Majority Voting in a Model of Means Testing*

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Abstract

We study a model of means testing where the subsidy received by each household is a linearly decreasing function of income. Public policy, which is determined by majority voting, consists of two dimensions: the overall funding level (or the tax rate) and the slope of the means testing function. We solve the model, establishing the existence of a majority voting equilibrium, when the political decisions are sequential – households vote first on the tax rate and then on the extent of means testing. In characterizing the equilibrium relationship between the means testing and tax rates, it is found that the means testing rate is a linear function of the tax rate. The fraction of the population receiving subsidies is independent of the funding level for the subsidy.

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1 Introduction

Many public subsidies around the world are targeted or means–tested (see van de Walle and Nead, 1995). Means-tested welfare payments in several rich economies are large. According to Moffitt (2002), expenditures on means tested transfer programs in the US climbed from \$400 per capita (in 1998 dollars) in 1968 to \$1400 in 1998. These expenditures represent 2.3% of per capita GDP in 1968 and 4.4% in 1998. In 2005, means tested government programs in Australia accounted for 6.3 percent of GDP; see Immervoll (2009). This growth can be attributed to a preference among voters and governments to means test or target transfers rather than provide unrestricted cash transfers; see Moffitt (2002).

Examples of means tested programs in the US include Medicaid, food stamps, school lunch programs, housing vouchers, the Low-Income Home Energy Assistance Program (LIHEAP), Pell grants, a wide range of Child Care Subsidy programs including the Child Care Development Fund (CCDF) and many others. Food stamps paid for by the United States Supplemental Nutrition Assistance Program (SNAP) decline by 30 cents for each additional dollar of net monthly income. The CCDF varies between states but an important feature is that the program provides a sliding fee scale dependent on income, essentially providing a means tested subsidy.

In this paper, we develop a model of means testing where the size of the means testing program and the degree of means testing are endogenously determined through majority voting. In our model, households prefer a consumption good and an in-kind subsidy good (e.g., health care, food, education, etc.). The only heterogeneity is in household income. A government finances the provision of the subsidy good through income taxation. Households can supplement the publicly provided subsidy good with their own expenditures on the good. The subsidy is means tested. We assume a linear means testing specification: the amount of subsidy good declines linearly with income.

The public policy in our model is two-dimensional. One dimension is the funding level for the program, i.e. the tax rate. The other dimension is the slope of the means testing function, i.e. how quickly the subsidy is phased out with increasing income. We determine both policy dimensions through majority voting. We determine political outcomes sequentially in order to avoid the well known existence problems associated with multidimensional voting.[1] We assume households first vote on the tax rate, followed by a vote on the slope of the means testing function. When households vote on the tax rate, they anticipate the majority *decision rule* for means testing that follows in the sequence.[2] (Note that the uniform subsidy is a special case of our model where one of the policy dimensions, the slope of the means testing function, is zero.)

Our results are as follows. First, we prove the existence and uniqueness of a majority

[1] It is well known that majority voting equilibria may not exist under multidimensional voting; see Plott (1967) and Ordeshook (1986, Chapter 4.7). One way to avoid the non-existence problem is to consider voting by *not* taking into consideration how voting in an early stage influences voting outcomes in later stages. This approach has been utilized by Alesina, Baqir and Easterly (1999), Alesina, Baqir and Hoxby (2004), Cremer, De Donder and Gahvari (2004), Cremer et al. (2007), De Donder, Le Breton and Peluso (2009), Etro (2006), Gregorini (2009), and Haimanko, Le Breton and Weber (2005).

[2] When voters are strategic and anticipate how early votes influence voting outcomes in later stages, a voting equilibrium may not exist for some preferences. This is illustrated in Ordeshook (1986, chapter 6.1 – 6.3).

voting equilibrium. We show that at each stage households' preferences over the policy are single-peaked, allowing us to invoke the Black (1958) median voter theorem. Second, at the stage of voting on the slope of the means testing function, the decisive voter is the household with the median income. Third, the majority preferred slope of the means testing function is proportional to the tax rate. Fourth, the cut-off income level above which households receive a zero voucher is independent of the tax rate. That is, the fraction of households that receive the voucher is independent of the size of the welfare program. None of these results rely on any specific properties of the utility function or the income distribution, except for skewness. Fifth, when the utility function is of the CES variety, at the stage of voting on tax rate, the decisive voter is the household with median income if the elasticity of substitution between the consumption good and the subsidy good is high; when the elasticity is low, the majority voting equilibrium is of the ends-against-the-middle variety as in Epple and Romano (1996).

A literature on the political economy of means-tested or targeted transfer payments has developed since the late 1990. Currie and Gahvari (2008) contains a brief survey of this literature. DeDonder and Hindriks (1998) were among the first, if not the first, to contribute to this literature by introducing a model of voting on targeted welfare (in cash) payments with endogenous labor supply. They find that at high targeting rates, political support for such transfer payments may collapse. Moreover, in their simultaneous voting there are serious non-existence issues so they employ other weaker solution concepts. Gelbach and Pritchett (2002) show that given a choice of uniform and targeted transfers, the choice of the targeted regime may actually decrease welfare of the poor. In the context of education Chen and West (2002) study uniform and targeted education vouchers. However, in their model of targeting, the vouchers are in a fixed and constant amount below an exogenous cut-off income level and zero above that level, rendering their political economy problem one dimensional. Piolatto (2010) extends Chen and West to allow for congestion of public schools. Moene and Wallerstein (2001) study how changes in income inequality determine redistribution when both the level of funding and targeting are determined either simultaneously or through two-party competition. They find that the effect of inequality on redistribution depends on which group is targeted. Casamatta, Cremer and Pestieau (2000) in the context of pension funding use a two stage procedure to determine the nature of redistribution in the first stage and the size of the coverage in the second stage.

The structure of our paper is the following. In Section 2, we develop our model of means testing. Uniform subsidies are a special case of this model. In Section 3 we prove that a sequential majority voting equilibrium exists for our model. We also prove the existence of and characterize the majority voting equilibrium for the case of uniform subsidies. Concluding remarks are contained in Section 5. Proofs are relegated to Appendix A.

2 Model

In this section, we describe a model of means testing. The economy is populated by a large number of households. We normalize the size of the population to 1. Households differ only by income, y, which is exogenously endowed across households according to the c.d.f. F (p.d.f. f); the p.d.f. is assumed to be continuously differentiable. We label households by their income and refer to a household with income y as "household y". The support of F is $\mathbb{R}_+$ and mean income, Y, exceeds median income, y_m.

Households derive utility from a numeraire consumption good c and another good d. We will refer to good d as the subsidy good. The common utility function is $u(c, d)$ which is strictly increasing in both arguments, strictly quasiconcave, and twice continuously differentiable. We also impose the following boundary condition:

Assumption 1. *For $c_1 > 0$, $d_1 > 0$, $c_2 \geq 0$, and $d_2 \geq 0$,*

$$u(c_1, d_1) > \max\{u(c_2, 0), u(0, d_2)\}.$$

The markets for c and d are assumed to be perfectly competitive with a large number of producers facing identical technologies exhibiting constant marginal and zero fixed costs. We measure units of d so as to normalize its price to one unit of consumption. Both consumption and the subsidy good are assumed to be normal goods.

