Should I kill Claudius?
On the impossibility of constructing rational preferences

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Abstract

In standard models of rational choice it is typically taken for granted that preferences are given and defined over the alternatives alone, and the possibility of making a rational choice is simply a matter of assumption. In this paper I generalise this aspect of the economic model so that preferences over alternatives are constructed from given preferences defined over various characteristics of the alternatives under consideration. I characterise the decision problem before investigating what conditions a procedure for aggregating preferences over attributes into preferences over alternatives must satisfy in order for the latter to be rational. I then consider what the implications of these conditions for the procedural rationality of the aggregation process.

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1 Choice in the presence of multiple attributes

To be, or not to be, that is the question

Whether 'tis Nobler in the mind to suffer

The Slings and Arrows of outrageous Fortune,

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Or to take Arms against a Sea of troubles,

And by opposing end them?

So speaks Hamlet, wrestling with the question of whether or not he must avenge his father by killing the King. If he makes the wrong decision, he will have contravened the law of God either by murdering the rightful King of Denmark, or by failing to take action against a usurper who has violated the divine right of Kings and their scion to rule. His agonising comparison of the various merits and detriments of "take Arms against a Sea of troubles" or "in the mind to suffer" consume him, driving his beloved Ophelia into the madness that will eventually lead to her death.

What Shakespeare shows us here is how the process of coming to a rational decision by weighing up the various costs and benefits of the alternatives under consideration can be utterly exhausting, even destructive. But, according to the economic theory of decision Hamlet should have simply have made a rational choice guided by his given preference between the two alternatives. In this paper I extend the economic theory of decision so that we can understand why the process of constructing a rational preference relation from the various attributes of the alternative choices at hand may become so difficult as to become irrational.

The existence of a well behaved preference relation guiding rational choice provides the foundation for nearly every standard model of economic behaviour. Even the increasingly prominent "behavioural" economics research program takes rational choice to be fundamental (see Rabin, 2013a,b). Typically, we simply assume that such choice is possible by constructing "given" binary preferences between alternatives under consideration for any individual which conform to the axioms of completeness and transitivity or by specifying a utility function representing them (Mas-Collel et al., 1995, Ch.2). The existence of preference between the alternatives is taken to be given, primitive to the model, a fact.

The assumptions implicit in taking such preferences to be primitive were relaxed briefly by economists developing the now somewhat neglected "New Consumer Theory". This theory was initially developed in ground-breaking work by Lancaster (1966) and Ironmonger (1972), who realised that what distinguishes one alternative from another is the differentiation of their characteristics and the preferences over these characteristics. Preferences for alternatives will therefore be determined by preferences for their mix of attributes. The new consumer theorists developed this idea into a theory of demand by defining utility functions over "characteristics" spaces and, via a "technology" representing the characterisation of alternatives by points in that space, utility functions over the alternatives themselves.

A cursory Google Scholar search reveals that Lancaster's original paper in the *Journal* *of Political Economy* has at the time of writing[1] received almost 8000 citations[2]. It provides a theoretical basis for the hedonic pricing literature built around Rosen (1974) which estimates demand as a function of objective attributes (Ratchford, 1975, p.10), as well as the multiple-attribute approach common in marketing (for the extensive early literature see Wilkie and Pessemier (1973)). It is curious that in spite of its massive influence (Lancaster (1990) provides a survey of the influence within economics alone), the concept of consumer preferences for product attributes has not become a feature of the major textbooks on economic theory (the most prominent being, arguably, Mas-Collel et al. (1995) and Jehle and Reny (2011)). Even Deaton and Muellbauer (1980) takes only 29 pages (including chapter exercises and bibliography) of a 450 page book within Chapter 10 of 14, to the subject, though even there the discussion of the original Lancaster/Ironmonger variant extends only to four pages. It is perhaps appropriate as the 50th anniversary of the publication of the seminal 1966 paper approaches to revisit the benefits which the New Consumer Theory brought to economics, and investigate some implications of the concept that the New Consumer Theorists themselves did not explore.

The greatest beauty of the "New Consumer Theory" was that it provided a means by which the new goods and services generated by a constantly evolving economy could be incorporated into the preference structure in a fairly systematic manner. If a standard approach is taken to preference then any theory of demand in an evolutionary economy must make an outright assumption about the integration of new alternatives into the preferences of individuals. For instance, in the theory of "growing awareness" of new elements of state space, von Neumann-Morgenstern utility can be extended to the new state space domain (Karni and Viero, 2013), but there is no theory governing the value the extended utility function assigns to those new elements. In the New Consumer Theory, the only assumptions required for a new alternative to be incorporated into the preference structure is that it be characterised by attributes, and that the consumer has some manner of transforming comparisons of attributes into preferences over alternatives.

Another major benefit of the New Consumer Theory was that it explained substitutability and product differentiation. The substitutability of two products depends on how close they are to each other in "characteristic space", while the demand for differentiated products depends on their differentiation in this space and preferences over that space (Ratchford,

[1] August 2014.

[2] By way of contrast, a similarly cursory Google Scholar search reveals that his arguably better known paper on *The general theory of second best* with Richard Lipsey has received at the same time a little over 2700 citations, and more in fact than either of John Nash's seminal papers in 1950 on equilibria in games *The bargaining problem* and *Equilibrium points in n-person games* (a little over 6500 and a little over 4700 respectively).

1975). The closer two products in characteristic space, the more indifferent the consumer will be between them.

The new consumer theory therefore gave us a more satisfactory theory of consumer demand for novel products, and for why certain products were substitutable for one another. However, in defining utility functions over "characteristics" spaces, and keeping the theoretical discourse in the Euclidean space realm of utility functions which impose rational preference relations by the transitivity of the real numbers Lancaster (1966), Ironmonger (1972)[3] imposed possibility of constructing rational preferences over alternatives from the utilities of attributes. They neither characterised their problem on the preference-axiomatic level nor investigate what is required for rational preferences over alternatives to be constructed from preferences over attributes.

