which is true as conditioned in section 3. Thus, $E2 < 1$.

5. **Proof of $E2$ increasing in ϵ**

$$E2 = \frac{\sqrt{(a-c-4\epsilon)^2 + 3\epsilon^2} - (a-c-4\epsilon)}{3\epsilon}$$
$$\Leftrightarrow \frac{\delta E2}{\delta \epsilon} = \frac{1}{3}\left[\sqrt{(a-c-4\epsilon)^2 + 3\epsilon^2} - (a-c-4\epsilon)\right] + \frac{1}{3}\epsilon\left[\frac{4\sqrt{(a-c-4\epsilon)^2 + 3\epsilon^2} - 4(a-c-4\epsilon) + 3\epsilon}{\sqrt{(a-c-4\epsilon)^2 + 3\epsilon^2}}\right] > 0.$$

Thus, $E2$ increases in ϵ.

6. **Proof of** $0 < E4 < 1$

$$E4 = \frac{\sqrt{(a-c-3\epsilon)^2 + 6\epsilon^2} - (a-c-3\epsilon)}{3\epsilon}$$

It is clear that $E4 > 0$

Assume that $E4 < 1$

$$\Leftrightarrow \sqrt{(a-c-3\epsilon)^2 + 6\epsilon^2} - (a-c-3\epsilon) < 3\epsilon$$
$$\Leftrightarrow \sqrt{(a-c-3\epsilon)^2 + 6\epsilon^2} < (a-c)$$
$$\Leftrightarrow 15\epsilon^2 - 6\epsilon(a-c) < 0$$
$$\Leftrightarrow a - c > 15/6\epsilon.$$

which is true for all $a - c > 3\epsilon$ following the condition in Section 3 (Pre-merger section).

7. Proof of $E4$ increasing in ϵ

$$E4 = \frac{\sqrt{(a-c-3\epsilon)^2 + 6\epsilon^2} - (a-c-3\epsilon)}{3\epsilon}$$
$$\Leftrightarrow \frac{\delta E4}{\delta \epsilon} = \frac{2(a-c)[\sqrt{(a-c-3\epsilon)^2 + 6\epsilon^2} - (a-c-3\epsilon)]}{3\epsilon^2\sqrt{(a-c-3\epsilon)^2 + 6\epsilon^2}} > 0.$$

Thus, $E4$ increases in ϵ.

8. **Proof of** $\hat{W^N} \geq W^N \Leftrightarrow \alpha \geq A4$

Assume that

$$\begin{aligned}
&\hat{W^N} \geq W^N \\
&\Leftrightarrow [\alpha(a-c) - (1+4\alpha)(1-\theta)\epsilon]^2 \geq [(1+2\alpha)(1-\theta)\epsilon]^2 \\
&\Leftrightarrow (a-c) \geq \frac{2(1+3\alpha)}{\alpha}(1-\theta)\epsilon \quad or \quad a-c \leq 2(1-\theta)\epsilon.
\end{aligned}$$

The latter cannot be true following the conditions in Section 4.2. Thus, the only feasible condition is $a-c \geq \frac{2(1+3\alpha)}{\alpha}(1-\theta)\epsilon \Leftrightarrow \alpha \geq \frac{2\epsilon(1-\theta)}{a-c-6\epsilon(1-\theta)} \Leftrightarrow \alpha \geq A4$.

9. **Proof of** $0 < A4 < 1$

$$A4 = \frac{2\epsilon(1-\theta)}{a-c-6\epsilon(1-\theta)}$$

Assume that $A4 > 0 \Leftrightarrow a-c-6\epsilon(1-\theta) > O \Leftrightarrow \epsilon < \frac{a-c}{6(1-\theta)}$

Given that $\theta \geq E1 \Leftrightarrow \epsilon \leq \frac{2\theta(a-c)}{5-4(1-\theta)^2}$

Simple calculations show that $\frac{2\theta(a-c)}{5-4(1-\theta)^2} < \frac{a-c}{6(1-\theta)}$. Thus, $\epsilon < \frac{a-c}{6(1-\theta)}$. Therefore, $A4 > 0$.