The government collects a tax on income at the rate $\tau \in [0, 1]$. Total tax revenue is given by τY. All tax revenue is used to finance d. The distribution of the tax revenues to the households is means-tested in the sense that the amount received by the household depends inversely on income and there is an income threshold above which a household receives no subsidy. Formally, the amount for household y is given by

$$s(y; \alpha, \beta) = \max\{\alpha - \beta y, 0\}, \quad \alpha \geq 0, \ \ \beta \geq 0. \tag{1}$$

Under this specification, the severity of means-testing is determined by β. A higher β implies more severe means testing, while $\beta = 0$ implies a uniform subsidy. We assume that the government runs a balanced budget; i.e.,

$$\int_0^\infty s(y; \alpha, \beta) f(y)\, dy = \tau Y.$$

Since the subsidy is 0 for a household with income larger than $\frac{\alpha}{\beta}$, we can write the balanced budget restriction as

$$\alpha F\left(\frac{\alpha}{\beta}\right) - \beta \int_0^{\frac{\alpha}{\beta}} y f(y)\, dy = \tau Y. \tag{2}$$

We refer to (2) as the Government Budget Constraint (GBC) and let $\widetilde{\alpha}(\tau, \beta)$ be the value of α satisfying (2) given (τ, β).

2.1 Household Optimization

Each household treats α, β, and τ as given and chooses the pair (c, d) so as to maximize utility $u(c, d)$ subject to the budget constraint

$$c + d \leq (1 - \tau) y + s(y; \alpha, \beta), \ \ c \leq (1 - \tau) y. \tag{3}$$

Denote the optimal choices of household y by $\widehat{c}(y; \alpha, \beta, \tau)$ and $\widehat{d}(y; \alpha, \beta, \tau)$ and the indirect utility of household y by $V(y; \alpha, \beta, \tau) \equiv u\left(\widehat{c}(y; \alpha, \beta, \tau), \widehat{d}(y; \alpha, \beta, \tau)\right)$.

Remark 1. *When $\alpha = 0$ or $\beta = \infty$, no household obtains a subsidy and all expenditures are privately financed. When $\tau = 0$, the GBC requires $\alpha = 0$; no household receives a subsidy. When $\tau = 1$, equations (2) and (3) imply that all households get zero consumption. Assumption 1 will rule this out as a potential equilibrium.*

Household y supplements its subsidy if and only if

$$\left.\frac{\partial u\left(\left(1-\tau\right)y+s\left(y;\alpha,\beta\right)-d,d\right)}{\partial d}\right|_{d=s(y;\alpha,\beta)}>0$$

or, equivalently,

$$R\left(y\right)\equiv\frac{u_1\left(\left(1-\tau\right)y,s\left(y;\alpha,\beta\right)\right)}{u_2\left(\left(1-\tau\right)y,s\left(y;\alpha,\beta\right)\right)}<1$$

where the subscripts here denote partial derivatives of u. Put differently, there exists a threshold income such that household y supplements its subsidy if and only if y exceeds the threshold. We further restrict preferences to ensure that, for income distributions with support on the real line, there exist low income households who do not supplement their subsidy as well as rich households who do supplement their subsidy.

Assumption 2. *For all* $\alpha>0,\ \beta\in(0,\infty)$, *and* $\tau\in(0,1)$,

(i) $\lim_{y\searrow 0}R\left(y\right)>1,$ *and,*
(ii) $\lim_{y\nearrow\infty}R\left(y\right)<1.$

The voting problem in this means-tested regime involves two variables, τ and β. Once these are determined, the value of α is pinned down by GBC (2). We determine the pair (τ,β) through majority voting in two stages. In the first stage, individuals vote on the tax rate τ anticipating how the means testing parameter β will be chosen in the second stage and how β might depend on τ. In the second stage, β is voted on taking τ from the first stage as given. We define a politico-economic equilibrium for this voting sequence as follows.

Definition 1. *A* ***politico-economic equilibrium*** *for the means-tested economy is an allocation* (c,d) *across households and a public policy* (α,β,τ) *satisfying* ***(i)*** *Each household's choice of* (c,d) *is individually rational given public policy* (α,β,τ)*;* ***(ii)*** *Given* τ, β *is a majority winner in the second stage;* ***(iii)*** *Anticipating how* τ *affects voting over* β, τ *is a majority winner in the first stage; and,* ***(iv)*** *The government runs a balanced budget; i.e.,* $\alpha=\widetilde{\alpha}\left(\tau,\beta\right)$.

We define majority voting in the usual sense of binary comparisons between all policies. We treat voters as sincere in that they will vote for the policy that yields a higher utility in any binary comparison between policies. We say that a policy is a *majority winner* at each stage if and only if no other policy satisfying the *GBC* at that stage is strictly preferred by a strict majority of the population.

3 Majority Voting Equilibrium

As noted earlier, the households in our model have to collectively choose two policy variables – τ and β – sequentially. In this section, we establish the existence of a majority voting equilibrium and characterize the equilibrium.[3]

[3] We establish the existence of and characterize the majority voting equilibrium for a uniform subsidies model in the context of education vouchers in Bearse et al. (2013).

3.1 Second Stage: Voting Over Means Testing

Here we solve the problem of voting over β. At this stage, the households have already voted on a tax rate τ and the problem is to choose (collectively) a means testing rate. To this end, we have to determine the majority preferred β for each τ. Household y chooses β to maximize the indirect utility subject to (2). Formally, given $\tau \in (0,1)$, her optimal β is $\arg\max\ V\left(y; \widetilde{\alpha}\left(\tau, \beta\right), \beta, \tau\right)$.

Remark 2. *When $\tau = 0$, there are no subsidies and the choice of β is irrelevant. The case $\tau = 1$ can never occur in equilibrium since every household would get zero consumption and, as noted before, this is ruled out by Assumption 1.*

The following lemma establishes properties of the GBC, $\widetilde{\alpha}\left(\tau, \beta\right)$, which are helpful in solving the decisive voter's optimization problem.

Lemma 1. *(Concavity of GBC) At any given tax rate, τ, the GBC, $\widetilde{\alpha}\left(\tau, \beta\right)$, is monotonically increasing and strictly concave in β, with a maximum gradient of $\frac{\partial \widetilde{\alpha}(\tau,\beta)}{\partial \beta} = Y$ at $\beta = 0$.*

As τ is given, the size of the pie to be redistributed is fixed. Since our means testing formula bestows larger amounts to poorer households, no household above mean income Y will benefit from this formula and they will all prefer $\beta = 0$. Political support for a positive means testing rate must then come from households whose incomes are below the mean. The following proposition characterizes the extent of means testing preferred by the majority.

Proposition 1. *(Majority preferred β) The household with the median income (y_m) is the decisive voter on the means testing rate, β. Given $\tau \in (0,1)$, $\widehat{\beta}\left(\tau\right)$ is the majority winner where $\widehat{\beta}$ solves*

$$y_m = \frac{\partial \widetilde{\alpha}\left(\tau, \beta\right)}{\partial \beta}.$$

In deciding the means testing rate, the decisive voter has to choose a pair (α, β) among the pairs that satisfy the GBC. The concave GBC is illustrated in the (α, β) space in Figure 1. To understand the trade-offs faced by each voter receiving a subsidy, consider an arbitrary household y whose income is less than Y (recall that households with $y \geq Y$ prefer $\beta = 0$). Suppose that household y is constrained by the subsidy amount in the sense that its consumption is $(1-\tau)y$ and its expenditure on the subsidy good is $\alpha - \beta y$. Then, on the margin, any increase in β has to be offset by an increase in α to keep this household indifferent. More precisely, every unit increase in β would require an increase in α by y units. So, in the (α, β) space, the slope of the indifference curve for household y is y. Now, suppose that household y is not constrained by the subsidy amount in which case the household will optimally allocate its resources $(1-\tau)y + \alpha - \beta y$ between consumption and the subsidy good. (Recall that the optimal choices are denoted by $\widehat{c}$ and $\widehat{d}$.) On the margin, a unit increase in β decreases this household's utility by $y \times \left\{u_1\left(\widehat{c}, \widehat{d}\right) + u_2\left(\widehat{c}, \widehat{d}\right)\right\}$ whereas a unit increase in α increases this household's utility by $\left\{u_1\left(\widehat{c}, \widehat{d}\right) + u_2\left(\widehat{c}, \widehat{d}\right)\right\}$. To keep the household indifferent, α has to increase by y units for every unit increase in β. Again, in the (α, β)