If we are to accept that preferences over alternatives are not in themselves primitive but arise from more deeply primitive given preferences defined over the *attributes* which characterise those alternatives, we cannot *prima facie* say that it is always possible to construct a rational preference relation over the alternatives themselves. An interesting paper by Kapeller et al. (2013, p.43) has given a simple, plausible example where the aggregation of otherwise rational preferences over attributes of alternatives aggregates to irrational preferences over the alternatives themselves. It has been demonstrated elsewhere[4] that their claim that rational choice is impossible is based on an erroneous assertion of mathematical equivalence between the problem of choice in the context of multiple attributes and the problem of social choice. Nonetheless, their example, in conjunction with others provided by Anand (1982), serves to show that the possibility of rational choice in the presence of multiple attribute alternatives is not immediately apparent. I endeavour here to characterise conditions under which rational choice is possible in the presence of multiple attribute alternatives.

Accounting for multiple attributes in the process of constructing preferences and making choices places the problem of choice here at the nexus of problems of "procedural rationality" and "substantive rationality". Simon (1976) distinguished "procedural" rationality from "substantive" rationality, which concerns itself whether individuals are making a choice consistent with a given objective (in economics, the maximisation of utility). Procedural rationality, on the other hand, concerns the process of thought in which an individual engages before coming ultimately to a decision. Procedural rationality requires that this process be reasonable not only given the basic objectives of the individual, but also given the computational demands placed on the individual constrained by cognitive ability (Simon, 1976, 1978b,a).

[3] And with a slightly different emphasis, Stigler and Becker (1977), Michael and Becker (1973), Rosen (1974) and Baumol (1967)

[4] Markey-Towler (2014a).

In the problem under consideration here procedural rationality concerns the complexity of the process of constructing preferences over alternatives from preferences over attributes[5]. Substantive rationality concerns the possibility of constructing a rational preference relation between the choice alternatives.

In the next section I formally define the choice problem in the context of multiple attribute alternatives on the level of preference relations. I then provide two rather trivial theorems for conditions which guarantee a rational choice always exists, before providing some more informative and interesting necessary and sufficient conditions for the existence of rational choice. I discuss the meaning of these conditions and argue that constructing a substantively rational preference relation may require such involved procedures for aggregating attribute preferences that substantive rationality comes at the price of procedural irrationality. That is to say, the conditions required of the choice environment and the process itself for constructing substantively rational preferences imply that there is something of a trade-off between substantive and procedural rationality. This is not a new suggestion by itself, it characterises much of Herbert Simon's work, but the presence of this trade-off within the problem of choice in the presence of multiple attributes has not, to my knowledge, been hitherto identified explicitly.

2 The choice problem

In any choice problem the most basic of primitives is a set X of alternatives amongst which a choice must be made. I will assume that this set is finite for simplicity[6]. In economics it is standard to suppose that choice is then guided by a preference relation $\succeq$ which is defined over the set X, such that there exists a set $\succeq(X)$ consisting of the elements of X pre-ordered according to $\succeq$. I will assume throughout that $|X| \geq 3$ so that rationality is never a trivial and uninteresting matter of finding a single binary preference. While it is perfectly possible that such preferences relate only to the *gestalt* of the alternative at hand, it is equally possible that each alternative has various attributes which serve to distinguish it in a more "objective" sense from the others and which influence the determination of $\succeq$. If this is indeed the case then each alternative under consideration will be characterised by its attributes.

[5]It should be stated from the outset however, that the notion of procedural rationality is not as formally clear cut as that of the notion of substantive rationality, so it is difficult to make definitive absolute statements of a "this is procedurally rational solution" nature. Hence the discussion of procedural rationality here will be conducted in a relatively informal manner.

[6]The alternatives set could be extended to incorporate infinitely many alternatives, though additional technical assumptions (either compactness or "single-peaked" preferences) would be required to guarantee the existence of a maximal element, which would complicate the exposition for little gain in generality.

Generalising the concept of attribute spaces suggested by Lancaster (1966) and Ironmonger (1972), if alternatives $\{x\}$ are characterised by their attributes then there exists an arbitrary characteristic space $A = \Pi_{i=1}^{N} A_i$ within which the attributes of x are defined. This space is taken to consist of the cross-product of multiple subspaces which divide A into "classes" represented by one subspace A_i indexed by $\{1, \dots, N\}$, each of which we could say represents a particular "characteristic"[7]. One could say an attribute $a_i \in A_i$ defines an alternative's attribute "for that particular characteristic".

If we were to go further and specify that A were a collection of metric subspaces rather than arbitrary, such that characteristic attributes were contained within the set of real numbers $\mathbb{R}$, we could think of these attribute subspaces as *dimensions* of the characteristic space A, each dimension representing the quantities an alternative may have of a particular characteristic - a "quantity of a quality" if you will. However, for what follows I will leave A to be an arbitrary space[8].

Formally, we can now define an alternative to be characterised by the set of its attributes, $x = \{a_i\}_{i=1}^{N}$, organised into a vector $\{a_i\}_{i=1}^{N} \in \Pi_{i=1}^{N} A_i$ within the characteristic space. This definition can be reversed to define $a_i(x)$ as the attribute of x within the characteristic subspace A_i. Implicit in this specification is the assumption, carried throughout what follows, that every alternative x is well defined in every characteristic subspace A_i by an attribute $a_i(x)$. This is largely a philosophical distinction to state that the lack of a particular characteristic in itself constitutes an attribute, and hence a point within a characteristic subspace[9].

If $A = \Pi_{i=1}^{N} A_i$ is a space which characterises alternatives X and their differentiation here has meaning, it follows that an individual will have preferences $\succeq_i$ defined over each characteristic subspace A_i in much the same manner as over the alternatives space. Then we have a set of attribute preferences $\{\succeq_i\}_{i=1}^{N}$ defined over each individual attribute subspace $A_i \subset A$ which pre-order the characteristics space A into a set $\Pi_{i=1}^{N} \{\succeq_i (A_i)\}$ of pre-ordered attribute vectors. These preferences I take to be much the same as standard preferences and to have the same interpretation such that we can read $a_i(x) \succeq_i a_i(x')$ as "attribute a_i of x is at least as preferred as attribute a_i of x'".

