Adaptation to catastrophic risk under climate change uncertainty

Chi Truong[1], Stefan Trück[1]

[a]*Faculty of Business and Economics, Macquarie University, NSW, Australia, 2109*

Abstract

We introduce a modeling framework to determine the optimal investment time for catastrophic risk reduction project. The model allows to incorporate the impact of uncertainty and continuous information updating into investment decision. We illustrate the application of the model with a case study of bushfire risk management in a local government area in Sydney, Australia. We found that ignoring uncertainty would results in significant loss. The loss is proportional to the initial belief in climate change, uncertainty, investment cost, discount rate and inversely proportional to the seriousness of climate change. Optimal investment threshold and option values are found to be sensitive to investment cost and discount rate but not sensitive to changes in uncertainty.

Keywords: Climate change adaptation, Incomplete information, Catastrophic risk, Bayesian, Real option, Climate change uncertainty

JEL Codes: Q54, D81, H54.

---

1. Introduction

A major concern with global warming is that the hotter climate system will be more energetic and raise the frequency and severity of catastrophic events. In many regions of the world, catastrophes of various kinds are predicted to be more active under climate change. For Australia, Queensland is predicted to observe increased flooding and storm surges while the South eastern Australia is predicted to see more bushfires in the coming years (Garnaut, 2011; Murphy and Timbal, 2008). Adaptation to climate change could potentially be one of the most challenging tasks in environmental management and will

---

☆We would like to thank

Email addresses: chi.truong@mq.edu.au (Chi Truong), stefan.trueck@mq.edu.au (Stefan Trück)

require input from all levels of government. While it has often been argued that action is most effective at the local level, local government is confronted with the complex and problematic task of planning and implementing mitigation and adaptation actions within existing budget constraints. This paper provides an economic framework to help local governments evaluate potential climate change adaptation options under climate change uncertainty.

We are concerned with the problem of climate change adaptation to reduce losses from catastrophes such as bushfire, flood and storm surge. An important characteristic of the problem is that there is a large uncertainty regarding the local impact of climate change, but that uncertainty decreases over time as more data on the changed climate are available. The decision maker has to determine the time to invest in adaptation projects so as to balance between the gain from waiting for uncertainty to reduce and the loss due to deferred adaptation. Furthermore, at the local level, there are usually a few observations of catastrophic events and the challenge presented by the rarity of catastrophic events needs to be overcome when implementing an investment model.

A number of studies have examined the problem of evaluating catastrophic risk reduction projects under the impacts of climate change. Kirshen et al. (2008a,b); Michael (2007); Symes et al. (2009); West et al. (2001) evaluated projects to reduce storm surge risk in coastal regions and Brouwer and van Ek (2004); Waters et al. (2003); Zhu et al. (2007); Bouwer et al. (2010); Mathew et al. (2012) evaluated adaptation to reduce flood risk in riverine areas. These studies assumed that future risk is known and follow a deterministic path. The impact of uncertainty is not incorporated into the investment decision. To our knowledge, this is the first study that incorporates the impact of climate change uncertainty for the evaluation of catastrophic risk reduction projects.

Some studies in climate change adaptation have adopted Bayesian updating in a discrete time modelling framework to endogenize the impact of uncertainty. Chao and Hobbs (1997) examined the investment of a shoreline protection project when there are two climate change scenarios and the decision maker's belief about climate change is updated each time a new observation on local climate becomes available. Optimal investment decisions are found by applying dynamic programming techniques to a finite time horizon problem. Venkatesh and Hobbs (1999) examined a similar problem using Bayesian Monte Carlo technique that allows them to examine more than two climate change scenarios.

New information, however, can be incorporated only every 20 years. An apparent constraint of these studies is that the value of the option to invest is only approximated and it is difficult, if not impossible, to evaluate the magnitude of approximation errors. As a consequence, it is difficult to determine whether some anomaly is due to approximation error or other issues.

In this paper, we introduce a modelling framework that allows new information to be incorporated continuously and gives rise to closed form solution in a special case. Our model is built upon recent works in the field of investment under incomplete information (Décamps et al., 2005; Klein, 2009). Different from these studies that take the value of the project as the primary process (so that the decision to invest is analogous to the decision to exercise a perpetual American call option), we develop our model based on a process of catastrophic risk and derive the value of the project from that process. This is an important extension that widens the applicablity of the model. In reality, it is often the catastrophic risk process that is estimated and observed, not the value of the project. In the implementation of the investment model, we introduce the GAMLSS framework developed by Rigby and Stasinopoulos (2005) to quantify catastrophic risk at a local area. With GAMLSS framework, not only the mean but also the variance and possibly the shape parameter of the loss distribution can be modelled to depend on climate and other variables. This flexibility proved valuable in exploring the complex relationships between climate, adaptation and catastrophic loss events.

2. Modeling framework

2.1. Frequency and Severity of Climate Impacted Hazards

Cumulative loss over a period $(0, t]$ is modeled as a compound Poisson process:

$$S_t = \sum_{n=1}^{N_t} X_n, \tag{2.1}$$

where N_t is the number of catastrophic events that occurs during period $(0, t]$ and X_n is the loss caused by the n^{th} event. Loss severity has an expected value β and the number of catastrophic events, N_t, follows a Poisson process that has intensity Λ_t. We assume that Λ_t follows a Geometric Brownian Motion

$$d\Lambda_t/\Lambda_t = \mu dt + \sigma dB_t,$$

where the growth rate μ is either μ_H or μ_L, but the decision maker does not know which one is the true value. She has an initial belief p_0 that the growth rate is μ_H and updates her belief upon observing Λ_t using Bayes' rule. The sigma field generated by the process Λ_t up to time t is denoted by $\mathscr{F}_t$ and the posterior probability of event $\mu = \mu_H$ at time t is denoted by P_t, i.e. $P_t = P[\mu = \mu_H|\mathscr{F}_t]$, with the initial condition $P_0 = p_0$. Upon applying Bayes' rule, the posterior probability P_t can be expressed as

$$P_t = \left[1 + \frac{1-p_0}{p_0}\left(\exp\left((\ln \Lambda_t - \ln \Lambda_0 - \frac{\mu_H + \mu_L - \sigma^2}{2} t)\right)\right)^{-\omega/\sigma}\right]^{-1}, \tag{2.2}$$

where $\omega = \frac{\mu_H - \mu_L}{\sigma}$. Equation (2.2) implies that P_t is revised upwards whenever $\ln \Lambda_t$ is higher than its corresponding expected value with μ takes the average level, $\mu = \frac{\mu_H + \mu_L}{2}$. The extent of revision is proportional to the signal to noise ratio ω and inversely proportional to the noise σ. Given (2.2), P_t can be expressed in terms of P_s for any $s \in (0, t)$,

$$P_t = \left[1 + \frac{1-P_s}{P_s}\left(\exp\left(\ln \Lambda_t - \ln \Lambda_s - \frac{\mu_H + \mu_L - \sigma^2}{2}(t-s)\right)\right)^{-\omega/\sigma}\right]^{-1}. \tag{2.3}$$