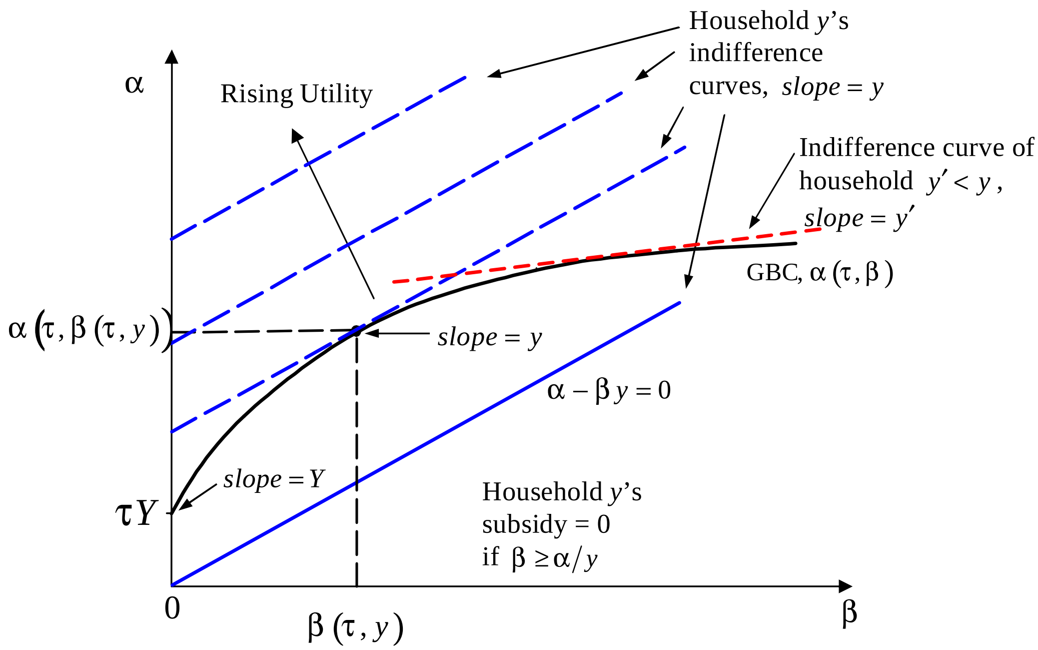

Figure 1: GBC and Indifference Curves of households.

space, the slope of the indifference curve for household y is y, as illustrated in Figure 1. The preferred (α, β) pair for household y is clearly where its indifference curve is tangent to the GBC. (Note from Figure 1 that the tangency point implies $\alpha - \beta y > 0$.) Proceeding along the same lines, a poorer household $y' < y$ would have a flatter indifference curve in the (α, β) space and would, hence, prefer a higher (α, β) pair. Thus, the preferred means testing rate is a decreasing function of income – richer households prefer a lower β, with households above the mean income preferring $\beta = 0$. This monotonicity allows us to invoke the median voter theorem and establish Proposition 1.

To determine the majority preferred tax rate, we have to characterize the function $\widehat{\beta}(\tau)$ in Proposition 1 i.e., we have to understand how the majority preferred β changes as the tax rate changes. To this end, the following properties of the GBC are helpful.

Lemma 2. *(Properties of GBC) Fix $\tau \in (0,1)$ and let (α, β) be a point on the GBC. Denote this GBC as GBC_1. Consider another GBC_j with tax rate $\tau_j = j\tau \in (0,1)$. Then, (i) the pair $(j\alpha, j\beta)$ is on GBC_j and (ii) the slope of GBC_j at $(j\alpha, j\beta)$ is the same as the slope of GBC_1 at (α, β).*

Remark 3. *This result does not depend on the linearity in y of the subsidy function, $s(y;\alpha,\beta) = \alpha - \beta y$ for $s > 0$, but rather on $s(y;\alpha,\beta)$ being homogeneous of degree 1 in (α, β). Consider the case $s_1(y, \theta; \alpha, \beta) = \alpha - \beta y^{\theta}$ for some exogenous θ. Inspection of the proof of Lemma 2 shows that both parts (i) and (ii) of the Lemma continue to hold for this alternative* *subsidy function which is nonlinear in y but is homogeneous of degree 1 in (α, β). However, Lemma 2 fails to hold for alternative subsidy functions such as $s_2(y; \alpha, \beta) = \alpha - y^{\beta}$. In this case, the GBC in equation (2) is given by $\alpha F\left(\alpha^{\frac{1}{\beta}}\right) - \int_0^{\alpha^{\frac{1}{\beta}}} y^{\beta} f(y)\, dy = \tau Y$. If we scale up τ, $\tau_j = j\tau \in (0,1)$ and consider the pair $(j\alpha, j\beta)$, we find $j\alpha F\left(j\alpha^{\frac{1}{j\beta}}\right) - \int_0^{j\alpha^{\frac{1}{j\beta}}} y^{j\beta} f(y)\, dy \neq j\tau Y$, implying part (i) of the Lemma fails to hold. The asymmetric manner in which α and β enter $s_2(y; \alpha, \beta) = \alpha - y^{\beta}$ implies that part (ii) of the Lemma also fails to hold, emphasizing the homogeneity of s in (α, β) is critical to the result.*

Lemma 2 characterizes the set of feasible (α, β) pairs that constrains household y_m for each τ. For every change in τ, proportionate changes in α and β are in the feasible set, according to part (i). Part (ii) implies that household y_m would indeed *choose* the proportionate change. That is, if (α, β) was the most preferred pair for household y_m on GBC_1, then $(j\alpha, j\beta)$ is its most preferred pair on GBC_j. Thus, the functional relation between the majority preferred β and the tax rate can be characterized as follows.

Proposition 2. *(Decision rule for the majority preferred β and α)*

$$\widehat{\beta}(\tau) = k_{\beta} \cdot \tau$$

for some constant $k_{\beta} > 0$ for all $\tau \in (0,1)$. Furthermore, associated with $\widehat{\beta}(\tau)$, the unique $\widehat{\alpha}(\tau) \equiv \widetilde{\alpha}(\tau, k_{\beta}\tau)$ that satisfies the GBC can be written as

$$\widehat{\alpha}(\tau) = k_{\alpha} \cdot \tau \text{ for some constant } k_{\alpha} > 0.$$

Among the pairs (α, β) that satisfy the GBC, the majority preferred pair establishes an extensive margin that excludes some households from receiving any portion of the pie (households with $y \geq \frac{\widehat{\alpha}}{\widehat{\beta}} = \frac{k_{\alpha}}{k_{\beta}}$). A natural question then is, does the extensive margin change as the size of the pie, i.e. the tax rate, and hence the GBC changes?

