Are Poor Countries Above Their Steady-State Income Levels?*

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Abstract

It has been argued that the neoclassical growth model implies that less-developed countries are, on average, far above their steady-state levels of income per capita, whereas richer countries are below their steady states. This paper contends that this surprising result is unreliable because it ignores the variability in the base-period ‘level of technology’ and is, therefore, not necessarily an argument against the plausibility of the neoclassical growth model. It does, however, illustrate a drawback of using cross-section data to examine the convergence implications of the neoclassical growth model.

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I Introduction

The neoclassical growth model (Solow 1956, Swan 1956) predicts conditional (beta) convergence. Each economy converges to its own steady-state value of capital and output per worker (usually in efficiency units), with a speed of convergence inversely related to the (absolute) distance from the steady state (e.g., Barro and Sala-i-Martin 1995, Ch. 1). In principle, at any particular time, an economy (whether high- or low-income) can be above or below its steady-state position. However, it has recently been argued that variation in different economies’ positions, relative to their steady states, is systematic: “lower income countries are far above their steady-state positions, while higher income countries are below their steady states. These results ... are the unexpected, but inevitable, implications to preserve the conventional neoclassical model” (Cho and Graham (hereafter CG) 1996, p.289, emphasis added). This has been interpreted as casting doubt on the validity of the neoclassical model (e.g., McCallum 1996, Aghion and Howitt 1998), particularly if, as CG (1996, p.286) claim, “a common conjecture rooted in the neoclassical growth model ... is that lower income countries converge to their steady states from farther below ...” (emphasis in original).

In this paper, we argue that the ‘over-developed’ poor countries result (i.e., poor countries having levels of output per effective worker above their steady states) is not reliable because the variability of country-specific factors, especially the base-period ‘level of technology’, is usually ignored in cross-section empirical work. Thus, it is not necessarily an argument against the plausibility of the neoclassical growth model. However, to the extent that cross-section empirical work on convergence can yield estimates of the crucial ‘convergence gaps’ between steady-state and actual income levels
that are not necessarily even of the correct sign, it adds further to the view that such studies are likely to give misleading results.

II How Can Less-Developed Countries Be ‘Over Developed’?

The argument that poorer countries are, on average, well above their steady-state positions can be outlined by considering an aggregate constant-returns-to-scale, Cobb-Douglas production function, in log form, at time \( t \):

\[
\ln(Y(t)) = \alpha \ln(K(t)) + \beta \ln(H(t)) + (1 - \alpha - \beta)[\ln(A(t)) + \ln(L(t))],
\]

where \( Y \) is aggregate real output, \( K \) is the physical capital stock, \( H \) is the human capital stock, \( L \) is labour input, and \( A \) is the level of technology. \( \ln \) denotes natural logarithms and \((t)\) represents period-\(t\) values. Assuming \( \alpha + \beta < 1 \), Mankiw Romer and Weil (hereafter MRW) (1992) derive an expression for steady-state real income per capita:

\[
\ln(y^*(t)) = \ln(A(t)) + \ln \hat{y}^* = \ln(A(0)) + gt - \alpha_1 \ln(n + g + \delta) + \alpha_2 \ln(s_k) + \alpha_3 \ln(s_h),
\]

where \( y \) is real income per capita (which MRW treat as equivalent to real income per worker), \( \hat{y} = y/A \) is real income per effective unit of labour, \( A(0) \) is the base-period level of technology, and \( * \) denotes steady-state values. \( n \) and \( g \) are, respectively, the (assumed) exogenous growth rates of the labour force and technology, \( \delta \) is the common depreciation rate of human and physical capital, and \( s_k \) and \( s_h \) are, respectively, the fractions of income
invested in physical and human capital.\(^1\) In terms of the elasticities in equation (1), \(\alpha_1 = (\alpha + \beta)/(1 - \alpha - \beta)\), \(\alpha_2 = \alpha/(1 - \alpha - \beta)\) and \(\alpha_3 = \beta/(1 - \alpha - \beta)\).