To make the problem more tractable, I will assume for what follows that the pre-orderings $\{\succeq_i\}_{i=1}^{N}$ of the attribute spaces $\{A_i\}_{i=1}^{N}$ are rational, hence complete and transitive. This is not as strong an assumption as might appear at first glance and it gives us a good starting

[7] In this case, $|A| = \sum_{i=1}^{N} |A_i|$.

[8] We need no restrictions on finiteness here because we do not require a maximal element to exist in sets which merely serve to characterise points in another set.

[9] Again, if we were to take $A_i \subset \mathbb{R}$, the implicit assumption I make here is that $0 \in A_i$ so that having zero "quantity" of a particular characteristic is the attribute 0

point for analysing under what conditions underlying attribute preferences aggregate to preferences over alternatives. If we were to have monotonic preferences over a quantitative attribute space A_i (not an entirely unrealistic assumption *ceteris paribus*) then the resulting preference relation between alternatives will be transitive, and if this space were quantised (discrete) and had few points not much information would be required for the preferences to be complete.

It is important here to note an assumption made in specifying each of these preferences $\succeq_i$ over A_i alone, this assumption being that preferences over a particular characteristic subspace are defined independently of attributes in *other* subspaces. This means that if a particular alternative x performs particularly strongly on a particular attribute $a_i(x)$, the preferences $\succeq_{-i}$ over the rest of its attributes $\{a_{-i}(x)\}$ are not affected. This assumption is justifiable on the grounds (other than parsimony) that we are not particularly interested in the attribute preferences *per se*, but rather the manner in which they contribute to preferences over alternatives. This assumption appears much less strong when we recognise choice as a process of locating an alternative within a pre-ordered space of attributes prior to aggregating this location into overall preferences. The aggregation process can vary the importance of certain regions of each space A_i in the determination of $\succeq$, which is in keeping with the idea that preferences over attributes are defined for the attributes alone, and these preferences are used to construct preferences over alternatives.

Now, to obtain a well-defined solution to the problem of choice amongst alternatives in the presence of multiple attribute alternatives, we require some means of aggregating preferences over each attribute subspace A_i into preferences over the alternatives space X. A process, or schema, described by a mapping $f : \Pi_{i=1}^{N} \{\succeq_i (A_i)\} \rightarrow \succeq (X)$, takes the pre-ordered characteristic subspaces and transforms them into a pre-ordered space of alternatives characterised by their attributes within those spaces. This process involves giving a "weight" or "importance" or "priority" to the preferences $\succeq_i$ over attributes in the characteristics subspaces in the determination of preferences $\succeq$ over the alternatives which they characterise. This is a special type of the broad class of "procedures" for thinking about a particular problem of choice which Herbert Simon (1976; 1978b; 1978a) named as the subject matter of the study of "procedural rationality"[10].

A strong assumption will be made concerning the process f in what follows. I take f to be defined across each pre-ordered subspace $\succeq_i (A_i)$ alone, and invariant for all points within this subspace, so that each attribute space is given a certain importance in the

[10]It should be noted that the study of procedural rationality is distinct from the study of "procedural utility", which defines hedonic utility functions over some quantitative representation of the process by which an outcome is arrived at (Benz, 2007), (Frey, 2008, pp.107-126).

determination of preferences. This imposes an ordinal interpretation on preferences and avoids thorny questions of cardinality of preferences within subspaces and across them, so that the individual is simply asking "do I prefer the attributes of x or x' with respect to characteristic A_i?", "is characteristic A_i more important than A_j?" and so on and so forth rather than "*by how much* do I prefer this attribute to that attribute?" and "*how much* more important is this characteristic than that?". This assumption of ordinality makes f a far less cognitively taxing procedure in any case, and if substantive rationality can be found with such a procedure it would rest on fairly reasonable psychological assumptions[11].

Taken together, the objects $\{X \quad A \quad f\}$ make up the primitives of the choice problem, specifying the primitives of a procedure for aggregating preferences over the attributes of various alternatives under consideration into preferences over the alternatives themselves. Choice then consists of selecting an alternative x from X according to some criterion. If this choice is to be "rational" in the substantive, economic sense (Simon, 1976), this criterion requires that the choice, x^*, will be the maximally preferred element in X, or $x^* = \max_{x \in X} \succeq (X)$. However, for this solution to be well-defined (as either a point-valued or set-valued solution) requires that maximal elements exist, and for this to be true of all sets within the power set 2^X we require the preference relation $\succeq$ itself be rational. If this were not true of all sets within 2^X then there would be a subset of alternatives $Y \subset X$ for which no $\succeq$ maximal elements exist due to intransitivity.

Following standard conventions in economics[12], we say that any preference relation $\succeq$ or $\succeq_i$ is rational if it is complete and transitive, that is, either $x \succeq x'$ or $x \preceq x'$ or both $\forall x, x' \in X$ and for any $x, x', x'' \in X$, $x \succeq x' \,\&\, x' \succeq x'' \implies x \succeq x''$ respectively. But given that $\succeq$ is emerging from a procedure f aggregating attribute preferences$\{\succeq_i\}_{i=1}^N$ it is also interesting whether the resultant preference relation is a "trivial" aggregation of these sub-preferences. By a "trivial" preference relation I mean one where a single characteristic subspace has *absolute* supremacy in the determination of the aggregated preference relation over the alternatives characterised by those attributes, so that preferences over alternatives are entirely determined by preferences over attributes in that characteristic subspace. Formally, a *non*-trivial preference relation can be defined thus:

[11] A possible avenue for future research would be to relax this assumption so that weights can vary across the possible attributes within the subspace A_i, and make these weights contingent upon the general location of alternatives in characteristic space, reflecting the idea that in certain regions of the attribute subspace A_i that particular subspace may be of less importance than in other regions, especially given the overall position of the alternative.

