Uncertainty, delegation and communication*

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May 28, 2009

Abstract

When communicating with an uninformed decision maker, the motives behind an expert’s message are often unclear. To explore this situation and investigate its impact on organizational design we extend the cheap-talk model of Crawford and Sobel (1982) to allow for uncertainty over the expert’s bias. We find that, in contrast to Dessein (2002), it is possible that the decision maker prefers communication to delegation; that is, it can be optimal for a decision maker to retain control and to solicit advice from the expert.

Key words: delegation, communication, uncertainty, bias, cheap talk.

JEL classifications: D23, D83, L23.

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*We would like to thank Oleksii Birulin, Wouter Dessein, Maxim Ivanov, Kunal Sengupta and participants at the 27th Australasia Economic Theory Workshop 2009. This paper represents the views of the authors and does not necessarily represent the views of the Reserve Bank of Australia or the University of Sydney. The authors are responsible for any errors.

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1 Introduction

Having the relevant specific knowledge is critical for effective making (Jensen and Meckling (1995)). However, those responsible for this task often lack key information. A decision maker might be uninformed because of the highly specialized knowledge needed to choose between potential projects. In such situations, the uninformed decision maker may seek the advice of an informed expert about what actions she should take – for example, the senior manager of a plant might ask a shop-floor manager advice on a new project or a CEO of a software firm could solicit a recommendation of its research department regarding the development of a new product.

In these situations, more often than not, the project selected affects the welfare of both the principal and the agent. For instance, a middle manager’s division is likely to be affected by a CEO’s decision; similarly, shareholders often rely on the advice of financial analysts, who in turn may have a vested interest in the prevailing stock price. Where the preferences of the agent and the principal do not coincide, the agent will have an incentive to behave strategically by distorting or obfuscating the truth in any message they communicate. This paper analyzes the strategic communication process between an expert advisor and an uninformed principal and its impact on an organization’s decision making and communication protocols.

Crawford and Sobel (1982) (hereafter CS) analyzed strategic communication in a cheap-talk game between a perfectly-informed sender (or agent) and a receiver (principal). The sender observes the state, which takes the value of a random variable, before sending a message to the receiver. Upon observing the message, the receiver takes an action. The state and the receiver’s action determine the payoffs for both players. The preference divergence, or the sender’s bias, is captured by a constant parameter $b$, which is known by both agents.

When the preferences of the two agents differ, CS found that regardless of the size of the bias, there cannot be an equilibrium in which the sender reveals the true state. Instead, the equilibrium involves a partition of all of the states of nature and
the agent’s message only identifies a partition element that includes the true state of nature. As the sender’s noisy signal is credible, the receiver in turn chooses the action that maximizes his expected utility given his (correct) probabilistic belief of the distribution of the state. Moreover, CS show that as noisy communication is always costly, more informative equilibria are preferred by the receiver.

As an alternative that avoids strategic communication, the decision maker in an organization could delegate his decision-making rights to the expert. Extending Crawford and Sobel (1982), Dessein (2002) found that with standard assumptions over the decision maker’s best actions, delegation is optimal ‘whenever informative communication is possible’ (p.822). In general, the loss of information in communication leads to a greater reduction in welfare for the receiver than the loss of control under delegation, especially when biases are small (Dessein, 2002, p.812).

In reality, however, the choice between communication and delegation is often much more complex. The experience of Nestlé highlights this point. Nestlé initially decided to delegate more decisions; this strategy, however, was not successful and the company decided to revert to centralized decision making, relying on the transfer of the requisite information to these key decision makers (The Economist, 5 August 2004). Similarly, the marketing divisions of General Motors and Dell Computers have made seemingly conflicting choices regarding delegation and communication:

General Motors announced this summer that it will merge its 5 marketing divisions into one [communication], ... Meanwhile, Dell Computer actively decentralizes [delegates] its marketing by assigning fewer market segments to divisions as they grow. Dell has 12 marketing divisions now, compared with 4 in 1994. (Donath, 1998, p.9)

One of the key assumptions of CS and Dessein (2002) is that the preference divergence between the two parties is common knowledge; that is, while the decision maker is uninformed about the appropriate project to implement for the realized state of the world, he has perfect knowledge about the expert’s bias. In the language of Dessein (2002), there is a systematic and predictable difference between the prefer-
ences of the principal and the agent. In many contexts, however, the decision maker will be unsure of the underlying objectives of the expert – that is, the bias of the agent is not systematic and predictable. Again, examples abound: a political leader may be uncertain about the political leaning of an advisor; the CEO might be unsure whether the manager is an empire builder or whether he is effort averse; and only some financial analysts wish to short sell a stock.

We model the situation when: the principal is uninformed; the principal is uncertain regarding the bias of the expert; and the action taken affects the utility of both the principal and the agent. To this end we extend CS to allow for the receiver to be uncertain about the sender’s bias; specifically, we allow the sender’s bias, which is private information, to take on two possible values. We also assume, partly to reflect the potential coarseness and imprecise nature of communication in real organizations, that only two possible messages can be sent by the sender (either Low or High).