MRW (1992, pp.422-3) examine the dynamics of adjustment to steady states by taking a Taylor series expansion in physical capital per effective worker around the steady state, obtaining:

\[
\ln\left(\frac{y(t)}{y(0)}\right) = (1-e^{-\lambda t})[\ln\left(\hat{y}^*(0)\right) - \ln\left(\hat{y}(0)\right)],
\]

(3)

where \(\lambda\) is interpreted as a speed-of-convergence parameter. The definition of \(\hat{y}^*\) and equation (3) implies (e.g., CG 1996, equation (3a)):

\[
\ln(y(t)) - \ln(y(0)) = gt + (1-e^{-\lambda t})[\ln\left(\hat{y}^*(0)\right) - \ln\left(\hat{y}(0)\right)]
\]

(4)

Equation (4) implies that countries with slow growth rates, i.e., less than \(gt\) (but not necessarily negative), over the period from 0 to \(t\) will be above their steady states (CG 1996, p. 289, fn. 6; Temple 1998b). With \((1-e^{-\lambda t}) > 0\), \([\ln(y(t)) - \ln(y(0))] - gt < 0\) implies \(\ln(\hat{y}^*) < \ln(\hat{y}(0))\) and \(\ln(y^*(0)) < \ln(y(0))\). To see which countries have slow growth rates, some analysts (e.g., McCallum 1996, Temple 1998b) examine the implications of absolute divergence in living standards across countries, corresponding to \(b_1 > 0\) in the regression

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\(^1\) \(y^*\), which is the focus of CG’s discussion, is the level of real income per capita along a steady-state growth path, consistent with the steady-state level of real income per effective unit of labour, \(\hat{y}^*\), and the
ln(y(t)) − ln(y(0)) = b_0 + b_1 ln(y(0)) + \zeta. \ (5)

Equating the RHS of equations (4) and (5) (and ignoring the error term, \zeta), if (1−e^{−\lambda t}) > 0 and \( g \) is constant across countries then absolute divergence, \( b_1 \) > 0, implies that initially richer countries have larger ‘convergence gaps’, ln(y*'(0)) − ln(y(0)), than poorer countries. From equation (4), poor countries will therefore grow slower than rich countries, at least on average. This, combined with the characteristics of slow-growth countries noted above, implies that initially poor countries are, on average, above their steady states.

Note, however, that if equation (4) is correct then equation (5) is misspecified because the determinants of y*(0) are omitted. Substituting from equation (2) in equation (4):

\[
\ln(y(t)) − \ln(y(0)) = \gamma_0 + \gamma_1 \ln(y(0)) + \gamma_2 \ln(n + g + \delta) + \gamma_3 \ln(s_k) + \gamma_4 \ln(s_h), \quad (6)
\]

where \( \gamma_0 = gt − \gamma_1 \ln(A(0)), \gamma_1 = −(1−e^{−\lambda t}), \gamma_2 = \alpha_1 \gamma_1, \gamma_3 = − \alpha_2 \gamma_1 \) and \( \gamma_4 = − \alpha_3 \gamma_1 \). The evidence from MRW (1992) and many other studies strongly suggests that \( \gamma_2, \gamma_3, \) and \( \gamma_4 \) in equation (6) are not all zero, so that imposing these restrictions will, in general, give biased estimates of \( b_1 \).

MRW’s (1992, Tables III and V) estimates for equation (6), for their ‘non-oil’ sample are consistent with conditional convergence, i.e., \( \gamma_1 \) is negative and statistically exogenously determined value of A. Because y* = \( \hat{y} \ast A \), if A (= A*) grows at the exogenously determined rate g then so does real income per capita along the steady-state path.

\(^2\) While it is convenient to use the regression in equation (5) as a rough descriptive device to judge whether initially rich countries have grown faster than poor ones, it is not valid to then interpret parameter estimates or implied steady-state values from equations (4) or (6) using estimates from equation (5), as the latter is not consistent with the underlying model. It should also be noted, as McCallum (1996) points out, that estimates of \( b_1 \) are not very precise and are often not statistically significantly different from zero. In this latter case, the argument suggests that “low-income countries are on average neither closer to (proportionately) nor farther from their steady-state positions than are rich countries” (McCallum 1996, p.65). This is a much less controversial result, especially considering that the steady-state growth path is country specific.
significant. Based on equations (2) and (6), CG use estimates of the $\gamma$s, based on MRW’s data, to obtain estimates of the base-period (1960) steady-state levels of income per capita by assuming $g = 0.02$ (to “identify $A(0)$ from $g$” (p.287)) and (with $t = 1985$) calculating (see their equation (3b')):

$$\ln(y^*(0)) = \{\gamma_0 - (0.02)(25) + \gamma_2 \ln(n + g + \delta) + \gamma_3 \ln(s_k) + \gamma_4 \ln(s_h)\}/(-\gamma_1). \quad (7)$$

Plotting $\ln(y^*(0))$ against $\ln(y(0))$, where ‘0’ denotes 1960 values, CG find that a fitted regression line has a slope significantly greater than unity, with poorer countries highly represented in situations where $\ln(y^*(0)) < \ln(y(0))$. CG’s results therefore appear to imply that, even when the determinants of $y^*(0)$ are included, “for the model to fit the data, one corollary is that many countries (especially poor ones) have been converging to their steady states from above” (Aghion and Howitt 1998, p.35).

