Abstract: Finance has been shown to influence economic growth. One of the sources of growth is innovation, which is an intrinsically risky process and requires financiers willing to bear this risk. This paper uses an endogenous growth model, where agents differ in their risk aversion and time preference, to explain the coexistence of banks and financial markets, and their impact upon economic development. The model is able to replicate the results in the existing literature that a negative shock affects bank equity overproportionately, causes a widening of the loan-deposit spread, and diminishes technology growth. Further, it offers an explanation for prolonged recovery periods of technology growth. In contrast to previous results, decreasing steady state loan-deposit spreads can be associated with lower economic growth.

Keywords: banks, endogenous growth, innovation, financial intermediation, portfolio choice, continuous time stochastic optimization

JEL Classification: G11, G21, O16, O33, O41
The essential function of credit ... consists in enabling the entrepreneur to withdraw the producers' goods which he needs from their previous employments, by exercising a demand for them, and thereby to force the economic system into new channels. ... The entrepreneur is never the risk bearer. ... The one who gives the credit comes to grief if the undertaking fails.” Schumpeter (1934, pp. 106, 137)

1 Introduction

The Asian- and the Russian-Crisis as well as the prolonged Japanese economic slowdown have revived the interest in the impact of bank-finance and bank health upon economic growth and economic fluctuations. Banks are vulnerable to economic volatility, due to their low capital ratios. For example in Japan negative shocks have reduced banks' profitability and equity. In order to reduce the exposure to insolvency risk, banks curtailed their lendings and thereby may have accelerated the economic downturn (e.g. Bernanke 1983; Bayoumi and Towe 1998; Kanaya and Woo 2000; Country Profile Japan 2003).

Two branches of the literature are relevant to explain the underlying economic mechanisms. Firstly, the importance of financial development\textsuperscript{1} and, in particular financial intermediaries for long-term economic growth, has been identified empirically (e.g. Levine 1997; Beck, Levine and Loayza 2000; Lucchetti, Papi and Zazzaro 2001). Existing theoretical explanations for the positive growth effect emphasis banks' ability to offer insurance against illiquidity risk (e.g. Bencivenga and Smith 1991; Vollmer 1999) and banks' monitoring and screening services (e.g. Greenwood 1990; King and Levine 1993; de la Fuente 1996; Gries, Sievert and Wieneke 2004). These services increase the return on savings and thus foster capital accumulation, which increases the growth rate in an AK model (Romer 1986), or as in the case of King and Levine (1993) improve the allocation of resources towards a technology producing sector. Secondly, the monetary policy literature highlights the importance of banks in the process of monetary transmission, i.e. the bank-lending channel (Bernanke and Blinder, 1988) and more specifically the so-called bank-capital channel (e.g. Kishan and Opiela 2000; Aikman and Vlieghe 2004; Gambacorta and Mistrulli 2004). Tight monetary policy reduces bank equity because banks’ financing costs increase (liability side, short term), while the banks cannot immediately adjust their loan interest rates (asset side, long-term). In order to maintain their capital ratios, new lending is curtailed and bank dependent firms suffer financing problems.

This paper suggests to combine the findings of this literature and provides a formal model

\textsuperscript{1}Financial development denotes "improvements in the extent or efficiency of the financial system” (Khan 2000, S. 4).
explaining a causal link of bank capital, bank lending and economic growth. Here the focus is upon the banking activity of transforming risky assets into risk free deposits, using bank equity for intertemporal smoothing of occasional debt defaults. Considering that the idea of banks as savings allocators and risk bearer originates from Schumpeter, and Schumpeterian growth models have been formalized (Aghion and Howitt, 1992 ), implementing risk averse bankers into their model is adjacent. In the original Aghion and Howitt growth model the incentive to innovate is the expected net present value of a resulting licence monopoly, and households are assumed to have a linear utility function, i.e. they are risk neutral. The net present value can only be calculated for the steady state, whereby the analysis of presumably interesting dynamics due to bank equity fluctuations are excluded. Wälde (2002) examines the effect of risk averse households and altered the Aghion and Howitt model elegantly by assuming tangible R&D output in the form of a prototype machine, which embodied technology is a public good. He is thereby able to solve the dynamics and to explain economic cycles via stochastic innovation successes that induce a reallocation of productive resources.

The endogenous growth model used in this paper is a variation of Aghion and Howitt’s (1992) Schumpeterian growth model, and Romer’s (1986) AK model. Similar to the Schumpeterian model, there is a risk free production sector that takes technology as given, and produces deterministically, as well as an innovative sector that produces stochastic technology improvements. The model is altered insofar as the incentive to employ resources in the innovative sector is not a resulting technology asset and a monopoly position, but potentially higher productivity than in the risk free production sector. Technology improvements themselves are considered pure externalities as in the AK model, yet produced with a risky technology as in the aforementioned variation of Wälde (2002). Further, it is assumed that the innovative sector requires external finance of its wage bill, as the workforce does not accept uncertain wages. Therefore, the availability of finance determines the labor allocation towards innovative production and thus technology growth.

Finance is supplied by two types of agents that differ in their risk aversion. The agent with low risk aversion chooses risky investment in excess of her wealth and covers the financial gap by short-selling risk free deposits to the highly risk averse agent. Thus the low risk averse agent is denoted as ’bank’ and the high risk averse agent is denoted as ’household’. This idea is adopted from Neuberger (1991) and offers an endogenous explanation for banking activities in the sense of risk shifting. The more the agents differ in respect to their risk aversion, the more risk will be shifted towards the bank via deposit short-selling, and saving respectively.

Thereby, an immediate result of the model is that differences in the risk aversion of citizens
may explain why some economies rely on bank-finance, while others rely more heavily on market-finance (e.g. Germany versus USA, Allen and Gale 1995). It is noteworthy that the model does not require transaction costs or asymmetric information to obtain this result. Further, technology-dependent wages imply a ratchet effect, if the technology does not depreciate (e.g. knowledge). In combination with financial asset fluctuation, this may explain prolonged setbacks of technology progress following a negative economic shock. Finally, the model shows that - contrary to intuition - a decreasing steady state interest rate spread can be associated with lower economic growth. The risk averse household reacts to the decreasing spread with an overproportionate portfolio shift away from market investment, towards safe, but low yielding deposits. A decreasing return on wealth outweighs increased savings, whereby wealth accumulation and economic growth settle at a lower steady state growth rate.