By allowing for uncertainty with respect to the sender’s bias, we increase the range of biases for which communicative equilibria are possible. In order to further investigate these equilibria, we define two different types of expert/sender: (1) an informative sender, who in equilibrium is willing to send both types of messages depending on the realized state of the world; and (2) an uninformative sender, who only sends one message across all states of the world. Further to that, even in situations when the sender will always be informative, a sender with a small – or moderate – bias compared to the principal will behave differently to an informative sender with a larger – or extreme – bias, affecting the relative performance of communication to delegation.¹

We find that if the two potential biases have the same sign, we generalize the result of Dessein (2002) that delegation is always preferred to communication when there is receiver uncertainty over the sender’s bias.² However, if the seller’s potential biases have opposite signs it is possible that the principal prefers communication

¹These two terms, moderate and extreme, are defined precisely in Section 3.
²To be precise, when the two potential biases are the same sign, delegation is preferred for the range of biases that communication is possible in Dessein (2002).
over delegation. The intuition for this result is that uncertainty over the expert’s preferences can mute the strategic effects of communication, encouraging a biased expert to send more informative messages. The reduction in the loss of information can be sufficient to allow communication to dominate delegation. The model also shows as the biases of both types increase and become extreme that it is better to communicate. Moreover, in the case in which one of the types of expert is moderate and one is extreme, there is a threshold probability (for the type being extreme) above which the principal will prefer to communicate (and maintain centralized decision making). Intuitively, by retaining control rights under communication the decision maker can maintain incentives for information transmission from unbiased experts, whilst insuring against (very biased) experts that might want to implement a project not in his interest.

Several other papers have also addressed the communication-or-delegation question. Ivanov (2008) showed that if the receiver can ‘optimally’ restrict the sender’s information, communication always dominates delegation. Krahmer (2006) compared communication and delegation when utility is transferable between the sender and receiver and contracts are only partially incomplete. We take a different but complementary approach. Rather than allowing the receiver to have some control on communication, we focus on the effect of introducing greater uncertainty in terms of the sender’s bias.

In another related study, Blume et al. (2007) examined information transmission when, with positive probability, the message sent is misinterpreted by the receiver and, as a result, is uninformative. They find that adding some noise to the sender’s signal can almost always improve the welfare of all parties as the noise creates incentives for the sender to reveal more information, and the value of this additional information outweighs that utility loss from misinterpretation.

Our paper builds on the existing literature that has focused on uncertainty regarding the bias of the sender. Several papers have shown that the partition equilibria in the CS model are robust to the inclusion of uncertainty about the sender’s bias. Hughes and Sankar (2006) found that the equilibria in a CS model involve partition
messages when the sender’s bias can take one of two possible values. Dimitrakas and Sarafidis (2005) generalized the CS model to allow for uncertainty with respect to a sender’s bias and they found that all equilibria are partition equilibria. Moreover, they found that a partitional equilibrium with a very limited number of partitions (for example 2 partitions) comes very close to the limit equilibrium, suggesting little information is lost when the sender is restricted to a small number of messages.³ Our focus here is not the existence of partition equilibria, but rather the relative advantage of communication over delegation. The results in these papers, however, allow us to simplify our analysis by focusing on a 2-message model, which is akin to a 2-partition equilibrium.

Li and Madarasz (2008) examined whether, prior to communication, disclosure by the expert of their bias should be mandatory. As all equilibria when the sender’s bias is known are partitional equilibria, ex ante mandatory reporting of the sender’s bias can make subsequent communication less informative, decreasing the welfare of both the expert and the decision maker.⁴ While Li and Madarasz (2008) do not specifically address the question of the expert’s preferences on organizational structure, some of the intuition for their results is similar to ours; uncertainty regarding her bias alters the incentives for the sender to engage in strategic communication (or obfuscation).

In a paper closely related to ours, Ottaviani (2000) constructed a model where the sender’s bias is symmetric. The sender’s bias can take a positive or negative value with equal probability (b or −b) — capturing situations where preference divergences are random rather than systematic. He found that the receiver’s welfare is always higher under communication than it is under delegation. In an important departure from Ottaviani (2000) we take a more general approach in that we do not require that the biases are symmetric. Moreover, we also consider an extension (Section 5 to

³ Morgan and Stocken (2003) analyzed communication when an investor/receiver is uncertain as to whether the stock analyst (sender) is unbiased (has a bias equal to zero) or has a positive bias, in that she wishes to inflate the value of the stock. They also show that institutional restrictions on the communication process, in that the analyst can only send a broad messages that rank the stocks (for example, sell, hold or buy) arise endogenously in equilibrium.

⁴ Agastya et al. (2009) also find a similar result.
our model by allowing any finite number of messages to be sent by the expert (rather
than just 2 possible messages) assuming symmetric biases, as in Ottaviani (2000).

2 The Model

Consider, in turn, the model under: (a) communication, when the principal opts to
retain her decision-making rights and consults an informed agent or expert; and (b)
delegation, when the principal relinquishes her decision-making rights regarding the
project selection to the agent.

Communication

Here, the underlying problem is that an uninformed decision maker must choose a
project. Given she is uninformed, the decision maker can elicit a costless message
from an informed expert. However, this expert is biased in that his preferences over
these projects differ to the preferences of the decision maker.

