A Theory of Monitoring and Internal Labor Markets

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Abstract

This paper analyses a firms strategies for monitoring and evaluation of workers, in a scenario where workers differ in their abilities to handle crises that occur periodically on the job. Good workers resolve crises with no loss of productivity, while bad workers fail in the face of crises and thus impose a cost on the firm. The cost of failure is large in high-skill jobs and small in low-skill jobs, so the firm wants to place only good workers in high-skill jobs. At each time the firm has an assessment of the probability that the worker is good. It promotes the worker only when the assessment is sufficiently high.

The firm can use one of two evaluation methods. It can either passively update its assessment as long as the worker continues without failure, or it can actively observe the worker until a crisis arises, and accurately learn the workers type. We show that the firms optimal strategy towards a worker at any point of time depends on its current assessment. For some ranges of assessments the firm passively updates, while for others it actively observes the worker. This yields consequences for wage-increases and promotions that accord well with observed behaviour.
The literature on monitoring in the labor market almost universally assumes that its sole purpose is to allow firms to punish workers who shirk, cheat or steal. Yet, this assumption both generates empirical predictions that are, at the least, problematic and is inconsistent with our everyday experience. We propose a simple theory in which the employer uses monitoring to learn about worker quality and to catch potentially costly mistakes. In the model, monitoring is costly and therefore not always profitable. When monitoring is too costly, in a sense that we will make precise later, in our model firms infer worker quality based on whether or not a worker has been revealed to have made mistakes. Workers in such jobs will either separate from the firm, receive automatic pay increases up to some maximum wage or be promoted. Thus the model generates an internal labor market with many of the characteristics associated with pay and promotion in large firms.

From a theoretical perspective, modeling monitoring as designed to enforce good behavior is problematic. The standard result from the literature on crime is that since detection is costly and deterrence depends on the probability of detection multiplied by the cost of punishment, fines should be as large as possible and monitoring should be minimal. Dickens, Katz, Lang and Summers (1989) refer to this as the monitoring puzzle. Akerlof and Katz show that the only solution to models of this sort is the one derived by Becker and Stigler (1974), which is to have workers “buy” their jobs and to have the purchase price returned to them when they retire. If workers’ ability to purchase their job is limited, firms may require them to engage in rent dissipating behavior (Murphy and Topel, 19??). Neither purchase of jobs nor obviously rent-dissipating requirements are a common feature of job contracts. If bonding is costless, more general earnings profiles are possible, but the logic of the argument requires that wages be less than value of marginal product early in seniority and more than VMP later (Lazear, 1979, 1981). The efficient contract sets hours so that VMP equals the worker’s marginal value of leisure, while workers would wish to choose to set the marginal value of leisure equal to their wage. This implies that junior workers will want to work less than required by the optimal contract while senior workers will want to work more. In fact, the desire to work less increases with seniority (Kahn and Lang, 1992, 1996).

Moreover, the view that monitoring serves primarily as a discipline device violates everyday experience. We do not check our research assistant’s work mainly to deter him from shirking, nor do we fire him if we catch an error. We want to avoid the cost of a mistake. The frequency with which we find errors will affect our assessment of the research assistant and thus our decision to rehire him. But in this sense monitoring is part of our process of
evaluating the research assistant. The threat that we may fire or not rehire the assistant may have the additional effect of encouraging the assistant to work harder in order to reduce the frequency of errors, but our monitoring decision is mostly affected by our need to catch errors and to know whether the assistant is competent. As we become more convinced that the assistant can handle the tasks to which he has been assigned, we are likely to reduce our monitoring. In this respect we resemble private sector managers who often monitor workers as part of an evaluation process and review work to determine that it has been done correctly.

In this paper, we focus on monitoring for the purpose of evaluation. Formal modeling of monitoring solely for the purpose of catching and correcting mistakes has not provided us with any nontrivial insights and combining the possibility of productive monitoring with evaluative monitoring changes the mathematics but not the essential message. Our model is closest in spirit to Bjerk (2008) who assumes that firms learn less about worker productivity in low-level jobs where productivity is less responsive to ability. However, presumably because firms cannot capture rents from their knowledge of worker productivity and they cannot commit to a job assignment, variation in learning is an exogenous response to a job assignment decision made on the basis of where the worker’s expected productivity is highest.

The way we model monitoring reflects our view that the issue is the ability to do the difficult parts of a job or, in our terminology, respond to crises. There is a well-known, story in which Albert Einstein complains to his chauffeur that he is tired and does not want to give another speech that evening. The chauffeur, who looks somewhat like Einstein, says, “I’ve heard you give that talk so many times, I bet I could give it for you.” They switch clothes and Einstein sits in the back of the lecture hall pretending to be the chauffeur. For most of the evening, the chauffeur performs excellently. He gives the speech fluently. He handles questions with aplomb until he is finally faced with a pompous questioner who asks something obscure about anti-matter. The chauffeur points to Einstein and says “That question is so trivial I am going to let my chauffeur answer it.”

