Incompleteness of the New Classical General Equilibrium Model and Its Implications

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Abstract

To provide a rigorous formulation of the notion of invisible hand, economists have constructed various general equilibrium models (GEMs). While these GEMs vary in many aspects, they share the same structure, viz., each GEM is a model world consisting basically of three elements: the environment, the mechanism, and the optimality criterion. Within such a model world, the invisible hand metaphor can be translated into a proposition that in each environment, the mechanism can finalize a resource allocation that satisfies the given optimality criterion. A scientifically tractable GEM therefore must be able to either verify or falsify this proposition in its analytical framework. Such a fundamental requirement, called "completeness," however, may not be satisfied by the inframarginal analysis, a nascent GEM that was introduce in the economic literature in 1990s'. This indicates that inframarginal analysis, which is the first GEM that incorporates Smithian division of labor, may not succeed in formulating the notion of the invisible hand.

Keywords: General Equilibrium Models; Pareto-Walrasian tradition; Infra-Marginal analysis (new classical economics); Invisible hand

JEL Classifications: B16, B23, C62, D50

1. Introduction

In a decentralized and competitive economy where the flow of resources is dispersedly decided by individuals in pursuit of different values, can the market coordinate the decisions of individuals to finalize an efficient resource allocation for the whole economy? This question has been at the heart of economics since its inception as a distinct discipline.[1] Adam Smith (1976), though did not offer anything resembling a rigorous analysis of resource allocation in his work, proposed an affirmative answer to this question. This idea, known as Smith's notion of the invisible hand, is regarded as one of "the most important intellectual contributions that economic thought has made to general understandings of social processes" (Arrow and Hahn 1980, p.1).[2]

Walras (1900) was the first economist who systematically attempted the construction of an analytical framework to discuss the determination of resource allocation of the whole economy (Arrow and Hahn 1980, 3). His analytical framework was then developed by McKenzie (1954), Arrow and Debreu (1954), Arrow and Hurwicz (1958), Debreu (1959), Hurwicz (1969), Arrow and Hahn (1970), Yang and Ng (1993), and Sun, Yang, and Zhou (2004), resulting in the emergence of

[1]See Arrow and Hahn (1980, 1-15) and Weintraub (1991) for a historical review.

[2]There are many and diverse interpretations of the invisible hand (Khalil, 2000a). Definitely, though, Adam Smith employs the term "invisible hand" only once in the *Wealth of Nations*, and this usage is actually an embarrassment (Khalil, 2000b). Smith (1976, p. 456) argues that policy makers should not worry about open markets and free international trade because, anyhow, local capital has "home bias." That is, "friction" would keep local capital at home, which is beneficial for the welfare of the home country—and this is an expression, as Smith states, of the "invisible hand."

various GEMs. These GEMs are typified by different scenarios of resource allocation, such as the pure exchange economy (Walras 1900), the private ownership economy (Arrow and Debreu 1954), and the little-familiar new classical economy (Yang and Ng 1993; Sun, Yang, and Zhou 2004).[3] Despite their diversity, all these GEMs are inspired by the invisible hand metaphor.

Given that the notion of invisible hand is one of the most important ideas underlying the whole general equilibrium theory (Arrow and Hahn 1980, 3), it is natural to ask: Has this notion been successfully formulated by the existing GEMs? The present paper is devoted to answering this question. Some economists might not think this is a "real" question as they believe the existence and the optimality (typically stated in the First Fundamental Theorem of Welfare Economics) of the Walrasian equilibrium have provided a formal answer to it (this is even written in some popular textbooks of economics, e.g., Mas-Collel et al 1995, p.327 and Jehle and Reny 2011, 183). This point is not agreed by the present paper. As shown below, a successful formulation of the notion of invisible hand requires not only the existence of a resource allocation that is optimal but also the attainability of such an allocation. Existence of the equilibrium of a GEM only means that the dynamic process of market mechanism – an information exchange process that coordinates individuals' dispersed decisions – has some stationary points; however, it does not guarantee that these stationary points can be approached. Therefore, even one has established the existence theorem and optimality theorem of Walrasian equilibrium in a GEM, she

[3]The new classical economics, inspired by Smithian concern with division of labor, is known also as the infra-marginal approach. The new classical model deals with a special type of environments in which, unlike other GEMs, each individual is a consumer-producer who needs to make decisions on labor specialization to determine what to produce and sell.

still needs to carefully check whether the equilibrium can be attained by the market mechanism from any initial state of the economy before she can say the model formulates “invisible hand” or the opposite.