The paper is structured as follows: The next section introduces the endogenous growth model including portfolio optimizing banks. Section three provides the solution to the model and discusses the comparative statics as well as the dynamic analysis. Section four discusses potential policy implications and concludes.

2  The Model

The model depicts a closed economy. The two factors of production are labor and technology knowledge, which resembles a public good. Output is produced by a risk free production process and a risky, innovative production process. The latter offers higher average returns and increases the level of available technology. There are two types of agents, which differ only in their risk aversion and time preference and optimize via their savings and portfolio choice. The risky asset is the wage bill of the innovative sector, while the safe asset will be shown to evolve due to the portfolio choices.

2.1  The Risk free Production Sector

In the risk free production sector, existing technology \( (A) \) is combined with labor input \( (L_S) \) to produce output \( (y_S) \). This output is also the numeraire of the whole model. The production function is linear in labor input, whereby long term growth requires technology improvements, if the total labor supply is constant.

\[
y_S = A L_S
\]  

(1)
Profit maximization equates wage \((w)\), with the marginal product of labor, which, in this case, is the technology level \(A\).

\[
w = A
\]  

(2)

Improvements in the technology are assumed to be an externality of 'innovative production'.

### 2.2 The Innovative Production Sector

An alternative use for labor \((L_R)\) is to combine it with new ideas to attempt new production technologies. However, this attempt is risky insofar as output obeys a Poisson process \(q\).

\[
y_R = \begin{cases} 
(1 + r_R) AL_R & \text{if } dq = 0 \\
(1 + r_R) AL_R - \beta AL_R & \text{if } dq = 1 
\end{cases}
\]

(3)

This innovative production requires an 'investment' in the extent of the wage bill \(wL_R\). Applying the equilibrium wage rate (2), it can be seen that the stochastic return on the investment \(K\) is \(dK(t) = r_R K(t) dt - \beta K(t) dq\). If the project has been successful \(dq = 0\), and the rate of return is \(r_R\). An unsuccessful attempt is formally described by \(dq = 1\) where a fraction \(\beta \in [0, 1]\) of the initial investment is lost. Thus the remaining fraction \((1 - \beta)\) can be interpreted as collateral (Neuberger 1991, p. 287). The likelihood of a new production technology attempt failing is given by the Poisson arrival rate \(\lambda\).

The technological knowledge obtained during this innovative production process is a public good and, as such, enhances the technology stock as an externality.

\[dA = Af(L_R)dt - Af(L_R)dq\]

(4)

Technology growth is thus similar to Romer (1990)\(^3\). However it is stochastic and the incentive to allocate resources to this sector is the expectation of higher productivity, rather than a resultant monopoly position. The monopoly setup must be avoided, because the net present value of monopoly profits can only be calculated for the steady state only, whereby dynamic analysis is excluded. Wälde (2002) solved the monopoly issue for his

\(^2\)Instead of working with upwards jumps due to a success in research (eg. Aghion and Howitt, 1998; Wälde, 1999 and 2002) this model use downwards jumps in case of a failing innovative investments. The important characteristic that new technologies cannot be produced deterministically remains. However the downward jump allows the interpretation of the innovative investment as a risky loan. This enables the utilization of existing literature and methods for banking activity and portfolio optimization.

\(^3\)This formulation includes a scale effect, i.e. a larger population would result in higher growth rates. The interested reader is referred to Jones (1995, 1998, 1999).
model, maintaining creative destruction, by assuming that the result of - and incentive for - research is a tangible prototype machine. In order to avoid an additional state variable (capital) in the present model, the output of 'research' has been altered to final goods, whereby technological progress becomes a pure externality. The economic concept that technological progress results from entrepreneurs diverting resources from known production processes, in an attempt to earn extra profits, remains. However, the factor labor does not accept uncertain wages and thus financiers, who outlay the 'investment' of the wage bill and bear the down- and upside risk of innovative production, must be found. This is also in line with Schumpeter’s (1934) concept of an entrepreneur who does not bear the economic risk of new ventures.

2.3 The Low Risk Averse Agent: Banker

This subsection describes, in detail, the behavior of the representative agent, with relatively low risk aversion. It demonstrates under which conditions she will become active in the banking activity of investing in risky assets, in excess of her (equity) capital. The financial gap is closed by short-selling safe deposits, whereby risk is transferred from the relative high risk averse agent to the banker. Bank equity is used to smooth the stochastic fluctuations of the risky asset. Because of the banker’s own risk aversion, negative realizations of innovative production and the according loss of equity have an immediate impact upon bank lending and deposit short-selling.

The representative risk averse banker maximizes utility from consumption \( u(c_b) \). To allow for constant steady state leverage, constant relative risk aversion is required \( u(c_b) = \frac{1}{\gamma} c_b^{\gamma} \) (Hakansson 1996, p. 917). The objective function with \( \rho_b \) denoting the personal discount rate of the banker is

\[
\int_t^\infty e^{-\rho_b s} u(c_b(s)) ds
\]

and the banker’s intertemporal budget constraint is the following stochastic differential equation

\[
dE = (r_R \omega_b E + r_D (1 - \omega_b) E - c_b) dt - \beta \omega_b E dq. \tag{5}
\]

The banker is thereby modeled as portfolio manager, who can invest a fraction \( (\omega_b) \) of her equity \( (E) \) in the innovative production sector at the risky return \( (r_R) \) and a fraction \( (1 - \omega_b) \) in risk free deposits \( (D) \) with the rate of return \( r_D \). Risk averse portfolio optimizing bank managers have already been examined within a partial static model by Pyle (1971), and Hart and Jaffee (1974). The latter have treated equity as an additional liability, though they have already acknowledged that for banks, this is a negligible source
of finance (Hart and Jaffee 1974, p. 130). It can be argued that bank equity is held as a buffer, reducing the risk of bankruptcy over time. Thus O’Hara (1983) extended the portfolio choice with a retained earnings choice, in the form of optimal consumption. The incentive for the manager to retain some expected profits instead of consuming them, is the risk of bankruptcy, which will cost his job (O’Hara 1983, p. 131). The consumption choice $c_b$ in the intertemporal budget constraint (5) can thus be interpreted as retained earnings choice, and further depicts banks’ reliance upon retained earnings to increase equity. For simplicity sake, in this paper it has been assumed that the banker acts as if she is fully liable.