This, admittedly apocryphal, anecdote captures the tendency for much work in even highly skilled jobs to be routine. The chauffeur can give the speech and answer routine questions. Only when a special circumstance arises, a “crisis” in our terminology, does skill matter. Except for the chauffeur’s deftness, the audience would have realized that it had been duped when he was asked a nonroutine question. But had no such question been asked, the chauffeur would have performed admirably. The audience would not have recognized the difference between Einstein and his chauffeur.
We do not wish to imply that there are no specialized skills. Many of us do not how to change the oil on a car, but for anyone trained as a mechanic, this is usually routine. Physicians frequently see and recognize the same cluster of symptoms, making certain diagnoses straightforward for anyone who has been properly trained. Most real estate transactions are sufficiently simple that in some U.S. states, no lawyer need be involved.

However, sometimes something nonroutine arises: the drain plug will not loosen; the symptoms do not quite fall into the usual cluster; previous sale of the property was mishandled. If the individual faced with the nonroutine task is skilled, she may be the only one who is ever aware of it: she finds an appropriate torque wrench and loosens the plug without stripping it; he diagnoses the condition accurately and prescribes the proper treatment; she contacts the lawyer who handled the previous sale and has the problem corrected. However, if the worker is unskilled at his job, the outcome may be very noticeable: he strips the plug; oil leaks out and the engine is seriously damaged; she produces a diagnosis of asthma when the problem is heart disease; the patient soon dies of a heart attack; the prior error goes unnoticed; the new owner later incurs considerable expense to establish rightful ownership.

We consider the case of a firm that has two types of jobs on which crises may arise. In the low-tier job, the cost of failing to address such crises is small. In the high-tier job, it is large. In routine settings, all workers are more productive in the high-tier job, but crises arise sufficiently frequently and failure to address them is sufficiently costly that the firm does not want to assign bad workers to the high-tier job. It will only assign workers to the high-tier job if it is sufficiently certain that they are good workers.

Firms can use two approaches to determining whether a worker is likely to be good. First, it can simply assign her to a low-tier job, in which the cost of failing to address a crisis is small. If the mechanic strips the oil plug, the customer will notice that his car is leaking oil and bring it back to the shop, and a skilled mechanic will fix the problem at little cost to the repair station. If there have been no customer complaints about a mechanic, then either she has solved all the crises or she has been lucky and not faced any. As the number of oil changes she has performed increases, it becomes increasingly unlikely that she did not have to address any crises and that, instead, she has solved the ones she has faced.

Alternatively, the firm can assign someone to monitor the employee’s work. She checks the medical records and notes that the case is somewhat puzzling. She checks the diagnosis. If it is correct, she concludes that the physician is good and can safely be placed in a setting where the cost of an
error is high. If it is incorrect, she reaches the opposite conclusion.

Initially we abstract from wages and the presence of an external labor market. We consider only the optimal allocation of workers to task and optimal monitoring of their performance. We show that if the cost of monitoring is finite but sufficiently small, then there are four assignment tiers. Those the firm believes are very likely to be good are placed in the high-tier job (group A). Those with a somewhat lower probability of being good are not monitored. After some time, if they have not been observed to fail to handle a crisis, they are placed in the high-tier job (group B). Below this tier, workers are monitored (group C). Note that, unlike the medical example above, the monitoring does not affect the outcome. It simply provides more accurate information because the monitor observes whether a crisis arose, but allowing the monitor to correct the mistake would not change the results in any important way. If no crisis has arisen, the firm’s beliefs are unchanged. If a crisis arises, it learns whether the worker is good or bad. Finally, if the probability that the worker is good is sufficiently low, the firm again does not monitor him (group D). However, unless its prior that the worker is good is 0, after some time, if the worker has not faced a crisis and failed, the probability that the worker is good rises to a sufficient level that the firm begins to monitor him.

The model has a number of nontrivial and, we believe, interesting implications. The following predictions hold provided that assignment is efficient:

1. The probability of promotion rises discontinuously as one moves from below to above the cutoff between group C and group B.

2. Mean time to promotion, conditional on promotion, may rise or fall as one passes this threshold. The variance in promotion time is greater below the threshold than just above it.