[12] See Mas-Collel et al. (1995, p.6) or Jehle and Reny (2011)

Definition. A preference relation is *non-trivial* if and only if

$$\nexists i \in \{1, \ldots, N\} : x \succeq x' \iff a_i(x) \succeq_i a_i(x')$$

In a sense, a trivial preference relation is similar (but *not* equivalent as Kapeller et al. (2013) argue) to a dictatorial social choice rule, in that one preference relation out of many determines the overall preference between alternatives. When a preference relation $\succeq$ is trivial in this manner, one characteristic subspace is dictatorial in the construction of preference between alternatives. But such preference relations $\succeq$ are not particularly interesting in the context of multiple attributes, as the characteristic space is not taken into account in any meaningful way given that the preference relation on alternatives is determined by only one of the characteristics. There is little difference in such cases between choice based on the *gestalt* of the alternative and choice based on consideration of attributes. Choice in the presence of multiple attributes then becomes a distinction without a difference. Hence we can define a class of preference relations of interest according to the following criterion:

Definition. A preference relation $\succeq$ constructed by f from preferences over attributes $\{\succeq_i\}_{i=1}^{N}$ is *non-trivially rational* if and only if

$$\begin{array}{ll} (1) & \forall x, x' \in X \text{ either } x \succeq x' \text{ or } x' \succeq x \text{ or both} \\ (2) & x \succeq x' \text{ and } x' \succeq x'' \text{ imply that } x \succeq x'' \\ (3) & \nexists i \in \{1, \ldots, N\} : x \succeq x' \iff a_i(x) \succeq_i a_i(x') \end{array}$$

The question of central importance then is: under what conditions will rational choice be impossible in any non-trivial sense due to the non-existence of a procedure f which can construct a rational preference relation over alternatives $\succeq$? Or, to put the point more bluntly, under what conditions can we find a procedure f such that substantively rational choice will be possible? What are the requirements upon the underlying preferences over attributes $\{\succeq_i\}_{i=1}^{N}$ and the procedure f by which these preferences are aggregated for the resulting preference relation defined over the alternatives to be non-trivially rational?

Of course, we could guarantee the rationality of $\succeq$ by mapping A into the real numbers using a utility function $u : A \to \mathbb{R}$ representing $\succeq$ directly on A and, by extension X. This is precisely what the New Consumer theorists did. But as should be somewhat clear from the specification above, doing so fails to address questions about what procedures f may be required to construct substantively rational preference. To simply map characteristics into the real numbers imports implicit assumptions concerning the process f of aggregating $\{\succeq_i\}_{i=1}^{N}$ into $\succeq$ which are not at all transparent. Simply assuming substantive rationality precludes us from asking interesting questions about what kind of processes may be required for this and whether they are likely to be procedurally rational.

3 Possibility theorems

It will be useful to construct some additional formal concepts before presenting theorems on the requirements for f and $\{\succeq_i\}_{i=1}^N$ such that$\succeq$ will be non-trivially rational. If f consists of a weighting system for the dimensions on which the attributes characterising the alternatives and preferences over these are defined then the set of dimensions on which an alternative dominates another becomes critical to determining overall preference. Hence we can define a set $S_x(x')$ consisting of a collection of the characteristic subspaces for which the attributes of alternative x weakly dominate those of x'.

Definition. The set of attribute dimensions for which x is at least as preferred as x' is given by $S_x(x') = \{A_i \subset A : a_i(x) \succeq_i a_i(x')\}$.

Note that with the traditional definition of preference relations (Mas-Collel et al., 1995, p.7), if $A_i \in S_x(x') \cap S_{x'}(x)$ then the alternatives are indifferent within that subspace and $A_i \in I_x(x') = \{A_i \subset A : a_i(x) \sim_i a_i(x')\} \subset S_x(x')$.

As per the choice problem outlined above this pre-ordering of A according to $\{\succeq_i\}_{i=1}^N$ must be aggregated by f into $\succeq$, and this requires that f assign dominance of certain characteristic subspaces over others in the aggregation of attribute preferences to determine preference over the alternatives. Informally, a set of attribute subspaces can be said to dominate another when agreement of preferences between the attributes of two alternatives in the former determines the preference between the two alternatives. This dominance relation I will denote as $d.(f)$, so that we say $A'd.(f)A''$ reads as "$A' = \{A_i \in A'\} \subset A$ dominates $A'' = \{A_i \in A''\} \subset A$ in the aggregation by procedure f of preferences over attributes into preferences over alternatives". Formally dominance is defined as follows

Definition. For any two mutually exclusive collections of characteristic subspaces $A', A'' \subset A$, $A' \subset A$ dominates $A'' \subset A$ when agreement of preferences within the first set determines the preference between alternatives, that is, $A'd.(f)A'' \iff A' \subset S_x(x') \implies x \succeq x'$.

Note that non-triviality can be restated using this definition, for if it is the case that no single preference relation over attributes has absolute supremacy in the determination of preferences over alternatives all the others for some f, then no one attribute subspace dominates all the others combined. However, if one attribute subspace dominates all others in the aggregation it will dominate any single one of them also. Hence $\succeq$ is non-trivial if and only if $\nexists A_i \subset A : A_i d.(f)\{A \setminus A_i\}$.

This notation is convenient for establishing three preliminary theorems specifying conditions under which a rational preference over alternatives can be constructed from rational preferences over their attributes. The first two almost need no proof and confirm trivial conditions under which the preference over alternatives will be rational. However, they enhance our understanding of the problem presented by the need to construct substantively rational preferences from underlying preferences over attributes. The first confirms that trivial preference relations will be rational, while the second confirms that if the spread of attributes is "trivial" in the sense that the attribute preferences between any two alternatives agree then the preferences over all alternatives will be rational.