The notion of the bank equity as a risk buffer has also been used in a partial equilibrium model by Neuberger (1991). Risk averse households optimize via their continuous time portfolio and savings decisions. Without financial intermediaries, the household can invest firm equity (Wiener process) and firm lending (Poisson process). Neuberger shows that the combination of firm collateral and bank equity can result in risk free deposits. Bankers are assumed to be less risk averse than the ‘regular’ household and insure with their equity and borrowers’ collateral against deposit default. However, Neuberger’s analysis is limited to the steady state situation, and does not explicitly explain the links with economic growth and cycles.

Similar continuous time stochastic portfolio and consumption optimization problems have also been examined by Merton (1969, 1971), Neuberger (1991), and Sennewald and Wälde (2005, p. 17).

Applying the Bellman Principal of Optimality, the objective function of the banker can be written (see appendix 5)

$$0 = \max_{\omega_b,c_b} \left\{ u(c_b(t)) - \rho_b J^* + J^*_E (r_R \omega_b E + r_D (1 - \omega_b) E - c_b) + \lambda [J^* (E - \beta \omega_b E) - J^* (E)] \right\},$$

where $J^*(E) \equiv \int_0^\infty e^{-\rho_s s} u(c_b(s), s) ds$. From the according first order conditions the optimal portfolio and consumption choice follow. The optimal portfolio choice is (see appendix 5)

$$\omega^*_b = \left[ 1 - \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{\gamma_b}} \right] \frac{1}{\beta},$$

with $1 - \gamma_b \in [0, 1]$ denoting the Pratt (1964) measure of relative risk aversion, and $\lambda$ denoting the Poisson arrival rate, i.e. the likelihood of a credit default. For positive risky investment, the expected rate of return on risky investments must exceed the safe rate of

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4Issues of portfolio managing banks with limited liability within a partial equilibrium context are discussed by Gollier et al. (1997).
return \( r_R - \lambda \beta > r_D \), in order to compensate for the utility loss due to the risk. To allow for a useful equilibrium, it is assumed that \( r_R \) is sufficiently high to fulfill this inequality. The economics are according to intuition. Higher leverage requires a higher risk premium \((r_R - r_D)\), decreased risk, i.e. lower likelihood of default \( \lambda \), higher collateral \((1 - \beta)\), or decreased risk aversion\(^5\). High leverage is typical for a bank. In fact, the equity ratios are so low that equity cannot be regarded as an important source of finance (Hart and Jaffee 1974, p. 130). The function of bank equity is rather to serve as a buffer\(^6\) against bankruptcy due to the variance in returns (O’Hara 1983).

It is assumed that the banker’s risk aversion is small\(^7\)

\[
1 - \gamma_b < \frac{\ln \left( \frac{\lambda \beta}{r_R - r_D} \right)}{\ln (1 - \beta)} \iff \omega_b^* > 1
\]

and therefore the banker allocates ‘loans’ to the innovative production sector in excess of her equity, and is financing the according gap by short-selling the safe assets as deposits. Risk aversion and positive collateral assures that the bank remains solvent even in case of loan default\(^8\). Hence, bank deposits are, in fact, safe assets.

The optimal consumption to equity ratio \( \tilde{c}_b \) is given by (see appendix 5)

\[
\tilde{c}_b = \frac{c_b}{E} = \frac{\rho_b + \lambda - \gamma_b \left( \frac{r_R - r_D}{\lambda \beta} + r_D \right) - (1 - \gamma_b) \lambda \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\omega_b^*/\gamma_b}}{1 - \gamma_b}.
\] (7)

The consumption choice is included to allow for optimal endogenous savings i.e., in the case of the bank, optimal retained earnings. This consumption ratio is increasing in the personal discount rate \( \rho_b \), and for the banker\(^9\) also in \( r_D \). The latter can be understood by acknowledging that \( r_D \) is a cost factor for the bank and reduces the mean rate of return on equity. Therefore, the incentive to retain profits is reduced and it is optimal to substitute current consumption for future consumption.

### 2.4 The High Risk Averse Agent: Regular Household

The representative household differs from the banker only in its risk aversion and time preference. The rationale for this symmetry is to show that heterogenous preferences

\(^5\)See appendix 6.1 for the derivatives.

\(^6\)“In most G-10 countries banks now hold more capital than the regulatory minimum. This may stem from the need for banks to preserve their credit ratings, in which case the capital requirement is immaterial. However, it seems more likely that the surplus capital reflects a perceived need to maintain a buffer to avoid the need for costly adjustments.” (Brealey 2001, p. 149)

\(^7\)See appendix 6.2.

\(^8\)See appendix 6.3.

\(^9\)\( \partial (\tilde{c}_b) / \partial r_D = \frac{\lambda \beta}{1 - \gamma_b} (\omega_b^* - 1) > 0 \), see appendix 6.7.
suffice to explain the coexistence and different reliance upon bank- versus market-finance.

\[ u(c) = \frac{1}{\gamma_h} c_h^{\gamma_h} \]

is the instantaneous utility function and assumed CRRA. The intertemporal budget constraint is

\[ dW = (r_R \omega_h W + r_D (1 - \omega_h) W - c_h) dt - \beta \omega_h W dq, \quad (8) \]

where \( W \) denotes household wealth and \( \omega_h \) is the fraction of wealth allocated towards the risky investment. The household is optimizing via its consumption (savings) choice \( c_h \) and portfolio choice \( \omega_h \). The optimization problem is according to the aforementioned optimization of the banker, and the first order conditions are

\[ \omega_h^* = \left[ 1 - \left( \frac{\lambda \beta}{r_R - r_D} \right)^{1/\gamma_h} \right] \frac{1}{\beta}, \quad (9) \]

\[ \tilde{c}_h \equiv \frac{c_h}{W} = \frac{\rho_h + \lambda - \gamma_h \left( \frac{r_R - r_D}{\beta} + r_D \right) - (1 - \gamma_h) \lambda \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\gamma_h}}{1 - \gamma_h}. \quad (10) \]

As usual, with an expected positive risk premium \( r_R - \lambda \beta - r_D > 0 \), it is always optimal to invest at least some wealth into the risky asset \( \omega_h > 0 \). For the household \( r_D \) increases the return on wealth and thus decreases the optimal consumption to wealth ratio. A high relative risk aversion \( 1 - \gamma_h > \ln \left( \frac{\lambda \beta}{r_R - r_D} \right) / \ln (1 - \beta) \) is typical for households, and assures that only a small fraction of wealth is invested in the risky asset. It is assumed that labor income must be instantaneously consumed \((c_L)\) and is not part of the optimization problem. This assumption is necessary, because otherwise the consumption and portfolio choice is influenced by the net present value of future labor income (Sennewald and Wälde 2005, p. 17), whereby the model could have been solved only for the steady state.