Consequently, in the model there are two players, a sender and a receiver. The
sender observes the state $s$ that is a random variable drawn from the interval $S = [0, 1]$.
The distribution of the state is assumed to be uniform. The sender also observes her
bias, $b$, that may take two values, $b_1$ or $b_2$ where $-b_1 \leq b_2 \leq b_1$. There is a probability
of $p \ (1 - p)$ that the sender’s bias is $b_1$ ($b_2$). The prior distributions of the sender’s
bias and the state are common knowledge to both sender and receiver. In this paper,
we will refer to the sender’s bias as her type. Note that as there are two types of
uncertainty in the model (over the state and the sender’s bias) this labeling is made
for the purpose of clarity.\footnote{It is also worth mentioning that the majority of the literature uses the term ‘type’ to describe
the sender’s observation of the state.}

The timing of the game is as follows. The sender, of type $b_i$ where $i = 1, 2$, observes
$s$, and sends a message, $m \in M$, to the receiver. After receiving $m$, the receiver
chooses an action $a$ that determines the utilities of both players. For simplicity, the
sender’s utility from action $a$ is:

$$U^S = -|a - (s + b)|,$$

while the receiver’s utility from action $a$ is:

$$U^R = -|a - s|.$$  

Note that here, for ease of exposition, we depart from many of the applications of the CS mode in that we assume linear utilities; for example, the leading example of CS (p.1440) uses a quadratic utility. This departure is not critical as it is the difference between a party’s preferred action and the implemented action that matters.

The following standard assumptions are made: sending a message is costless for the sender; the message cannot be verified by the receiver; and the receiver also cannot commit to a decision rule ex ante. We also make one additional assumption. Upon observing the state, the sender can send one of only two messages - Low or High, specifically $M = \{\text{Low}, \text{High}\}$. That is, while the state of nature and the number of actions that can be taken is infinite, the number of reports that can be sent is restricted. There are several reasons as to why the message space might be restricted. As noted above, Morgan and Stocken (2003) found that a sender may wish to implement a restriction on the possible messages that can be sent to simple ranking (Low or High for example). This result arises endogenously in their model and, furthermore, does not necessarily result in any loss if information. Moreover, even when the message space is unrestricted in the original CS model, there is an equilibrium in which only two distinct messages are sent in equilibrium — a two-partition model — that is equivalent to the equilibrium in our model when $b_1 = b_2$.\footnote{We utilize this equivalence to compare the results from this model to the two-partition CS model (hereafter the benchmark model).}

Another reasons for our binary message space could be that the communication technology is coarse. It is often not possible to precisely describe exactly the type of
project required, particularly as we are assuming a costless communication process. Dwyer (1999, cited in Joiner et al., 2002) argued that if the sender must resort to technical language to be more precise, this might actually reduce the clarity of the message. Others have also made a similar assumption: for example Takáts (2007) assumes that the sender cannot report all her information. This coarseness could arise from the sender’s inability to precisely articulate or from the receiver’s lack of expertise of comprehension, leading the receiver to understand the rough gist of the message, while being cloudy on the detail. However, it might be that case that the sender can communicate in broad terms about the project she recommends, be it ‘small’, ‘large’, ‘locate in region X’, etc. Note that we relax this assumption in Section 5 to allow for any finite number of messages to be sent.

The solution concept we shall employ is the Bayesian Nash Equilibrium. The decision maker’s beliefs, \( P(\cdot | \text{Low}) \) and \( P(\cdot | \text{High}) \), be formed using Bayes’ rule for possible messages \( \text{Low} \) and \( \text{High} \). The decision maker’s actions, \( X_L \) and \( X_H \), maximize his expected utility given his beliefs \( P(\cdot | \text{Low}) \) and \( P(\cdot | \text{High}) \). The expert’s messages, \( \text{Low} \) or \( \text{High} \), maximize her utility for all \( s \) given the decision maker’s strategy.

Our model satisfies the properties of Dimitrakas and Sarafidis (2005), who found that all equilibria of the model are partitional equilibria. As our purpose in this paper is to generate a counter example to the delegation result of Dessein (2002), we use their results to focus on the simple 2-message equilibrium that is equivalent to a two-partition equilibrium. This means that we are comparing the optimality of delegation to communication in the least informative (and least advantageous) communicative equilibrium, suggesting any dominance of communication would only be enhanced with more refined communication.

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7One could argue that the uninformed receiver could expend effort to understand the message, but this would be equivalent to investing effort into becoming informed, as in Aghion and Tirole (1997), possibly eliminating the need to communicate with an expert. Here we follow CS and Dessein (2002) in assuming the information structure as exogenous.
Delegation

The model of delegation is as follows. Before the state of nature is revealed, the receiver delegates control of the action to the sender. Upon observing the state of nature, the sender chooses an action that determines the utility of both players.

Utility functions for sender and receiver are unchanged under delegation. As the sender’s signal is perfect, it is assumed she can choose any action with complete precision under delegation. Thus, the sender will choose her optimal action, $s + b$, yielding utilities of $U^S = 0$ and $U^R = -b$ for the sender and receiver respectfully.

3 Solving the model

Upon receiving the report Low (High), the receiver chooses the action $X_L$ ($X_H$) to maximize his utility. Without loss of generality, $X_L < X_H$. A sender of type $b_i$ reports Low if $U^S_i(X_L) \geq U^S_i(X_H)$ and reports High if $U^S_i(X_H) > U^S_i(X_L)$, where $i = 1, 2$. As noted by Dimitrakas and Sarafidis (2005), the modified CS model when the receiver is uncertain about the sender’s bias that all of the equilibria are partitional; here we focus on the outcome with two messages which is equivalent to a 2-partition equilibrium. This equilibrium has the feature that in a state of the world $s'$, if a sender prefers to send Low, she will also prefer to send Low for any state $s \in [0, s']$. Similarly, if a sender of a given bias prefers to send High for a signal $s''$, she will send High for any state $s \in [s'', 1]$.