Since P_t depends on P_s and what happens between s and t, but not on the history of P before s, P_t is a Markov process. The evolution of the process P_t can be described by a stochastic differential equation with respect to a Brownian motion adapted to the filtration generated by Λ_t. To derive this differential equation, we define a new Brownian motion[1]

$$\bar{B}_t \equiv \sigma^{-1}\left(\ln \Lambda_t - \ln \Lambda_0 - \int_0^t E(\mu|\mathscr{F}_s)ds + \frac{1}{2}\sigma^2 t\right), \tag{2.4}$$

from which, the dynamics of Λ_t can be expressed as

[1]Note that the Brownian motion B_t is not measurable with respect to filtration $\{\mathscr{F}_t, t \geq 0\}$, since μ is unknown and therefore, knowing the history of Λ_t up to time t is not sufficient to know the history of B up to time t.

$$d\Lambda_t/\Lambda_t = [\mu_L + P_t(\mu_H - \mu_L)]dt + \sigma d\bar{B}_t. \tag{2.5}$$

Upon applying the Ito's Lemma to (2.2), we obtain

$$dP_t = P_t(1 - P_t)\frac{(\mu_H - \mu_L)}{\sigma}d\bar{B}_t. \tag{2.6}$$

2.2. Investments into Climate Change Adaptation

We consider an investment project with investment cost I that is sunk once committed. The project reduces the frequency of catastrophic events by a proportion k from the investment time until infinity. At the discount rate r, the expected net present value (NPV) of investing in the project at time τ is given by

$$E\left[k\beta \int_\tau^T e^{-rs}\Lambda_s ds - e^{-r\tau}I|\mathscr{F}_0\right], \qquad \text{where } T \to \infty. \tag{2.7}$$

The decision maker seeks the investment strategy that maximizes the expected NPV. Investment strategies may depend on all the information available at the investment time but cannot depend on the information observed after the investment time. This requirement is satisfied when τ is a stopping time. The investment problem is

$$\max_\tau E\left[k\beta \int_\tau^T e^{-rs}\Lambda_s ds - e^{-r\tau}I|\mathscr{F}_0\right] \tag{2.8}$$

subject to

$$d\Lambda_t/\Lambda_t = [\mu_L + P_t(\mu_H - \mu_L)]dt + \sigma d\bar{B}_t, \tag{2.9}$$

$$dP_t = P_t(1 - P_t)\frac{(\mu_H - \mu_L)}{\sigma}d\bar{B}_t. \tag{2.10}$$

This problem has two state variables, P_t and Λ_t that are interrelated. It is not easy to calculate the NPV of a project invested at a given state and determining the value of the option is even more difficult. In what follows, we use measure transform method to simplify the problem.

2.3. Transform problem using measure change

The investment problem is simplified by changing measure P to measure $\tilde{P}$ under which Λ_t has a known and constant growth rate of μ_H. Measure change is achieved by using the Radon Nickodym derivative Z such that

$$d\tilde{P} = ZdP, \tag{2.11}$$

where $Z_t = \exp\left(-\int_0^t \theta_s d\bar{B}_s - \frac{1}{2}\int_0^t \theta_s^2 ds\right)$, and $\theta_t = -(1-P_t)\omega$.
Under $\tilde{P}$, $\tilde{B}_t = \bar{B}_t + \int_0^t \theta_s ds$ is a Brownian motion. The state variable P_t can be replaced by the likelihood ratio $\phi_t = \frac{1-P_t}{P_t}$ that evolves according to the differential equation $d\phi_t = -\omega\phi_t d\tilde{B}_t$.
The investment problem then becomes

$$F(\phi_0, \Lambda_0) = \max_{\tau} \tilde{E}\left[\frac{1}{Z_T}\left(k\beta\int_{\tau}^{T} e^{-rs}\Lambda_s ds - e^{-r\tau}I|(\phi_0, \Lambda_0)\right)\right] \tag{2.12}$$

subject to

$$d\Lambda_t/\Lambda_t = \mu_H dt + \sigma d\tilde{B}_t, \tag{2.13}$$

$$d\phi_t/\phi_t = -\omega d\tilde{B}_t. \tag{2.14}$$

Note that in (2.12), $\eta_T \equiv Z_T^{-1}$ has initial starting point $\eta_0 = 1$ and evolves according to the differential equation $d\eta_t/\eta_t = \theta_t d\tilde{B}_t = d\phi_t/(1+\phi_t)$. Therefore, $\eta_T = \frac{1+\phi_T}{1+\phi_0}$ and ϕ_T is related to Λ_T according to a time dependent relation (which is obtained by solving differential equation 2.13 and 2.14),

$$\frac{\phi_T}{\phi_0} = \left(\frac{\Lambda_T}{\Lambda_0}\right)^{-\omega/\sigma} \exp\left[\frac{\omega T}{2\sigma}(\mu_H + \mu_L - \sigma^2)\right]. \tag{2.15}$$

At state (ϕ_0, Λ_0), the value of the option to invest is

$$F(\phi_0, \Lambda_0) = \frac{1}{1+\phi_0}\max_{\tau} \tilde{E}\left[(1+\phi_T)\left(k\beta\int_{\tau}^{T} e^{-rs}\Lambda_s ds - e^{-r\tau}I\right)|(\phi_0, \Lambda_0)\right], \tag{2.16}$$

and when the state changes to (ϕ_t, Λ_t), the value of the option is

$$F(\phi_t, \Lambda_t) = \frac{1}{1+\phi_t} \max_\tau \tilde{E} \left[(1+\phi_T) \left(k\beta \int_\tau^T e^{-r(s-t)} \Lambda_s ds - e^{-r(\tau-t)} I \right) |(\phi_t, \Lambda_t) \right]. \quad (2.17)$$

To find the option value $F(\phi_t, \Lambda_t)$, we solve the problem

$$G(\phi_t, \Lambda_t) = \max_\tau \tilde{E} \left[(1+\phi_T) \left(k\beta \int_\tau^T e^{-rs} \Lambda_s ds - e^{-r\tau} I |(\phi_t, \Lambda_t) \right) \right] \quad (2.18)$$

subject to (2.13) and (2.14).