Theorem 1. *If $\succeq$ is trivial such that $\exists i \in \{1 \quad N\} : x \succeq x' \iff a_i(x) \succeq_i a_i(x')$ (there is an absolutely dominant attribute subspace), then $\succeq$ will be rational.*

Proof. If $\exists A_i \subset A : A_i d.(f)\{A \setminus A_i\}$ then preferences over A_i absolutely dominate preferences over all other alternatives taken together, and hence $\succeq$ is completely determined by $\succeq_i$ and $\{a_i(x)\}_{x \in X}$, so since we have assumed that all alternatives are well defined in all characteristic subspaces, for any two alternatives either $a_i(x) \succeq_i a_i(x')$ or $a_i(x') \succeq_i a_i(x)$ or both, and accordingly $x \succeq x'$ or $x' \succeq x$ or both. Similarly, suppose we have for any three alternatives $a_i(x) \succeq_i a_i(x')$ and $a_i(x') \succeq_i a_i(x'')$, so that by the definition of dominance, $x \succeq x'$ and $x' \succeq x''$. Since $\succeq_i$ is rational by assumption it follows that $a_i(x) \succeq a_i(x'')$ and thus by the triviality of $\succeq$ that $x \succeq x''$. Since A_i was selected arbitrarily, any trivial preference relation will be rational. □

This is a slightly more important result than its "triviality" would suggest. It demonstrates that as long as the preferences with respect to attributes within each characteristic subspace are rational, it is *always* possible to find a procedure with which a rational preference relation can be constructed when considering the attributes of alternatives.

Remark. Substantively rational choice in the presence of multiple attribute alternatives is always possible if we do not exclude trivial procedures for determining preferences $\succeq$.

However, such a preference relation over alternatives is not particularly interesting insofar as the guarantee of substantive rationality comes at the cost of triviality in the consideration of the attributes of the alternatives. When the preference relation$\succeq$ is trivial there is no meaningful consideration of multiple attributes, as the alternative is in effect equated with its attributes in the "dictatorial" characteristic subspace.

But if we are to consider a wider range of aggregations f than trivial ones, then what conditions guarantee that the preference over alternatives will be substantively rational? This is the content of the second "trivial" theorem concerning rational choice in the presence of multiple attribute alternatives.

Theorem 2. *If* $\forall x, x' \in X$ *all attribute preferences "agree" such that* $a_i(x) \succeq_i a_i(x') \, \forall i \in \{1, \ldots, |A|\}$ *or* $a_i(x) \preceq_i a_i(x') \, \forall i \in \{1, \ldots, N\}$ *or both, then* $\succeq$ *will be rational for any aggregation procedure* f.

Proof. Completeness of $\succeq$ follows immediately from the fact that for any two $x, x' \in X$ either $a_i(x) \succeq_i a_i(x') \, \forall i \in \{1, \ldots, N\}$ or $a_i(x) \preceq_i a_i(x') \, \forall i \in \{1, \ldots, N\}$ or both due to the agreement of attribute preferences, and so for *any* aggregation scheme f either $x \succeq x'$ or $x \preceq x'$ or both. Now without loss of generality arbitrarily select any two alternatives $x, x' \in X$ and assume that $a_i(x) \succeq_i a_i(x') \, \forall i \in \{1, \ldots, N\}$ so that $x \succeq x'$ for *any* aggregation scheme f. Since it was assumed above that $|X| \geq 3$ there must exist another $x'' \in X$. Let us suppose we have selected $x, x', x'' \in X$ so that for any aggregation scheme f we have $x' \succeq x''$, which, since attributes are all in agreement for any two alternatives across all characteristic subspaces, implies that $a_i(x') \succeq_i a_i(x'') \, \forall i \in \{1, \ldots, N\}$. It follows from the transitivity of $\succeq_i \, \forall i \in \{1, \ldots, N\}$ that $a_i(x) \succeq_i a_i(x'') \, \forall i \in \{1, \ldots, N\}$ and so $x \succeq x''$, and the preference relation $\succeq$ is transitive. Thus for any aggregation f we have a rational preference relation $\succeq$. □

Note that Theorem 2 tells us that when alternatives are characterised by attributes for which the preferences between those alternative's attributes are in the same direction, we can guarantee that a rational preference relation over alternatives exists independently of the procedure f for aggregating attribute preferences. Indeed, *any* aggregation scheme f will generate a rational preference relation $\succeq$, provided attributes in *all* characteristic subspaces agree on the preference between the attributes of two alternatives. This result confirms that the possibility of non-trivial rational choice will depend in part on variables which lie beyond the individuals' conscious control. The possibility of non-trivial rational choice, and the aggregation scheme f which will support it, depends on the position of the various alternatives in the characteristic space A, and the degree of agreement of preferences between alternative attributes across the various subspaces of A.

While these theorems add to our understanding to the problem they are still trivial in that they provide rather extreme conditions on the aggregation mapping f and the attributes $\left\{\{a_i(x)\}_{i=1}^N\right\}_{x \in X}$ to guarantee the existence of a rational preference relation over alternatives. What they serve to demonstrate is that the possibility of rational choice will depend on the choice of the particular procedure f for aggregating attribute preferences (under the individuals' control) as well as the positions of the various alternatives in the attribute space A (outside the individuals' control). However, they provide unsatisfactorily extreme sufficient conditions for substantive rationality. It is possible to determine far weaker necessary and sufficient conditions for non-trivial rational choice to be guaranteed by the aggregation scheme.

Theorem 3. *Define $A' = \{A_i \in A'\} \subset A$ to be a collection of subspaces of A. Suppose that for some arbitrary alternatives $x, x', x'' \in X$ we have a non-trivial aggregation mapping f such that $x \succeq x'$ and $x' \succeq x''$. Then $x'' \succ x$ (a violation of transitivity) if and only if*

$$\exists A' \neq \emptyset : \begin{cases} A_i \notin S_x(x') \cap S_{x'}(x'') \; \forall A_i \in A' & (1) \\ \& \, A' d.(f) \, S_x(x'') & (2) \end{cases}$$

Intuitively, conditions (1) and (2) together require the existence of a set of dominant characteristic subspaces which do not conform to a “linear” pattern (in a topological rather than traditional sense) in the spread of attributes across the characteristic space. Condition (1) requires non-conformance “linearity” in that it states those characteristics in the set A' must not be in the set of those spaces on which the third alternative is preference-dominated by the attributes of both of the others. Condition (2) requires that this set of subspaces which go “against the grain” dominate those which would guarantee transitivity.