We avoid making strong assumptions about market structure, but two relatively weak assumptions give us some additional implications. We assume that if the worker has been revealed to be bad (at this firm), he separates from the firm. And we assume that the wage is an increasing function of the probability that the worker is a good worker. This generates the result that at least some workers receive scheduled pay increases unless they separate. Moreover, some workers reach the “top of the pay scale” until they are either promoted or separate from the firm. Thus the model predicts some of the features of “bureaucratic pay scales.”

If we are willing to make some additional assumptions about the wage-setting mechanism, we have some additional predictions. In particular, if all
information is public and labor markets are competitive, then workers are paid their expected value of marginal product net of any monitoring cost. In this case

1. If failures are costly in the low-tier job (but $b > \gamma c$ also probably want to assume that expected output of bad workers in low jobs is 0; so workers are never fired), wages drop discontinuously at the boundary between group D and group C and rise again discontinuously at the boundary between group C and group B.

2. If failures are costly in the low-tier job, wages rise with seniority for workers in groups D (provided $\theta \neq 0$), group B and some workers in group A but not for workers in group C. We need to check whether this implies serial correlation of wages increases.

3. If the cost of failures in the high-tier job is high, then demotions from the high-tier job are rare. An individual who is promoted to a high-tier job is less likely to be demoted than that individual was ex ante to experience a wage decrease within the low-tier job.

4. Wages of individuals promoted from group B are continuous at promotion. Those promoted from group C receive a discontinuous wage increase.

5. The average wage after promotion is less than the average wage of incumbents in high-tier jobs.

Such an equilibrium can be sustained if all information about performance is public and workers are risk neutral. In this case, when it is efficient for workers to be monitored, they willingly accept lower wages in ordered benefit from the monitoring. Alternatively, if firms are able to capture all the rents from information about worker productivity, they will also make efficient decisions. Other settings in which workers and firms share the costs and rents are also possible, but for the moment we exclude environments in which it is profit-maximizing for firms to make decisions that do not maximize the expected present value of the worker’s lifetime output.

1 Workers and jobs

An employer hires a worker whom he can put in a high job (H) or a low job (L). The worker’s productivity in a given job depends on his type, which
may be good (G) or bad (B). Both types of workers produce a flow output normalized to zero per unit time in the low job and \( g > 0 \) per unit time in the high job. For the moment it is irrelevant whether type is general or firm-specific, but it may be helpful to think of it as firm-specific.

*Crisis* occur in both types of jobs with a Poisson arrival rate \( \lambda \). This assumption ensures that job assignment is unaffected by its impact on learning about productivity. Assumptions of this nature are common in the literature on internal labor markets (e.g. Gibbons and Waldman, 2004). It might be more natural to assume that crises are more common in the high job, but it greatly complicates the model. We leave this extension for future work.

Bad workers *fail* when a crisis occurs. Failure generates negative output of \(-c_l\) in the low job and \(-c_h < -c_l\) in the high job. If a worker is bad and a crisis occurs, then the failure is immediately observed and the worker’s type is revealed. Good workers resolve crises when they occur, with no impact on productivity. Thus if the worker is good, the occurrence of a crisis can be known only if the worker is actively observed, in which case the worker’s type is revealed. We assume that \( g - \lambda c_h < -\lambda c_l \) so that the expected flow of output net of costs associated with crises is more negative when a bad worker is placed in an H job than when he is placed in an L job.

Time is continuous and the future is discounted at a rate \( r \).

Under complete information, it is clear that good workers will be put in H-jobs and bad workers will leave the firm since their productivity is negative. We assume that if a worker is revealed to be bad, she separates from the firm immediately. This assumption is natural if type is firm-specific. If type is general, whether it makes sense will depend on how the labor market is structured.

## 2 Monitoring

The employer can use one of two strategies to assess workers: observation (M) or no-observation (N). Under the no-observation strategy N he puts the worker in a job and does not observe him. Thus he only gets confirmation of the worker’s type if the worker is bad and a crisis occurs, in which case the worker fails. If the worker is good, or until a crisis arrives, the employer observes nothing, and can update his beliefs about the worker as time passes.

Under the observation or monitoring strategy M, the employer observes the worker until a crisis occurs, at which time he learns the worker’s type. There is a flow cost of \( b \) per unit time of observing, and the cost must be borne until a crisis occurs. Note that under the observation strategy the
employer’s beliefs remain unchanged until a crisis occurs, at which time the employer knows the worker’s type.

3 Optimal promotion without observation

Consider a new worker at time $t = 0$ with prior probability $\theta_0$ of being good. We will assume that $\theta_0$ is sufficiently high that it makes sense for the worker to be with the firm. The critical value at which this occurs will depend on the structure of the labor market and is not derived here, but one sensible interpretation is that $\theta_0$ is the probability that the worker is good at a randomly chosen job. Under this interpretation, the firm’s prior that the worker is good at this job is updated based on experience at the current firm but the market prior that the worker is good is unaffected by the outcome of this particular employment relationship. But nothing in this section relies on this interpretation.