The remainder of the paper is organized as follows: **Section 2** discusses how GEMs are constructed. In this section, we show that all GEMs, when they differ in many aspects, share a common construct in the sense that each of them is a model world composed of three elements: a set of environments, an optimality criterion and a mechanism of market coordination. **Section 3** clarifies how the invisible hand metaphor is translated into a rigorous proposition in the GEM. The concepts of “competitive economy” and “decentralized economy” are defined and a formal expression of the “invisible hand” is presented. **Section 4** gives an explanation of the “two-level structure” of analysis in the general equilibrium literature and clarifies its relation to the completeness requirement. **Section 5** illustrates that while Pareto-Walrasian approach is a success story within the neoclassical tradition, such success is actually very limited. The Pareto-Walrasian approach cannot meet the completeness requirement once it is incorporated with the new classical environment of Smithian division of labor (infra-marginal analysis). **Section 6** provides concluding remarks.

In the general equilibrium theory, economists typically organize their analysis in a two-level structure, in which they traditionally The fAn analysis of the construct of GEMs shows that the notion of invisible hand has been translated into a proposition that. and, hence, are devoted to the proof of propositions – such as the existence, optimality, and stability of equilibrium –which express this metaphor

We find that a GEM can be seen as theoretically successful only if it meets a very basic requirement: Can the axiomatic system of the GEM under focus verify or falsify the proposition that expresses the invisible hand—viz., a competitive market always leads to resource allocations that satisfy the welfare (efficiency) criterion defined by the axiomatic system. We characterize a GEM as "complete" if it satisfies this requirement. Based on this, we then turn to discuss whether all existing GEMs meet this requirement. The present paper shows that the completeness requirement cannot be satisfied by a particular GEM, namely, the nascent new classical approach introduced in the early 1990s (i.e., infra-marginal analysis). This approach is a GEM within the context of the famous, but notoriously neglected, Smithian division of labor, which Smith illustrated by the story of the pin factory (Smith 1976, pp. 14-15).

2. Construction of GEMs

It is widely agreed that general equilibrium analysis starts with Walras (1900). As a leader of the marginal revolution, Walras believes that the prices of goods must be determined at a state of equilibrium, in which demand equals supply in all markets at the same time. To defend this idea, he established a model in which a set of individuals, whose characteristics, such as preferences[4] and endowments, are given, make decisions on demand and supply according to the price signals, while the price signals are sent to individuals by an auctioneer who adjusts prices of goods to minimize the difference between demand and supply of each good. Walras calls this dynamic process of communication or coordination "tâtonnement." He believes that it captures the core of market competition. Within this framework, Walras discusses whether the equilibrium of such economy exists and, in addition, whether it can be reached.

[4] Individual preferences were specified by the changes in marginal utility in Walras (1900).

Although the main motivation of Walras is to provide a theory of the value of goods, his theory inadvertently provides a formal analytical framework for resource allocation at the economy level. However, Walras does not define any criterion of optimality and thus fails to provide a theory concerning the optimality of the final allocations, which is a necessary part of the notion of invisible hand. This gap is filled by one of his students, Pareto. Inspired by Edgeworth (1881), Pareto (1909) defines a criterion of optimality that is now known as the "Pareto criterion." Pareto proves that each equilibrium in Walras' model satisfies this criterion.

