Should single-equation dynamic gasoline demand models include moving average terms?

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Abstract

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Abstract

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1 Introduction

This paper has three objectives. The first is to argue that the standard way in which economists specify the partial adjustment model used in single-equation dynamic demand estimation fails to account for unobservable effects which might influence demand and which may only evolve slowly over time. This is hardly a new observation, see Houthakker and Taylor (1966), but it seems to have since been forgotten in much of the single-equation demand estimation literature. The second objective of this paper is to make the point that including unobservable effects which do not adjust instantaneously implies that ordinary least squares estimation, by far the dominant approach in the literature, is inconsistent. Lastly, we explore the empirical consequences of these first two observations.

One important advantage of inclusion of unobservable effects which might evolve slowly over time is that the resulting estimation model is a more flexible one in which short- and long-run elasticities do not depend upon each other. A second advantage of the unobservable effects model that we propose is that it nests the standard partial adjustment model. We propose two straightforward tests to see if the restrictions imposed by the standard partial adjustment model hold in the data. We illustrate the model using examples from Australia and the United States. For both of these countries, we can reject the standard partial adjustment model. Inclusion of unobservable effects leads to much less precise estimates of the long-run price elasticity. While this makes the model less attractive to seekers of absolute truth, it provides evidence that empirical work using ordinary least squares tends to over-estimate the precision of our knowledge of the long-run price elasticity. While having unbiased estimates of
point parameters is important for policy, it is also essential to understand the degree of precision of those estimates.

Current conventional wisdom is that gasoline price elasticities are quite low. This paper implies that they may in fact be higher than previously estimated. In a time of record gasoline prices, it may turn out that consumers reduce their gasoline consumption more than previous research has suggested. This also has obvious important implications for the consumer response to climate change policies such as carbon taxes.

2 Dynamic demand models

Aggregate studies of gasoline demand commonly adopt a single-equation approach to estimation based upon some type of partial adjustment model. The single-equation approach is simple to implement and requires only a limited amount of data, but does not allow for the imposition of cross-equation restrictions and realistic modeling of the inter-relationship between consumption of different goods. Our purpose in this paper is not to argue for the superiority or inferiority of this approach. The approach is widely used and our critique relates to the internal consistency of the approach.

A commonly used form of the partial adjustment model is based upon an equation for desired gasoline consumption per person \(v^*_t\)

\[
v^*_t \equiv \left( \frac{V^*}{N} \right)_t^\kappa_1 \left( \frac{P_g}{P} \right)_t^\kappa_2 \left( \frac{Y}{N} \right)_t^\kappa_3 \epsilon_t
\]

and an equation which allows the adjustment of actual to desired consumption to take one period

\[
v_t = (v^*_t)^\theta (v_{t-1})^{1-\theta}
\]

\(V^*\) is desired gasoline consumption, \(N\) is population, \(P_g\) is the price of gasoline, \(P\) is the overall price level, and \(Y\) is income.

Combining equations (1) and (2) and taking logs provides the model which is commonly estimated in the literature,

\[
\ln(v_t) = \beta_0 + \beta_p \ln(p_t) + \beta_y \ln(y_t) + \beta_v \ln(v_{t-1}) + \epsilon_t
\]

where \(y_t\) is per-person income, \(p_t\) is the real price of gasoline, and \(\epsilon_t\) is an i.i.d. random variable. The short-run income and own-price elasticities are \(\beta_y\) and \(\beta_p\). The long-run elasticities are given by \(\frac{\beta_y}{1-\beta_v}\) and \(\frac{\beta_p}{1-\beta_v}\). This model thus imposes that the ratio of the short- and long-run price elasticities is equal to the ratio of the short- and long-run income elasticities. (This restriction can be relaxed by adding lags of price and income to the model (as in distributed lag models) but in the gasoline demand literature,
authors often only add lags of \( v_t \). In section 2.1 below, we present a model which allows testing of this restriction.