If $\theta_0$ is not too high, the employer will place the worker in the L-job. If the worker fails at some time, his type is revealed to be bad, and he will leave the firm. At any time $t$, if he has not failed, then the employer updates his belief about the worker’s type to $\theta(t, \theta_0)$. As time passes and $\theta(t, \theta_0)$ becomes sufficiently high, the employer may promote the worker to the H-job. Similarly, a worker who comes in with a sufficiently high prior at time 0 will be placed in the H-job.

Given Poisson arrival, the density function for the arrival of the first crisis is $\lambda e^{-\lambda t}$, hence the probability that the first crisis arrives by time $\tau$ is $p(\tau) = 1 - e^{-\lambda \tau}$. Thus the probability that a bad worker does not fail by time $\tau$ is $1 - p(\tau) = e^{-\lambda \tau}$.

We prove the following theorem in the appendix.

**Theorem 3.1.** If the firm does not monitor the worker, then it promotes the worker when its assessment of the probability that the worker is a good worker reaches

$$\theta^* = \frac{\lambda (c_h - c_l) - g}{\lambda (c_h - c_l)}$$

provided that

$$\theta_0 < \theta^*$$

and the value of this strategy is given by

$$U^*(\theta_0) = \frac{\lambda}{r(\lambda + r)} \theta_0 g \left[ \frac{\theta_0}{1 - \theta_0} \frac{g}{\lambda (c_h - c_l) - g} \right]^{\frac{\lambda}{\lambda + r}} - (1 - \theta_0) c_l \frac{\lambda}{\lambda + r} \text{ for } \theta_0 \leq \theta^*.$$
If $\theta_0 \geq \theta^*$, the firm places the worker in the H job immediately.

Note that (1) has a natural interpretation. The worker is promoted when the expected flow payoffs in the L and H jobs are equal, that is

$$-(1 - \theta^*)\lambda c_L = g - (1 - \theta^*)\lambda c_h.$$  

This follows from the assumption that learning about productivity is independent of job placement. Therefore the assignment decision is determined solely by the effect on expected output.

4 Payoff with the observation strategy

When the employer observes the new worker he knows when the first crisis arrives, and immediately identifies the worker’s type. Before the arrival of the first crisis no new information is generated, so there is no continuous updating of beliefs.

Let $\theta_0$ be the prior that the worker is good. When the first crisis arrives, with probability $\theta_0$ the (good) worker resolves the crisis and is promoted to the H-job, with the complementary probability he fails and leaves the firm. In either case the employer ceases to monitor him. Recall that observation has a flow cost of $b$ per unit time. Suppose the first crisis arrives at time $t$, then the employer’s payoff is:

$$\int_0^t -be^{-r\tau}d\tau + \theta_0 \int_t^\infty ge^{-r\tau}d\tau - (1 - \theta_0)c_l \frac{\lambda}{\lambda + r} =$$

$$-\frac{b}{r} + \frac{1}{r}(\theta_0 g + b)e^{-rt} - (1 - \theta_0)c_l \frac{\lambda}{\lambda + r}$$

Since the density function of the arrival of the first crisis is $\lambda e^{-\lambda t}$, the expected value of the strategy is:

$$\tilde{U}(\theta_0) = \int_0^\infty \{-\frac{b}{r} + \frac{1}{r}[\theta_0 g + b]e^{-r\tau}\} \lambda e^{-\lambda t} dt - (1 - \theta_0)c_l \frac{\lambda}{\lambda + r}$$

$$= \frac{1}{r(\lambda + r)}[\lambda \theta_0 g - rb] - (1 - \theta_0)c_l \frac{\lambda}{\lambda + r}. \quad (4)$$

We assume that

$$\frac{g}{r} - \frac{b}{\lambda} > 0. \quad (5)$$

If not, even if the firm knew that the worker was good and even if the only way it could assign the worker to the H-job was by monitoring and observing him solve a crisis, it would prefer not to do so.
5 Optimal monitoring

Next we compare the two monitoring strategies to determine which one yields the greater expected payoff. For ease of notation, we define

\[ c = c_h - c_l \]

Using (2) and (4), we have that monitoring will be more profitable than waiting is if

\[ \lambda \theta_0 g [1 - \left( \frac{\theta_0}{1 - \theta_0} \right) \frac{\hat{g} (\frac{g}{\lambda c - g}) \hat{r}}{\hat{r}}] \geq r b. \]  \hspace{1cm} (6)

Write the expression on the left-hand side of (6) as:

\[ Z(\theta_0) = \lambda \theta_0 g [1 - \left( \frac{\theta_0}{1 - \theta_0} \right) \frac{\hat{g} (\frac{g}{\lambda c - g}) \hat{r}}{\hat{r}}] \] \hspace{1cm} (7)

**Theorem 5.1.** There is always a range \([0, \theta_a]\) and a range \((\theta_b, \theta^*)\) in which it is efficient not to monitor the worker.