The pioneering work of Walras and Pareto implies that the most meaningful propositions about resource allocation can be scientifically discussed only in a model that rigorously and precisely specifies the economic environment under which resource allocation is considered, how individuals' decisions are coordinated, and what allocations can be regarded as socially optimal, i.e., efficient. These specifications lead to the three essential elements of a GEM: a set of *environments* E, a *mechanism* $\mathcal{M}$, which is a generalization of the tâtonnement, and an *optimality criterion* φ. Economists familiar with general equilibrium theory acknowledge that all GEMs share a similar structure in which a series of assumptions enter the analysis at the beginning, followed by propositions about the existence, stability, and optimality of the equilibrium (e.g., Arrow and Debreu 1954; Debreu 1959; Sun, Yang, and Zhou 2004). The purpose of these assumptions is to specify the above-mentioned three elements.

An environment $e \in E$ is a complete description of the necessary parameters that are related to resource allocation, such as the number of individual decision makers and the characteristics of each individual, including the choice set, preferences, and initial endowment.[5] When an economist constructs a GEM, she must introduce some assumptions to specify how many individuals are in the economy and what properties their choice sets, preferences, and endowments satisfy. These assumptions specify the range in which the value of each parameter can take and thus define a class of environments under consideration.

Given the choice set of each individual, we can define a resource allocation as a collection of the action of each individual in the economy. While many resource allocations are feasible in each environment e, only some of them can be regarded as socially optimal. To locate these socially optimal allocations, we need an optimality criterion φ to clarify what properties an allocation must satisfy so that it can be regarded as socially optimal. In economics, various optimality criteria of resource allocation have been proposed. But the Pareto criterion of optimality is the most broadly accepted, though it fails to include any consideration for matters of "equity" or "justice" in the concept of social optimality.

While the environment in which resource allocation is to be considered and the optimality criterion has been clearly specified, we still need to clarify how decisions of individuals are coordinated to finalize an allocation. This gap is filled by the introduction of a mechanism. A mechanism $\mathcal{M}$, as defined by Hurwicz (1960, 1973), consists of four components. The first component is a participant set H that indicates who are involved in the determination of resource allocation. According to Hurwicz (1973), H usually constitute a broad group of individuals, not only consumers and producers this setup is reasonable as the additional participants may include governments,

[5]See Hurwicz (1973) for a detailed formulation of the environment. If firms are considered in the model, then production sets are their choice sets.

planning agencies, central banks, or unions. (, etc.) in the environment.[6] The second component is a message space Ω that contains all messages which individuals can use in the dialogue. The third component is a function M that specifies for each participant some response rules according to which, given the information available to him at a given time,[7] he sends messages to others. In each environment e, these response rules can determine a particular communication (i.e. information exchange) process $\mathrm{M(e)}$ in Ω. The last component is an outcome rule r which specifies what allocations are to be realized given the course of the communication and therefore transform the communication into actions. We can denote a mechanism as $\mathcal{M} = (H, \Omega, M, r)$.

that specify, for each environment, "a period of dialogue" or an information exchange process according to which decisions concerning the resource flow (production and exchange) are determined

Walrasian tâtonnement is a specific case of Hurwicz's mechanism. In the tâtonnement, participants of the communication involve the individuals (consumers and producers) in the environment and an auctioneer. The message space consists of vectors of prices and excess demands. Therefore, if there are n kinds of commodities trade in the economy, the message space is $\mathbb{R}^n$. The auctioneer's response rule makes him send out price signals which change proportional to aggregate excess demand, while the response rules of the other participants require them to send their excess

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7 The information consists of messages previously received, as well as some (direct) knowledge of environment.

demands back to the auctioneer, given that the excess demand vector of each individual is determined by his utility or profit maximization under the given price. These response rules together determines a communication process consisting of the adjustment of price

(1) $\dot{P} = \lambda \sum_{i \in I}(z_i(p))$

where $z_i(p)$ is the excess demand vector of individual i given by

(2) $z_i(p) = x_i^*(p) - s_i$

With $x_i^*(p)$ being the optimal choice maximizing i's utility under price p and s_i the endowment of individual i. Finally, the outcome rule is defined as follows: it does not assign any real allocations until the communication reaches its end, i.e., everyone is repeating his previous message, where it carries out exchanges according to the bids proposed by individuals.