If adjustment is instantaneous, then \( \theta = 1 (\beta_v = 0) \) and there is no lagged dependence. To account for slower adjustment processes, authors generally add additional lags of \( v_t \), making decisions about the number of appropriate lags using standard macro-econometric model selection criteria, so that the model becomes

\[
\ln(v_t) = \beta_0 + \beta_p \ln(p_t) + \beta_y \ln(y_t) + \beta_{v1} \ln(v_{t-1}) + \ldots + \beta_{vk} \ln(v_{t-k}) + \varepsilon_t
\]

(4)

Short-run income and price elasticities are again provided by the parameters \( \beta_y \) and \( \beta_p \). Long-run elasticities are those parameters scaled by \( 1 - \sum_{l=1}^{k} \beta_{vl} \).

This model clearly has limitations. Perhaps the most important limitation is the treatment of price as being exogenous. One can justly ask whether estimation of equation (4) recovers demand function parameters or some complicated function of demand and supply parameters. Nonetheless, this model remains the workhorse of government transportation bureaus world-wide and is widely estimated in the academic literature. The majority of the studies discussed in the reviews and meta-analyses of Brons et al. (2008), Graham and Glaister (2002), Espey (1998) and Dahl and Sterner (1991) use this basic approach. Furthermore, more recent empirical literature using panel data techniques (Pock (2009), Dargay (2004) and Baltagi and Griffin (1997)), allowing for price reversibility (Dargay and Gately (1997), Dargay (1992), and Gately (1991)), using co-integration techniques (Karathodorou et al. (2009), Hunt and Ninomiya (2003) and Alves and Bueno (2003)) and allowing for the effect of income distribution (Storchmann (2005)) are all based upon this underlying model. The critique in this paper applies equally to all of these extensions.

2.1 An alternative specification

Obviously, controlling for partial adjustment is important when modeling gasoline demand. Many of the factors which affect demand are fixed in the short term. These include the type of vehicle that one owns, the relative location of one’s school, work, and home, city layouts and traffic densities, and psychological attachment to the perceived freedom afforded by a car.\(^1\) We argue that the model of equation (3) does not achieve this objective as well as it might. In fact, equation (3) is a restricted version of the more general model we consider below.

Consider an alternative model for gasoline consumption at time \( t \) which depends upon relative prices and real income, as above, and also depends upon an unobservable

\(^1\)See Pakes et al. (1993) for a detailed discussion of how demand for different types of cars adjusted in response to the oil crisis of the early 1970s.
stock variable, \( s^*_t \) which captures simultaneously physical (efficiency of current vehicle stock, physical dispersion of commuters) and psychological (automobile utilization rates, habits) aspects of fuel consumption

\[
\ln(v_t) = \alpha_1 + \alpha_2 \ln(p_t) + \alpha_3 \ln(y_t) + \alpha_4 s^*_{t-1} + w_t
\]

(5)

\( w_t \) is an i.i.d. error term (which will generally be different than \( \varepsilon_t \) above) and the other variables are as defined above.

Following the formulation proposed by Houthakker and Taylor (1966), habits and other unobserved or hard-to-measure factors evolve according to

\[
\Delta s^*_t \equiv s^*_t - s^*_{t-1} = \ln(v_t) - \delta s^*_{t-1}
\]

(6)

If this were a standard capital equipment model, \( \delta \) would be the depreciation rate. Here it captures how quickly the determinants of fuel consumption change. Were \( \delta = 1 \), adjustment would be instantaneous and hence the current stock of vehicles and habits have no impact on next period’s fuel consumption. Note further that if \( \delta = 1 \), equation (5) collapses to equation (3). Estimating equation (3) (or equation (4)) is thus imposing an assumption that there are no ‘stock effects’ (either physical or psychological) related to past consumption which affect current consumption. For gasoline consumption in particular, as noted above, this assumption is far too strong.