The intrinsic value at state (ϕ_t, Λ_t) is the value given when the investment time τ is equal to t in (2.18),

$$V(\phi_t, \Lambda_t) = \tilde{E} \left[(1+\phi_T) \left(k\beta \int_t^T e^{-r(s-t)} \Lambda_s ds - I \right) |(\phi_t, \Lambda_t) \right] \quad (2.19)$$

$$= \tilde{E} \left(k\beta\lambda_t/(r-\mu_H) - (1+\phi_T)I + \phi_T k\beta \int_t^T e^{-r(s-t)} \Lambda_s ds |(\phi_t, \Lambda_t) \right). \quad (2.20)$$

Since ϕ_T is a product of ϕ_t and a martingale (which is obtained using (2.15)),

$$\phi_T = \phi_t \exp \left(-\frac{1}{2}\omega^2 (T-t) - \omega \int_t^T d\tilde{B}_s \right), \quad (2.21)$$

the last component in (2.20) can be written as

$$\phi_t \hat{E} \left(k\beta \int_t^T e^{-r(s-t)} \Lambda_s ds |(\phi_t, \Lambda_t) \right), \quad (2.22)$$

where $\hat{E}$ is the expectation under measure $\hat{P}$ given by

$$d\hat{P} = \hat{Z} d\tilde{P}$$

and

$$\hat{Z} = \exp \left(-\frac{1}{2}\omega^2 (T-t) - \omega \int_t^T d\tilde{B}_s \right).$$

Under measure $\hat{P}$, process Λ_s has growth rate μ_L and the intrinsic value becomes

$$V(\phi_t, \Lambda_t) = (k\beta\Lambda_t/(r-\mu_H) - (1+\phi_t)I + \phi_t k\beta\Lambda_t/(r-\mu_L)). \quad (2.23)$$

At a given state (ϕ_t, Λ_t), the decision is whether to invest and get the intrinsic value $V(\phi_t, \Lambda_t)$ or to defer investment. Deferring investment to the next instant $t + \Delta t$ gives a value $e^{-r\Delta t}\tilde{E}\left[G(\phi_{t+\Delta t}, \Lambda_{t+\Delta t})|\mathscr{F}_t\right]$. The value $G(\phi_t, \Lambda_t)$ at state (ϕ_t, Λ_t) is the larger of intrinsic value obtained by immediate investment and the value obtained by deferring,

$$G(\phi_t, \Lambda_t) = \max\{V(\phi_t, \Lambda_t), e^{-r\Delta t}\tilde{E}\left[G(\phi_{t+\Delta t}, \Lambda_{t+\Delta t})|\mathscr{F}_t\right]\}. \tag{2.24}$$

With Ito Lemma, we can express (2.24) as

$$\max\left(V(\phi_t, \Lambda_t) - G(\phi_t, \Lambda_t), \frac{1}{2}\sigma^2\Lambda_t^2 G_{22} + \frac{1}{2}\omega^2\phi_t^2 G_{11} - \omega\sigma\phi\Lambda G_{12} + \mu_H \Lambda G_2 - rG\right) = 0, \tag{2.25}$$

where $G_1 = \partial G(\phi_t, \Lambda_t)/\partial\phi_t$, $G_2 = \partial G(\phi_t, \Lambda_t)/\partial\Lambda_t$ and $G_{22} = \partial^2 G(\phi_t, \Lambda_t)/\partial\Lambda_t^2$. The value of the option can be found by solving the stochastic differential equation

$$\frac{1}{2}\sigma^2\Lambda_t^2 G_{22} + \frac{1}{2}\omega^2\phi_t^2 G_{11} - \omega\sigma\phi\Lambda G_{12} + \mu_H\Lambda G_2 - rG = 0, \tag{2.26}$$

subject to boundary conditions

$$G(\phi_t^*, \Lambda_t^*) = V(\phi_t^*, \Lambda_t^*) \tag{2.27}$$

$$G_1(\phi_t^*, \Lambda_t^*) = V_1(\phi_t^*, \Lambda_t^*) \tag{2.28}$$

$$G_2(\phi_t^*, \Lambda_t^*) = V_2(\phi_t^*, \Lambda_t^*) \tag{2.29}$$

$$G(0, \Lambda) = \frac{I}{\alpha_H - 1}\left(\frac{\alpha_H - 1}{\alpha_H}\frac{k\beta\Lambda/(r - \mu_H)}{I}\right)^{\alpha_H} \tag{2.30}$$

$$G(\infty, \Lambda) = \frac{I}{\alpha_L - 1}\left(\frac{\alpha_L - 1}{\alpha_L}\frac{k\beta\Lambda/(r - \mu_L)}{I}\right)^{\alpha_L}. \tag{2.31}$$

In general cases, the stochastic differential equation (2.26) does not have a closed form solution. The option value can be found by applying finite difference methods to (2.26) or lattice method to (2.24). Note that for lattice method, starts from an initial state (ϕ_0, Λ_0), the value of ϕ_t can be determined based on Λ_t, and the binomial tree is a simple one state tree. This is much simpler than solving (2.26) with two state variables. We therefore use binomial lattice method to calculate option values and the expected waiting time. We use 100 years for investment time horizon. Further increases in time horizon were found not to change the calculated investment thresholds.

2.4. Special case

As shown in (2.15), when $\mu_H + \mu_L = \sigma^2$, the two state variables of the investment problem maps one to one in a time homogeneous relation. The problem has effectively one state variable Λ_t and the optimal stopping time takes the form of stopping when Λ_t exceeds the optimal threshold Λ^*. The stochastic diffferential equation (2.26) is reduced to an ordinary differential equation and the optimal threshold Λ^* is given by

$$k\beta\Lambda^* = \frac{p_0 \Lambda^{*\omega/\sigma}\alpha_H + (1-p_0)\Lambda_0^{\omega/\sigma}\alpha_L}{p_0(\alpha_H - 1)\Lambda^{*\omega/\sigma}/(r-\mu_H) + (1-p_0)(\alpha_L - 1)\Lambda_0^{\omega/\sigma}/(r-\mu_L)} I, \tag{2.32}$$

where $\alpha_i = \frac{1}{2} - \mu_i/\sigma^2 + \sqrt{(\mu_i/\sigma^2 - \frac{1}{2})^2 + 2r/\sigma^2}, \qquad i \in \{H, L\}$.
The value of the option is given by

$$F(\Lambda_t) = P_t \left(\frac{\Lambda_t}{\Lambda^*}\right)^{\alpha_H} \left(\frac{k\beta\Lambda^*}{r-\mu_H} - I\right) + (1-P_t)\left(\frac{\Lambda_t}{\Lambda^*}\right)^{\alpha_L}\left(\frac{k\beta\Lambda^*}{r-\mu_L} - I\right). \tag{2.33}$$

For planning purposes, it may be useful to know the expected waiting time. When $(\mu_H - \frac{1}{2}\sigma^2) > 0$, the expected waiting time is given by (see Appendix for proof)

$$E\tau_{\Lambda^*} = (\mu_H - \frac{1}{2}\sigma^2)^{-1} \ln \frac{\Lambda^*}{\Lambda_0}. \tag{2.34}$$

3. Empirical Application

To illustrate the application of the model, we use the model to determine the optimal time to invest in a bushfire risk reduction project in Ku-ring-gai, a local area of Sydney, Australia. The area has residential properties in close proximity to bushland and ranks third in bushfire vulnerability among the 61 local government areas in the Greater Sydney Region.