3. Completeness of GEMs

The three components of a GEM, the environment, the mechanism and the optimality criterion, constitute a model world in which a story about the use of resources to satisfy human demand can be analyzed. Now we would ask: How the notion of invisible hand is presented in such a model world?

Let look again at the "invisible hand". It says,

First, we need to clarify what one means by "competitive economy."Since the 19th century, economists have reached an agreement that the basic property of a competitive economy is that no individual buyer or seller can influence the price. This idea is captured in the Walrasian tâtonnement where prices are assumed to be the decisions of the auctioneer, where consumers and producers are assumed to regard such prices as given parameters of their decisions. We can generalize this assumption to the broader concept of mechanism to obtain a definition of "competitive economy":

Definition 1: *A mechanism* $\mathcal{M} = (H, \Omega, M, r)$ *is said to be competitive if, when price signals are available in* Ω*, they can only be the received messages of individual decision makers and the sent-out messages of the information center in the communication process* $M(e)$ *in each environment* e.

Second, we need to clarify what one means by a "decentralized economy." Hayek (1945) is the first economist who provides a concrete explanation of this concept. In this seminal paper, an important feature of the decentralized economy is that the knowledge that is necessary to optimize resource allocation is dispersed across individuals and, more importantly, can never be wholly centralized in the hand of any person or any organization. Hurwicz (1972) concluded from Hayek's explanation two requirements that Hurwicz thought a decentralized economy must satisfy, as stated in the definition below.

Definition 2: *A mechanism* $\mathcal{M}=(H, \Omega, M, r)$ *is said to be decentralized if (1) each participant is initially the only one with information about her own characteristics, and (2) information about any participant's characteristics cannot be exchanged during the communication process* $M(e)$ *in any environment* e*, i.e., such information is not available in* Ω.

Tâtonnement is a good example of the competitive and decentralized mechanism.[8] In tâtonnement, only the individual knows about his or her own

[8] There is another interpretation of Hayek's 1945 paper (see Khalil 2002; Klein 2012). Namely, Hayek is stating that agents cannot send messages to each other via "an information center" as

characteristics at the beginning. The messages that can be used in communication are price vectors and personal net demand vectors, and only the auctioneer can adjust price and send out price messages. These assumptions guarantee that Walrasian tâtonnement is both competitive and decentralized. Due to this reason, most contributors to general equilibrium theory, such as Arrow and Debreu (1954), Arrow and Hurwicz (1958), Hurwicz (1969), Arrow and Hahn (1970), Yang and Ng (1993), and Sun, Yang, and Zhou (2004), have retained Walras' tâtonnement as a standard element of their models.

Thirdly, to he notion of invisible hand asserts that at least one resource allocation can be finalized in a competitive and decentralized economy, where each GEM provides a particular set of specifications. To ensure an allocation can be attained in a GEM, we need that the communication process $M(e)$ has some "outcomes" in each environment $e \in E$. Only when this is true, an allocation can then be determined by the rule r according to the outcome of the communication. The requirement that $M(e)$ has some "outcomes" in each environment $e \in E$ is mathematically equivalent to the global stability of $M(e)$, i.e., $M(e)$ has some stationary points and each of its trajectories converges to one of these stationary points (Arrow and Hurwicz 1958). We call a stationary point an attractor if there is at least one trajectory converges at it. Obviously, each attractor of $M(e)$ represents a possible "outcome" of the communication process $M(e)$.

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conceived by Hurwicz (1960, 1973). That is, Hayek's emphasis is not exclusively about the private knowledge of each agent, but also on the impossibility of having a clearing house as imagined by Hurwicz. While Hayek's thesis is primarily a critique of socialist planning, as the case in the Soviet Union, it is equally a critique of the conception of competitive economy á la Hurwicz. But for the purpose of this paper, we may ignore this interpretation.

Based on these clarifications and concepts, we can now transfer the notion of the invisible hand, which states that an efficient resource allocation can always be attained by competitive market system in a decentralized economy, into the following proposition.