The state where Type $b_i$ is indifferent between reporting Low and High is $T_i$. Type i’s indifference implies that at state $T_i$

$$(T_i + b_i) - X_L = X_H - (T_i + b_i)$$

where the payoffs to type $b_i$ from sending message Low or sending message High are displayed on the left-hand side and the right-hand side, respectively. Solving for $T_i$:

$$T_i = \frac{1}{2}(X_L + X_H) - b_i. \quad (3)$$
As noted above, equation (3) implies that Type i reports \textit{Low} for all states less than \(T_i\), and \textit{High} for all states greater than \(T_i\). Given that \(b_2 \leq b_1, T_1 \leq T_2\). Intuitively, Type 1’s higher bias implies that she prefers the higher action at a lower state of nature.

The following definitions are introduced to describe the reporting behavior of the two types:

\textbf{Definition 1. } \textit{Informative} - A sender type is \textit{informative} if she reports, depending on the state of the world, both \textit{Low} and \textit{High} in equilibrium. That is, her indifference point lies between 0 and 1.

\textbf{Definition 2. } \textit{Uninformative} - A sender type is \textit{uninformative} if in equilibrium she only sends one message across all states - that is, her reports are uninformative.

Two further definitions are introduced to describe the reporting behavior of \textit{informative} types. These will be useful in organizing and explaining the results.

\textbf{Definition 3. }\textit{Moderate} - An \textit{informative} type is classed as moderate if her indifference point lies between \(X_L\) and \(X_H\). Intuitively, if a sender’s bias is small than her indifference point is close to the midpoint of \(X_L\) and \(X_H\).

\textbf{Definition 4. }\textit{Extreme} - An \textit{informative} type is extreme if she reports both \textit{Low} and \textit{High} in equilibrium, but reports \textit{High} when \(s = X_L\), or \textit{Low} when \(s = X_H\). As a type’s bias increases, her indifference point moves further from the midpoint of \(X_L\) and \(X_H\).

Obviously, communication is only possible when at least one type is informative. For \(-b_1 \leq b_2 \leq b_1\), at least one of the following conditions must hold:

1. Both types are informative, i.e. \(0 < T_1 < T_2 < 1\);
2. Type 1 is uninformative and Type 2 is informative, i.e. \(T_1 < 0 < T_2 < 1\);
3. Both types are uninformative, that is no effective communication is feasible.

We investigate equilibria in each of the following conditions in turn.
3.1 Both types informative

There are 3 potential subcases where both types are informative.

3.1.1 Subcase 1 — $b_1$ and $b_2$ moderate

If $b_1$ and $b_2$ are close to zero, then $T_1$ and $T_2$ are close to the midpoint of $X_L$ and $X_H$. Both players are moderate when $0 < X_L < T_1 < T_2 < X_H < 1$. The sender’s problem is shown in Figure 1. As preferences are linear and because the receiver takes $T_1$ and $T_2$ as given, he solves the following problem upon observing a Low signal:

Figure 1: Both sender types ($b_1$ and $b_2$) are moderate

\[
\min_{X_L} \frac{p}{2} \left[ X_L^2 + (T_1 - X_L)^2 \right] + \frac{1 - p}{2} \left[ X_L^2 + (T_2 - X_L)^2 \right].
\] (4)

That is, if the receiver observes a Low signal, with probability $p$ the state is between 0 and $T_1$ and with probability $1 - p$ the state is between 0 and $T_2$. The receiver then chooses $X_L$ to minimise his welfare loss, given the distribution of states in which Low may be sent.

The receiver’s problem upon receiving High is:

\[
\min_{X_H} \frac{p}{2} \left[ (X_H - T_1)^2 + (1 - X_H)^2 \right] + \frac{1 - p}{2} \left[ (X_H - T_2)^2 + (1 - X_H)^2 \right].
\] (5)

If the receiver is sent a High signal, with probability $p$ the state is between $T_1$ and 1 and with probability $1 - p$ the state is between $T_2$ and 1. Similarly, the receiver
chooses $X_H$ to minimize his welfare loss over the distribution of states in which $High$ is reported.

The receiver’s optimal actions as a function of each type’s indifference points is obtained by finding the FOCs for (4) and (5) respectively:

$$X_L = \frac{p}{2} T_1 + \frac{1-p}{2} T_2,$$
$$X_H = \frac{1}{2} + \frac{p}{2} T_1 + \frac{1-p}{2} T_2 = \frac{1}{2} + X_L.$$ (7)

We derive the optimal actions by solving the receiver’s optimization problems, using equations (6), (7) and (3). From this we derive the following results.

$$X_L = \frac{1}{4} - pb_1 - (1-p)b_2, \quad \text{and} \quad X_H = \frac{3}{4} - pb_1 - (1-p)b_2,$$ (8)

$$T_1 = (1-p)(b_1 - b_2) + \frac{1}{2} - 2b_1, \quad \text{and} \quad T_2 = (2-p)(b_1 - b_2) + \frac{1}{2} - 2b_1.$$ (9)

The restrictions on the equilibrium in the subcase, where both types are moderate, are $0 < X_L < T_1 < T_2 < X_H < 1$. From $0 < X_L$ and $X_H < 1$ it follows that

$$-\frac{1}{4} < pb_1 + (1-p)b_2 < \frac{1}{4}. \quad (10)$$

From $X_L < T_1 < T_2 < X_H$ it follows that

$$-\frac{1}{4} < b_2 < b_1 < \frac{1}{4}. \quad (11)$$

It is easy to see that restrictions (10) follow from restrictions (11). The following result summarizes the above discussion.