Proof. Sufficiency: It follows from the transitivity of $\succeq_i$ that $a_i(x) \succeq a_i(x')$ and $a_i(x') \succeq a_i(x'')$ implies $a_i(x) \succeq_i a_i(x'')$. Then, by the definition of $S_x(x')$ and $S_{x'}(x'')$ as the sets of attribute subspaces in which one alternative dominates another, for any $A_{-i} \in S_x(x') \cap S_{x'}(x'')$ it is true that $A_{-i} \in S_x(x'')$, because

$$S_x(x') \cap S_{x'}(x'') = \{A_i : a_i(x) \succeq a_i(x')\} \cap \{A_i : a_i(x') \succeq a_i(x'')\}$$

$$\implies S_x(x') \cap S_{x'}(x'') = \left\{ A_i : \begin{array}{c} a_i(x) \succeq a_i(x') \\ \& \, a_i(x') \succeq a_i(x'') \end{array} \right\}$$

and transitivity therefore implies

$$S_x(x') \cap S_{x'}(x'') = \{A_i : a_i(x) \succeq a_i(x'')\} = S_x(x'')$$

so it is the case that $A_i \notin S_x(x'')$. Now, since each preference relation for attribute subspaces is complete, and there is one defined for each subspace, we have that $S_x(x'') \cup S_{x''}(x) = A$. So because $A_i \notin S_x(x'')$, it follows not only that $A_i \in S_{x''}(x)$, but that $A_i \in S_{x''}(x) \setminus I_{x''}(x)$ and so $a_i(x'') \succ a_i(x)$. Hence if $\exists A' \neq \emptyset : A_i \notin S_x(x') \cap S_{x'}(x'') \; \forall A_i \in A'$, this implies that $\exists A' \neq \emptyset : A_i \in S_{x''}(x) \setminus I_{x''}(x) \; \forall A_i \in A'$, and then by the definition of the dominance relation $d.(f)$ and the content of the set A', $A' d.(f) \, S_x(x'') \implies x'' \succ x$.

Necessity: Suppose that we have a non-trivial aggregation mapping f such that for some arbitrary alternatives $x \succeq x'$ and $x' \succeq x''$ and $x'' \succ x$. Now, by the definition of the dominance relation $d.(f)$, and the definition of $S.(\cdot)$, $x'' \succ x$ only if $S_{x''}(x) \setminus I_{x''}(x)\, d.(f)\, S_x(x'')$. But by the argument employed in the proof of sufficiency above, $S_x(x'') = S_x(x') \cap S_{x'}(x'')$, so $x'' \succ x$ presupposes the existence of a non-empty collection of attribute subspaces $A' \subset A$ such that $A_i \notin S_x(x'') = S_x(x') \cap S_{x'}(x'')\ \forall A_i \in A'$ and $A' d.(f)\, S_x(x'')$. □

The "inverse" of this Theorem provides necessary and sufficient conditions for rational choice to be possible in the multiple attribute alternative environment. Completeness is here a fairly trivial aspect of the problem due to the completeness of the underlying preferences vis-a-vis attributes, which guarantees that a binary preference can always be constructed provided the aggregation schema is defined for this region of characteristic space. If irrationality is to be present in the preference relation $\succeq$ then it will be due to a violation of transitivity, the conditions for which are outlined by Theorem 3. Hence we have an possibility result which can be summarised thus:

Corollary. *Non-trivial rational choice out of a set of three or more alternatives is possible if and only if conditions (1) and (2) of Theorem 3 do not hold for any three alternatives selected arbitrarily from X and the aggregation scheme f is well defined for all points in $\Pi_{i=1}^{N} \{\succeq_i (A_i)\}$.*

Proof. Rational choice is defined as selecting the maximally preferred element, $x^* = \max_{x \in X} \succeq (X)$, but, given that X is countably finite, and given that f is well defined for all points in $\Pi_{i=1}^{N} \{\succeq_i (A_i)\}$ this problem is only well defined for each subset of 2^X if the conditions of Theorem 3 fail to hold. □

Alternatively we can restate this corollary in a more intuitive, though less formal manner:

Remark. Non-trivial rational choice is only possible if and only if there is a sufficient degree of linearity in the position of alternatives in pre-ordered characteristic space across its characteristic subspaces.

4 Discussion

The conditions which are necessary and sufficient for substantively rational choice to be possible are not particularly neat and do not give us a particularly quick means of classifying the class of aggregations schemes which support non-trivial rational preference relations. This is because there is an interaction between the aggregation scheme to be employed and the spread of alternatives across characteristic space for any given choice problem. However, these conditions have topological properties which have interesting implications. As I have pointed out, conditions (1) and (2) indicate that for non-trivial rational choice we need the spread of the attributes of alternatives in characteristic space to be somewhat "linear", so that preferences between attributes tend towards agreement between any three alternatives. Stated differently, non-trivial rational choice requires that alternatives tend to perform either strong or weak across a sufficiently broad range of characteristics, rather than performing strongly in a certain class and weakly on others.

Non-trivial rational choice is therefore possible if and only if the process f gives sufficiently little weight to characteristic subspaces where alternatives which are attribute preference-dominated on other characteristic subspaces are performing strongly. As more alternatives which are otherwise generally dominated across characteristics space become strong in a particular set of characteristic subspaces, this mapping must give increasingly little relative weight to those subspaces to maintain transitivity at the level of alternatives. Preserving substantive rationality may thus require the procedure f to become "dictatorial" with respect to attribute subspaces which "go against the grain" of the general spread of attributes in characteristics space.