Corollary 1. *(Extensive margin invariant to changes in τ) Changes in the tax rate τ do not change the extensive margin for the means tested subsidy, leading the set of subsidy recipient households to remain unchanged.*

Intuition can be gained from Figure 2 where the GBC's associated with two different tax rates are illustrated together with the median income household's optimal choices. Recall from Proposition 1, the median income household (y_m) is decisive on β at all tax rates. Corollary 1 implies at points E_1 and E_2, the same households receive a positive subsidy. This is confirmed by the fact that both E_1 and E_2 are on the line $\frac{\alpha}{\beta} y = 0$, implying the same income threshold for positive subsidies at both points. If the median income household were to choose a point to the left of E_2 on GBC_2, it would put them on a lower indifference curve than at E_2 and the income threshold would be higher than at E_1 and E_2. They are worse off because the new fixed budget is being allocated over more subsidy recipients providing household y_m with the same after tax income and less subsidy. This rules out increasing the income threshold. Choosing a point to the right of E_2 on GBC_2 would also put them on a lower indifference curve than at E_2 and the income threshold would be lower than at E_1 and E_2. The base subsidy (α) would be higher but the means testing rate (β) will increase by

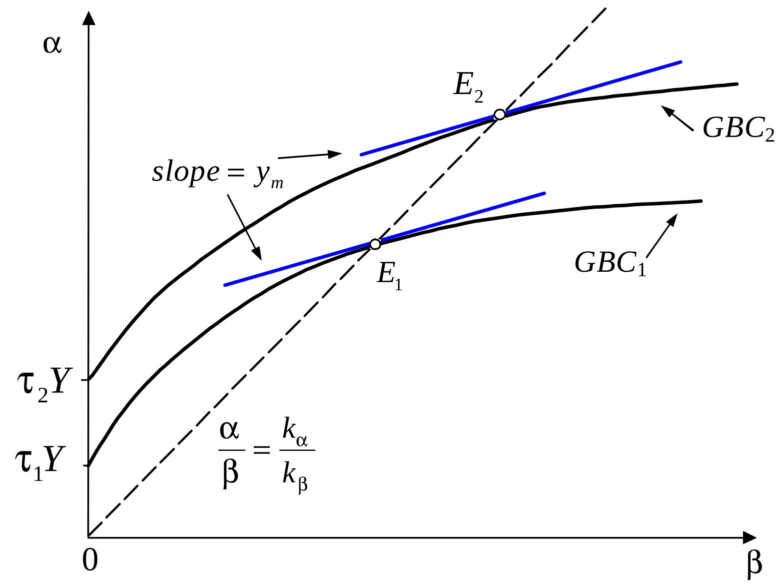

Figure 2: Median income household (y_m) preferred β at two different tax rates showing that extensive margin does not change as tax rate changes.

proportionally more, reducing the number of recipients. Household y_m, along with higher income positive subsidy recipients at E_2, are worse off because the net position is a lower overall subsidy, s. This rules out a lower income threshold and fewer subsidy recipients with a change in the tax rate.

This corollary implies the threshold income where households receive no subsidy, $y = \frac{\widehat{\alpha}(\tau)}{\widehat{\beta}(\tau)}$, does not change with the equilibrium tax rate. It also implies the extensive margin does not vary with the size of the pie and the identities of the households that are excluded from receiving subsidies are invariant to the tax rate. Put differently, no matter what the size of the pie, it is always distributed between the *same* households, those below the income level $\frac{k_\alpha}{k_\beta}$.

The corollary draws on results from Proposition 1 and Lemma 2. First, from Proposition 1, the median income household (y_m) is decisive on β at all tax rates. At any given tax rate, the median income household will choose a pair (α, β) where the slope of the GBC, $\frac{\partial \widetilde{\alpha}(\tau,\beta)}{\partial \beta} = y_m$. Second, based on Lemma 2, a change in the tax rate will lead to proportionate changes in α and β. Thus changing τ will not change the equilibrium value of $\frac{\alpha}{\beta}$.

Chen and West (2000) study an alternative form of targeting where they assume an exogenous income threshold above which no subsidy is received. We endogenize this threshold below. Surprisingly, we find this threshold is fixed independently of the tax rate.

3.2 Voting over the Tax Rate

To determine the majority preferred τ, each household takes as given the majority preferred *functions* $\widehat{\beta}(\tau)$ and $\widehat{\alpha}(\tau)$ given by Proposition 2 and chooses its most preferred τ. With a slight abuse of notation, household y chooses τ to

$$\max V(y;\tau) \equiv V(y; k_\alpha\tau, k_\beta\tau, \tau).$$

We will first establish that the households' preferences over τ are single-peaked. It is easy to see that households above the income level $\frac{k_\alpha}{k_\beta}$ would prefer a tax rate of zero since they do not receive any subsidy at all. Furthermore, their utility is monotonically declining in the tax rate, so $V(y;\tau)$ peaks at $\tau = 0$ for $y > \frac{k_\alpha}{k_\beta}$. The set of households whose utilities are declining in the tax rate is, in fact, larger. Consider all households who receive non-negative subsidies. The critical household for which the subsidy exactly offsets taxes satisfies the condition: $\alpha - \beta y < \tau y$ or $k_\alpha\tau - k_\beta\tau y = \tau y$, so the critical household is described $y = \frac{k_\alpha}{1+k_\beta}$. For $y > \frac{k_\alpha}{1+k_\beta}$, the household receives less in subsidy than what it pays in taxes, so (i) the most preferred tax rate of such households would be zero and (ii) the household's utility is monotonically declining in τ since the gap between taxes and benefits $(\tau y - (k_\alpha\tau - k_\beta\tau y))$ is increasing in τ (or, the total resources available to household y are decreasing in τ). In Appendix A, we prove that households with $y < \frac{k_\alpha}{1+k_\beta}$ also have single-peaked preferences, so we have the following proposition.

Proposition 3. *(Majority preferred τ existence) Given $\widehat{\beta}(\tau)$ and $\widehat{\alpha}(\tau)$ from Proposition 2, households' preferences over τ are single-peaked and, hence, there exists a majority voting equilibrium tax rate.*

Since the households with incomes above $\frac{k_\alpha}{1+k_\beta}$ prefer a zero tax rate, for the equilibrium tax rate to be positive, the decisive voter must come from the group $y < \frac{k_\alpha}{1+k_\beta}$. Denote the decisive voter's income by y_d. The lemma below states necessary conditions for the majority voting equilibrium tax rate to be positive.

Lemma 3. *(Properties of τ) Suppose that the majority preferred tax rate is positive. Then, (i) the decisive voter's income $y_d \in \left[0, \frac{k_\alpha}{1+k_\beta}\right]$, (ii) The most preferred tax rate $\widehat{\tau}(y_d)$ of the decisive voter is such that the decisive voter is constrained i.e., his consumption and subsidy good expenditure are given by*

$$\widehat{c} = (1 - \widehat{\tau}(y_d))\, y_d, \quad \widehat{d} = k_\alpha\widehat{\tau}(y_d) - k_\beta\widehat{\tau}(y_d)y_d,$$

(iii) The most preferred tax rate $\widehat{\tau}(y_d)$ is the unique solution to

$$\frac{u_2\left((1-\tau)\, y_d, k_\alpha\tau - k_\beta\tau y_d\right)}{u_1\left((1-\tau)\, y_d, k_\alpha\tau - k_\beta\tau y_d\right)} = \frac{y_d}{k_\alpha - k_\beta y_d}.$$

The decisive voter, if unconstrained by the subsidy amount, can make himself better off with a higher tax rate. Higher τ implies more resources, but a tighter constraint on d (or c). For an increase of $\Delta\tau$, he gains $k_\alpha\Delta\tau - k_\beta y\Delta\tau - y\Delta\tau > 0$ units of resources, which translates into higher c and d since he is not constrained, as illustrated in Figure 3. He can increase the tax rate until he is constrained, at which point the increase in τ would imply less consumption and his marginal rate of substitution of consumption for the subsidy good is no longer equal to 1. However, he can continue to increase the tax rate and make himself better off until he reaches the equality in part (iii) of Lemma 3.