**Result 1.** An equilibrium where both types are informative and moderate can be supported iff $|b_i| < \frac{1}{4}$ for $i = 1, 2$.

**Remark 1.** *Benchmark Case.* CS have shown that with linear quadratic preferences, when there is certainty over the sender’s bias, informative communication is only
possible if the sender’s bias is less than $\frac{1}{4}$. As way of comparison, in our model in the benchmark case when $b_1 = b_2 = b$, from equation (8)

$$X_L = \frac{1}{4} - b, \quad X_H = \frac{3}{4} - b, \quad T_1 = T_2 = \frac{1}{2} - 2b$$

and communication can only be supported if $b < \frac{1}{4}$.

### 3.1.2 Subcase 2 — $b_1$ extreme, $b_2$ moderate

As Type 1’s bias increases above $\frac{1}{4}$, $T_1$ decreases below $X_L$ and $b_1$ becomes an extreme sender. The restrictions for this case are $0 < T_1 < X_L < T_2 < X_H < 1$.

![Figure 2: $b_1$ extreme, $b_2$ moderate](image)

The receiver solves the following minimization problems for Low (see Figure 2):

$$\min_{X_L} \frac{p}{2} \left[ X_L^2 - (X_L - T_1)^2 \right] + \frac{1-p}{2} \left[ X_L^2 + (T_2 - X_L)^2 \right], \quad (12)$$

and High:

$$\min_{X_H} \frac{p}{2} \left[ (X_H - T_1)^2 + (1 - X_H)^2 \right] + \frac{1-p}{2} \left[ (X_H - T_2)^2 + (1 - X_H)^2 \right]. \quad (13)$$

The receiver’s optimal actions as a function of each type’s indifference points is
obtained by finding the FOCs for (12) and (13) respectively:

\[ X_L = \frac{1}{2}T_2 - \frac{p}{2(1-p)}T_1, \quad (14) \]

\[ X_H = \frac{1}{2} + \frac{p}{2}T_1 + \frac{1-p}{2}T_2. \quad (15) \]

The optimal actions are obtained by solving first-order conditions to the receiver’s optimization problems (equations (14) and (15)) and each type’s indifference point (equation (3)) to yield:

\[ X_L = (1-p)(b_1-b_2) + \frac{(1-4b_1)(1-2p)}{2(2-p)}; \quad X_H = \frac{1}{2} + (1-p)(b_1-b_2) + \frac{(1-4b_1)(1-p)}{2(2-p)}, \quad (16) \]

\[ T_1 = (1-p)(b_1-b_2) + \frac{(1-4b_1)(1-p)}{2-p}; \quad T_2 = (2-p)(b_1-b_2) + \frac{(1-4b_1)(1-p)}{2-p}. \quad (17) \]

The restrictions on the equilibrium in this subcase where type one is extreme and type 2 is moderate are \(0 < T_1 < X_L < T_2 < X_H < 1\).

\( T_1 > 0 \) is satisfied if

\[ b_1 < \frac{1}{2+p} - \frac{2-p}{2+p}b_2. \]

\( T_1 < X_L \) if

\[ b_1 > \frac{1}{4}. \]

\( T_2 < X_H \) if

\[ b_1 < \frac{1}{2p} + \frac{2-p}{p}b_2. \]

The following result provides a summary of when this communicative equilibrium is possible.

**Result 2.** When both types of sender are informative with \( b_1 \) extreme and \( b_2 \) moderate a communicative equilibrium exists if \(-\frac{1}{4} < b_2 < \frac{1}{4}\) and \( \frac{1}{4} < b_1 < \min\left(\frac{1}{2+p} - b_2 \frac{2-p}{2+p}, \frac{1}{2p} + b_2 \frac{2-p}{p}\right)\).

Although Type 1’s bias is greater than \( \frac{1}{4} \), communication is still possible from both types as the receiver is maximizing expected utility over the reporting strategies of all sender types. In contrast to the original CS model, uncertainty over the sender’s
bias allows sender types with larger biases to still be informative.

3.1.3 Subcase 3 — $b_1$ extreme, $b_2$ extreme

As the first type’s bias increases and the second type’s bias decreases, communication is still possible with $T_1 < X_L$ and $X_H < T_2$. That is, if the biases of both types spread apart, communication can still be sustained when both types’ biases are greater in magnitude than $\frac{1}{4}$. Type 1 reports Low in extremely low states, and Type 2 reports High in extremely high states. The restrictions for this case are $0 < T_1 < X_L < X_H < T_2 < 1$. The receiver solves the following minimization problems (see Figure 3):

$$\min_{X_L} \frac{p}{2} [X_L^2 - (X_L - T_1)^2] + \frac{1-p}{2} [X_L^2 + (T_2 - X_L)^2]; \quad (18)$$

and:

$$\min_{X_H} \frac{p}{2} [(X_H - T_1)^2 + (1 - X_H)^2] + \frac{1-p}{2} [(1 - X_H)^2 - (T_2 - X_H)^2]. \quad (19)$$