A number of options has been identified by Ku-ring-gai Council to reduce the risks from bushfires. These include, among others, building new fire-trails, constructing new fire-stations and rezoning land, see KC (2010). Fire trails allow for controlled hazard reduction burning, break wild fire transition and potentially allow more time for fire brigades to respond to bushfires. Constructing more fire stations reduces the response time and, thus, may also significantly reduce the risk of a fire to become more severe. In the following, we will focus on evaluating an adaptation project that involves the construction of additional fire trails to illustrate the proposed framework and provide economic insights on the optimal timing of the investment.

3.1. Parameter Estimation

Bushfires are rare events, especially at the local level. This makes the task of relating fire risk to climate conditions especially challenging. To establish a reliable relationship between climate conditions and fire risk, we extend the scope of the statistic model to the national level so that all bushfire events that have occurred in five states of Australia(NSW, VIC, TAS, SA, ACT) since 1970 can be used. These events are reported in media together with details of the number of houses lost, the location where they occurred and the associated weather conditions. They are collated in Blanchi et al. (2010).Together with these data, we use daily climate variables provided by Bureau of Meteorology (Lucas, 2010) and the number of houses in each state provided by Australian Bureau of Statistics in estimating bushfire frequency and severity.

In the frequency estimation, we use Poisson regression in which the logarithm of Poisson intensity depends linearly on covariates. We hypothesize that the occurrence of a fire event in a given state depends on weather conditions (maximum temperature of the day, windspeed measured at 3pm, cumulated amount of rainfall, humidity measured at 3pm), the number of houses in the state[2], risk mitigation activities proxied by national gdp, and their lags. In addition, dummies for state are included to reflect the impact of factors not observed in this model. After excluding in-significant variables (except for lnhouse and gdp), we are left with seven variables as shown in Table 1.
Weather variables that have important impacts on bushfire risk include maximum temperature, rainfall accumulated over the last six days, and wind speed. We found that with all factors being the same, the risk that a fire event occurring in NSW or TAS is higher than in VIC while the risk in SA and ACT are the same as in VIC. The number of houses is found to increase bushfire risk although the relationship is not quite strong. Mitigation

[2] We use the lnhouse instead of house variable since with lnhouse, the risk is zero when house is zero

activities represented by GDP is found to reduce bushfire risk, but the relationship is not significant.

Table 1: Estimation Results of Loss Frequency

	Estimate		p value
(Intercept)	-25.4667	***	0
dnsw	0.8519	**	0.03
dtas	2.5561	***	0
tmax	0.3419	***	0
rainc6	-0.0727	*	0.07
wind	0.0959	***	0
lnhouse	0.4054		0.18
gdp	0		0.85
Pseudo R-squared	0.46		

Bushfire risk for the study region is computed by substitute local data (dnsw = 1, house = 30000) into the regression model. Daily risk is then aggregated to give annual Poisson intensity. We then use maximum likelihood to estimate the drift and local standard deviation of $\ln \Lambda_t$. The estimated drift is taken as μ_H and the zero drift corresponding to no climate change is taken as μ_L, i.e. $\mu_H = 0.015$, $\mu_L = 0$. The estimated standard deviation is $\sigma = 0.16$

For loss severity, we model the number of houses lost in a bushfire event to follow a Lognormal distribution whose parameters are dependent on covariates. The model is estimated using the GAMLSS framework developed by Rigby and Stasinopoulos (2005). The set of covariates used in the frequency model is also used in the severity model and the results are shown in Table 2. We found that GDP and wind speed have significant impacts on the location parameter while maximum temperature and rainfall accumulated over the recent five days have significant impact on the scale parameter. Often, the scale parameter has higher impact on the high quantiles of the distribution while the location parameter has higher impact on the body. Therefore, the results suggest that maximum temperature and cumulating rainfall are significant factors driving large loss events.

Table 2: Estimation Results of Loss Severity

	Estimate		p-value
Mu Coefficients			
(Intercept)	2.56	**	0.03
house	0.37		0.26
gdp	-7.41	**	0.04
wind	0.04	***	0
Sigma Coefficients			
(Intercept)	-3.77	***	0
tmax	0.11	***	0
rainc5	-0.03		0.26
Pseudo R-squared	0.19		

The number of houses in Ku-ring-gai is then used with the regression results to downscale loss distributions to the conditions of Ku-ring-gai. To determine the expected loss for Ku-ring-gai, we use the median of the location and scale parameter. This gives an expected house loss of 39.

Other parameters relating to the investment project, including investment cost, loss mitigation effectiveness and project life, are estimated by expert elicitation. Expert elicitation method is an effective way to overcome data scarcity problems and has been used in many previous climate adaptation studies, see e.g. Baker and Solak (2011); Mathew et al. (2012). The expert specifies that the conducted project is expected to reduce the frequency of house damaging bushfire events by 20%. The estimated costs for a finite lifetime project can be used to calculate the investment cost of an infinite lifetime project as follows. First, we convert the investment cost I_M of a project that lasts M years into an annuity flow, A:

$$A = I_M \frac{1-\beta}{1-\beta^{M+1}},$$

where $\beta = 1/(1+r)$. The annuity A is then used to calculate the investment cost of an infinite life project:

$$I = A(1+r)/r. \tag{3.1}$$

Thus, at a 5% discount rate, the present value of building a bushfire trail every 50 years, each costing \$1.5 million to build is \$1.64 million.

Table 3: Information

Parameters	Value
Current Poisson intensity ($\lambda(0)$)	0.06
Risk mitigation by project (*1-k*)	20%
Lifetime of the project (M)	50 years
Investment cost per project (I_M)	\$1.5 million
Project maintenance cost (C)	\$50,000
Discount rate (r)	5%

3.2. Empirical Results

3.2.1. Baseline case

Figure 1 provides the plot of the option value $F(\Lambda_0)$ for the baseline set of parameters where the initial belief is $p_0 = 0.5$. For this case, at the current level of Poisson intensity $\Lambda_0 = 0.06$, the option value is \$2,145,759 and the optimal investment threshold for the initial period is $L^* = 0.062$. For future periods, the belief P_t is perfectly correlated with L_t and can be determined based on the value of observed L_t using (2.15). Investment decision can be made based on the investment boundary shown in Figure 2 below.

The optimal investment threshold $L^* = 0.062$ is substantially higher than the threshold given by the NPV rule ($\Lambda_0 = 0.0385$). Investing at the NPV rule threshold results in a loss of \$519,648.