### 2.5 Markets

The wage bill in the innovative sector must be balanced by the risky investment of the representative banker and household.

\[ wL_R = \omega_b E + \omega_h W \quad (11) \]

Equilibrium in the deposit market is given by the balance of the household’s deposit investment and bank’s deposit short selling.

\[ (1 - \omega_h)W = -(1 - \omega_b)E \quad (12) \]
The fixed labor supply $L$ is allocated via the labor market towards risky innovative production and risk free production.

$$L = L_R + L_S$$

(13)

The aggregate constraint always holds.

$$y_S + y_R = c_L + c_b + c_h + dE + dW$$

3 Solution

The equilibrium interest rate $r_D^*$ and thus the spread $r_R - r_D^*$ is determined by the deposit market equilibrium (12) including the optimal portfolio choices of the banker and the household (6, 9).

$$\frac{E}{W} = \frac{(1 - \omega^*_h(r_D^*))}{(1 - \omega^*_b(r_D^*))}$$ and \( \frac{\partial r_D^*}{\partial \frac{E}{W}} > 0 \)

(14)

A relative increase of bank equity to household wealth implies ceteris paribus a relative increase of the bank’s deposit short-selling to household deposit demand. The deposit interest rate rises and balances supply and demand of deposits as the banker will decrease her leverage (6) and thus her need for short-selling, while the household will increase its wealth allocation towards deposits (9). For notational convenience $r_D^*$ and $\omega^*_i(r_D^*)$ are used in the following equations, keeping in mind their dependency upon the equity to wealth ratio $E/W$. By applying the optimal leverage (6) and consumption ratio (7) in the intertemporal budget constraint (5) the motion of bank equity is derived:

$$\frac{dE}{E} = \left[ \frac{r_R - \rho_h - \lambda + (1 - 1/\beta) r_D^*}{(1 - \gamma_h)} \right] dt - \beta \omega^*_D dq$$

(15)

Accordingly, the motion of household wealth is determined (8, 9, 10):

$$\frac{dW}{W} = \left[ \frac{r_R - \rho_h - \lambda + (1 - 1/\beta) r_D^*}{(1 - \gamma_h)} \right] dt - \beta \omega^*_h dq$$

(16)

Thus the growth rates of bank equity and household wealth can be depicted in a growth rate - deposit rate diagram. Figure 1 shows the deterministic component of the stochastic differential equations (15) and (16), i.e. the growth rates for times without default $dq = 0$. The situation of default $dq = 1$ is discussed in the dynamics section. Both growth rates are decreasing in $r_D$ although at different slopes. For the bank the negative slope is intuitive,
Figure 1: The deterministic component of equity and wealth growth is a function of the equilibrium deposit rate \( r_D^* \).

since liability costs increase in \( r_D \), whereby the return on equity decreases. Further, lower returns decrease the incentive to retain profits\(^{13}\). For the household the economic intuition is less straightforward: The return on deposits is increasing, which induces a portfolio shift towards safe deposits, away from the high yielding risky investment. Risk aversion causes this shift to outweigh the positive impact upon deposit earnings so that the return on wealth decreases. As opposed to the bank, the household’s consumption ratio is decreasing in \( r_D \)^{14}, whereby the negative impact upon the wealth accumulation is dampened. Therefore, the bank’s rate of equity accumulation is more sensitive to changes in the deposit rate than the household’s rate of wealth accumulation.

The technology growth rate depends upon the labor allocated to the innovative production sector, which in itself depends upon financial funds allocated to this sector and the wage rate. Since there is no intrinsic risk free asset in the economy, all assets end up in the risky sector. Analytically, the deposit market equilibrium (12) nets the safe investment of households with the ‘negative’ safe investment of the bank: \( (1 - \omega_h)W = -(1 - \omega_b)E \Rightarrow W + E = \omega_bE + \omega_hW \). These financial resources divided by the equilibrium wage rate (2) gives the equilibrium labor allocation (11) towards the innovative sector

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\(^{13}\)See appendix 6.7.

\(^{14}\)See appendix 6.7.
\[ L_R = (W + E) / A. \] This amount can be used in (4) to depict the equilibrium technology growth

\[ \frac{dA}{A} = f \left( \frac{W + E}{A} \right) dt - f \left( \frac{W + E}{A} \right) dq \]

These three differential equations (15, 16, 17) fully describe the model. It can be seen that ongoing technology growth requires household wealth and bank equity accumulation, and that fluctuations in the accumulation feed back to technology growth. However, fluctuations of technology growth do not have an impact upon the deterministic component of the wealth and equity growth rate. This makes the dynamics tractable and is a result of the assumption, that labor income does not enter the portfolio and savings choice. Before proceeding to the dynamics, the model is solved for the steady state and the comparative statics are discussed.

### 3.1 Steady State and Comparative Statics

In the steady state, all state variables grow at the same rate. The growth rates of equity (15) and wealth (16) unambiguously determine the steady state deposit rate \( \bar{r}_D \), and thus the steady state equity to wealth ratio (14), as shown in figure 1. Steady state technology growth requires that technology \( A \) grows at the same rate as equity \( E \) and wealth \( W \). Due to the focus of this paper, the discussion of comparative statics is limited to the 'individual' parameters which alter the financial decisions, namely the relative risk aversions \( (1 - \gamma_h, 1 - \gamma_b) \) and time preferences \( (\rho_h, \rho_b) \), which affect the accumulation schedules in figure 1.