Note that CG’s identification of $A(0)$ requires a stronger assumption than MRW. For estimation, MRW assume that $\ln(A(0)) = a + \varepsilon$, where $a$ is a constant and $\varepsilon$ is a country-specific error term assumed, controversially, to be independent of the explanatory variables. So, whereas CG assume $A(0)$ is constant across countries, MRW assume only that $a$ is constant. On MRW’s assumptions, CG identify $a$, not $\ln(A(0))$, with the implication that their estimates of $\ln(y^*(0))$ omit the country-specific $\varepsilon$ terms and are therefore not correct.

III Country-Specific Levels of Technology and Measures of Convergence Gaps
The conventional analysis of the ‘over-developed’ less-developed country assumes that: (i) \(A(0)\) is constant across different countries (or, at least, that any deviations are random) and (ii) \(g\) is constant across all countries for all time periods. These assumptions imply that, for any time period, all countries have a common level of technology. Even if \(g\) can be treated as constant across different countries (which is dubious, but common in most empirical studies of the neoclassical growth model), it is implausible that \(A(0)\) is invariant across all the countries in a cross section spanning a wide range of levels of development (as MRW 1992, p.411 admit; see also Caselli, Esquivel and Lefort 1996, p.380). The implications of the variation across countries in \(\ln(A(0))\) for the properties of OLS estimation of growth equations like equation (6) have been discussed by, for example, Islam (1995), Sims (1996) and Caselli, Esquivel and Lefort (1996). Here we focus on the implications for the measurement of convergence gaps and the ‘over-developed poor countries’ result.

Assumption (i) is important because the level of the steady-state growth path and the size and sign of the convergence gap between \(\ln(y^* (t))\) and \(\ln(y(t))\) depend on \(\ln(A(0))\).

Following Durlauf and Quah (1999), from equation (3), and using \(\ln(y(t)) \equiv \ln(A(t)) + \ln(\hat{y}(t))\):

\[
\ln(y(t)) = \ln(A(0)) + gt + (1-e^{-\lambda t})[\ln(\hat{y}^*) - \ln(\hat{y}(0))] + \ln(\hat{y}(0))
\]

\[
= \ln(\hat{y}^*) + \ln(A(0)) + gt - e^{-\lambda t}[\ln(\hat{y}^*) - \ln(\hat{y}(0))].
\]

\[\text{(8)}\]

\(^3\text{CG rank the 98 countries by 1960 GDP per adult and split them into four approximately equal groups. The geometric mean GDP per adult in the 24 countries with the lowest values of GDP per adult in 1960 is, based on their calculations, approximately twice their mean steady-state GDP per adult.}\]
The first three terms in equation (8) determine the steady-state growth path for income per capita, with the first two terms determining the level and $gt$ determining the slope. The last term represents transitional growth. Note, in particular, that if two economies, B and C, have identical values of the determinants of $\hat{y}^*$ and the same rate of growth of technology, $g$, but economy B has a higher level of $A(0)$ than economy C, then B will be on a higher steady-state growth path and, for any given level of $y(0)$, will have a numerically larger convergence gap. The latter can be seen directly by subtracting $\ln(y(0))$ from both sides of equation (2), with $t = 0$:

$$\ln(y^*(0)) - \ln(y(0)) = \ln(A(0)) + \ln(\hat{y}^* - \ln(y(0))).$$

(9)

Any meaningful attempt to estimate the size or sign of the convergence gaps for different countries must therefore allow for cross-country variation in $A(0)$; otherwise it is likely that base-period income per capita will be compared (explicitly or, as with CG, implicitly) with an inappropriate steady-state growth path. For example, if economies B and C have the same value of $y(0)$ but C has a sufficiently lower value for $A(0)$ compared to economy B then it is feasible, at time 0, for C to be above and B to be below their respective steady-state growth paths, i.e., if

$$[\ln(\hat{y}^*) + \ln(A(0)) + gt]_B > \ln(y(0))_{B,C} > [\ln(\hat{y}^*) + \ln(A(0)) + gt]_C,$$

where subscripts $B$ and $C$ identify the two countries. It is not an inevitable implication of the neoclassical growth model that low-income countries are above and high-income countries are below their steady states; to test this would require information on the cross-
country variation in $A(0)$.\textsuperscript{4} From equation (9), if high-income countries have higher $A(0)$ values they will, other things equal, have larger convergence gaps and, hence, higher growth rates. Conversely, low growth rates may be partly due to low values of $A(0)$.