**Proof:** \(Z(\theta_0)\) is zero and thus less than \(rb\) when \(\theta_0 = 0\) which proves the existence of the first range. Using (1) it is readily verified that \(Z(\theta_0)\) is zero again when \(\theta_0 = \theta^*\) which proves the existence of the second range.

Theorem 5.1 proves that workers who are very unlikely to be good and workers who are close to promotion will not be monitored, but it does not establish that firms will ever monitor workers in order to determine their quality.

Under what circumstances will the firm engage in monitoring? It can be verified that \(Z(\theta_0)\) is concave. Hence the Condition 6 is satisfied for \(\theta_0\) in some compact interval strictly contained in \([0, \theta^*]\) if at its maximum \(Z(\theta_0)\) exceeds \(rb\).

In the appendix we prove the following

**Theorem 5.2.** \(Z(\theta_0)\) exceeds \(rb\), that is there is a range in which monitoring is preferred to no-monitoring if

\[ \left( \frac{g (\lambda + r)}{g \lambda - br} \right)^{\frac{\lambda + r}{r}} < \frac{\lambda c - g}{b}. \]

Condition (5.2) is not particularly informative. We can derive somewhat more informative conditions. Recall that, by assumption, both sides of the inequality are positive.

As \(rb \to g \lambda\), the left hand side goes to infinity while the right hand side remains finite. Thus when monitoring costs are high, not surprisingly the
firms never finds it efficient to monitor. On the other hand, when monitoring costs are low, there is always a range in which monitoring is efficient.

The gain from monitoring is that the firm is assured that it never places a bad worker in the H job. The cost of doing so is given by $\lambda c$. Again not surprisingly, as this term gets large, there is always a range in which monitoring is efficient and when it gets small or, equivalently, when the benefit from placing a good worker in the high job gets small, monitoring is never efficient.

An increase in the frequency of crises, $\lambda$, lowers the left-hand-side and increases the right-hand-side of inequality (5.2). Thus more frequent arrival of crises is associated with a larger range of other parameters consistent with monitoring. Conversely, a higher rate of time discounting is in many ways similar to a lower rate of arrival of crises and thus is associated with a more restricted set of parameters consistent with some monitoring.

Notes for showing this:

\[
\frac{d}{d\lambda} \left( \frac{g(\lambda + r)}{g\lambda - br} \right)^{\frac{\lambda + r}{r}} =
- g \left( \frac{\lambda + 1}{\lambda + r} \right)^{\lambda + r} \left( g\lambda + gr - g\lambda \ln \frac{g\lambda + gr}{g\lambda + gr + br} + br \ln \frac{1}{g\lambda + gr} \right)
\]

This will have the same sign as

\[-g(r + \lambda) \left( \frac{g(\lambda + r)}{g\lambda - br} \right)^{\lambda + r} \left( b + g \right) r + \left( br - g\lambda \right) \ln \frac{g(\lambda + r)}{g\lambda - br} \]

which has the same sign as

\[-(b + g)r + (g\lambda - br) \ln \frac{g(\lambda + r)}{g\lambda - br} < -(b + g)r + \left( g\lambda - br \right) \frac{g + br}{g\lambda - br} < 0\]

\[
\frac{d}{dr} \left( \frac{g(\lambda + r)}{g\lambda - br} \right)^{\frac{\lambda + r}{r}} = \frac{1}{r^2} \frac{\lambda}{(g\lambda - br)^2} g \left( \frac{1}{g\lambda - br} \right)^{\lambda + r} \left( \lambda + r \right)
\cdot \left( br + gr - g\lambda \ln \frac{1}{g\lambda - br} \left( g\lambda + gr \right) + br \ln \frac{1}{g\lambda - br} \left( g\lambda + gr \right) \right)
\]

which has the opposite sign from $d/d\lambda$. 