3.2.2. Impacts of initial belief

The impact of initial belief is examined by comparing the case of a low initial belief, $p_0 = 0.1$, with the case of high initial belief, $p_0 = 0.9$, see Figure 2. With a higher initial belief, both the project value and the option value increase. The investment threshold, on the other hand, is lowered from 0.0645 to 0.059. The reduction in the investment threshold can be explained by the perfect, positive correlation between P_t and L_t. This means that if we are initially at the investment threshold and L_t decreases, then there will be an increase in P_t that can perfectly compensate for the decrease in L_t to put us on the investment threshold again. In the (P, L) space, the investment threshold is a decreasing line. This applies to any t and so applies to $t = 0$.
To enable comparison with other factors, we also examine the impact of an increase in initial belief by 10%. For this higher belief, the investment threshold is 0.0555 and the

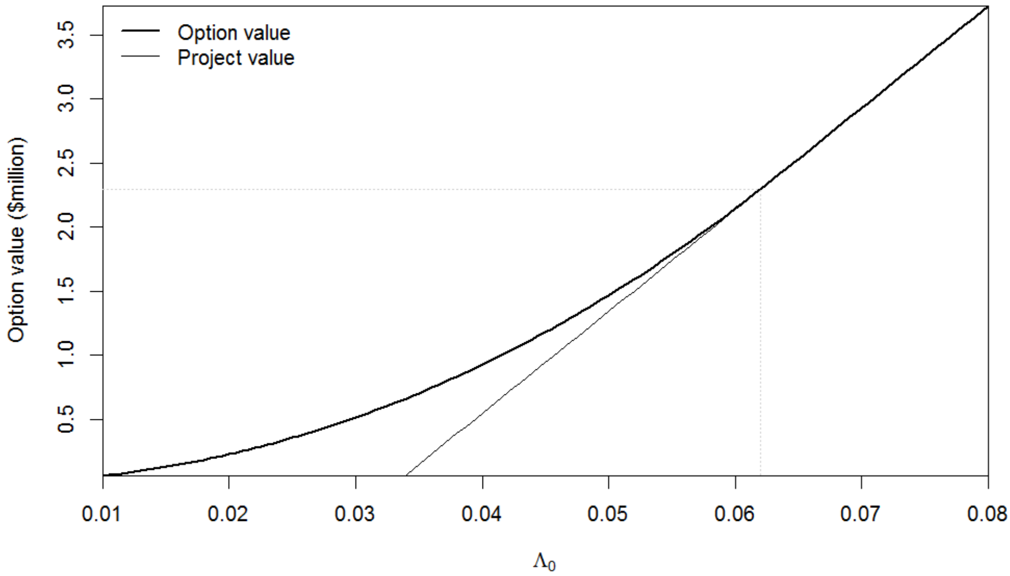

Figure 1: The value of investment option versus the project value for different values of Poisson intensity Λ_0. The marked value of Λ_0 where the option value is first equal to NPV is the optimal investment threshold.

option value at $\Lambda_0 = 0.06$ is increased by 3.85%.

3.2.3. Impacts of uncertainty

Uncertainty about climate change and therefore the growth of Poisson intensity Λ is represented by $\mu_H - \mu_L$ in our model. Analytical examination of uncertainty impact is possible for the special case where the investment threshold for a period t with state (ϕ_t, Λ_t) is given by (2.32) and can be rewritten as

$$k\beta\Lambda^* = \frac{P^*\alpha_H + (1-P^*)\alpha_L}{P^*(\alpha_H - 1)/(r-\mu_H) + (1-P^*)(\alpha_L - 1)/(r-\mu_L)} I \tag{3.2}$$

$$= \frac{IE\alpha(\mu)}{E[(\alpha(\mu)-1)(r-mu)]}, \tag{3.3}$$

where $\alpha(\mu) = \frac{1}{2} - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - \frac{1}{2})^2 + 2r/\sigma^2}$, $P^* = 1/(1+\phi^*)$ and $\phi^* = \phi_0 \left(\frac{\Lambda^*}{\Lambda_0}\right)^{-(\omega/\sigma)}$.

Uncertainty affects the investment threshold in two ways. On the one hand, for a given P^*, since $\alpha(\mu)$ is a convex function, Λ^* decreases when uncertainty in μ increases. On the other hand, with an increase in uncertainty, the signal to noise ratio ω increases and P^* increases. The increase in P^* results in a higher investment threshold. Whether the

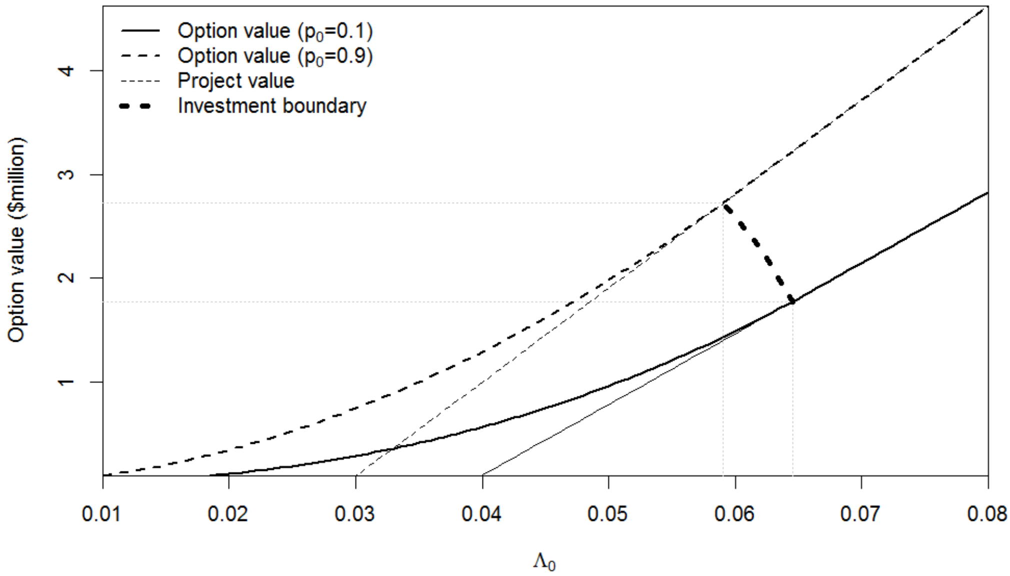

Figure 2: The value of investment option versus the project value for a low initial belief ($p_0 = 0.1$) and a high initial belief ($p_0 = 0.9$). The low initial belief case is drawn with continuous line while the high initial belief is drawn with dashed lines. The heavy dotted curve is the investment boundary that contains the optimal thresholds defined in Λ_0 when p_0 varies from 0.1 to 0.9.

investment threshold increases or decreases with uncertainty depends on actual parameter values.