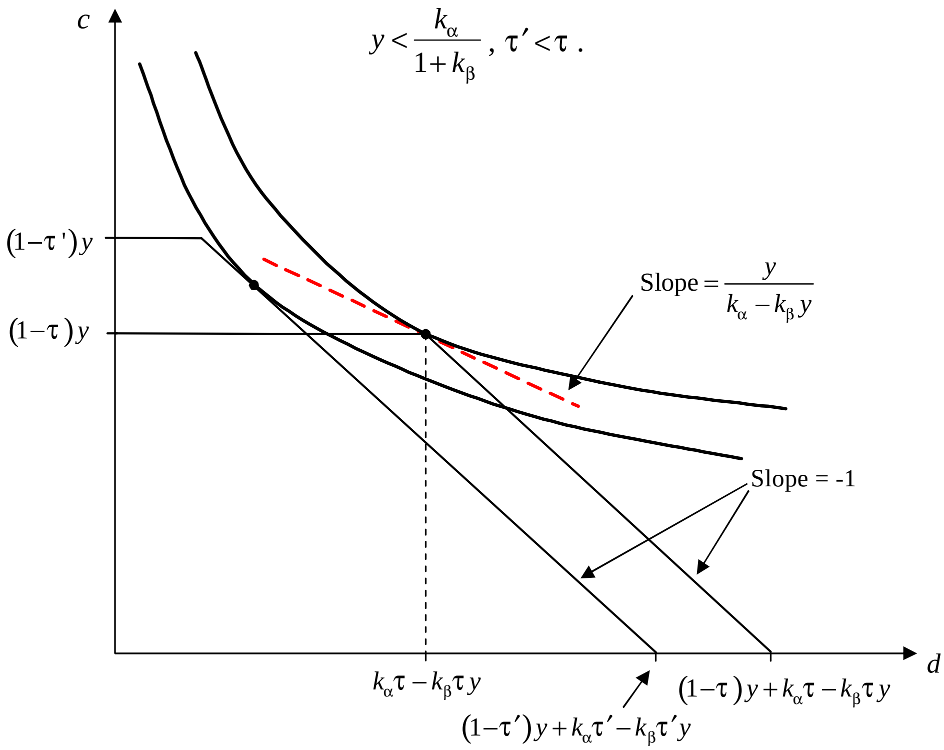

Figure 3: Tradeoffs for the decisive voter

What remains to be determined is who is the decisive voter. To this end, we restrict the preferences further to the constant relative risk aversion (CRRA) class. Let

$$u(c,d) = \begin{cases} \frac{1}{1-\sigma}\left(c^{1-\sigma} + \delta d^{1-\sigma}\right), & \sigma > 0, \sigma \neq 1, \quad \delta > 0, \\ \ln c + \delta \ln d, & \sigma = 1. \end{cases} \tag{4}$$

For this class of preferences, the proposition below pins down the decisive voter and the majority voting equilibrium.

Proposition 4. *(Majority preferred τ for CRRA utility) Let $u(c,d)$ be specified according to (4). If $\sigma \leq 1$, then the decisive voter is household y_m and the majority preferred tax rate is given by*

$$\frac{\delta\left(k_\alpha\tau - k_\beta\tau y_m\right)^{-\sigma}}{\left((1-\tau)\, y_m\right)^{-\sigma}} = \frac{y_m}{k_\alpha - k_\beta y_m}. \tag{5}$$

If $\sigma > 1$, then the decisive voter is implicitly determined by

$$1 - F\left(\frac{k_\alpha}{1+k_\beta}\right) + F\left(y_d\right) = 0.5 \tag{6}$$

and the majority preferred tax rate is given by

$$\frac{\delta\left(k_\alpha\tau - k_\beta\tau y_d\right)^{-\sigma}}{\left((1-\tau)\, y_d\right)^{-\sigma}} = \frac{y_d}{k_\alpha - k_\beta y_d}.$$

4 Uniform Subsidy with Endogenous Entitlement Threshold

An alternative means testing mechanism was proposed by Chen and West (2000) where households receive a uniform subsidy if income is below a predetermined entitlement threshold. Here, consistent with our analysis above, we endogenize the determination of the entitlement threshold with the majority voting mechanism. The subsidy for household y is given by:

$$s = \begin{cases} \frac{\tau Y}{F(\widehat{y})} & \text{if } 0 \leq y \leq \widehat{y}, \\ 0 & \text{if } \widehat{y} < y. \end{cases}$$

The means testing here follows a step function. All households $y_i \leq \widehat{y}$ receive the full uniform subsidy while households $y_i > \widehat{y}$ receive no transfers. We maintain the balanced budget assumption:

$$\int_0^\infty s(y;\tau) f(y)\, dy = \int_0^\infty \frac{\tau Y}{F(\widehat{y})} f(y)\, dy = \tau Y.$$

With the exception of the subsidy structure and political choices, the household optimization problem is almost identical to that outlined in Section 2.1. The controls in this social choice problem are the tax rate, τ, and the entitlement threshold, $\widehat{y}$. This multidimensional choice problem is solved through majority voting in two sequential stages. Similar to the means tested model analyzed above, households vote on the tax rate τ in the first stage anticipating the choice of $\widehat{y}$ in the second stage. In the second stage, the entitlement threshold, $\widehat{y}$ is determined with all households treating the choice of τ from the first stage as given. A politico-economic equilibrium for this voting sequence is as follows.

Definition 2. *A **politico-economic equilibrium** for the endogenous entitlement uniform subsidy economy is an allocation (c,d) across households and a public policy $(\tau,\widehat{y})$ satisfying (**i**) Each household's choice of (c,d) is individually rational given public policy $(\tau,\widehat{y})$; (**ii**) Given τ, $\widehat{y}$ is a majority winner in the second stage; (**iii**) Anticipating how τ affects voting over $\widehat{y}$, τ is a majority winner in the first stage; and, (**iv**) The government runs a balanced budget.*

We first establish a voting equilibrium in the second stage, given the tax rate chosen in the first stage.

Proposition 5. *(Majority preferred entitlement threshold) For any given $\tau > 0$, every household y prefers the entitlement threshold to be $\widehat{y} = y$. The median income household is decisive in the choice of the entitlement threshold which is given by $\widehat{y} = y_m$.*

The equilibrium entitlement threshold is independent of the tax rate chosen in the first stage vote. The result means that whatever budget is chosen for this subsidy scheme, it is always shared solely among the poorest half of the population. These households receive a subsidy given by $s = 2\tau Y$ while all other households receive $s = 0$. As in the linear means testing case, the fraction of the population receiving a subsidy here is fixed and independent of the tax rate as well.

We now turn to the determination of the equilibrium tax rate or first stage voting. We define the lowest income household to be y_l and follow a similar approach to that in our means testing model above, where households maximize the indirect utility $V(y;\tau)$.

Proposition 6. *(Endogenous threshold, uniform subsidies majority preferred τ) (i) Households' preferences over τ are single-peaked and there exists a majority voting equilibrium tax rate. (ii) For the equilibrium tax rate to be positive, the decisive voter's income $y_d^E \in [y_l, y_m]$ and the most preferred tax rate $\widehat{\tau}(y_d^E)$ of the decisive voter is such that*

$$\widehat{c} = \left(1 - \widehat{\tau}(y_d^E)\right) y_d^E, \quad \widehat{d} = 2\widehat{\tau}(y_d^E)Y.$$

(iii) If $\frac{\partial^2 V(y;\tau)}{\partial y \partial \tau} < 0\,(>0)$, the decisive voter is the median (lowest) income household $y_d^E = y_m\,(y_l)$ and the majority preferred tax rate is the unique solution to

$$\frac{u_2\left(\left(1-\tau\right) y_d^E, 2\tau Y\right)}{u_1\left(\left(1-\tau\right) y_d^E, 2\tau Y\right)} = \frac{y_d^E}{2Y}.$$

(iv) If $\frac{\partial^2 V(y;\tau)}{\partial y \partial \tau} = 0$, then all households $y \in [y_l, y_m]$ vote for the same positive tax rate which is the solution to

$$\frac{u_2\left(\left(1-\tau\right) y, 2\tau Y\right)}{u_1\left(\left(1-\tau\right) y, 2\tau Y\right)} = \frac{y}{2Y}, \ \forall y \in [y_l, y_m].$$

These results imply that in the endogenous entitlement, uniform subsidy scheme, the subsidy will be shared by half the population and will be funded at the lowest possible levels without shutting down.