5.1 Example

Suppose that \( r = 0.05 \), \( g = 1 \), \( \lambda = 0.2 \), and \( c = 10 \), which implies that workers are promoted to the high job when there is a 50\% probability that they are good. Then provided \( b \) is less than about 0.24, there will be a range in which there is monitoring. When \( b = 0.2 \), the firm does not monitor any worker for whom its estimate of \( \theta \) is less than about 0.13, monitors those for whom its estimate of \( \theta \) falls between about 0.13 and 0.35 and does not monitor those above this range.

\[
\theta^* = 1 - \frac{g}{c \lambda} \\
r = 0.05 \\
g = 1 \\
\lambda = 0.2 \\
c = 10 \\
b = 0.2
\]

6 Implications for Internal Labor Markets

A surprising feature of many jobs is that they have a wage/seniority profile that appears to be unrelated to performance. Until they reach the top of the scale for their occupation, workers receive predetermined wage increases at regular intervals unless they separate from the firm or are promoted.\(^1\)

Some models “explain” this phenomena by assuming that wages must be determined by contract before uncertainty is realized (Hashimoto, 1979), but such models lead to inefficient separations. It would be natural for either the employer or the worker to reopen negotiations when either the employer is willing to pay more to retain the worker or the worker is willing to accept a lower wage in order to avoid being dismissed.

In contrast, bureaucratic internal wage scales arise naturally in this model. We do not fully model wage determination. The precise wage will depend on the institutional and informational assumptions we make. How-

\(^1\)See Doeringer and Piore (1971) for a discussion of internal labor markets. Medoff and Abraham (1980) document a strong positive relation between wages and seniority within a job classification but not between performance evaluation and seniority.
ever, we assume that the wage is an increasing function of $\theta$. We find this assumption plausible.

Workers who are hired with a low probability of being good are not monitored. Except when a bad worker faces a crisis and separates from the firm, the firm’s assessment of $\theta$ rises continuously for such workers, and the rate of change depends only on the current value of $\theta$ for a given arrival rate of crises. Therefore all such workers see their wages rise in lockstep. Once the firm’s estimate of $\theta$ has risen sufficiently high, the firm monitors the worker and does not revise its estimate and therefore the worker’s wage until it observes that he has successfully responded to a crisis, in which case he is promoted, or fails to solve a crisis, in which case he exits the firm.

Workers who are hired with a sufficiently high probability of being good also receive automatic pay increases unless they separate. However, in contrast with workers at the low end of the probability spectrum who enter the monitoring range and therefore have fixed wages within a finite amount of time, workers with a high initial skill achieve their maximum wage only in infinite time.

.1 Proof of theorem 3.1

Obviously to be shortened:

Suppose the worker with prior $\theta_0$ has been put in some job at time 0 and has not failed until time $t$. If the worker is good, then non-failure occurs with probability 1, and if he is bad then the probability of non-failure is equal to the probability that a crisis has not occurred by time $t$. Thus the employer’s updated belief about the worker’s type (i.e., the updated probability that the worker is good) is:

$$\theta(t, \theta_0) = \frac{\theta_0}{\theta_0 + [1 - p(t)](1 - \theta_0)}$$

which for future reference we rearrange as

$$1 - p(t) = \frac{\theta_0[1 - \theta(t)]}{\theta(t)(1 - \theta_0)}$$

Let $\bar{\theta}$ be the threshold such that a worker who was initially placed in an L-job is promoted to the H-job when $\theta(t) \geq \bar{\theta}$. We will show below that $\bar{\theta}$ is independent of $\theta_0$. Define $\bar{\ell}(\theta_0)$ such that $\theta(\bar{\ell}(\theta_0), \theta_0) = \bar{\theta}$. Below we will suppress the arguments in $\bar{\ell}(\cdot), \theta(\cdot)\)$ etc.

If $\theta_0 < \bar{\theta}$, then the employer puts the worker in the L-job, and promotes him if he has not failed by time $\bar{t}$. Thus a good worker produces nothing
between times 0 and \( \bar{t} \), and thereafter produces a flow output of \( g \). A bad worker fails before promotion with probability \( p(\bar{t}) \). With probability \( 1 - p(\bar{t}) \) he produces nothing until \( \bar{t} \), and thereafter produces \( g \) until the first crisis arrives, at which time he produces \( -c_h \) and is fired. Hence we can write down the expected payoff from the \( N \)-strategy with prior \( \theta_0 \) and threshold \( \bar{\theta} \). Noting that \( \bar{\theta}, \, \bar{t}, \, p(\bar{t}) \) etc. are all strictly nonmonotonically related, we write this expected payoff as a function of \( 1 - p(\bar{t}) \)

\[
U(\theta_0, [1 - p(\bar{t})]) = \theta_0 \int_{\bar{t}}^{\infty} ge^{-rt} dt + (1 - \theta_0)[1 - p(\bar{t})]\left[\int_{\bar{t}}^{\infty} \lambda e^{-\lambda(t-\bar{t})} e^{-rt} [-c_h] dt + e^{-rt} \frac{g}{\lambda + r}\right] - (1 - \theta_0)c_l \int_{\bar{t}}^{\infty} \lambda e^{-(\lambda+r)t} dt
\]

\[
= e^{-rt}\left\{ \frac{1}{r} \theta_0 g - \frac{1}{\lambda + r} [1 - p(\bar{t})] (1 - \theta_0) (\lambda(c_h - c_l) - g) \right\} - (1 - \theta_0)c_l \frac{\lambda}{\lambda + r}
\]