For the case of bushfire risk, the impact of increasing the uncertainty by 10% is quite small and to provide visible impact, we examine the case in which $\mu_H - \mu_L$ doubles while the average growth rate $\frac{\mu_H+\mu_L}{2}$ remains the same. As shown in Figure 3, given the higher uncertainty, the optimal investment threshold increases to 0.0635 and the option value at $\Lambda_0 = 0.06$ increases by 24%.

3.2.4. Impacts of volatility

When μ is known and constant, it is well known from the real option literature that the value of the option increases with volatility σ. For the case of incomplete information where the true level of drift μ is unknown, volatility has two opposite impacts. On the one hand, a higher volatility will increase the value of the option in extreme cases i.e. $\mu = \mu_H$ or $\mu = \mu_L$. On the other hand, a higher volatility will reduce the signal to noise ratio ω and reduce the value of the option. The net impact will depend on the empirical set of parameters. For the current application, when volatility increases by 10%, the

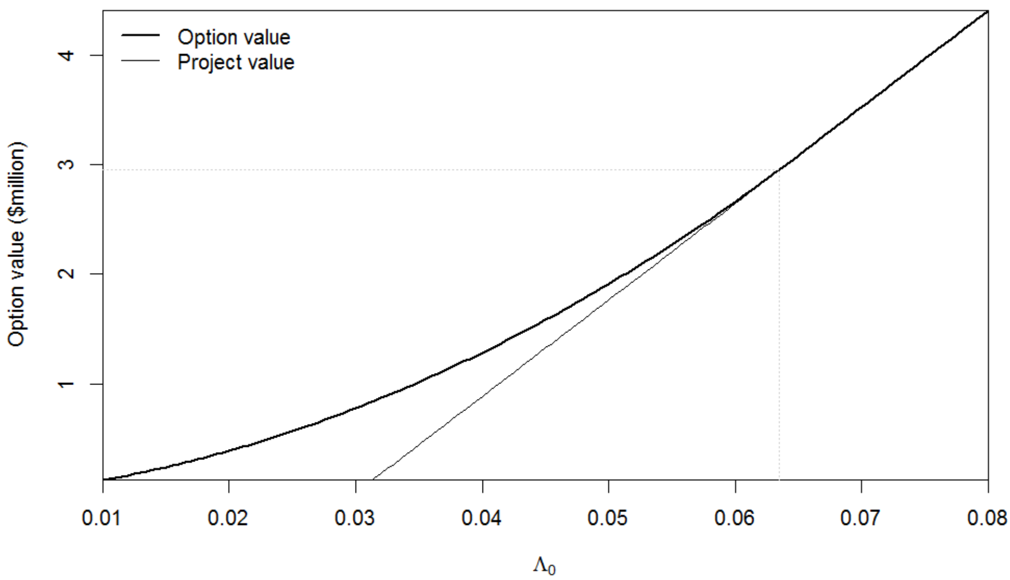

Figure 3: Option value and project value when uncertainty represented by $\mu_H - \mu_L$ doubles while average growth rate $(\mu_H + \mu_L)/2$ remains the same.

optimal investment threshold increases significantly to 0.065 while the option value at $L_0 = 0.06$ increases mildly by 0.8% (Figure 4).

3.2.5. More serious climate change

Many climate change studies, see e.g. Weitzman (2009); Keller et al. (2004), have suggested that the magnitude of change may be larger than predictions by statistical models. We examine a more serious climate change scenario in which the high growth rate is increased by 10%. As a result, the option value at the current level of Poisson intensity, $\Lambda_0 = 0.06$, is increased by 6% (from \$2,145,759 in the basecase) and the investment threshold is reduced to 0.0615. The mild increase in the option value is partly because the intial belief is set at the average level. With a higher initial belief, $p_0 = 0.9$, the increase in the option value would be 8 %.

3.2.6. Impacts of Investment cost

The impact of an increase in investment cost is shown in Figure 6. When investment cost increases by 10%, the investment threshold increases significantly to 0.068 and the option value at Λ_0 is reduced by 9.8%. Investment cost, therefore, has important impact on the value obtained from investing and the time when the project should be invested.

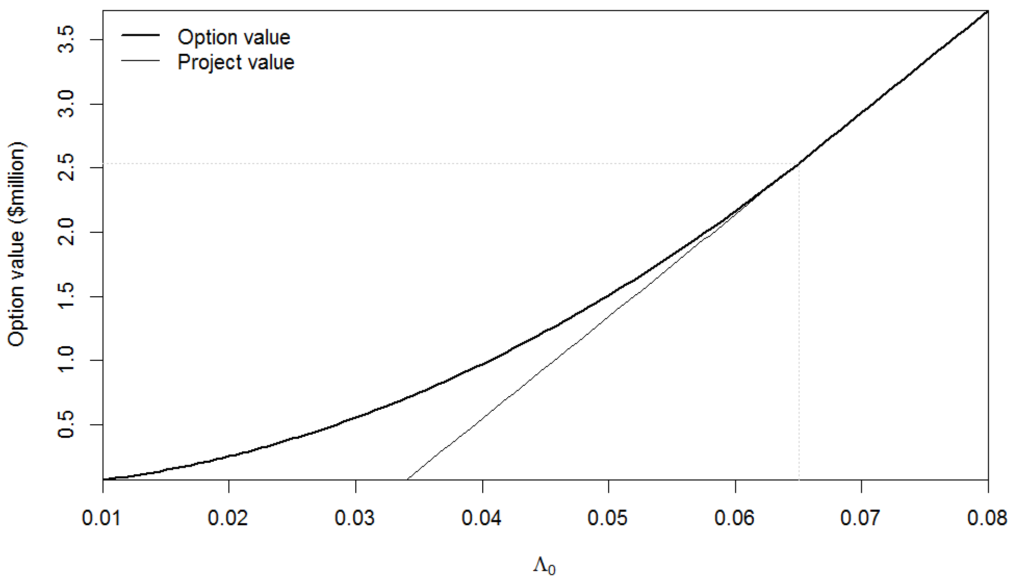

Figure 4: Option value and project value when volatility increase by 10% (from 0.16). The marked value of Λ_0 where the option value is first equal to NPV is the optimal investment threshold.