Using the same steps as in the previous subcases one can derive:

$$X_L = 2p(1-p)(b_1 - b_2) + \frac{(1 - 4pb_1)(1 - 2p)}{2}, \quad (20a)$$
\[ X_H = \frac{1}{2p} - 2(1-p)^2(b_1 - b_2) + \frac{(1-4pb_1)(2p-1)(1-p)}{2p}, \quad (20b) \]
\[ T_1 = (1-p)(1+b_2(1-2p) - b_1(1+2p)), T_2 = (1-p)(1+b_2(1-2p) - b_1(1+2p)) + b_1 - b_2. \quad (21) \]

The restrictions on the equilibrium in the subcase where both types are extreme are \(0 < T_1 < X_L < X_H < T_2 < 1\). It will be the case that \(T_1 < X_L\) if
\[ b_1 > \frac{1}{2(1+p)} + \frac{1-p}{1+p}b_2. \]
\(T_1 > 0\) is satisfied if
\[ b_1 < \frac{1}{2p+1} - \frac{2p-1}{2p+1}b_2. \]
\(T_2 < 1\) if
\[ b_2 > -\frac{1}{3-2p} + \frac{2p-1}{3-2p}b_1. \]
\(T_2 > X_H\) if
\[ b_2 < -\frac{1}{2(2-p)} + \frac{p}{2-p}b_1. \]

The following result provides a summary of when this communicative equilibrium is possible.

**Result 3.** If both types are extreme, a communicative equilibrium exists if \(\frac{1}{2(1+p)} + \frac{1-p}{1+p}b_2 < b_1 < \frac{1}{2p+1} - \frac{2p-1}{2p+1}b_2\) and \(-\frac{1}{3-2p} + \frac{2p-1}{3-2p}b_1 < b_2 < -\frac{1}{2(2-p)} + \frac{p}{2-p}b_1\).

### 3.2 One type informative and one type uninformative

If a sender’s bias is sufficiently large, she will always send the same message in equilibrium - that is, she is uninformative. In this section, we examine the two subcases in which type 1 is uninformative but type 2 is still informative.

#### 3.2.1 Subcase 4 — \(b_1\) uninformative, \(b_2\) moderate

Type 1 is the uninformative type. As \(b_1 > b_2\), this implies only Type 2 sends the message *Low* in equilibrium. Here, Type 2 is a moderate sender (see Figure 4).
The receiver’s minimization problems are:

\[
\min_{X_L} \frac{p}{2} [0] + \frac{1-p}{2} [X_L^2 + (T_2 - X_L)^2] \quad (22)
\]

and

\[
\min_{X_H} \frac{p}{2} [X_H^2 + (1 - X_H)^2] + \frac{1-p}{2} [(X_H - T_2)^2 + (1 - X_H)^2] \quad (23)
\]

Following the same steps as in the previous cases to solve for \( X_L \) and \( X_H \) yields

\[
X_L = \frac{1}{2(2+p)} - \frac{2}{(2+p)} b_2, \quad X_H = \frac{3}{2(2+p)} - \frac{2(1-p)}{(2+p)} b_2 \quad (24)
\]

and

\[
T_1 < 0, \quad T_2 = \frac{1 - 4b_2}{2 + p}. \quad (25)
\]

Notice that as she only sends one message, the uninformative type’s bias is irrelevant to the choice of \( X_L \) and \( X_H \). However, the probability of her type is relevant: as \( p \) increases, \( X_H \) converges to \( \frac{1}{2} \).

In terms of restrictions on equilibrium in this subcase, type 1 is uninformative if
$T_1 < 0$, which results in the following constraint:

$$b_1 > \frac{1}{2 + p} - \frac{(2 - p)b_2}{2 + p}.$$

Type 2 is moderate if $X_L < T_2 < X_H$.

$T_2 < X_H$ places the lower bound on $b_2$:

$$b_2 > -\frac{1}{4(1 + p)}.$$

$T_2 > X_L$ places the upper bound on $b_2$:

$$b_2 < \frac{1}{4}.$$

This analysis is summarized by the following result.

**Result 4.** If Type 1 is uninformative and Type 2 is informative and moderate, a communicative equilibrium is supportable if $b_1 > \frac{1}{2 + p} - \frac{(2 - p)b_2}{2 + p}$ and $-\frac{1}{4(1 + p)} < b_2 < \frac{1}{4}$.

### 3.2.2 Subcase 5 — $b_1$ uninformative, $b_2$ extreme

Now consider the case when Type 1 is uninformative and Type 2 is an extreme sender, as shown in Figure 5.

The receiver’s minimization problems are:

$$
\min_{X_L} \frac{p}{2} [0] + \frac{1-p}{2} [X_L^2 + (T_2 - X_L)^2];
$$

and

$$
\min_{X_H} \frac{p}{2} [X_H^2 + (1 - X_H)^2] + \frac{1-p}{2} [(1 - X_H)^2 - (T_2 - X_H)^2].
$$

In this subcase, the receiver’s optimal equilibrium actions are:

$$X_L = \frac{1 - 4pb_2}{2(2p + 1)}, \quad X_H = \frac{1}{2p} - \frac{1 - p}{p} \frac{1 - 4pb_2}{2(2p + 1)}.$$
Figure 5: Type 1 is uninformative, Type 2 is extreme

where

\[ T_1 < 0, \quad T_2 = \frac{1 - 4pb_2}{2p + 1}. \] (29)

In terms of restrictions on the equilibrium, Type 1 is uninformative if \( T_1 < 0 \), which results in the following constraint:

\[ b_1 > \frac{1}{2p + 1} - \frac{2p - 1}{2p + 1} b_2. \]

Type 2 is an extreme sender if \( X_H < T_2 < 1 \). \( T_2 > X_H \) is satisfied if:

\[ b_2 < -\frac{1}{4(1 + p)}. \]

Type two is informative if \( T_2 < 1 \):

\[ b_2 > -\frac{1}{2}. \]

This analysis is summarized by the following result.