5 Concluding Remarks

We studied publicly funded means-tested subsidies when funding decisions are made through majority voting. The voting problem has two policy variables – the tax rate and the means testing rate. We assume that households first vote on the tax rate anticipating how it will affect the means testing rate and then vote on the means testing rate. We show that a majority voting equilibrium exists and solve for the majority preferred tax rate and means testing rate. We establish that the means testing rate is a linear function of the tax rate and that the fraction of the population receiving the subsidy is independent of the tax rate used to fund the subsidy.

We have assumed a particular order of the vote, first the tax rate, then the slope of the means testing function. When the order of the vote is reversed so that the tax rate is voted on second, preferences over both the slope of the means testing function and over the overall tax rate are no longer single-peaked and majority voting fails to exist. This is in sharp contrast to De Donder, Le Breton and Peluso (2010) who find that in their set-up, a voting equilibrium exists, regardless of the order of voting.

An alternative method that can be used to avoid the non-existence issue is due to Kramer ((1972) and Shepsle (1979). In this approach it is assumed that voting takes place separately on each issue, but not in a sequential fashion. De Donder, Le Breton and Peluso (2010) show that under certain conditions, namely a strategic complementarity between policy dimensions, these two approaches generate the same voting equilibrium. In our paper the Kramer-Shepsle equilibrium exists, but does not coincide with the outcome from sequential voting. We leave further investigation of these issues for future work.

Appendix A Proofs

Proof of Lemma 1. Holding τ fixed and applying the implicit function theorem to (2), the slope and curvature of the GBC are given by

$$\frac{\partial \widetilde{\alpha}(\tau,\beta)}{\partial \beta} = \frac{\int_0^{\widetilde{\alpha}/\beta} y f(y)\, dy}{F(\widetilde{\alpha}/\beta)} > 0, \quad \frac{\partial^2 \widetilde{\alpha}(\tau,\beta)}{\partial \beta^2} = -\frac{\tau^2 Y^2 f(\widetilde{\alpha}/\beta)}{\beta^3 F(\widetilde{\alpha}/\beta)^3} < 0 \tag{7}$$

so that the GBC is increasing and strictly concave. Furthermore, taking the limit as β decreases to zero, $\lim_{\beta\searrow 0} \frac{\partial\widetilde{\alpha}(\tau,\beta)}{\partial\beta} = Y$, which is the maximum slope of the GBC given strict concavity. ■

Proof of Proposition 1. We prove the statement in three steps. First, we show that the indifference curves for household y are linear in the (β, α) plane. Second, we show that the most preferred β is a decreasing function of household income. Finally, we show that the majority preferred β is chosen by the household with median income, y_m. See Figure 1.

Indifference curves of household y: Recall that the indirect utility of the household is

$$V(y,\alpha,\beta,\tau) \equiv u\left(\widehat{c}(y,\alpha,\beta,\tau), \widehat{d}(y,\alpha,\beta,\tau)\right).$$

For those points in the (β, α) plane $\frac{\alpha}{\beta} \geq y$, the slope of household y's indifference curve is

$$\left.\frac{\partial\alpha}{\partial\beta}\right|_{V(y,\alpha,\beta,\tau)=const.} = -\frac{\partial V(y,\alpha,\beta,\tau)/\partial\beta}{\partial V(y,\alpha,\beta,\tau)/\partial\alpha} = y > 0.$$

Thus, the indifference curves for household y are of the form $\alpha - \beta y = $ constant.

Most preferred β *on the GBC for each household*: Since lemma 1 provides $\lim_{\beta\searrow 0} \frac{\partial\widetilde{\alpha}(\tau,\beta)}{\partial\beta} = Y$, indirect utility $V(y, \widetilde{\alpha}, \beta, \tau)$ is maximized at $\beta = 0$ for all households $y \geq Y$. For $y < Y$, the indirect utility $V(y, \widetilde{\alpha}, \beta, \tau)$ is maximized at a unique

$$\left\{\beta > 0 : y = \frac{\partial\widetilde{\alpha}(\tau,\beta)}{\partial\beta}\right\}.$$

Denote the most preferred β of household y as $\widehat{\beta}(\tau, y)$. It is easy to see from Figure 1 that for $y < Y$, $\widehat{\beta}(\tau, y)$ is decreasing in y. For $y \geq Y$, $\widehat{\beta}(\tau, y) = 0$.

To be internally consistent, we have to verify whether households with $\widehat{\beta}(\tau, y) > 0$ do indeed receive positive subsidies i.e., does $\widehat{\beta}(\tau, y)$ satisfy the inequality $\widetilde{\alpha}\left(\tau, \widehat{\beta}(\tau, y)\right) - \widehat{\beta}(\tau, y)\, y > 0$ for all $y < Y$? It is easy to see from Figure 1 that every household $y < Y$ will choose a β such that it gets a positive subsidy. This is because $\alpha - \beta y = 0$ is a lower indifference curve for household y than the indifference curve that is tangent to the GBC.

Majority preferred β: Let $\widehat{\beta}(\tau, y_m)$ be the most preferred β on the GBC for the household y_m. (Recall that our income distribution has $y_m < Y$.) Consider a candidate $\beta_c < \widehat{\beta}(\tau, y_m)$ on the GBC. All households with $y \leq y_m$ strictly prefer $\widehat{\beta}(\tau, y_m)$ to β_c since $\widehat{\beta}(\tau, y)$ is decreasing in y. Consequently, no feasible β less than $\widehat{\beta}(\tau, y_m)$ can garner a majority. Next, consider a candidate $\beta_c > \widehat{\beta}(\tau, y_m)$ on the GBC. All households with $y \geq y_m$ prefer $\widehat{\beta}(\tau, y_m)$

to β_c. Consequently, no feasible $\beta > \widehat{\beta}(\tau, y_m)$ can get a majority who strictly prefer it to $\widehat{\beta}(\tau, y_m)$. Thus, $\widehat{\beta}(\tau, y_m)$ is the majority preferred β on the GBC. ■

Proof of Lemma 2. (i) The left hand side of GBC (2) is homogeneous of degree 1 in (α, β) and the right hand side of GBC is homogeneous of degree 1 in τ. Hence, proportionate increases in α, β and τ will satisfy the GBC.