Note that \( e^{-rt} = e^{[-\lambda\bar{t}]} = [1 - p(\bar{t})]^{\bar{t}} \), which substituted in (10) yields

\[
U(\theta_0, [1 - p(\bar{t})]) = \frac{1}{r} [1 - p(\bar{t})]^{\bar{t}} \theta_0 g - \frac{1}{\lambda + r} [1 - p(\bar{t})]^{\bar{t}} (1 - \theta_0) (\lambda(c_h - c_l) - g) - (1 - \theta_0)c_l \frac{\lambda}{\lambda + r}
\]

(11)

The employer maximizes this payoff by choosing \( \bar{\theta} \), or equivalently \( \bar{t} \) or \( p(\bar{t}) \). Maximizing \( U(\theta_0, [1 - p(\bar{t})]) \) in (11) with respect to \( [1 - p(\bar{t})] \) we obtain the first order condition:

\[
0 = \frac{1}{r} \frac{\lambda}{\lambda + r} [1 - p(\bar{t})]^{\bar{t}-1} \theta_0 g - \frac{1}{\lambda + r} \frac{\lambda + r}{\lambda} [1 - p(\bar{t})]^{\bar{t}} (1 - \theta_0) (\lambda(c_h - c_l) - g)\]

(12)

\[
[1 - p(\bar{t})]^{-1} \theta_0 g = (1 - \theta_0) (\lambda(c_h - c_l) - g)
\]

(13)

Let (12) be solved at \( \bar{t} = t^* \), and correspondingly \( \bar{\theta} = \theta^* \) etc. Using (9), (12) simplifies to

\[
\frac{1}{r} \frac{\lambda}{\lambda + r} [1 - p(t^*)]^{t^*-1} \theta_0 g - \frac{1}{\lambda + r} \frac{\lambda + r}{\lambda} [1 - p(t^*)]^{t^*} (1 - \theta_0) (\lambda(c_h - c_l) - g)\]

(14)
\[ g = (\lambda(c_h - c_l) - g) \frac{[1 - \theta^*]}{\theta^*} \]
\[ \theta^* = \frac{\lambda(c_h - c_l) - g}{\lambda(c_h - c_l)} \]

It can be checked that the second derivative of \( U(\theta_0, [1 - p(\tilde{t})]) \) in (11) is strictly negative at the solution, so this is indeed a strict maximum.

Note that the condition for \( \theta^* \) has a simple interpretation. It is the \( \theta \) that satisfies
\[ \theta g + (1 - \theta)(g - \lambda c_h) - (1 - \theta)\lambda c_l = 0 \]
that is the \( \theta \) at which the expected increased flow of output from placing the worker in the H job is exactly 0.

Note also that the optimal threshold \( \theta^* \) is independent of the prior \( \theta_0 \), from which it follows that a worker entering with prior \( \theta_0 \geq \theta^* \) will be placed directly in the H-job. At the optimum, the employer’s expected payoff from a new worker with prior \( \theta_0 \leq \theta^* \) can be obtained by making the appropriate substitutions in (11) to give:

\[ U^*(\theta_0) = \frac{\lambda}{r(\lambda + r)} \theta_0 g \left[ \frac{\theta_0}{1 - \theta_0} \frac{g}{\lambda(c_h - c_l) - g} \right]^\frac{r}{\lambda} - (1 - \theta_0) c_l \frac{\lambda}{\lambda + r} \quad \text{for} \quad \theta_0 \leq \theta^* \]

(14)

It follows directly that \( U^* \) is increasing in \( \theta_0 \). For \( \theta_0 \geq \theta^* \) the employer places the worker directly in the H-job. It is straightforward to check that the expected payoff is then

\[ U^*(\theta_0) = \frac{1}{r} \theta_0 g - \frac{(1 - \theta_0)(\lambda c_h - g)}{\lambda + r} > U^*(\theta^*) \quad \text{for} \quad \theta_0 > \theta^* \]