The approach of including an unobservable stock variable may look quite similar to the model of equations (1) and (2) above, however, it produces a very different estimating equation. Using equation (5) to write \( \Delta \ln(v_t) \), substituting equation (6) into that expression, and using equation (5) lagged one period to replace \( s^*_{t-2} \) in the expression for \( \Delta \ln(v_t) \) provides, after some re-arrangement,

\[
\ln(v_t) = \alpha_1 \delta + \alpha_2 \Delta \ln(p_{g,t-1}) + \alpha_3 \Delta \ln(y_{t-1}) + \alpha_4 \Delta \ln(y_t) + (1 + \alpha_4 - \delta) \ln(v_{t-1}) + w_t - (1 - \delta) w_{t-1}
\]

(7)

or,

\[
\ln(v_t) = \gamma_0 + \gamma_p \ln(p_{g,t-1}) + \gamma_{\Delta p} \Delta \ln(p_t) + \gamma_y \ln(y_{t-1}) + \gamma_{\Delta y} \Delta \ln(y_t) + \gamma_v \ln(v_{t-1}) + u_t
\]

(8)

Importantly, the error term in (8) is a moving average. The presence of the lagged dependent variable implies that this equation can not be estimated consistently by ordinary least squares.\(^2\) (The standard approach in the papers cited in section 2 above is to estimate 4 by ordinary least squares.) Modern econometric software makes

\(^2\)See, for example, Wooldridge (2009, Chapter 11). The one exception to this statement would be if \( w_t \) followed an AR(1) process \( w_t = (1 - \delta) w_{t-1} + \nu_t \) where \( \nu_t \) were i.i.d.
it easy to estimate equation (8) by maximum likelihood (see Lütkepohl (2006) and Stata Corporation (2009)). Generalized method of moments (GMM) can also be used, but is less efficient.

A further advantage of estimating equation (8) is that the short-run elasticity is captured by $\gamma_{\Delta p}$ and the long-run elasticity is determined by $\gamma_p$ and $\gamma_v$. Thus the restriction that the long-run elasticity is a scaled amount of the short-run elasticity which is imposed in equations (3) and (4) is relaxed in this model. Furthermore, the ‘stock’ effects imply that past income and prices are related to current consumption, unlike in the standard model.

Stocks may take more than one period to adjust in which case we would replace equation (6) with

$$s_{t}^* = \ln(v_t) + (1 - \delta_1) s_{t-1}^* + \sum_{j=2}^{q} \delta_j s_{t-j}^*$$  \hspace{1cm} (9)

Combining equations (5) and (9), we would estimate

$$\ln(v_t) = \gamma_0 + \sum_{j=1}^{q} \gamma_{p_j} \ln(p_{t-j}) + \gamma_{\Delta p} \Delta \ln(p_t) + \sum_{j=1}^{q} \gamma_{y_j} \ln(y_{t-j})$$

$$+ \gamma_{\Delta y} \Delta \ln(y_t) + \sum_{j=1}^{q} \gamma_{v_j} \ln(v_{t-j}) + u_t$$  \hspace{1cm} (10)

$u_t$ is now a $q$th-order moving average. Again, ordinary least squares is inconsistent and maximum likelihood estimation will be the most efficient approach to estimation.

Note that an additional time period of adjustment implies additional lags of demand, price, and income and an additional moving-average term. For a particular period, if lagged adjustment matters, all of these terms should appear in the model. If none of them are statistically significant, we can infer that there is no additional lag in the adjustment process for that period. In terms of model specification then, these lags and the moving average term should all be included (if jointly significant) or all excluded (if jointly insignificant) based upon joint hypothesis testing. This provides a model specification which is consistent with the underlying adjustment process that is proposed, rather than one that just adds lags of $v_t$ in an ad hoc manner.

As we show below, there are important substantive differences in the estimates from these two models. The approach of adding ad hoc lags of $\ln(v_t)$ to the partial adjustment model to eliminate residual serial correlation is not theoretically equivalent to correct estimation of equation (10). One important distinction is the presence of the moving average. The other important distinction is that past income and prices now enter directly in determining current consumption. This is a direct consequence of the habit formation model—for example, past decisions driven by income (type of car, style of living, etc.) lock one into current and future consumption patterns.
2.2 Testing between the two specifications

This approach allows for two simple specification tests to distinguish between the two models. The first is the joint test of the $q$ moving average components from equation (10). Since this is equivalent to testing $\delta_1 = 1$ and $\delta_j = 0$ (for $j > 1$) in equation (9), it provides a test of the model of equation (4) against the model of equation (10). If the moving average terms are not significantly different than zero, ordinary least squares will be appropriate.