(ii) Given $\tau \in (0,1)$, the right hand side of GBC (2) is fixed. Totally differentiating the left hand side w.r.t. to α and β, it is easy to show using Leibniz rule that

$$\frac{d\alpha}{d\beta} = \frac{\int_0^{\frac{\alpha}{\beta}} y dF(y)}{F\left(\frac{\alpha}{\beta}\right)}.$$

Clearly, proportionate increases in α and β have no effect on the slope. ■

Proof of Proposition 2. For $\tau \in (0,1)$, let $\widehat{\alpha}$ and $\widehat{\beta}$ be the most preferred pair of household y_m i.e., the majority preferred pair on the GBC satisfies

$$\left.\frac{d\alpha}{d\beta}\right|_{\left(\widehat{\alpha}, \widehat{\beta}\right)} = y_m.$$

Consider an arbitrary j such that $j\tau \in (0,1)$. For the tax rate $j\tau$, Lemma 2 establishes that the pair $\left(j\widehat{\alpha}, j\widehat{\beta}\right)$ satisfies the GBC associated with $j\tau$ and that

$$\left.\frac{d\alpha}{d\beta}\right|_{\left(j\widehat{\alpha}, j\widehat{\beta}\right)} = y_m.$$

Hence, the most preferred pair on the new GBC is $\left(j\widehat{\alpha}, j\widehat{\beta}\right)$. Properties of the most preferred α and β follow immediately. ■

Proof of Corollary 1. Consider a tax rate τ_1. Let the corresponding majority preferred $\beta = \beta_1$ and the value of α implied by the GBC (2) be α_1. Households with $y \leq \frac{\alpha_1}{\beta_1}$ will receive a subsidy $s_1(y) = \alpha_1 - \beta_1 y$. If we consider an alternative tax rate $\tau_j = j\tau_1 \in (0,1)$, we know from Lemma 2 that the corresponding majority preferred $\beta_j = j\beta_1$ and $\alpha_j = j\alpha_1$. Households with $y \leq \frac{\alpha_j}{\beta_j} = \frac{j\alpha_1}{j\beta_1} = \frac{\alpha_1}{\beta_1}$ will receive a subsidy $s_j(y) = \alpha_j - \beta_j y = j\alpha_1 - j\beta_1 y = j(\alpha_1 - \beta_1 y) = j \times s_1(y)$. Consider a 10% increase (decrease) in the tax rate as an example. This increases (decreases) the subsidy budget by 10%. However, all existing subsidy recipients now receive a 10% larger (smaller) subsidy. As a consequence, the whole increase (decrease) in the budget is allocated solely to existing subsidy recipients and no new (existing) recipients are added (cut). ■

Proof of Proposition 3. For households with $y \geq \frac{k_\alpha}{1+k_\beta}$, the utility is monotonically declining in τ and their most preferred tax rate is zero. We will show that the utility of households with $y < \frac{k_\alpha}{1+k_\beta}$ are also single-peaked. Existence of the majority voting equilibrium then follows immediately from Black (1958).

To establish single-peakedness for households with $y < \frac{k_\alpha}{1+k_\beta}$, define two functions – $\underline{V}$ where household y is constrained by the voucher for all τ and $\overline{V}$ where the household is never constrained by any τ.

$$\overline{V}(y;\tau) \equiv u\left(\overline{c}\left((1-\tau)y + k_\alpha\tau - k_\beta\tau y\right), \overline{d}\left((1-\tau)y + k_\alpha\tau - k_\beta\tau y\right)\right) \quad (8)$$

$$\underline{V}(y;\tau) \equiv u\left((1-\tau)y, k_\alpha\tau - k_\beta\tau y\right) \quad (9)$$

where the functions $\overline{c}$ and $\overline{d}$ describe interior solutions given resources $(1-\tau)y + k_\alpha\tau - k_\beta\tau y$ and no additional constraints. It is easy to see that $\underline{V} \leq \overline{V}$ since the resource constraint is the same, but $\underline{V}$ has an additional constraint on educational expenditure. Define $\overline{\tau}(y)$ such that

$$\underline{V}(y;\overline{\tau}) = \overline{V}(y;\overline{\tau})$$

i.e., at $\overline{\tau}$ household y's interior choice of educational expenditure is exactly the same as the voucher amount or the voucher constraint is just barely binding. It is easy to see that there is a unique $\overline{\tau}(y)$ (set $(1-\tau)y = \overline{c}\left((1-\tau)y + k_\alpha\tau - k_\beta\tau y\right)$ and solve for τ). Clearly, for a tax rate higher than $\overline{\tau}$, household y would be constrained. We can then write the indirect utility of household y as

$$V(y;\tau) = \begin{cases} \overline{V}(y;\tau) & \text{if } \tau < \overline{\tau}(y) \\ \underline{V}(y;\tau) & \text{if } \tau \geq \overline{\tau}(y) \end{cases}. \quad (10)$$

For household $y < \frac{k_\alpha}{1+k_\beta}$, $\overline{V}$ is increasing in τ since $k_\alpha\tau - k_\beta\tau y > \tau y$. For this household, it is also easy to see that $\underline{V}$ is strictly concave in τ. Thus, the indirect utility for household y, $V(y;\tau)$, is (i) the same as $\overline{V}(y;\tau)$ for $\tau < \overline{\tau}(y)$ and, hence, increasing and (ii) the same as $\underline{V}(y;\tau)$ for $\tau \geq \overline{\tau}(y)$ and, hence, strictly concave. At $\overline{\tau}(y)$, by construction, $\underline{V} = \overline{V}$, so there is no discontinuity in $V(y;\tau)$ at $\overline{\tau}(y)$.

Now, $\underline{V}$ is single-peaked at $\widehat{\tau}(y)$ where $\widehat{\tau}(y)$ is the unique solution to

$$yu_1\left((1-\tau)y, k_\alpha\tau - k_\beta\tau y\right) = (k_\alpha - k_\beta y)\, u_2\left((1-\tau)y, k_\alpha\tau - k_\beta\tau y\right).$$

Furthermore, $\widehat{\tau}(y) > \overline{\tau}(y)$. This is because (i) at $\tau = \overline{\tau}(y)$, $u_1\left((1-\tau)y, k_\alpha\tau - k_\beta\tau y\right) = u_2\left((1-\tau)y, k_\alpha\tau - k_\beta\tau y\right)$ since household y's optimal choice (based on $\overline{V}$) of consumption is exactly the after-tax income and educational expenditure is exactly the voucher amount and (hence) (ii) $\left.\frac{\partial \underline{V}}{\partial\tau}\right|_{\tau=\overline{\tau}(y)} = u_1\left((1-\overline{\tau}(y))y, k_\alpha\overline{\tau}(y) - k_\beta\overline{\tau}(y)y\right)\{k_\alpha - (1+k_\beta)y\} > 0$ for all $y < \frac{k_\alpha}{1+k_\beta}$.

Thus, $V(y;\tau)$ is single-peaked for all households. ■

Proof of Lemma 3. (i) If the decisive voter's income is greater than $\frac{k_\alpha}{1+k_\beta}$, then his most preferred tax rate is zero.

(ii) Suppose, to the contrary, that the decisive voter is not constrained. Then, $u_1(c,d) = u_2(c,d)$ where $c < (1-\tau)y_d$ and $d > k_\alpha\tau - k_\beta\tau y_d$. Consider an increase in τ. With a higher τ, the decisive voter gets more resources since $-\tau y_d + k_\alpha\tau - k_\beta\tau y_d > 0$ and increasing in τ for all $y_d < \frac{k_\alpha}{1+k_\beta}$. Consequently, the decisive voter would be better off with a higher and higher τ, as long his choice of consumption is not constrained by his after-tax income. Hence, his most preferred tax rate has to satisfy the equations in part (ii).