\textbf{Result 5.} If Type 1 is uninformative and Type 2 is informative and extreme, a communicative equilibrium exists if \( b_1 > \frac{1}{2p+1} - \frac{2p-1}{2p+1} b_2 \) and \(-\frac{1}{2} < b_2 < -\frac{1}{4(1+p)}\).
3.3 No Communication

If the biases of both types are too large the only equilibrium is no communication. As the distribution of the state is uniform, the receiver’s utility maximizing action is $\frac{1}{2}$, regardless of the message sent. This occurs in two distinct subcases.

3.3.1 Subcase 6

Type 1 sends the message High, and Type 2 the message Low. This implies $T_1 < 0$ and $T_2 > 1$, and $X_L = X_H = \frac{1}{2}$. This is the unique equilibrium iff $b_1 \geq \frac{1}{2}$ and $b_2 \leq -\frac{1}{2}$.

3.3.2 Subcase 7

Both types send the message High. $T_1, T_2 < 0$ and $X_H = \frac{1}{2}$. $X_L$ is unrestricted. This is the unique equilibrium iff $b_2 \geq \frac{1}{4}$.

All equilibria of the model with two sender types have been dealt with. In the next section, a summary of the set of equilibria under uncertainty is provided.

3.4 Characterization of the Message Space

Using the three previous subsections, Figure 6 presents, for $-b_1 \leq b_2 \leq b_1$, a complete graphical characterization of the message space for $b_1$, $b_2$ and $p = 0.5$. Note that the areas not characterized are symmetric around the diagonals to the region $-b_1 \leq b_2 \leq b_1$ that is illustrated. While the exact characterization depends on the probability of each type, the layout remains qualitatively unchanged for different values of $p$.

Areas 1, 2 and 3 corresponds to a communicative equilibrium where both types are informative (Subcases 3.1.1, 3.1.2 and 3.1.3); areas 4 and 5 correspond to the cases where Type 1 is uninformative and Type 2 is informative and moderate (Subcase 3.2.1) and Type 2 is informative and extreme (Subcase 3.2.2), respectively. Communication is not possible in regions 6 and 7 (Subcases 3.3.1 and 3.3.2).

From section 3.3, the values of $b_1$ and $b_2$ under which communication is impossible
never change. In areas 1-5 where communication is informative the critical restriction relates to the smaller bias, specifically $\frac{1}{4} \leq b_2 \leq \frac{1}{4}$.

The following result summarizes the effect that uncertainty over the sender’s bias has on informative communication.

**Result 6.** Under uncertainty, there is no longer a unique upper bound on the bias that can support informative communication. In contrast to Crawford and Sobel (1982), it is now possible for informative signalling if a sender’s bias is $|b_1| > \frac{1}{4}$.

**An Example**

The previous subsections have provided a complete characterization of communication when there is uncertainty about the sender’s bias. An example is now provided to more clearly show that communication can be supported for a wider range of biases under uncertainty.
Example 1. Suppose $b_1 = 0.4$, $b_2 = -0.1$ and $p = 0.5$. The communicative equilibrium corresponds to subcase 3.1.2. The receiver’s equilibrium actions are $X_L = 0.25$ and $X_H = 0.65$. Type 1 reports Low in states $s \in (0, 0.05)$ and High for states $s \in (0.05, 1)$. Type 2 reports Low for $s \in (0, 0.55)$ and High over $s \in (0.55, 1)$.

4 Communication versus delegation

The uninformed principal faces two choices. On one hand, he may elicit advice from an expert with uncertain bias. On the other hand, he may avoid strategic communication and delegate the responsibility for making the decision directly to the expert. This choice for the principal is the focus of this section. To help answer this question we first establish that the receiver is better off with (informative) communication than without it, outlined in the following result.

**Result 7.** The receiver’s expected utility under communication is always greater than without communication.

*Proof.* The proof is a revealed-preference argument. If the receiver believes that the signal is uninformative with probability 1, then his utility-maximizing actions are $X_L = X_H = \frac{1}{2}$. This set of actions will always yield the receiver an expected utility of $-\frac{1}{4}$.

Under informative communication, the receiver’s utility maximizing actions are $X_L < \frac{1}{2}$ and $X_H > \frac{1}{2}$. As the receiver is still able to choose the set $X_L = X_H = \frac{1}{2}$ and obtain on average a utility of $-\frac{1}{4}$, not doing so must yield a strictly higher expected utility.