This reflects the intuition of Theorem 1 that it is *always* possible to construct a rational preference amongst alternatives if we include the possibility of trivial, "dictatorial", aggregation schemas. But if a non-dictatorial aggregation is to be found then the possibility of rational choice depends in large part on the nature of the differentiation of the alternatives under consideration along the pre-ordered characteristic subspaces. It is not hard to imagine that in the presence of multiple attribute alternatives, the procedure which constructs the preference required for non-trivial rational choice can become quite precise and intricate, as a schema f must be found which preserves completeness and transitivity over *all* alternatives while also not being so "dictatorial" as to be trivial. If the differentiation of alternatives by their spread across characteristics space is fairly chaotic and unordered (i.e. highly differentiated) then the schema f may need to be quite carefully tailored to give just the right amount of weight to each characteristic subspace that a substantively rational non-trivial preference emerges.

Hence substantive non-trivial rationality of preferences and choice in the context of multiple attribute alternatives requires either a significant degree of transitivity across many characteristic subspaces in which the attributes of the alternatives are located, or potentially a highly selective and intricate procedure for aggregating these attributes which conforms to conditions (1) and (2) of Theorem 3 for *every* triad of alternatives. It seems reasonable to assert that as the procedure f becomes more and more intricate, the task of constructing preferences over alternatives from preferences over attributes becomes computationally more complex, and hence more cognitively taxing. If this is indeed the case, then substantive rationality comes at the price of sacrificing procedural rationality.

Herbert Simon's (1976; 1978b; 1978a) notion of procedural rationality places a premium on the computational simplicity of the process of thought which the cognitively constrained individual must engage in. The argument that substantive rationality can come at the expense of procedural rationality can be maintained even without applying the argument of Shugan (1980) that the costliness of making a decision increases with the number of attributes to be compared. Even with a fixed number of characteristics, a high degree of differentiation by attributes across alternatives means that it may take some time before an individual finds an aggregation scheme f from which a preference between alternatives emerges which does not have the individual "going in circles" due to intransitivity.

So while substantively rational choice may be possible, the procedure which supports it may be procedurally irrational due to its placing a computational demand on the individual made impossible by the cognitive and time constraints of human beings, and hence there is to some degree a trade-off between substantive and procedural rationality. It becomes rational, in fact, to use "fast and frugal" heuristics for f, for it is well known (Gigerenzer and Goldstein, 1996; Gigerenzer, 1999) that many simple heuristics which are not cognitively taxing, still come close to selecting the "best" outcome even while they may satisfy the conditions 3 for irrationality[13]. Indeed, the trade-off between substantive non-trivial rationality and procedural rationality may help to explain the "organisation corollary" of George Kelly's (1963) personal construct psychology, which states that there are "core" and "peripheral" needs contained within personal constructs. Rather than face the trade-off between a substantive rationality which accounts genuinely for multiple attributes and procedural rationality, the individual in question may prefer to adopt a dictatorial aggregation scheme f based upon which attribute lies closest to the "core" of the individuals' needs.

This is important in an economic setting, particularly when the problem of choice is that of the consumer, or buyer in general. In economies with profit-seeking producers, it is rational for producers to differentiate their products in order to generate profits from market power. Profit-seeking is the driver of evolutionary dynamics in the economy, with differentiation of products (alternatives from which to choose) being the cause of variety generation (or origination) on which replication-selection dynamics (or self-organisational dynamics) can operate through consumer choice (Dopfer et al., 2004). Producers in such an economy *ceteris paribus* should seek to differentiate their products by augmenting their attributes in subspaces where competitors are weak rather than strong (Markey-Towler, 2014b), "shifting the goalposts" in order to create market power rather than "playing catch-up" by trying to make their products at least as good as their competitor's. It is generally more profitable to create a new, differentiated product for which one will be a monopolist

[13]My thanks to Andreas Chai for making this clear to me.

than compete by imitating an established one.

In an evolutionary economy the drive amongst producers to differentiate products will thus tend to generate a fairly chaotic spread of alternatives in characteristic space. The argument above suggests then that the class of procedurally and substantively rational schemas f is fairly small, for the alternative attributes have a fairly chaotic spread and a small degree of transitivity across characteristic subspaces. In an evolutionary economy consumers may need fairly complicated aggregation schemes so that the weightings of the subspaces are calibrated just so that the preferences over alternatives constructed from preferences over their attributes are complete, transitive and non-trivial, i.e. substantively rational. Complicated aggregation schemes take considerable time and effort both to construct and to implement, and so the consumer in a typical economy will face (to some degree) a trade-off between substantive and procedural rationality as they engage in economic activity. This corroborates the truth within the argument of Kapeller et al. (2013) that rational consumer choice may be impossible, for guaranteeing substantive rationality may come at the cost of procedural rationality.

One interesting exception to the trade-off between guaranteeing substantively rational preferences and procedural rationality in this context is the lexicographic procedure for aggregating attribute preferences. I have been careful thus far to state only that a *potentially* computationally demanding procedure f might be required to aggregate the attribute preferences of highly differentiated alternatives to guarantee substantive rationality. As a matter of fact lexicographic preferences, it is well known, always generate a rational preference relation and are a very simple class of procedures to implement once a lexicographic ordering has been arranged.

Lexicographic preferences constitute an ordinal aggregation mapping f which can be specified as follows

$$f = \left\{ x \succeq x' \iff \begin{array}{cc} & a_1(x) \succeq_1 a_1(x') \\ \vee & a_1(x) \sim_1 a_1(x') \wedge a_2(x) \succeq_2 a_2(x') \\ \vdots & \vdots \\ \vee & \wedge_{i=1}^{N-1} \{a_i(x) \sim_i a_i(x')\} \wedge a_N(x) \succeq_N a_N(x') \end{array} \right\}$$

so clearly dominance of any one attribute subspace over another depends on its position in the index set, specifically, $A_i d.(f) A_j \iff i < j$, and given the lexicographic nature of the procedure, it is unnecessary to think of dominance in any other than a binary manner. Using this definition, we can recast the proof of the rationality of lexicographic preferences in terms of Theorem 3 but more importantly demonstrate their non-triviality:

Theorem 4. *Lexicographic preferences are non-trivially rational*

Proof. Non-triviality: Suppose that we have $k > 1 : a_k(x') \succ_k a_k(x)$ and that $a_i(x) \sim_i a_i(x') \,\forall\, i < k$. Now by the standard conceptualisation of $\succeq$, $a_i(x) \succeq_i a_i(x') \,\forall\, i < k$, but $x' \succ x$ by the definition of $\{\succeq (X)\} = f\left(\{\succeq_i (A_i)\}_{i=1}^{N}\right)$ so since k was arbitrarily chosen amongst the dimensions dominated by attribute subspace A_1 and $x' \not\succ x$, lexicographic preferences assign no absolute priority to a single attribute subspace.