(iii) At the constrained allocation $c = (1-\tau)y_d$ and $d = k_\alpha\tau - k_\beta\tau y_d$. A marginal increase in τ implies a loss of y_d in consumption and a gain of $k_\alpha - k_\beta y_d$ in the voucher amount. He will set the most preferred tax rate such that

$$y_d u_1\left(\left(1-\tau\right)y_d, k_\alpha\tau - k_\beta\tau y_d\right) = \left(k_\alpha - k_\beta y_d\right)u_2\left(\left(1-\tau\right)y_d, k_\alpha\tau - k_\beta\tau y_d\right),$$

so the utility loss on the margin is equal to the utility gain. ■

Proof of Proposition 4. For household $y < \frac{k_\alpha}{1+k_\beta}$, the most preferred tax rate, following part (iii) of Lemma 3, is the unique solution to

$$\frac{\delta\left(k_\alpha\tau - k_\beta\tau y\right)^{-\sigma}}{\left(\left(1-\tau\right)y\right)^{-\sigma}} = \frac{y}{k_\alpha - k_\beta y}.$$

Denote the solution as $\widehat{\tau}(y)$. Rewrite the above equation as

$$\begin{aligned} \frac{\delta^{\frac{1}{\sigma}}\left(1-\widehat{\tau}(y)\right)y}{k_\alpha\widehat{\tau}(y) - k_\beta\widehat{\tau}(y)y} &= \left(\frac{y}{k_\alpha - k_\beta y}\right)^{\frac{1}{\sigma}} \\ \text{or, } \frac{\delta^{\frac{1}{\sigma}}\left(1-\widehat{\tau}(y)\right)}{\widehat{\tau}(y)} &= \left(\frac{y}{k_\alpha - k_\beta y}\right)^{\frac{1}{\sigma}-1}. \end{aligned} \tag{11}$$

Now, $\frac{y}{k_\alpha - k_\beta y}$ is increasing in y and for $\sigma < 1$, the right hand side is increasing in y. Hence, $\widehat{\tau}(y)$ must be decreasing in y to preserve the equality for $y < \frac{k_\alpha}{1+k_\beta}$. For households with incomes above $\frac{k_\alpha}{1+k_\beta}$, the preferred tax rate is zero. Consequently, household y_m is the decisive voter and its most preferred tax rate is the unique solution to (5). For $\sigma = 1$, $\widehat{\tau}(y)$ is independent of y for $y < \frac{k_\alpha}{1+k_\beta}$. Again, household y_m is the decisive voter.

For $\sigma > 1$, the right hand side of (11) is decreasing in y, so $\widehat{\tau}(y)$ is an increasing function of y. Thus, the ordering of the preferred tax rate is as follows: households with $y \geq \frac{k_\alpha}{1+k_\beta}$ prefer a zero tax rate, households at lower end of the income distribution prefer a slightly higher tax rate and voters in the middle prefer an even higher tax rate. As a result, the decisive voter's income is less than y_m and the identity of the decisive voter is pinned down by (6). ■

Proof of Proposition 5.

(i) Since the decision to collect taxes has been made in the first stage vote, $\tau > 0$ and every household contributes to the government budget.

(ii) As a result, every household would rather receive the subsidy than not. This rules out household y voting for $\widehat{y} < y$ and excluding themselves from the subsidy.

(iii) Every household y also wants the largest possible subsidy s. Thus, household y would prefer not to vote for $\widehat{y} > y$ as this increases the number of beneficiaries to which the public budget is distributed, thereby reducing s.

(iv) Thus, each household y's most preferred $\widehat{y} = y$.

(v) However, values of $\widehat{y} < y_m$ will not garner majority support since a majority will be excluded from the subsidy. As a result, only values $\widehat{y} \geq y_m$ are feasible. Households $y < y_m$will have to choose between different values of $\widehat{y} \geq y_m$. Based on the reasoning in (iii), all households $y \leq y_m$ prefer $\widehat{y} = y_m$ to any values of $\widehat{y} > y_m$. Thus $\widehat{y} = y_m$ has support from all households $y \in [0, y_m]$ with mass $F(y_m)$ which makes it a majority winner. Higher values of $\widehat{y}$ will be defeated by $\widehat{y} = y_m$, supported by the coalition $y \in [0, y_m]$.

■

Proof of Proposition 6.

(i) All households with $y > \widehat{y} = y_m$ prefer a tax rate of $\tau = 0$ since they are not entitled to any subsidy payment. As they do pay taxes, the indirect utility for these households is declining in τ. Similar to the proof of Proposition 3, to establish the single-peakedness for households with $y \leq \widehat{y} = y_m$, define two functions – $\underline{V}$ and $\overline{V}$:

$$\underline{V}(y;\tau) \equiv u((1-\tau)y, 2\tau Y); \quad \overline{V}(y;\tau) \equiv u\left(\overline{c}(1-\tau)y + 2\tau Y, \overline{d}(1-\tau)y + 2\tau Y\right)$$

where the functions $\overline{c}$ and $\overline{d}$ describe interior solutions given resources $(1-\tau)y + 2\tau Y$ and no additional constraints. Define $V(y;\tau)$ in a manner similar to (10). Properties of $\underline{V}$ and $\overline{V}$ follow in a manner similar to the proof of Proposition 3 and $V(y;\tau)$ is single-peaked. Existence of a majority voting equilibrium follows from Black (1958).

(ii) We have already established that the most preferred tax rate of households with $y > \widehat{y} = y_m$ is zero. Thus, if the decisive voter $y_d^E > y_m$, then the equilibrium tax rate must be $\tau = 0$. Only subsidy recipients ($y \leq \widehat{y} = y_m$) will support a positive tax, implying the decisive voter must be $y_d^E \in [y_l, y_m]$.
Now, suppose that the decisive voter is not constrained. Then, $u_1(c,d) = u_2(c,d)$ where $c < (1-\tau)y_d^E$ and $d > 2\tau Y$. Consider an increase in τ. As in the proof of Lemma 3, the decisive voter would be better off with a higher and higher τ, as long his choice of consumption is not constrained by his after-tax income. Hence, his most preferred tax rate has to be such that $\widehat{c} = (1-\widehat{\tau}(y_d^E))y_d^E$ and $\widehat{d} = 2\widehat{\tau}(y_d^E)Y$.

(iii) Given the result in part (ii) of the proposition, the most preferred tax rate solves the problem of $\max u\left((1-\tau)y_d^E, 2\tau Y\right)$ where $y_d^E \in [y_l, y_m]$. At the constrained allocation $c < (1-\tau)y_d^E$ and $d > 2\tau Y$, a marginal increase in τ implies a loss of $\Delta\tau y_d^E$ in consumption and a gain of $\Delta\tau 2Y$ in the subsidy amount. He will set the most preferred tax rate such that

$$y_d^E u_1\left((1-\tau)y_d^E, 2\tau Y\right) = 2Y u_2\left((1-\tau)y_d^E, 2\tau Y\right), \tag{12}$$

so the utility loss on the margin is equal to the utility gain.
The key to identifying the decisive voter is to understand that all households $y > \widehat{y} = y_m$ form a voting block that wish to contribute as little as possible to the subsidy. That is, they prefer $\tau = 0$ and if $\tau > 0$, they support the lowest possible τ.
If $\frac{\partial^2 V(y;\tau)}{\partial y \partial \tau} < 0$, the τ at which (12) is satisfied decreases with household income y. Thus

the median income household y_m supports the lowest non-zero tax rate of all households $y \in [y_l, y_m]$. The $y > \widehat{y} = y_m$ households also support this tax rate, making the median income household decisive. Conversely, if $\frac{\partial^2 V(y;\tau)}{\partial y \partial \tau} > 0$, the reverse is true and the lowest income household, y_l, supports the lowest non-zero tax rate. The $y > \widehat{y} = y_m$ households also support this tax rate, making the lowest income household decisive.

(iv) Finally, if $\frac{\partial^2 V(y;\tau)}{\partial y \partial \tau} = 0$, the tax rate ($\tau$) at which (12) is satisfied is identical for all households $y \in [y_l, y_m]$, who form a decisive coalition in support of this tax rate.

■

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