Proposition S.[9] *Given a GEM=*$(E,\mathcal{M},\varphi)$*, if the mechanism* $\mathcal{M}=(H,\Omega,M,r)$ *is competitive and decentralized, then for any* $e\in E$*,*

(1) the communication process $M(e)$ *has at least one stationary point;*

(2) each trajectory of $M(e)$ *converges to some stationary point;*

(3) the finalized allocations of resources of each stationary point, to which some trajectory of $M(e)$ *converges, satisfy the optimality criterion* φ*.*

Proposition S states that if the mechanism of a GEM is competitive and decentralized, then three properties of the GEM hold, which are: *existence* of stationary points of the communication process in each environment, *stability* of communication process in each environment[10], and *efficiency* of the finalized allocation in each environment. These properties jointly guarantee that in each given environment, no matter how resources are initially allocated, the functioning mechanism in the economy will finalize an allocation that satisfies the optimality criterion. This is exactly what the notion of invisible hand is trying to say.

Proposition S can be true or false depending on what a set of environments E, what a mechanism $\mathcal{M}$ and what a criterion of optimality φ are combined together to construct a GEM. Despite this, however, any GEM that aims to formulate the

[9] The letter “S” here is chosen to suggest that the proposition is from Adam Smith.

[10] See Arrow and Hurwicz (1958) for a discussion of the implication of stability.

notion of invisible hand must satisfy a very fundamental scientific requirement, viz., it should be able to specify whether Proposition S is true or false. This results in the following definition of completeness of GEM:

Definition3: *A GEM=$(E, \mathcal{M}, \varphi)$ is said to be complete if Proposition S can be either verified or falsified within the axiomatic system of the GEM under focus.*

To be clear, Definition 3 does not require that Proposition S holds in a GEM. Rather, Definition 3 requires that if a GEM is complete, it can give a scientific judgment on whether Proposition S (i.e., the idea of invisible hand) is true or false. That is, the GEM must be able to verify or falsify each of the three properties stated in Proposition S. Obviously, the completeness requirement is a very basic condition that any GEM must meet before it can be regarded as a successful formulation of the idea of invisible hand.

4. The "two-level" structure of analysis

In the literature of general equilibrium theory, as an inheritance from Walras (1900) and Pareto (1909), proofs of the three properties in Proposition S are typically organized in a "two-level structure" that divides the analysis of a GEM into two parts: the static analysis and the dynamic analysis[11](Arrow and Hurwicz 1958). In the static analysis, economists focus on the static properties of the GEM, such as the existence of the stationary points and the efficiency of finalized allocations.[12] In the dynamic analysis, economists check the stability of the communication process i.e., whether

[11]This "two-level" structure can be seen in most formulations of neoclassical general equilibrium theory, e.g., Arrow and Hahn (1980) and Mas-Colell, Whinston and Green (1995).

[12] A typical static analysis of the neoclassical GEM is presented in Debreu (1959).

each trajectory of this dynamic system converges to some stationary point.[13]

The major benefit of using the two-level structure of analysis is that it can help economists to establish the existence of the stationary points and the efficiency of finalized allocations without looking into the dynamic details of the communication process. When the mechanism $\mathcal{M} = (H, \Omega, M, r)$ functions in an environment $e \in E$, the resulting communication process M(e) defines a dynamic system in space Ω. Therefore, as long as we can determine the sufficient and necessary conditions of the stationary points of M(e), such as the demand equals supply on every market, we can directly check the existence of the stationary points by simply checking whether those conditions a message in Ω can satisfy. We can also check the efficiency of the finalized allocation by simply checking whether allocations resulting from messages that satisfy those conditions meet the optimality criterion φ. In these analyses, only the sufficient and necessary conditions for a message $m \in \Omega$ being a stationary point are relevant. We do not need to provide any detailed information of the communication process in the static analysis. This largely simplifies and tidies the analytical framework of a GEM.