Secondly, we can test the restriction discussed above that the long-run and short-run income and gasoline elasticities are scaled versions of one another in equation (4) but not in equation (10). If $\sum_{j=1}^{q} \gamma_{pj} = \gamma_{\Delta p}$ and $\sum_{j=1}^{q} \gamma_{yj} = \gamma_{\Delta y}$, then the restriction of equation (4) holds. If not, we prefer the unobserved stocks model.

3 Empirical example

We estimate these models using publicly available data from the United States and Australia. For the United States, gasoline volume is from the Energy Information Administration of the Department of Energy, per-capita disposable income is from the U.S. Bureau of Commerce, prices (consumer price index and gasoline prices) are from the Bureau of Labor Statistics, and population over age 16 is from the Census Bureau. The quarterly data are seasonally adjusted and is available from third quarter 1967 to fourth quarter 2007.

All of the Australian data are from the Australian Bureau of Statistics except petrol (gasoline) volume which is from the Department of Industry, Tourism, and Resources’ Australian Petroleum Statistics Publication. Our Australian data are available from third quarter 1966 to fourth quarter 2006.

Hughes et al. (2006) and Graham and Glaister (2002) discuss the evidence that there has been a shift in gasoline elasticities during the period covered by our data. Looking at the univariate volume series it is quite evident that there are important structural breaks in the late 1970s or early 1980s. In our empirical example, therefore, we only present estimates for the period since the second oil shock of the late 1970s. For the U.S. data, we choose this second sample period from 1982 while for Australia we choose this second sample period from 1979. We chose these points on the basis of simple break tests applied to the univariate series of gasoline volumes. The substantive results presented below are not sensitive to these choices.

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3 The data and STATA .do files to reproduce these tables are available on the corresponding author’s web page. Link to be provided on acceptance.

4 The main conclusions of this section apply equally to models estimated on the pre-1980 data and to models estimated over the period 1966 to 2007. These results are available from the authors.
The second and third columns of table 1 present the results for the U.S. The last two columns present the results for Australia. The Australian models include time trends. Both models for both countries produce reasonable looking short- and long-run price and income elasticities.\(^5\)

Lag lengths for all models are chosen using the Akaike Information Criteria (AIC) and requiring that the residuals approximate white noise. The optimal lag length for the traditional model of equation (4) will not necessarily be the same as the optimal lag length for the unobserved habits model of equation (10) and vice versa. For the U.S. we find that the optimal lag length for the unobserved habits model is one quarter whereas for the traditional model it is four quarters. Interestingly, for Australia we find that a model which includes lags of one and four quarters is optimal for the traditional model but that including a lag at the third quarter is optimal for the unobserved effects model. Table 1 presents the ‘best’ models from the two approaches for each country.

Table 1: Price and income elasticity estimates from gasoline demand equations for U.S. and Australia

<table>
<thead>
<tr>
<th>Equation number in text</th>
<th>Estimates for United States</th>
<th>Estimates for Australia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time period</td>
<td>1982Q1 to 2007Q4</td>
<td>1979Q1 to 2006Q4</td>
</tr>
<tr>
<td>Lags of ( v_t )</td>
<td>1,2,3,4</td>
<td>1,4</td>
</tr>
<tr>
<td>Short-run price elasticity</td>
<td>-0.13 (0.0051)</td>
<td>-0.12 (0.024)</td>
</tr>
<tr>
<td>Long-run price elasticity</td>
<td>-0.083 (0.046)</td>
<td>-0.30 (0.093)</td>
</tr>
<tr>
<td>Short-run income elasticity</td>
<td>0.012 (0.0066)</td>
<td>0.21 (0.099)</td>
</tr>
<tr>
<td>Long-run income elasticity</td>
<td>0.075 (0.025)</td>
<td>0.53 (0.029)</td>
</tr>
<tr>
<td>AIC</td>
<td>-665.0</td>
<td>-555.3</td>
</tr>
<tr>
<td>p-value of test on MA terms</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Asterisks represent significance at 5% (**), 10% (*), and 90% (n/a) levels.