An increase in the household’s time preference \( \rho_h \), decreases its willingness to save and thus shifts the \( dW/W \) curve downwards. The according decrease of household’s deposit supply raises the equilibrium interest rate \( \bar{r}_D \), whereby the accumulation of equity decreases. Wealth accumulation also decreases, despite the household’s willingness to save, because the rising risk-free rate lures households away from the high-yielding risky investment. The overall effect is a decrease in steady state growth. Interestingly, whilst an increase in the bank’s time preference \( \rho_b \) also shifts down the \( dE/E \) curve, it causes an increase in the steady state growth. The bank is less inclined to retain profits and the resulting decrease of deposit short-selling causes the equilibrium deposit rate \( \bar{r}_D \) to decrease (14).

Accumulation increases because the lower \( \bar{r}_D \) not only raises the profitability of equity as well as that of wealth, but even overcompensates for the loss in retained earnings and savings respectively. An increase in the banker’s relative risk aversion \( (1 - \gamma_b) \) has a similar impact, however rather via the leverage (portfolio) choice instead of the retained earnings choice (figure 2). The new line \( dE_1/E_1 \) is less steep and the banker immediately cuts her
deposit short-selling, whereby the equilibrium interest rate is decreasing. As before, this raises the profitability and accumulation of equity, wealth and the steady state growth rate.

Figure 2: An increase in the banker’s relative risk aversion raises the steady state growth rate.

3.2 Dynamics

The economic system is stable, as equity grows faster than wealth if the deposit rate is falling short of its steady state level (figure 3). Further the deposit rate itself is rising in the equity-wealth fraction (14), and vice versa. The technology growth rate (17) adjusts to the steady state growth of equity and wealth, since above steady state technology growth increases wages faster than available finance; therefore labor employment in the innovative sector and accordingly technology growth decrease and vice versa.

The following illustrates the dynamics and economic intuition of the model for the negative realization of the Poisson process ($dq = 1$). Lending to the innovative sector is risky as the process of innovative production is not always successful. If the production is unsuccessful ($dq = 1$), the financiers can recover only a fraction $(1 - \beta)$ of their initial outlays and thus suffer a loss. From the accumulation equations (15 and 16) it can be seen that the bank’s equity growth rate is affected worse than the household’s wealth growth rate ($\beta \omega_b^* > \beta \omega_h^*$),
since the banker chose a much riskier portfolio. The "post-crisis" risk-free deposit rate \( r_{DC} \) falls short of the steady state rate, because the banker requires a higher interest rate spread to accept households deposit savings, due to her relative shortfall of equity. The decreased deposit rate induces higher leverage of the bank and riskier but higher bank profits and equity accumulation. As previously noted, a lower deposit rate also spurs the household's wealth accumulation. The equity-wealth ratio and thus the deposit rate tend towards their steady state values (figure 3). The impact of a substantial shock (low

Figure 3: At the post-crisis interest rate \( r_{DC} \), bank capital recovers faster than household wealth. The system is stable.

level of collateral \( 1 - \beta \) upon technology is depicted by figure 5. The levels have been drawn arbitrarily and the focus is upon the slopes, which equal the growth rates of the subsequent variables. Bank equity \( (E) \) takes the deepest dip, but recovers at the fastest rate. Since technology was assumed to be non-tangible, as for example knowledge, it usually does not decline. However, with a sufficiently negative shock upon equity and wealth, total finance does not suffice to attract the pre-crisis level of labor towards the innovative sector. Labor productivity and thus wages in the risk-free sector do not adjust downwards, whereby more labor is allocated into this risk-free, 'non-innovative' sector and the technology growth rate diminishes. Even if equity and wealth were to recover to their pre-crisis levels, the assets do not suffice to finance the former level of employment in the innovative sector, as the technological progress achieved in the meantime has raised
Figure 4: Bank equity is more sensitive to a negative shock than household wealth. The risk-free rate of return decreases and recovers inline with the equity/wealth ratio.

the equilibrium wage rate. The impact of temporary shocks upon technology growth can thereby be prolonged.

4 Conclusion

This paper has integrated the function of banks, transforming risky credits, by guaranteeing with their own equity, into safe deposits, in an endogenous growth model. Bank behavior and equity thereby immediately gain importance. The main results can be summarized as follows: The model explains differing financial structures through the portfolio choices of the bank and the household. The household is investing in deposits as a safe asset, while the banker is short-selling deposits in order to finance risky investments in excess of her capital. The higher (lower) the risk aversion of the household (bank), the more funds will be channeled indirectly via the banks instead of the financial market.

Another finding is that a narrowing interest rate spread does not result in higher growth rates. This surprising result stems from the household’s opportunity to invest in deposits as well as in risky direct investments. Risk aversion implies an overproportionate household portfolio shift towards the low-yielding deposits, as an reaction to the narrowing spread. Thereby, the return on the portfolio, savings and also wealth accumulation de-
Figure 5: Bank equity is most sensitive to shocks. Technology growth is affected for a prolonged period.

increases. The deposit rate keeps rising, until the new, lower steady state growth is reached. It is important to note that this is not a market failure, as the household trades off wealth growth in favor of safety.

The dynamics following an endogenous shock are intuitive, with banks suffering overproportionately, and initially wide but then narrowing interest rate spreads. An exception is the prolonged recovery period of the technology growth rate. The reason is that technology progress is always non-negative and thus is increasing labor productivity and wages in the risk-free sector. Therefore, additional equity and wealth must be accumulated in order to finance the pre-shock workforce.

What are the implications of these results for potential interventions on the financial sector? It goes without saying that the results obtained from highly stylized model can be suggestive, at most. For example the model implies that minimum capital ratios are not required as bankers will act sufficiently prudent. However, this finding was derived under the assumption of full liability of bankers. With limited liability the banker might choose to ‘overinvest’ in risky assets (gamble for redemption) once the capital ratio fell bellow a certain threshold (Gollier, Koehl and Rochet 1997). In this case minimum capital requirements or increasing the personal liability of bank managers is certainly useful.
Appendix

The solution for the stochastic portfolio and savings (retained earnings) optimization is obtained by applying Bellman’s Principle of Optimality, the change-of-variables formula and an educated guess (Sennewald and Wälde 2005, p. 2). Very similar continuous time stochastic portfolio and consumption optimization problems have been examined by Merton (1969, 1971) and Neuberger (1991).