The two-level structure of analysis entails the use of "equilibrium" in the analysis. In the static analysis, since no dynamic details of the communication process have been formally introduced, it is not proper to use "stationary point of the communication process" to describe the outcome of market coordination. Instead, we need another name for the "outcomes of communication", i.e., messages that satisfy the condition $F(m) = 0$.This leads to the use of "equilibrium" in general equilibrium

[13] Literature on dynamic analysis of the neoclassical model is collected in Arrow and Hurwicz (1977).

theory, a tradition originated from Walras (1900) as well. In Walras (1900), function of the market mechanism specifies a communication process, tâtonnement, in each environment, which reaches its end only when demand meets supply in all markets. Therefore, the sufficient and necessary condition for a price system being a stationary point of the tâtonnement is that it clears all markets simultaneously. Having this in mind, Walras (1900) introduces the word "equilibrium" into his analysis to refer to a price vector that satisfies this condition. Today, researchers of GEMs usually call the stationary point of tâtonnement the "Walrasian equilibrium" or the "competitive equilibrium".

While the two-level structure of analysis simplifies the proofs of the existence theorem and the efficiency theorem, it also has an disadvantage. That is, it isolates the stability theorem from these two theorems and hence makes people ignore the necessity of stability theorem in justify Proposition S.[14] However, theorems of the existence of equilibrium and the efficiency of equilibrium allocation actually only suggest that the communication process of market coordination has some stationary points and the allocations associated with these stationary points are socially optimal, but say nothing on whether each trajectory of the dynamic process will converge at one of these stationary points. In other words, the static analysis can only tell us whether equilibria exist and whether the associated allocations are optimal, but is unable to tell us if such equilibria can be attained by the market mechanism. Therefore, a stability theorem is indispensable to the verification of invisible hand in a GEM.

[14] For example, Mas-Colell, Whinston and Green (1995 p.545), claim that the first fundamental theorem of welfare economics is a formal expression for Adam Smith's claimed "invisible hand."

5. Complete and incomplete GEMs

Economists have developed various GEMs. Most of these GEMs preserve the Walrasian tâtonnement and Pareto optimality as two standard elements of the analysis, though they have imposed different assumptions on the environment of economy. We call such an approach to constructing GEMs the Pareto-Walrasian tradition (Lee 2006). GEMs constructed under the Pareto-Walrasian tradition fall into two categories: the neoclassical GEMs and the nascent, little-known new classical GEMs. The main conclusion drawn in this section is that while the neoclassical GEMs generically satisfy the completeness requirement, the new classical GEMs do not.

GEMs that include the pure exchange GEM and the private ownership GEM can be labeled as "neoclassical." The pure exchange GEM consists of the tâtonnement, the Pareto criterion, and a set of specific pure-exchange environments in which all market participants are pure consumers who exchange goods but do not produce.[15] Walras (1900) first considered the existence and the stability of market equilibrium (the stationary point of the tâtonnement) in the pure exchange environments, but he did not provide any analysis of the welfare aspects of the allocation at equilibrium. In the 1940s, the Pareto criterion was formally incorporate into the pure exchange GEM and this model became the benchmark and basis of the later mechanism design literature (e.g., Arrow and Hurwicz 1958; Hahn 1962; Hurwicz 1969; Mount and Reiter 1974; Wilson 1985). In this literature, economists (e.g., Arrow and Hurwicz 1958; Arrow, Block and Hurwicz1959; Arrow and Hahn 1980; etc.) have identified a number of pure exchange GEMs that satisfy the existence, optimality, and stability properties.

[15] A GEM usually involves three types of economic activities: consumption, trading, and production. The neoclassical pure exchange model (neoclassical model henceforth) is a special case of general equilibrium analysis, however. It ignores production.

In addition, the pure-exchange environment was replaced by private ownership GEM, developed by Arrow and Debreu (1954). This was further explored and developed by many economists. Interestingly, the more sophisticated and elaborated GEMs preserved the same outcome, namely, the completeness requirement, characterizing the simple pure-exchange GEMs. This should indicate that the Pareto-Walrasian tradition is scientifically successful in the sense defined here, i.e., the completeness sense.