\(^5\)A growing literature models co-integration between income and gasoline consumption. For Australia, Breunig and Gisz (2009) show that income and petrol prices are both best described as trend stationary, thus co-integration would be inappropriate. We include time trends in the Australian models for this reason. For the U.S. data, Wadud et al. (2007) show a co-integrating relationship between gasoline demand and income for the post-1978 period. We tested for, and failed to find, co-integration in our data. We think gasoline consumption and income can both reasonably be modeled as trend stationary (see figure 1 in Wadud et al. (2007)), however when we included time trends in our model they were insignificant. For the purposes of this paper, our central argument does not depend upon whether the data are co-integrated or not. Furthermore, the point elasticity estimates (see Wadud et al. (2007)) for the U.S. are not much affected by choice of model.
The last three rows of the table contain our tests of the partial adjustment specification. It is important to note that we are not testing the model of column 2 against the model column 3 of Table 1. (The different number of lags make them incomparable.) The tests which are summarized at the bottom of columns 3 and 5 are as discussed in the text. We choose the optimal lag structure for the unobserved habits model of equation (10) (column 3 for the U.S. and column 5 for Australia) and then test to see whether that model can be rejected in favor of the traditional model, as described in sub-section 2.2.6

For both countries, we reject that the moving average terms in the unobservable stocks model are zero. For the price elasticities, we can reject, at the five per cent level, that the short- and long- run price elasticities are scaled versions of one another in both countries. For the income elasticities, we never reject that the long-run and short-run values are scaled versions of each other. Our conclusion is that we reject the restrictions which are implied by the standard partial adjustment model of equation (4) in favor of the unobservable stocks model of equation (10). What implications does this have for our estimates?

The point estimates of the elasticities from these two approaches are not significantly different than one another. However, the model which accounts for unobservable habits produces estimates with much larger standard errors. For the U.S., the 95 per cent confidence interval for the short-run income elasticity goes from \((-0.001, 0.025)\) to \((-0.056, 0.432)\). The 95 per cent confidence interval for the long-run income elasticity goes from \((0.026, 0.125)\) to \((0.022, 0.20)\). For the short-run price elasticity, the confidence interval goes from \((-0.023, -0.003)\) to \((-0.075, -0.007)\). The 95 per cent confidence interval for the long-run price elasticity is \((-0.174, 0.008)\) in the standard model, which increases to \((-0.248, 0.035)\). In all cases the width of the confidence interval more than doubles.

In the Australian data, 95 per cent confidence intervals for short-run income and price elasticities go from \((0.011, 0.404)\) to \((0.046, 0.590)\) and \((-0.166, -0.068)\) to \((-0.224, -0.075)\), respectively. 95 per cent confidence intervals for long-run income and price elasticities go from \((-0.041, 1.11)\) to \((-1.32, 4.23)\) and \((-0.487, -0.117)\) to \((-0.971, 0.246)\), respectively. Compared to the U.S. data, we find smaller increases in the confidence intervals for the short-run elasticities, but large increases in the width of the confidence intervals for the long-run parameters.

That the unobservable habits model results in more uncertainty is intuitively com-

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6An alternative approach would be to take the optimal lag length for the traditional model of equation (4), adopt this same lag length for the unobserved habits model of equation (10), and then test to see if the unobservable habits model can be rejected. In both cases, we reject the traditional model in favor of the unobserved habits model of equation (10).
compatible with the way in which the model is specified. We would expect to observe this result for other time periods and other countries. The unobservable habits model, unlike the traditional model, models the adjustment process through an unobservable state variable. A model transformation allows for the underlying state variable to be inferred and incorporated into the estimation, but this provides a more complex estimating equation that also has more parameters. Uncertainty in the evolution of the unobserved state variable is directly translated into parameter uncertainty.

We also note, in passing, that the unobservable habits model results in larger (in absolute value) point elasticities, although the differences are not statistically significant. We have no reason to believe that this is a general rule. The unobservable habits model may produce either smaller or larger point elasticities.

4 Conclusion and discussion

In this paper, we propose that the original model of Houthakker and Taylor (1966), which allows for current demand to depend upon an unobservable, slowly evolving ‘stock’ or ‘habit’ parameter provides a better description of behavior than the standard partial adjustment model which is used in the literature on single-equation dynamic demand estimation. We show that the standard model is a restricted version of the unobservable habit model. In an empirical application on gasoline demand for the U.S. and Australia, we show that the standard model is rejected in favor of the unobservable habit model. The main empirical implication is that the confidence intervals for both short- and long-run price elasticities are much larger in our preferred model. We would argue that it is important to have an accurate idea of the true precision of one’s estimates for understanding behavior and for making policy recommendations.