Defining the so-called value function $J(E(t), t)$ as maximized expected lifetime utility

$$J(E(t), t) \equiv \max_{\omega, c} \varepsilon_t \int_t^\infty e^{-\rho_s \tau} u(c_b(\tau))d\tau$$

and applying the Bellman’s Principle of Optimality, the objective equation can be rewritten:

$$J(E(t), t) = \max_{\omega, c} \varepsilon_t \left\{ \int_t^{t+\Delta t} e^{-\rho_s \tau} u(c_b(\tau))d\tau + \int_{t+\Delta t}^\infty e^{-\rho_s \tau} u(c_b(\tau))d\tau \right\}$$

$$0 = \max_{\omega, c} \varepsilon_t \left\{ e^{-\rho_s t}u(c_b(t))dt + J(E + dE, t + dt) \right\}$$

The integral for infinitesimal small increments of time ($\Delta t \rightarrow 0$) has been solved via the mean value theorem. It is assumed that $J$ is a continuously differentiable function of $E$ and $t$. Then the change of the value function is (Sennewald and Wälde 2005, p. 4):

$$dJ(E, t) = [J_t + J_E (r_R \omega_b E + r_D (1 - \omega_b)E - c_b)] dt + [J (E - \beta \omega_b E, t) - J (E, t)] dq$$

The intuition is that with $dq = 0$ no jump occurs and the change results from the deterministic derivatives for the two arguments $(E, t)$. However, with $dq = 1$ the additional downwards jump $(-\beta \omega_b E)$ has to be considered. Since the change of utility and not equity itself is important for the agent, $J (E - \beta \omega_b E, t) - J (E, t)$ has to be used. The probability of the Poisson jump to occur is $\varepsilon dq = \lambda dt$ (Aghion and Howitt 1998, p. 55).

Thus the Bellman equation can be rewritten

$$0 = \max_{\omega, c} \left\{ e^{-\rho_s t}u(c_b(t))dt + [J_t + J_E (r_R \omega_b E + r_D (1 - \omega_b)E - c_b)] dt + [J (E - \beta \omega_b E, t) - J (E, t)] \lambda dt \right\}$$

$$= \max_{\omega, c} \left\{ e^{-\rho_s t}u(c_b(t)) + J_t + J_E (r_R \omega_b E + r_D (1 - \omega_b)E - c_b) \right\} + [J (E - \beta \omega_b E, t) - J (E, t)] \lambda$$

The objective function can be further simplified by defining $J^*(E(t), t) \equiv e^{\rho_s t}J(E(t), t)$,
whereby $J^*$ becomes independent of time\footnote{15} $J^*(E(t), t) = J^*(E)$ (Merton 1969, p. 252).

$$
0 = \max_{\omega_b,c_b} \left\{ \begin{array}{l}
\begin{align*}
&e^{-\rho t}u(c_b(t)) - \rho_b e^{-\rho t} J^* + e^{-\rho t} J^*_E (r_R \omega_b E + r_D (1 - \omega_b) E - c_b) \\
&\quad + [e^{-\rho t} J^* (E - \beta \omega_b E) - e^{-\rho t} J^* (E)] \lambda \\
\end{align*}
\end{array}
\right\}
0 = \max_{\omega_b,c_b} \left\{ \begin{array}{l}
\begin{align*}
u(c_b(t)) - \rho_b J^* + J^*_E (r_R \omega_b E + r_D (1 - \omega_b) E - c_b) &+ \lambda [J^* (E - \beta \omega_b E) - J^* (E)]
\end{align*}
\end{array} \right\} \tag{18}
$$

The first order conditions are:

$$
\begin{align*}
u' &= J^*_E \\
J^*_E (r_R E - r_D E) + \lambda J^*_{E - \beta \omega_b E} (-\beta E) &= 0
\end{align*}
$$

In order to find a closed form solution, the form of the value function has to be guessed and verified (Sennewald and Wälde 2005, p. 15). I follow Merton (1969, p. 250) guessing that the value function is of the same form as the instantaneous utility function $J^*(E) = \frac{b(t)}{\gamma_b} E^{\gamma_b}$ and thus $J^*_E (E) = b(t) E^{\gamma_b - 1}$. With this guess the optimal fraction of risky investment can be identified by the first order condition for $\omega_b^*$.

$$
\begin{align*}
J^*_E (r_R E - r_D E) + \lambda J^*_{E - \beta \omega_b E} (-\beta E) &= 0 \\
b(t) E^{\gamma_b - 1} (r_R E - r_D E) + \lambda b(t) (E - \beta \omega_b E)^{\gamma_b - 1} (-\beta E) &= 0 \\
(r_R - r_D) + \lambda (1 - \beta \omega_b^*)^{\gamma_b - 1} (-\beta) &= 0
\end{align*}
$$

$$
\begin{align*}
1 - \beta \omega_b^* &= \left( \frac{r_R - r_D}{\lambda \beta} \right)^{\frac{1}{\gamma_b - 1}} \\
\omega_b^* &= \left[ 1 - \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{\gamma_b - 1}} \right] \frac{1}{\beta}
\end{align*}
$$

Using the guess in the first order condition of consumption gives $b^{\frac{1}{\gamma_b - 1}}$ as consumption to equity ratio.

$$
\begin{align*}
c_b^{\gamma_b - 1} &= b(t) E^{\gamma_b - 1} \\
b^{\frac{1}{\gamma_b - 1}} &= \frac{c_b}{E}
\end{align*}
$$

$$
\begin{align*}
J^*(E, t) &= e^{\rho t} J(E, t) = e^{\rho t} \int_t^{\infty} e^{-\rho s} u(c_b(s), s) \, ds \\
&= \int_t^{\infty} e^{-\rho (s - t)} u(c_b(s), s) \, ds \\
&= \int_0^{\infty} e^{-\rho s} u(c_b(s), s) \, ds = J^*(E)
\end{align*}
$$
The ratio itself can be calculated by using the objective function (18)\(^\text{16}\) (Merton 1969; Sennewald and Wälde 2005).