Assume that $A4 <\Leftrightarrow 2\epsilon(1-\theta) < a-c-6\epsilon(1-\theta) \Leftrightarrow \epsilon < \frac{a-c}{8(1-\theta)}$

Given that $\theta \geq E1 \Leftrightarrow \epsilon \leq \frac{2\theta(a-c)}{5-4(1-\theta)^2}$

Simple calculations show that $\frac{2\theta(a-c)}{5-4(1-\theta)^2} < \frac{a-c}{8(1-\theta)}$. Thus, $\epsilon < \frac{a-c}{8(1-\theta)}$. Therefore, $A4 < 1$.

References

[1] Artz, B., Heywood, J.S. and McGinty, M. (2009), The merger paradox in a mixed oligopoly, *Research in Economics*, 63(1), pp.1-10.

[2] Barcena-Ruiz, J.C. and Garzon, M.G. (2003), Mixed duopoly, merger and multiproduct firms, *Journal of Economics*, 80(1), pp. 27-42.

[3] Coloma, G. (2006), Mergers and Acquisitions in Mixed-Oligopoly Markets, *International Journal of Business and Economics*, 5(2), pp. 147-59.

[4] DeFraja, G. and Delbono, F. (1990), Game theoretic models of mixed oligopoly, *Journal of Economic Surveys*, 4(1), pp. 1-17.

[5] Deneckere, R. and C. Davidson (1985), Incentives to Form Coalitions with Bertrand Competition *Rand Journal of Economics*, 16(4), pp. 473-86.

[6] Fjell, K. and Pal, D. (1996), A Mixed Oligopoly in the Presence of Foreign Private Firms, *Canadian Journal of Economics*, 29(3), pp. 737-43.

[7] Fauli-Oller, R. (2000), Takeover Waves, *Journal of Economics & Management Strategy*, 9(2), pp. 189-210.

[8] Granier, L. (2008), Endogenous mergers and cost heterogeneity, *Applied Economics*, 40(14), pp. 1865-71.

[9] Gelves, J.A. and Heywood, J.S. (2013), Privatizing by merger: The case of an inefficient public leader, *International Review of Economics and Finance*, 27, pp. 69-79.

[10] Kamaga, K. and Nakamura, Y. (2007), The Core and Productivity-Improving Mergers in Mixed Oligopoly, *International Journal of Business and Economics*, 6(3), pp. 181-98.

[11] Kamien, M. and Zang, I. (1990), The limits of monopolization through acquisition, *Quarterly Journal of Economics*, 105(2), pp. 465-99.

[12] Kamien, M. and Zang, I. (1993), Monopolization by sequential acquisition, *Journal of Law, Economics and Organization*, 9(3), 205-29.

[13] Kamijo, Y. and Nakamura, Y. (2009), Stable Market Structures from Merger Activities in Mixed Oligopoly with Asymmetric Costs, *Journal of Economics*, 98(1), pp. 1-24.

[14] Matsumura, T., (1998), Partial Privatization in Mixed Duopoly, *Journal of Public Economics*, 70, 473-483.

[15] Mendez-Naya, J. (2008), Merger Profitability in Mixed Oligopoly, *Journal of Economics*, 94(2), pp. 167-76.

[16] Merrill, W. and Schneider, N. (1966), Government firms in oligopoly industries: a shortrun analysis, *Quarterly Journal of Economics*, 80, pp. 400-412.

[17] Perry, M. K. and R. H. Porter (1985), Oligopoly and the Incentive for Horizontal Merger *American Economic Review*, 75(1), pp.219-27.

[18] Qiu, L.D. and Zhou, W. (2007), Merger waves: a model of endogenous mergers, *The Rand Journal of Economics*, 38(1), pp.214-26.

[19] Rodrigues, V. (2001), Endogenous mergers and market structure, *International Journal of Industrial Organisation*, 19(8), pp. 1245-61.

[20] Salant, S., Switzer, S. and Reynolds, R. (1983), Losses from Horizontal Merger: The Effects of an Exogenous Change in Industry Structure on Cournot Nash Equilibrium *Quarterly Journal of Economics*, 98(2), pp.185-99.

[21] Toxvaerd, F. (2008), Strategic merger waves: A theory of musical chairs, *Journal of Economic Theory* 140(1), 126.

[22] Zhou, W. (2008), Endogenous horizontal mergers under cost uncertainty, *International Journal of Industrial Organization*, 26(4), pp. 903-912.