Why has the literature ignored the original Houthakker and Taylor (1966) unobservable habits model in favor of the simpler partial adjustment model? We offer two (speculative) hypotheses. The first is that the hey-day of single-equation dynamic demand models was in the 1970s and 1980s when the availability of computer software to estimate non-linear models by maximum likelihood were, to say the least, limited. The simpler partial adjustment model, estimated by ordinary least squares, was easier to implement. Today’s practice is a relic of simpler times, even if modern econometric software make estimating the unobservable habits model by maximum likelihood just as easy as an ordinary least squares regression.

Our second hypothesis is that practitioners are attracted to more precise coefficient estimates. The single-equation dynamic demand models discussed here are primarily used in agricultural and transportation departments for policy analysis. Given the
choice between a small and a large confidence interval for an elasticity, the applied
economist tends to be (naturally) attracted to the more precise estimate. That this
precision may in fact be spurious is not a question that comes to mind.

From the point of view of forecasting, the critique presented in this paper may not
matter. Moving average terms will not essentially improve forecasting performance.
Distributed lag models such as those of Baltagi and Griffin (1997) may have equal or
better forecast performance. Our purpose in this paper is not to compare forecast
performance nor undertake a comprehensive comparison of alternative adjustment
models.

Lastly, we would point out that our critique is a broader one in that it applies
equally to the state-of-the-art estimation techniques which are being applied to ag-
gregate demand data. These include co-integration models, models which allow for
price reversibility, and panel data models applied to aggregate data. Application of
these models (see the examples cited in section 2) are based upon the simple partial
adjustment model and estimated using linear techniques. All of these models can,
and should, be augmented with the unobservable stock equation and estimated using
maximum likelihood or generalized method-of-moment techniques.

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## Appendix

Table A1: Estimates from gasoline demand equations for U.S.

Table entries are regression coefficients (standard errors in parentheses)

<table>
<thead>
<tr>
<th>Equation number in text</th>
<th>(4)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time period</td>
<td>1982q1 to 2007q4</td>
<td></td>
</tr>
<tr>
<td>Constant ($\beta_0, \gamma_0$)</td>
<td>-0.95** (0.36)</td>
<td>-0.44** (0.26)</td>
</tr>
<tr>
<td>$\Delta ln(p_t) (\gamma_{\Delta p})$</td>
<td>-0.013** (0.0051)</td>
<td>-0.041** (0.017)</td>
</tr>
<tr>
<td>$ln(p_t) (\beta_p)$</td>
<td>-0.0081** (0.0038)</td>
<td>0.19 (0.12)</td>
</tr>
<tr>
<td>$ln(p_{t-1}) (\gamma_{p1})$</td>
<td>-0.012* (0.0066)</td>
<td>0.0084* (0.0047)</td>
</tr>
<tr>
<td>$\Delta ln(y_t) (\gamma_{\Delta y})$</td>
<td>0.55** (0.099)</td>
<td>0.92** (0.044)</td>
</tr>
<tr>
<td>$ln(y_{t-1}) (\gamma_{y1})$</td>
<td>0.20* (0.11)</td>
<td>0.24** (0.11)</td>
</tr>
<tr>
<td>$ln(v_{t-1}) (\beta_{v1}, \gamma_{v1})$</td>
<td>0.24** (0.11)</td>
<td>0.24** (0.11)</td>
</tr>
<tr>
<td>$ln(v_{t-2}) (\beta_{v2})$</td>
<td>-0.15 (0.098)</td>
<td>-0.15 (0.098)</td>
</tr>
<tr>
<td>$ln(v_{t-3}) (\beta_{v3})$</td>
<td>-0.15 (0.098)</td>
<td>-0.15 (0.098)</td>
</tr>
<tr>
<td>$ln(v_{t-4}) (\beta_{v4})$</td>
<td>-0.15 (0.098)</td>
<td>-0.15 (0.098)</td>
</tr>
</tbody>
</table>

### Moving average parameters

| $\theta_1$ | -0.37** (0.12) |

Asterisks represent significance at 5% (**) and 10% (*) levels.