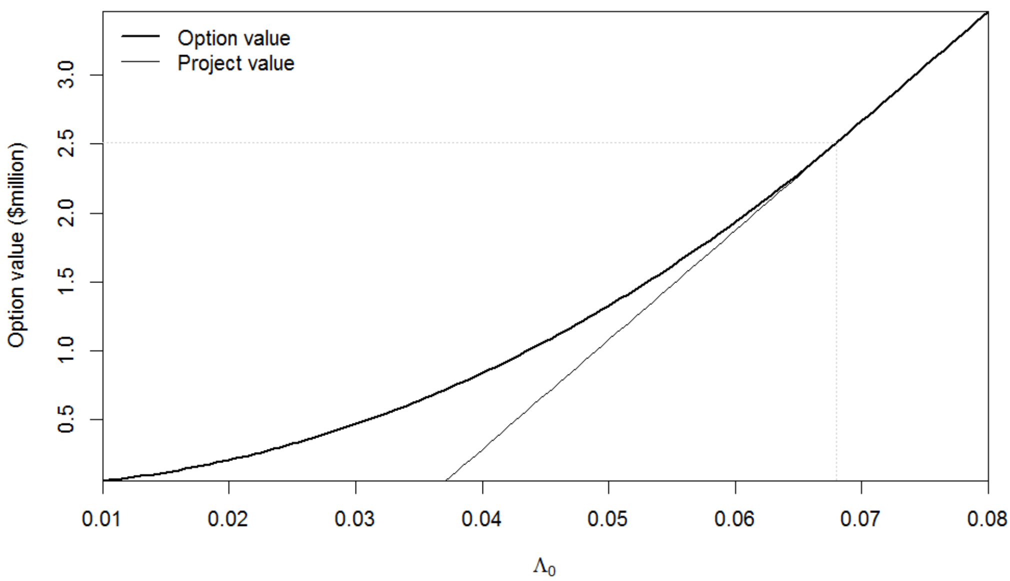

Figure 6: Option value and project value when investment cost increases by 10% (from 0.16). The marked value of Λ_0 where the option value is first equal to NPV is the optimal investment threshold.

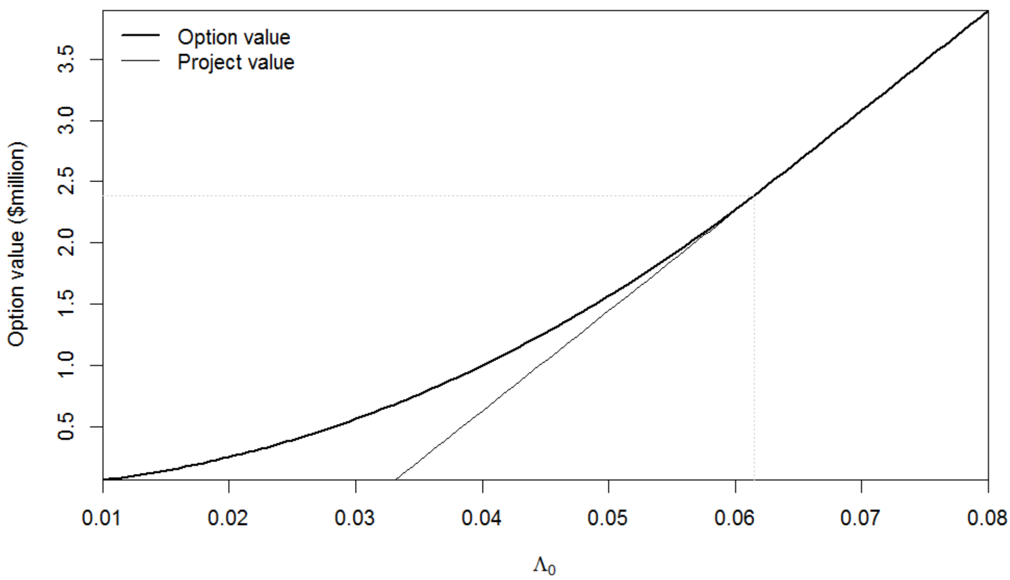

Figure 5: Option value and project value when the high growth rate μ_H increase by 10% (from 0.16). The marked value of Λ_0 where the option value is first equal to NPV is the optimal investment threshold.

3.2.7. Impacts of discount rate

The impact of discount rate on the results is examined by comparing the baseline scenario with the case where discount rate increases by 10%. As a result of the higher discount rate, investment threshold is increased to 0.0635 and the option value at $\Lambda_0 = 0.06$ is significantly reduced by 18.53%.

4. Conclusion

In this paper, we introduce a modeling framework to determine the optimal investment time for catastrophic risk reduction project. The model allows to incorporate the impact of uncertainty and continuous information updating into investment decision. In addition, the closed form solution to the investment problem is available when parameter values satisfy a certain requirement. In this framework, the impact of uncertainty on investment threshold is indeterminate. Uncertainty can accelerate or decelerate investment, depending on the actual values of parameters.

We illustrate the application of the model with a case study of bushfire risk management in a local government area in Sydney, Australia. Catastrophic risk is quantified using the GAMLSS framework that allows covariates to influence all parameters of loss distribu-

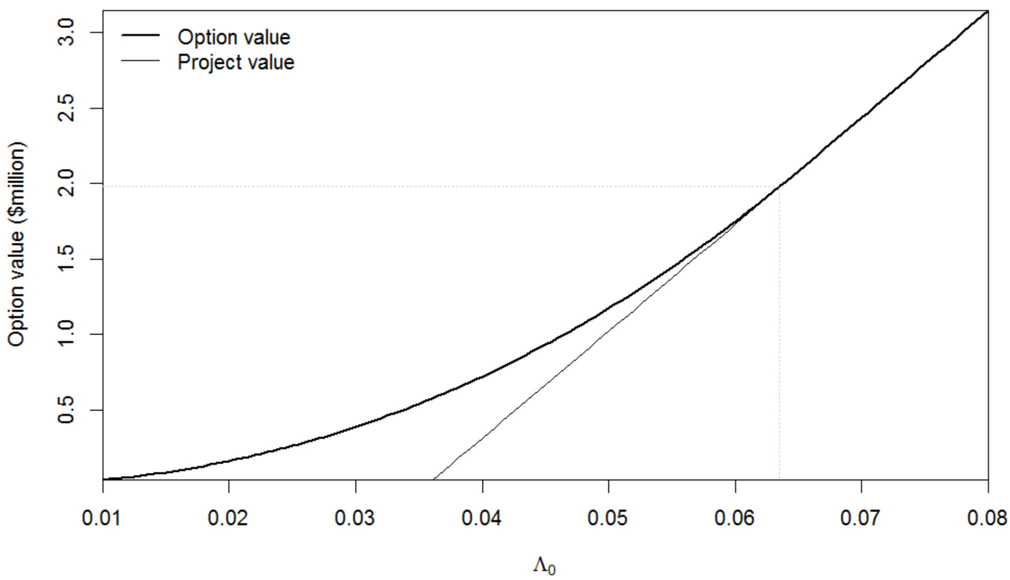

Figure 7: Option value and project value when discount rate increases by 10% (from 0.16). The marked value of Λ_0 where the option value is first equal to NPV is the optimal investment threshold.

tion, not only the mean. We found that ignoring uncertainty would results in significant loss. The loss is proportional to the initial belief in climate change, uncertainty, investment cost, discount rate and inversely proportional to the seriousness of climate change. Optimal investment threshold and option values are found to be sensitive to investment cost and discount rate but not sensitive to changes in uncertainty.