Rationality: The proof of completeness is trivial, simply note that by the definition of f, as long as a pair of attributes exists within every attribute subspace (which is the case by assumption), then the preferences over the alternatives they characterise is well-defined. But for $\succeq$ to be rational also requires transitivity. Suppose that f is lexicographic. Then $A_i d.(f) A_j \iff i < j$. Now if $x \succeq x' \,\&\, x' \succeq x''$ then by definition of f it must be the case that $\exists A_i \in S_x(x') : A_i d.(f) S_{x'}(x)$ and $\exists A_j \in S_{x'}(x'') : A_j d.(f) S_{x''}(x')$. But since $x \succeq x'$, $i < j$ must be the case, so $k > i \,\forall\, A_k \in S_{x''}(x') \cup S_{x'}(x)$. Hence by the definition of f, $A_i d.(f) A_k \,\forall\, A_k \in S_{x''}(x') \cup S_{x'}(x)$. Now, if (1) of Theorem 3 holds, then any $A_n \in A'$ is not in $S_x(x') \cap S_{x'}(x'')$. But notice that $\neg\{S_x(x') \cap S_{x'}(x'')\} \subset S_{x'}(x) \cup S_{x''}(x')$, so any $A_n \in A'$ is also in $S_{x'}(x) \cup S_{x''}(x')$ and therefore dominated by both A_i and A_j. Hence under lexicographic preferences, if (1) of Theorem 3 holds, then (2) does not, and preferences are rational. □

This is an interesting result because it implies that since lexicographic preferences are non-trivial, non-trivially rational preferences are always possible, and thus that rational choice is always possible in mathematically possible:

Remark. Non-trivial rational choice in the presence of multiple attribute alternatives is always possible if a lexicographic procedure f is adopted.

Given that lexicographic procedures can be computationally quite efficient, requiring considerations on the order of complexity as would a "checklist", this could partly explain why they are widely observed empirically in the psychological literature on decision theory (Earl, 1990). They provide a computationally efficient manner in which a substantively rational decision can be made. This further confirms the problem of aggregation in multiple attribute decision making as an explanation for the existence of core and peripheral needs within personal constructs (Kelly, 1963). The organisation of personal constructs can determine the order of characteristics within the lexicographic aggregation scheme and thereby sidestep the trade-off of substantive rationality against procedural rationality implied by the choice problem.

However, lexicographic procedures for constructing preferences are extremely problematic for economic theory, because if preferences $\succeq$ are determined by a lexicographic procedure, it is well known that they cannot be represented by a utility function (see Mas-Collel et al.

(1995, p.46) or Rubinstein (2006, p.15)). Hence the rules of differential calculus would no longer be available to represent choice elegantly as the solution of an optimisation problem and almost the totality of neoclassical economic decision theory would be undermined. We can also imagine that if we have a large number of attributes, lexicographic preference can become procedurally irrational as well as a large number of attributes must be factored into decision making as well as the order in which they figure in the aggregation of attribute preferences to alternatives preferences. In this case, the trade-off of procedural against substantive rationality is maintained.

Lexicographic preferences are a special class of procedurally rational heuristics which can be applied to the problem of choice. In general, these procedures are represented by simple mappings $g : A \rightarrow C(X)$ between the characteristic space A into a choice set $C(x)$, which need by no means be a singleton for any particular realisation $g(a \in A)$. The best known of these procedures is the simple multi-dimensional "satisficing" concept posited by Simon (1955) and extended on notably by Reinhard Selten (1998; 1999). I have explained above (p.4) how designing and following these procedures - *homo oeconomicus* becoming "rule user and rule follower" (Dopfer, 2004) - can be rational. However, while these procedures (and a vast number of other heuristics surveyed in Gigerenzer and Selten (1999) and Simon (1978a)) are computationally efficient and therefore procedurally rational, it is by no means assured that they are substantively rational. Indeed, both Simon and Selten explicitly differentiated their models from substantively rational models by pointing out that they were constructed to be *alternatives* to choice as solution to an optimisation problem. The benefit of having Theorem 3 above is that it provides us with a means by which to test whether these procedurally rational mappings give rise to a substantively rational choice. If a single example of alternative's locations conforming to the conditions outlined in Theorem 3 can be found then the procedure will be substantively irrational[14].

5 Conclusion

The theory of rational choice has typically been simply a matter of assuming the choice of the maximal element of a set of alternatives pre-ordered by a given rational preference relation. However, if we accept that alternatives are differentiated by their attributes as per Kelvin Lancaster and Duncan Ironmonger's New Consumer Theory, and that preferences over these characteristics influence preferences over alternatives, we can no longer simply assume a rational choice amongst alternatives exists. As I have shown, for attributes to be meaningfully considered in a substantively rational decision process we require a sufficient

[14]The development of this idea would be a fruitful topic for further research.

degree of regularity in the strength and weakness of alternatives across a broad range of characteristics if we are not to require either a rather intricate procedure for constructing preferences, one which does not give consideration to multiple attributes, or one which prohibits a utility representation of the problem. I have argued that these requirements are actually quite a stringent one in a typical (evolutionary) economic setting and that as a result, here as in general, guaranteeing the possibility of substantively rational choice may require procedurally irrational processes.

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