.2 Stuff on whether there is a range with monitoring

We now establish conditions under which \( Z(\theta_0) \) exceeds \( rb \), that is there is a range in which monitoring is preferred to no-monitoring. The first-order condition for maximizing \( Z(\theta_0) \) is:

\[ 0 = \frac{dZ(\theta_0)}{d\theta_0} \]
\[ = \lambda g[1 - (\frac{\theta_0}{1 - \theta_0})^\frac{r}{\lambda} \left( \frac{g}{\lambda c - g} \right)^\frac{r}{\lambda}] - \lambda \theta_0 g \left[ \frac{r}{\lambda} \left( \frac{\theta_0}{1 - \theta_0} \right)^{\frac{r}{\lambda} - 1} \left( \frac{1 - \theta_0}{1 - \theta_0} \right)^{1 - \frac{1}{(1 - \theta_0)^2}} \right] \]
\[ = \lambda g[1 - \{(\frac{\theta_0}{1 - \theta_0})^\frac{r}{\lambda} \left( \frac{g}{\lambda c - g} \right)^\frac{r}{\lambda}]\{1 + \frac{r}{\lambda} \frac{1}{1 - \theta_0} \}] \]

(15)
Let $\theta = \hat{\theta}$ solve the above. This implies that at the maximum

$$\left(\frac{\hat{\theta}}{1 - \hat{\theta}}\right) \left(\frac{g}{\lambda c - g}\right) = \left[\frac{\lambda(1 - \hat{\theta})}{\lambda(1 - \theta) + r}\right]^2$$ \hspace{1cm} (16)

Note that the right-hand side of (16) is less than unity, which in turn implies that

$$(\theta) \left(\frac{\lambda c}{\lambda c - g}\right) < 1$$

$$\hat{\theta} < \frac{\lambda c - g}{\lambda c} = \theta^*$$

substituting (16) in (7) gives

$$Z(\hat{\theta}) = \lambda \hat{\theta} g \left[\frac{r}{\lambda(1 - \theta) + r}\right]$$

which in (6) yields

$$\tilde{U}(\hat{\theta}) \geq U^*(\hat{\theta})$$

$$\Leftrightarrow \frac{\lambda \hat{\theta} g}{\lambda(1 - \theta) + r} > b$$

$$\Leftrightarrow \hat{\theta} > \frac{b(\lambda + r)}{\lambda(g + b)}.$$ \hspace{1cm} (17)

Equation (17) tells us that there is a region in which monitoring is optimal if the $\theta$ that maximizes the value of monitoring exceeds the term on the right hand side. A few points are immediately evident. If $b$ is very large, $\hat{\theta}$ must exceed $1 + r/\lambda$ and therefore monitoring will never be optimal. Likewise as $\lambda$ goes to zero, the requisite $\hat{\theta}$ goes to infinity. The condition is easier to satisfy as $r$ gets small and $g$ gets large. Not surprisingly, a necessary, but not sufficient condition is that

$$\frac{b}{\lambda} < \frac{g}{r}$$

which says that the expected cost of monitoring must be less than the present value of output if a good worker is assigned to the high job.

To get more precise conditions under which condition (17) is satisfied, note that $\hat{\theta}$ satisfies (16), which can be rewritten as

$$\left(\frac{\hat{\theta}}{1 - \hat{\theta}}\right) \left[\frac{\lambda(1 - \hat{\theta}) + r}{\lambda(1 - \hat{\theta})}\right]^2 = \left(\frac{\lambda c - g}{g}\right).$$ \hspace{1cm} (18)
Moreover, the left-hand side is increasing in $\theta$. Therefore condition (17) is satisfied if and only if the LHS is smaller than the RHS at $\theta = \frac{b(\lambda + r)}{\lambda (g + b)}$. Thus, we can write the condition as

$$
\left( \frac{g(\lambda + r)}{\lambda r} \right)^{\frac{\lambda + r}{r}} < \frac{\lambda c - g}{b}.
$$

Recall that we have already established that a necessary condition for the existence of a range in which firms monitor is that the denominator of the left hand side be positive.

We can consider some special cases:

$$
\lim_{\lambda \to \infty} \left( \frac{g(\lambda + r)}{\lambda r} \right)^{\frac{\lambda + r}{r}} = e^{\frac{g + b}{g}}
$$

so that the ratio of the expected flow loss of output to flow cost of monitoring must be greater than $e$. If $r$ goes to 0, we get an identical condition. We can rewrite this as

$$
\lambda c - g > be^{\frac{g + b}{g}}
$$

*but I’m not sure this means anything.* And, not surprisingly, as $b$ tends to 0, there will be a range in which monitoring is efficient provided there is an expected flow loss from placing a bad worker in a high job.

**References**