Appendix A. Expected waiting time

We will show that for a process $dX_t = adt + \sigma dB_t$, where B_t is a Brownian motion, $X_0 = x > 0$, a stopping time $\tau_m = \min\{t \geq 0 : X_t \leq m\}$, and a scalar $u > 0$,

$$Ee^{-u\tau_m} = \exp\left[\frac{-a + \sqrt{a^2 + 2u\sigma^2}}{\sigma^2}(x - m)\right]. \tag{A.1}$$

Then, taking the limit $\lim_{u\downarrow 0} \frac{\partial Ee^{-u\tau_m}}{\partial u}$ gives

$$E\tau_m = \frac{m - x}{a}. \tag{A.2}$$

This result holds only when $a > 0$ since if $a < 0$, it is well known that there is a chance the process X_t wanders off to $-\infty$ and $E\tau_m$ is infinite.
Applying the above result to the process $\lambda_t = \ln \Lambda_t$ that has drift $a = \mu_H - \frac{1}{2}\sigma^2$ and the hitting boundary $m = \ln \Lambda^*$, the expected waiting time becomes

$$E\tau_{\Lambda^*} = (\mu_H - \frac{1}{2}\sigma^2)^{-1} \ln \frac{\Lambda^*}{\Lambda_0}. \tag{A.3}$$

We now provide result (A.1). We begin by defining $f(x) = E(e^{-u\tau_m}|X_0 = x)$ and take u as a value of the discount rate. We then choose dt sufficiently small so that X is unlikely to hit the level m in the next short time interval dt. Then the expected discount factor after dt is $E(f(x + dx))$ and

$$f(x) = e^{-udt}E(f(x + dx)) = (1 - udt))(f(x) + f_x dx + \frac{1}{2} f_{xx}(dx)^2), \tag{A.4}$$

which implies that $f(x)$ satisfies the stochastic differential equation

$$\frac{1}{2}\sigma^2 f_{xx} + af_x - uf = 0. \tag{A.5}$$

The solution to this equation is

$$f(x) = Ae^{\psi_1 x} + Be^{\psi_2 x}, \tag{A.6}$$

where $\psi_1 = \frac{-a-\sqrt{a^2+2r\sigma^2}}{\sigma^2}$ and $\psi_2 = \frac{-a+\sqrt{a^2+2r\sigma^2}}{\sigma^2}$.
When x is very large, τ_m is likely to be small and $e^{-u\tau_m}$ is close to 0, i.e. $f(\infty) = 0$. For this to occur, $B = 0$. As x approaches m, τ_m is likely to be small and $e^{-u\tau_m}$ is close to 1, i.e. $f(m) = 1$. This means $A = e^{-\psi_1 m}$ and $f(x) = e^{\psi_1(x-m)}$, i.e.

$$Ee^{-r\tau_m} = \exp\left[-\frac{a + \sqrt{a^2 + 2r\sigma^2}}{\sigma^2}(x - m)\right]. \tag{A.7}$$

References

Baker, E., Solak, S., 2011. Climate change and optimal energy technology r&d policy. European Journal of Operational Research 213 (2), 442–454.

Blanchi, R., Lucas, C., Leonard, J., Finkele, K., 2010. Meteorological conditions and wildfire-related houseloss in australia. International Journal of Wildland Fire 19 (7), 914–926.

Bouwer, L. M., Bubeck, P., Aerts, J. C., 2010. Changes in future flood risk due to climate and development in a dutch polder area. Global Environmental Change 20 (3), 463–471.

Brouwer, R., van Ek, R., 2004. Integrated ecological, economic and social impact assessment of alternative flood control policies in the Netherlands. Ecological Economics 50 (1-2), 1–21.

Chao, P. T., Hobbs, B. F., 1997. Decision analysis of shoreline protection under climate change uncertainty. Water Resources Research 33 (4), 817–829.

Décamps, J.-P., Mariotti, T., Villeneuve, S., 2005. Investment timing under incomplete information. Mathematics of Operations Research 30 (2), 472–500.

Garnaut, R., 2011. The Garnaut review 2011: Australia in the global response to climate change. Cambridge University Press.

KC, K.-r.-g. C., 2010. Climate change adaptation strategy. Tech. rep., Ku-ring-gai Council.

Keller, K., Bolker, B. M., Bradford, D. F., 2004. Uncertain climate thresholds and optimal economic growth. Journal of Environmental Economics and Management 48 (1), 723–741.

Kirshen, P., Knee, K., Ruth, M., 2008a. Climate change and coastal flooding in metro boston: impacts and adaptation strategies. Climatic Change 90 (4), 453–473.

Kirshen, P., Ruth, M., Anderson, W., 2008b. Interdependencies of urban climate change impacts and adaptation strategies: a case study of metropolitan boston USA. Climatic Change 86 (1-2), 105–122.

Klein, M., 2009. Comment on investment timing under incomplete information. Mathematics of Operations Research 34 (1), 249–254.

Lucas, C., 2010. On developing a historical fire weather data-set for australia. Australian Meteorological and Oceanographic Journal 60 (1), 1.

Mathew, S., Trück, Henderson-Sellers, A., 2012. Kochi, india case study of climate adaptation to floods: Ranking local government investment options. Global Environmental Change 22 (1), 308 – 319.

Michael, J. A., 2007. Episodic flooding and the cost of sea-level rise. Ecological Economics 63 (1), 149–159.

Murphy, B. F., Timbal, B., 2008. A review of recent climate variability and climate change in southeastern Australia. International Journal of Climatology 28 (7), 859–879.

Rigby, R., Stasinopoulos, D., 2005. Generalized additive models for location, scale and shape. Journal of the Royal Statistical Society: Series C (Applied Statistics) 54 (3), 507–554.

Symes, D., Akbar, D., Gillen, M., Smith, P., 2009. Land-use mitigation strategies for storm surge risk in south east queensland. Australian Geographer 40 (1), 121–136.

Venkatesh, B. N., Hobbs, B. F., 1999. Analyzing investments for managing lake erie levels under climate change uncertainty. Water Resources Research 35 (5), 1671–1683.

Waters, D., Watt, E., Marsalek, J., Anderson, B., 2003. Adaptation of a storm drainage system to accommodate increased rainfall resulting from climate change. Journal of Environmental Planning and Management 46 (5), 755–770.

Weitzman, M. L., 2009. On modeling and interpreting the economics of catastrophic climate change. The Review of Economics and Statistics 91 (1), 1–19.

West, J. J., Small, M. J., Dowlatabadi, H., 2001. Storms, investor decisions, and the economic impacts of sea level rise. Climatic Change 48 (2-3), 317–342.

Zhu, T. J., Lund, J. R., Jenkins, M. W., Marques, G. F., Ritzema, R. S., 2007. Climate change, urbanization, and optimal long-term floodplain protection. Water Resources Research 43 (6).