Assumption (ii) is also important because variation in $g$ across countries can help to explain why richer countries are not necessarily further below their steady states and why poorer countries are not necessarily above their steady states. If richer countries tend to have higher values of $g$, absolute divergence in equation (5), even ignoring misspecification problems, does not necessarily imply that richer countries have larger values for $[\ln(y^{*}(0)) - \ln(y(0))]$ in equation (4) (Temple, 1998b). From equations (2) and (9), higher values of $g$, other things equal, lead to smaller values of the convergence gap.

Similarly, variation in $g$ means that, even if assumption (i) is accepted, a low value for $[\ln(y(t)) - \ln(y(0))]$ in equation (4) does not necessarily imply $[\ln(y^{*}(0)) - \ln(y(0))] < 0$ if $g$ is correspondingly lower (Temple, 1998b).\textsuperscript{5} In the conditional convergence regression in equation (6), variation in $g$ undermines any attempt to identify the steady-state values of $y$ by ‘fixing’ $g$, as in equation (7), in order to separate $A(0)$ from $g$.

The previous arguments imply that variation in $A(0)$ is potentially important in determining the signs and sizes of the convergence gaps in the neoclassical model. Unfortunately, empirically, the unobservable nature of $\ln(A(0))$ limits the scope for allowing for differences in country-specific technology shift terms in cross-section studies.\textsuperscript{6} However, there are a number of complementary pieces of evidence that suggest

\textsuperscript{4} It is, of course, possible that some low-income countries are above their steady states. For example, Ben-David (1998) suggests that this may well be the case for countries near subsistence level.

\textsuperscript{5} Temple (1998b) concentrates on the argument that cross-country variation in $g$ is positively correlated with capital-labour ratios, which can help explain a number of puzzling results in the empirical growth literature, including the ‘over-developed’ poor countries result. Our focus is on cross-country variation in $A(0)$ and, hence, assumption (i), whereas Temple focuses on assumption (ii). While, in principle, variation in $g$ and $A(0)$ could be independent, in practice they are likely to be complementary explanations.

\textsuperscript{6} A natural reaction might be to use panel-data methods to deal with unobservable cross-country variation in $A(0)$. However, as Durlauf and Quah (1999) emphasize, the use of panel-data techniques can hinder rather than help the interpretation of convergence regressions.
that measures of the sign or size of the convergence gaps are likely to be very sensitive to
cross-section variation in $\ln(A(0))$.

Proxy measures of $A$ obtained for a cross-section of countries (e.g., Islam 1995,
Klenow and Rodriguez-Clare 1997, Hall and Jones 1999) display significant cross-country
variation. Moreover, they strongly support the prior that countries with low income per
head tend to have low estimated values for $\ln(A)$.\textsuperscript{7} While any attempt to measure
unobservable technical efficiency is fraught with difficulties, for common samples the
correlations between different constructed proxies are reasonably high (de la Fuente and
Domenech 2000), despite the use of different methods (e.g., levels accounting or estimated
regression models).

Because proxies for $\ln(A(0))$ exist it seems reasonable to consider whether they can
be explicitly included in equation (6) in a CG-type analysis. Such an exercise can
demonstrate the sensitivity of CG’s results, e.g., the estimated slope of the plot of $\ln(y^*(0))$
on $\ln(y(0))$, to variation in $A$.\textsuperscript{8} For example, using MRW’s data, the Islam estimates of
$\ln(A(0))$ and corresponding point estimates from his Table IV to calculate estimates of
$\ln(y^*(0))$ we obtained a plot of $\ln(y^*(0))$ against $\ln(y(0))$ for which the slope coefficient is
not significantly different from unity ($t$-statistic = 1.24) after allowing for the variation in
$\ln(A(0))$; full results are available on request. However, while such an exercise can
demonstrate the sensitivity of CG’s results, the use of ‘imported’ $A(0)$ estimates can not
provide a ‘quick fix’ method of measuring the size and sign of convergence gaps because

\textsuperscript{7} For example, for Islam’s (1995) estimates, an index of the value of $\ln(A(0))$ relative to the minimum value
of $\ln(A(0))$, where 0 corresponds to 1960, varies from 1 to 38.48. The simple correlation between $\ln(A(0))$
and $\ln(y(0))$ is 0.866.