\[
\frac{c_b}{E} = \frac{\rho_b + \lambda - \gamma_b \left( \frac{r_R - r_D}{\beta} + r_D \right) - (1 - \gamma_b) \lambda \left( \frac{\lambda | \gamma_b |}{r_R - r_D} \right)^{\gamma_b}}{1 - \gamma_b}
\]

References


\(^{16}\)


6 Extended Appendix for the Referee

6.1 Appendix

Higher leverage requires a higher risk premium \((r_R - r_D)\):

\[
\omega_b^* = \left[ 1 - \left( \frac{\lambda \beta}{r_R - r_D} \right)^{1_{1-\gamma_b}} \right] \frac{1}{\beta} > 0 ,
\]

\[
\frac{\partial \omega_b^*}{\partial (r_R - r_D)} = \left[ \frac{1}{1 - \gamma_b} \left( \frac{\lambda \beta}{r_R - r_D} \right)^{1_{1-\gamma_b} - 1} \frac{\lambda \beta}{(r_R - r_D)^2} \right] \frac{1}{\beta} > 0
\]

, decreased risk, i.e. lower likelihood of default \(\lambda\)

\[
\frac{\partial \omega_b^*}{\partial \lambda} = \left[ \frac{1}{1 - \gamma_b} \left( \frac{\lambda \beta}{r_R - r_D} \right)^{1_{1-\gamma_b} - 1} \beta \right] \frac{1}{\beta} = - \frac{1}{1 - \gamma_b} \left( \frac{\lambda \beta}{r_R - r_D} \right)^{1_{1-\gamma_b} - 1} < 0
\]

and higher collateral \((1 - \beta)\)

\[
\frac{\partial \omega_b^*}{\partial \beta} = \left[ -\frac{1}{1 - \gamma_b} \left( \frac{\lambda \beta}{r_R - r_D} \right)^{1_{1-\gamma_b} - 1} \frac{\lambda}{r_R - r_D} \right] \frac{1}{\beta} - \left[ 1 - \left( \frac{\lambda \beta}{r_R - r_D} \right)^{1_{1-\gamma_b}} \right] \frac{1}{\beta^2} < 0,
\]

or decreased risk aversion. For risky investment to be considered by risk averse agents, its expected rate of return must exceed the safe rate of return \(r_R - \lambda \beta > r_D\). It is assumed that this condition always holds, whereby \(\ln (\lambda \beta / r_R - r_D) < 0\). Therefore, bank leverage is decreasing in the banker’s risk aversion.

\[
\frac{\partial \omega_b^*}{\partial (1 - \gamma_b)} = -\frac{1}{\beta} \left[ \left( \frac{\lambda \beta}{r_R - r_D} \right)^{1_{1-\gamma_b}} \left( -\frac{1}{(1 - \gamma_b)^2} \right) \ln \left( \frac{\lambda \beta}{r_R - r_D} \right) \right] < 0
\]

6.2 Appendix

A necessary condition for banking activity is sufficiently low risk aversion. The required level is derived in the following.

\[
\omega_b^* > 1 \iff \left[ 1 - \left( \frac{\lambda \beta}{r_R - r_D} \right)^{1_{1-\gamma_b}} \right] \frac{1}{\beta} > 1
\]

\[
1 - \beta > \left( \frac{\lambda \beta}{r_R - r_D} \right)^{1_{1-\gamma_b}}
\]

\[
\ln (1 - \beta) > \frac{1}{1 - \gamma_b} \ln \left( \frac{\lambda \beta}{r_R - r_D} \right)
\]

\[
1 - \gamma_b < \frac{\ln \left( \frac{\lambda \beta}{r_R - r_D} \right)}{\ln (1 - \beta)}
\]
The sign changes direction because $\ln (1 - \beta) < 0$. The impact of $r_D$ upon $\omega$ is according to intuition negative for households and banks.

$$\frac{\partial \omega^*_i}{\partial r_D} = \left[ -\frac{1}{1 - \gamma_i} \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{1 - \gamma_i}} \frac{\lambda \beta}{(r_R - r_D)^2} \right] \frac{1}{\beta}$$

Reshuffling provides the negative impact upon the rate of return caused by decreased risky investment due to a rise in $r_D$.

$$(r_R - r_D) \frac{\partial \omega^*_i}{\partial r_D} = \left[ -\frac{1}{1 - \gamma_i} \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{1 - \gamma_i}} \frac{1}{(r_R - r_D)} \right] \frac{1}{\beta}$$

$$\omega^*_i - \frac{1}{\beta} < 0$$

The inequality $\omega^*_i - \frac{1}{\beta} < 0$ can be derived from the first order condition (6).

### 6.3 Appendix

The solvency constraint in case of loan default is:

$$E + r_R E \omega^*_b - \beta \omega^*_b E + r_D E (1 - \omega^*_b) > 0$$

$$1 + r_R \omega^*_b - \beta \omega^*_b + r_D (1 - \omega^*_b) > 0$$

$$1 - \beta \omega^*_b + (r_R - r_D) \omega^*_b + r_D > 0$$

From the first order condition (6) it can be seen that $\beta \omega^*_b < 1$, whereby the solvency constraint always holds.

### 6.4 Appendix

The commodity market clears in a Walrasian manner.

$$y_S + y_R = AL_S + (1 + r_R) AL_R - \beta AL_R dq$$

$$= wL + r_R wL_R - \beta wL_R dq$$

$$= wL + r_R (\omega_b E + \omega_h W) - \beta (\omega_b E + \omega_h W) dq$$

$$= wL + r_R \omega_b E - \beta \omega_b E dq + r_R \omega_h W - \beta \omega_h W dq$$

$$= c_L + c_b + c_h + dE + dW$$

The equations have been used in the following order: 1 + 3, 2, 13, 11, 5 and 8, where the equality of bank’s deposit cost and household’s deposit return was utilized.
6.5 Appendix

The deposit market equilibrium balances the safe investment choice of the household and (short-selling) of the bank. It can be used to write the equity / wealth ratio as a function of the deposit rate $r_D$ and vice versa.

\[
(1 - \omega_h^*) W = - (1 - \omega_b^*) E \\
\frac{E}{W} = - \frac{(1 - \omega_h^*)}{(1 - \omega_b^*)}
\]