The new classical GEMs differ from neoclassical GEMs in one regard: The new classical GEMs aim to give a formal and theoretical expression of Smith's notion of division of labor. It was first proposed by Yang and Ng (1993) and then formally formulated in Sun, Yang, and Yao (2004) and Sun, Yang and Zhou (2004). These authors believed that the invisible hand of competitive markets does not only "efficiently allocate resources for a given network structure of division of labor," but also "coordinate individuals' decisions in choosing their patterns of specialization in order to fully exploit positive network effects of division of labor net of transaction costs" (Sun, Yang, and Yao 2004).

To justify their opinion, the contributors to the new classical approach construct a general equilibrium analytical framework that inherits the Walrasian tâtonnement and the Pareto criterion from the neoclassical GEMs but replaces the environments considered in the neoclassical GEMs by a new type of environments. In these environments, individuals are allowed to make decisions on the pattern and level of labor specialization to determine whether to specialize or remain autarchic and what to produce if they specialize.[16] We call these environments the new classical environment. Recently, this analytical framework has been applied to many economic topics (see Cheng and Yang 2004; Yang 2004; Yang and Ng 1998). Commenters believe that this analytical framework provides "a refreshing new approach to microeconomics, one that has the potential to address many issues that have long resisted formal treatments" (Smythe1994).

Aside from the fruitfulness and extensions of the new classical approach, though, and this is the core thesis of this paper, combining a set of new classical environments with the Walrasian tâtonnement and the Pareto criterion may result in losing the completeness requirement that has been a fundamental feature of GEMs. What is of interest, the originators and contributors to the literature on new classical GEMs have overlooked this glaring shortcoming in their approach.

To show the shortcoming, i.e., the incompleteness of the new classical GEMs, let us consider a specific new classical GEM that applies the Walrasian tâtonnement and the Pareto criterion to a set of very simple new classical environments. These environments are characterized by the following assumptions:

A1. There is a continuum of *ex ante* individuals, I, in each environment.

A2. In each environment, individuals have the identical preference which can be represented by a utility function $u(x,y,x^d,y^d)=(x+kx^d)(y+ky^d)$, where x and y are the self-provided quantities of the two consumer goods respectively, x^d and y^d are the quantities of the goods purchased from others, k represents the transaction efficiency.[17]

[16] Contributors of the new classical GEMs believe that the neoclassical GEMs have long ignored the issue of division of labor. The neoclassical approach simply regarded the structure of division of labor as exogenous in their environments (Yang and Ng 1993).

[17] The appearance of k in front of x^d and y^d implies the transaction costs are losses for the

A3. In each environment, individuals have the identical technology that can be represented by the production functions $x + x^s = l_x^a$ and $y + y^s = l_y^a$, where l_x and l_y are the amount of time spent in producing the two goods respectively, a is a parameter that measures the degree of economies of specialization.

A4. In each environment, individuals face the same recourse constraint $l_x + l_y = 1$

A5. $k \in (0,1), a > 1,$ and $k > 4^{1-a}$.

A1-A4 imply that each environment is characterized by four parameters: I, u, k and X_a, where $X_a \equiv \left\{(l_x, l_y, x^s, y^s, x^d, y^d) \in \mathbb{R}_+^6 : x + x^s = l_x^a; y + y^s = l_y^a; l_x + l_y = 1\right\}$ is the choice set of each individual. Therefore, we can denote each environment by $e_s = (I, u, k, X_a)$ and denote the set of all environments under assumptions A1 – A5 by $E_s = \left\{e_s = (I, u, k, X_a) : k \in (0,1), a > 1,\ k > 4^{1-a}\right\}$. We will show that although the existence and optimality of Walrasian equilibrium in each environment can be verified in this model, the stability of the communication process in some environments, however, can neither be verified nor falsified.