Greek letters ($\beta_0$, $\gamma_0$, etc.) refer to coefficients as defined in text in equations (4) and (10), respectively. For example, the constant in equation (4) is $\beta_0$ and the constant in equation (10) is $\gamma_0$.

**Notes:** Estimating a model with only three lags for equation (4) produces a model with much poorer fit and residuals with large auto-correlation.
Table A2: Estimates from gasoline demand equations for Australia
Table entries are regression coefficients (standard errors in parentheses)

<table>
<thead>
<tr>
<th>Equation number in text</th>
<th>(4)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time period</td>
<td>1979q1 to 2006q4</td>
<td></td>
</tr>
<tr>
<td>Constant ( (\beta_0, \gamma_0) )</td>
<td>1.45** (0.06)</td>
<td>0.11 (0.06)</td>
</tr>
<tr>
<td>Time trend</td>
<td>-0.00076** (0.00027)</td>
<td>-0.00078* (0.00040)</td>
</tr>
<tr>
<td>( \Delta \ln(p_t) (\gamma_p) )</td>
<td>-0.12** (0.024)</td>
<td>- 0.15 ** (0.038)</td>
</tr>
<tr>
<td>( \ln(p_{t-1}) (\gamma_{p1}) )</td>
<td>- 0.18 ** (0.048)</td>
<td></td>
</tr>
<tr>
<td>( \ln(p_{t-3}) (\gamma_{p3}) )</td>
<td>- 0.11 ** (0.042)</td>
<td></td>
</tr>
<tr>
<td>( \ln(p_{t-4}) (\gamma_{p4}) )</td>
<td>0.23** (0.046)</td>
<td></td>
</tr>
<tr>
<td>( \Delta \ln(y_t) (\gamma_{y}) )</td>
<td>0.32** (0.14)</td>
<td></td>
</tr>
<tr>
<td>( \ln(y_{t-1}) (\beta_{y}) )</td>
<td>0.21** (0.099)</td>
<td></td>
</tr>
<tr>
<td>( \ln(y_{t-3}) (\gamma_{y3}) )</td>
<td>0.50** (0.18)</td>
<td></td>
</tr>
<tr>
<td>( \ln(y_{t-4}) (\gamma_{y4}) )</td>
<td>- 0.25* (0.015)</td>
<td></td>
</tr>
<tr>
<td>( \ln(v_{t-1}) (\beta_{v1}, \gamma_{v1}) )</td>
<td>0.056 (0.076)</td>
<td>-0.063** (0.058)</td>
</tr>
<tr>
<td>( \ln(v_{t-3}) (\gamma_{v3}) )</td>
<td>0 ( \cdot )</td>
<td></td>
</tr>
<tr>
<td>( \ln(v_{t-4}) (\beta_{v4}, \gamma_{v4}) )</td>
<td>- 0.56** (0.075)</td>
<td>0.90** (0.056)</td>
</tr>
</tbody>
</table>

### Moving average parameters

| \( \theta_1 \) | 0.43** (0.16) |
| \( \theta_3 \) | 0 \( \cdot \) |
| \( \theta_4 \) | -0.56** (0.13) |

Asterisks represent significance at 5% (**) and 10% (*) levels.

Greek letters ((\( \beta_0, \gamma_0 \), etc.) refer to coefficients as defined in text in equations (4) and (10), respectively. For example, the constant in equation (4) is \( \beta_0 \) and the constant in equation (10) is \( \gamma_0 \).

**Notes:** Estimating a model with only one lag at the fourth quarter for equation (4) produces a model with much poorer fit and residuals with large auto-correlation. For the model of equation (10), we estimate a model with lags included at the first, third and fourth quarters. The coefficients on the third lag of income \( (\ln(y_{t-3})) \), the third lag of consumption \( (\ln(v_{t-3})) \), and the third moving average term \( (\theta_3) \) are all very close to zero. The test for joint significance of those three coefficients gives a p-value of 0.69, so we set those three coefficients equal to zero in the final model. This does not affect the substantive results. We cannot eliminate any other coefficients without large effects on model fit.