\textsuperscript{8} They can also be included in estimated neoclassical growth models to try to avoid the serious estimation
problems caused by omitting variation in $A$ (e.g., Knowles, Lorgelly and Owen 2002).
of the difficulties in ensuring compatible treatment of any scale factors in the measurement of all the relevant variables with the ‘imported’ $A(0)$ estimates.\(^9\)

Some cross-section studies (e.g., Temple, 1998a) proxy variation in \(\ln(A(0))\) by including regional dummy variables in the estimated model, on the basis that the variation in $A(0)$ across regional country groups is much greater than within these groups. The sensitivity of estimates of the convergence gaps is also apparent if dummy variables are used to proxy for variation in \(\ln(A(0))\). Including a 0-1 intercept dummy for sub-Saharan African countries, which, in a crude fashion, allows for a lower $A(0)$ value for these countries relative to the rest of the sample, gives a significant negative coefficient on the dummy variable in equation (6). For the plot of $\ln(y^*(0))$ against $\ln(y(0))$, the intercept term is positive (0.203), but not statistically significant, the slope term is 0.848 (but not significantly different from 1, on a 5% two-tailed test), and the estimated convergence gaps are negative for all but 10 of the countries in the sample. In contrast, including separate 0-1 intercept dummies for the OECD, Latin America and the Caribbean, Middle East/North Africa/Other Mediterranean, East Asia, and sub-Saharan Africa gives significant coefficients on all of these variables, other than the Latin America and Caribbean dummy. For the plot of $\ln(y^*(0))$ against $\ln(y(0))$, the intercept term is strongly positive (5.44 for the unrestricted model and 4.14 for the restricted model) and statistically significantly different from zero, while the slope term is significantly less than 1). Except for one high-income country in the restricted model, all the estimated convergence gaps are positive. Experimentation with different sets of regional dummy variables emphasizes the marked sensitivity of the estimated convergence gaps to the choice of regional dummy variables.

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\(^9\) This does not affect the estimated slope term in the plot but will (in log-linear models) affect the estimated intercept term and, hence, estimates of the steady-state values. For example, using the Islam estimates of $\ln(A(0))$ the implied convergence gaps turn out to be negative for all the countries in the sample.
The lack of robustness of estimates of the convergence gaps is reinforced by examining the effect on CG’s results of varying the (assumed common) value of $g$ used to identify $A(0)$. Increasing this value in equation (7), for given values of all the estimated parameters and variables, does not alter the estimated slope term in the plot of $\ln(y^*(0))$ against $\ln(y(0))$, but gives a larger negative estimated intercept term and a larger proportion of countries with negative convergence gaps. For example, for MRW’s 98-country non-oil sample, CG’s chosen value of $g = 0.02$ gives an estimated intercept term in the plot of $-2.658$ ($t$-value = 2.96) with 49 countries out of 98 above their steady state. Increasing $g$ to 0.03 gives an estimated intercept term of $-3.525$ ($t$-value = 3.92) with 85 countries out of 98 above their steady state. This suggests that any assumption to ‘anchor’ the steady-state values is somewhat arbitrary, yet is crucial to estimates of these steady-state values.

**IV Conclusion**

Existing evidence strongly suggests that there is significant variation in levels of $A(0)$, the base-period ‘level of technology’, across different countries; poorer countries generally have low values of $A(0)$, while rich countries have higher values. The neoclassical growth model predicts that variation in $A(0)$ will affect the level of an economy’s steady-state growth path, its convergence gap, $\ln(y^*(0)) – \ln(y(0))$, and hence its rate of economic growth. As a result, any meaningful attempt to evaluate the size or sign of the convergence gaps between actual and steady-state incomes in different countries must allow for this cross-country variation in $A(0)$. Omitting variation in $A(0)$ from the analysis (by assuming that it is constant across countries or that it varies randomly) suppresses a key determinant of the steady-state growth path. Consequently, for any particular country, it is likely that base-period income per capita will be compared with a steady-state growth
that is inappropriate because it omits this important country-specific shift factor. Restricting $A(0)$ to be constant (or random) across all countries will therefore bias any estimates of cross-country convergence gaps to the extent that they may not even be of the correct sign. Under such conditions any conclusion that poorer countries are systematically ‘over developed’ is dubious. More generally, because of the difficulties in allowing for variation in $A(0)$ in cross-country studies, this adds another drawback to an already long list of problems with using pure cross-section data to examine the convergence implications of the neoclassical growth model.
References