Because E_s is a subset of the environments discussed in Sun, Yang and Yao (2004) and Sun, Yang, and Zhou (2004), conclusions obtained in their models apply to E_s as well. This implies that in each environment $e_s \in E_s$, the Walrasian equilibrium exists and satisfies Pareto criterion. Specifically, as proved by Yang and Ng (1993), in each $e_s \in E_s$, the only Walrasian equilibrium is that the relative price between x and y equals to 1 and the finalized allocation of resources is that half of the population specialize in producing x and the other half in y. These outcomes

consumer.

indicate that the combination of E_S with the Walrasian tâtonnement and the Pareto criterion will constitute a complete GEM as long as the stability of the dynamic process of tâtonnement can be verified or falsified. Unfortunately, such stability condition cannot be satisfied.

Table 1. Two optimal choices in $e_S = (I, u, k, X_a)$

Optimal choice	Indirect Utility	Production-Trade plan
X/Y: specializing in x	$u_1(p) = 4^{-1}kp^{-1}$	$l_x = 1$, $l_y = 0$, $x = x^s = 1/2, y^d = x^s / p$
Y/X: specializing in y	$u_2(p) = 4^{-1}kp$	$l_x = 0$, $l_y = 1$, $y = y^s = 1/2, x^d = py^s$

Let p be the price of y measured in x. According to Yang and Ng (1993), under the assumption of tâtonnement, for each $e_S \in E_S$ and any $p > 0$ announced by the auctioneer, the optimal choice of an individual must be one of the two strategies – (X/Y) specializing in x and (Y/X) specializing in y – as listed in Table 1.

Obviously, for all $p < 1$, indirect utility of specializing in x is greater than that of specializing in y. Therefore, all individuals will choose to produce and supply x, resulting in an excess supply of x that forces the auctioneer to decrease p. In the other case, for all $p > 1$, there is an excess supply of y that forces the auctioneer to increase p. As a result, if the initial price announced by the auctioneer is not equal to 1, the price will approach to 1 gradually. However, when price reaches 1, the payoffs of two strategies – specializing in x and specializing in y – become equivalent and each individual will choose randomly between these two strategies.

In this case, if the number of individuals who choose X/Y happens to equal the number of individuals who choose Y/X, the market of each good is cleared and price sticks to 1, making the communication process a stable one. However, since no further mechanism is presented in the assumptions of the model to specify how individuals will choose between X/Y as opposed to Y/X when they are indifferent, we are unable to conclude here whether the number of individuals choosing X/Y is equal to that of Y/X. In another word, we can neither verify nor falsify the stability of the communication process of price adjustment. This violates the requirement of completeness.

6. Conclusion

One of the most celebrated achievements of economic science is the invisible hand notion as rigorously proven in a long string of neoclassical GEMs. This paper does not question this towering intellectual achievement. However, this paper shows the limits of this towering intellectual achievement in the sense that such achievement can be sustained as long as GEMs exclude Smithian division of labor. This paper shows that once a GEM in the Pareto-Walrasian tradition incorporates Smithian division of labor, what is known as new classical GEM, it ends up incomplete, i.e., unable to demonstrate the invisible hand notion. That is, the towering intellectual achievement of the neoclassical Pareto-Walrasian tradition is valid only insofar as the GEMs avoid the incorporation of Smithian division of labor.

For a GEM to be complete, it has to show whether the particular proposition concerning the invisible hand is defensible, i.e., true or false. This paper shows that neoclassical GEMs are complete for specified environments and mechanisms. But such completeness is attained at a price: namely, the GEM must exclude the Smithian division of labor. Once a GEM incorporates Smithian division of labor, i.e., as the case with new classical GEMs, it is no longer complete. The reason is that, once a GEM includes Smithian division of labor, we lose the ability to ensure definitely the stability of the communication process of price adjustment towards the equilibrium price. That is, we no longer can be assured that the communication process generated by tâtonnement can reach an outcome.

Given that division of labor is important for prosperity and wealth of nations, it should not be neglected by economists—despite the fact of the incompleteness result of new classical GEMs. This shortcoming in new classical GEMs can still be addressed. This shortcoming calls for a new agenda that requires a new sophisticated mechanism that replaces the Walrasian tâtonnement, while preserving the quest after Smithian division of labor in GEMs. The needed new mechanism would be of great interest, but it is outside the scope of this paper.

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