Using the implicit function theorem, it can be shown that the deposit rate is rising in the equity / wealth fraction:

\[
0 = \frac{E}{W} + \frac{(1 - \omega_h^*)}{(1 - \omega_b^*)} \\
\frac{\partial r_D}{\partial \frac{E}{W}} = - \frac{1}{\left[ \left( \frac{\partial \omega_i^*}{\partial r_D} \right) (1 - \omega_i^*) - (1 - \omega_i^*) \left( \frac{\partial \omega_i^*}{\partial r_D} \right) \right]^{\frac{1}{2}}} \\
= - \frac{(1 - \omega_b^*)^2}{\left( \frac{\partial \omega_i^*}{\partial r_D} \right) (1 - \omega_b^*) - (1 - \omega_b^*) \left( \frac{\partial \omega_i^*}{\partial r_D} \right)} > 0 \text{ if } 1 - \omega_b^* < 0
\]

6.6 Appendix

Using the optimal leverage (bank 6, household 8 respectively) and consumption ratio (7, 9) in the intertemporal budget constraint (5, 10) the equity and wealth growth rates can be depicted as functions of the deposit rate $r_D$. Since the calculations for the bank and household are alike $I$ represents $E$ or $W$ and $i$ represents $b$ and $h$. The stochastic part $\beta \omega_i^* dq$ is also a function of $r_D$, but since it cannot be further simplified it is not substituted.
\[
\frac{dI}{I} = \left( r_R - r_D \right) \left[ 1 - \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{\gamma_i}} \right] \frac{1}{\beta} + r_D \\
\frac{\rho_i + \lambda - \gamma_i \left( r_R - r_D \right) \left( 1 - \gamma_i \right) \lambda \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{\gamma_i}}}{1 - \gamma_i} \right) \right) dt - \beta \omega_i^* dq
\]

\[
= \left( r_R - r_D \right) \frac{1}{\beta} \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{\gamma_i}} \left( r_R - r_D \right) \frac{1}{\beta} + r_D \\
\frac{\rho_i + \lambda - \gamma_i \left( r_R - r_D \right) \left( 1 - \gamma_i \right) \lambda \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{\gamma_i}}}{1 - \gamma_i} \right) \right) dt - \beta \omega_i^* dq
\]

\[
= \left( 1 - \gamma_i \right) \left( r_R - r_D \right) + r_D - \rho_i - \lambda + \gamma_i \left( r_R - r_D \right) + r_D \right) \\
\frac{1}{1 - \gamma_i} \right) \right) dt - \beta \omega_i^* dq
\]

\[
= \frac{(r_R - r_D)}{1 - \gamma_i} \right) dt - \beta \omega_i^* dq
\]

### 6.7 Appendix

The reaction of the consumption ratio following a change in \( r_D \) is dependent upon the risk aversion.

\[
\tilde{c}_i = \frac{\rho_i + \lambda - \gamma_i \left( \frac{r_R - r_D}{\beta} \right) + r_D \left( 1 - \gamma_i \right) \lambda \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{\gamma_i}}}{1 - \gamma_i}
\]

\[
\frac{\partial (\tilde{c}_i)}{\partial r_D} = \frac{-\gamma_i \left( \frac{1}{\beta} + 1 \right) - (1 - \gamma_i) \frac{\gamma_i}{1 - \gamma_i} \lambda \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{\gamma_i}} - \frac{1}{\gamma_i} \frac{\lambda \beta}{r_R - r_D}}{1 - \gamma_i}
\]

\[
= \frac{-\gamma_i \left( \frac{1}{\beta} + 1 \right) - \gamma_i \lambda \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{\gamma_i}} - \frac{1}{\gamma_i} \frac{\lambda \beta}{r_R - r_D}}{1 - \gamma_i}
\]

\[
= \frac{\gamma_i \frac{1}{\beta} - \frac{1}{\gamma_i} \lambda \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{\gamma_i}} - \frac{1}{\gamma_i} \frac{\lambda \beta}{r_R - r_D}}{1 - \gamma_i}
\]

\[
= \frac{\gamma_i \frac{1}{\beta} - \frac{1}{\gamma_i} \lambda \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{\gamma_i}} - \frac{1}{\gamma_i} \frac{\lambda \beta}{r_R - r_D}}{1 - \gamma_i}
\]

\[
= \frac{-\gamma_i + \gamma_i \left( 1 - \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{1}{\gamma_i}} \right) \frac{1}{\beta}}{1 - \gamma_i}
\]

\[
= \frac{\gamma_i \left( \omega_i^* - 1 \right)}{1 - \gamma_i}
\]

This term is positive for the bank \((i = b)\) and negative for the household \((i = h)\). The result is in line with Merton’s (1969 , p. 254) analysis, that the 'substitution effect'
outweigh the 'income effect' for individuals with low relative risk aversion $0 < (1 - \gamma_i) < 1$, while the effects offset on another for the borderline case of logarithmic utility $\gamma_i = 0$. An increase in the interest rate changes the relative price of current versus future consumption in favor of future consumption from the households point of view. This is the substitution effect. However increased lifetime income due to rising interest income can induce the household to decrease savings (income effect). For the bank the effects have opposing signs, because the deposit rate is a cost and not return factor for the banker. Accordingly, the sign is unambiguous for the mean rate of return of the risky investment, which increases returns for households as well as banks.

$$\frac{\partial (\tilde{c}_i)}{\partial r_R} = -\gamma_i \frac{1}{\beta} - (1 - \gamma_i) \lambda \frac{\gamma_i}{1 - \gamma_i} \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{\gamma_i}{1 - \gamma_i} - 1} \frac{-\lambda \beta}{(r_R - r_D)^2}$$

$$= -\gamma_i \frac{1}{\beta} + \lambda \frac{\gamma_i}{1 - \gamma_i} \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{\gamma_i}{1 - \gamma_i} - 1} \frac{1}{(r_R - r_D)}$$

$$= -\gamma_i \frac{1}{\beta} \left( 1 - \left( \frac{\lambda \beta}{r_R - r_D} \right)^{\frac{\gamma_i}{1 - \gamma_i} - 1} \frac{\lambda \beta}{(r_R - r_D)} \right)$$

$$= -\frac{\gamma_i \omega_i^s}{1 - \gamma_i} < 0$$