Turning to the receiver’s choice of communication or delegation, for each communicative subcase of section 3 the receiver will compare his expected utility, $U^C$, with his expected utility under delegation. Given the assumptions over the distribution of biases and the sender’s action under delegation, the receiver’s expected utility under delegation, $U^D$, is

$$-p|b_1| - (1 - p)|b_2|. \quad (30)$$
4.1 Subcase 1 — $b_1$ and $b_2$ moderate

When both types of senders are moderate the expected $U^C$ is equal to sum of the expected losses given in (4) and (5):

$$U^C = -\frac{p}{2} \left[ X_L^2 + (T_1 - X_L)^2 + (X_H - T_1)^2 + (1 - X_H)^2 \right] - \frac{1-p}{2} \left[ X_L^2 + (T_2 - X_L)^2 + (X_H - T_2)^2 + (1 - X_H)^2 \right].$$

By substituting the values of $X_L, X_H$ from (8) and values of $T_1$ and $T_2$ from (9) into the equation above we derive

$$U^C = -\frac{1}{8} - pb_1^2 - (1 - p)b_2^2 - (pb_1 + (1 - p)b_2)^2.$$  \hspace{1cm} (32)

Communication is preferred by the receiver provided $U^C > U^D$. In this subcase, communication is optimal when:

$$\frac{1}{8} < p|b_1| + (1 - p)|b_2| - pb_1^2 - (1 - p)b_2^2 - (pb_1 + (1 - p)b_2)^2.$$  \hspace{1cm} (33)

Analyzing the above inequality, if $b_1$ and $b_2$ are biased in the same direction, then delegation always dominates communication. Note that if $b_1 \neq b_2$ it follows that $pb_1^2 + (1 - p)b_2^2 > (pb_1 + (1 - p)b_2)^2$. Labeling $\bar{b} = pb_1 + (1 - p)b_2$, the RHS of inequality (33) is less than $|\bar{b}| - 2\bar{b}^2$. On the other hand, $|\bar{b}| - 2\bar{b}^2 \leq \frac{1}{8}$.

If $b_1$ and $b_2$ are biased but in opposing directions, it is possible that communication dominates delegation. For example, if $p = 0.5, b_1 = 0.2$ and $b_2 = -0.2$ the inequality given in (33) is satisfied and communication yields higher expected utility than delegation for the receiver. This information is summarized in the following result.

**Result 8.** When both types have biases in the same direction, delegation dominates communication. When both types have biases in the opposite directions, it is possible that communication dominates delegation.

Sender types with opposing biases allows the receiver to: take a higher *Low* action relative to equilibrium action they would take with communication with only a Type
1 sender after receiving a *Low* message; and to take a lower *High* action relative to their equilibrium action following a message of *Low* when communicating only with a Type 2. As both types’ biases are relatively small, this shifts the indifference points for both sender types towards $\frac{1}{2}$, which reduces welfare loss under communication. If $b_1 > 0$ and $b_2 < 0$ then this effect can be strong enough for communication to be optimal.

**Remark 2. Benchmark Case.** Dessein (2002) showed that when there is certainty over the sender’s bias, delegation dominates communication whenever informative communication is possible. Our framework generalizes of this result of Dessein (2002) to allow for uncertainty about the sender where the potential biases have the same sign. The uncertainty about the bias also broadens the range of biases for which communication is feasible and, by changing the incentives for the sender during communication, creates an environment in which communication can be preferred by the receiver to delegation.$^8$

### 4.2 Subcase 2 — $b_1$ extreme, $b_2$ moderate

In this case $U^C$ is equal to minus sum of functions (12) and (13):

$$U^C = -\frac{p}{2} \left[ X_L^2 - (X_L - T_1)^2 + (X_H - T_1)^2 + (1 - X_H)^2 \right] - \frac{1-p}{2} \left[ X_L^2 + (T_2 - X_L)^2 + (X_H - T_2)^2 + (1 - X_H)^2 \right] .$$

(34)

Substitute values of $X_L$, $X_H$ from (16) and values of $T_1$ and $T_2$ from (17) to derive

$$U^C = -\frac{1}{4} + \frac{(1-p)(1-4b_1)}{(2-p)^2} + (1-p) \left( pb_1 + (2-p)b_2 + \frac{1}{2} \right) \left( b_1 - b_2 + \frac{(1-4b_1)(2-3p)}{2(2-p)^2} \right).$$

(35)

Communication is preferred to delegation if:

---

$^8$Our model also provides another perspective on the relationship between delegation and uncertainty. Prendergast (2002) suggested a possible negative relationship between uncertainty over the project and the probability of delegation; here, with uncertainty regarding the expert’s bias as well as the project leads to the prospect that communication (centralization of decision making) is preferred.
\[
\frac{1}{4} - p|b_1| - (1 - p)|b_2| < \frac{p(1-p)(1-4b_1)}{(2-p)^2} + (1-p) \left( pb_1 + (2-p)b_2 + \frac{1}{2} \right) \left( b_1 - b_2 + \frac{(1-4b_1)(2-3p)}{2(2-p)^2} \right).
\]  

(36)

Result 9. For any \( b_1 \) and \( b_2 \) where Type 1 is extreme and Type 2 moderate, there exists a \( p^* \) where communication is optimal for all \( p^* < p < 1 \).

Proof. Take a point in area 2. As \( p \) increases this point could either: (1) remain in area 2; or (2) become located in area 4 or 5, as the boundaries of the areas change. (1) If the point remains in area 2 we use the following argument. Type 1 is extreme only when \( b_1 > \frac{1}{4} \), implying that for sufficiently high \( p \) the payoff from delegation is lower than \(-\frac{1}{4}\). However, from Proposition 7, the receiver’s payoff from communication must be greater than \(-\frac{1}{4}\). Consequently, provided that inequality (36) is continuous, if \( p \) is sufficiently high communication is preferred to delegation.

(2) If the point becomes located in area 4 or 5, both boundaries \( b_1 = \frac{1}{2+p} - b_2 \frac{2-p}{2+p} \) and \( b_1 = \frac{1}{2p} - b_2 \frac{2-p}{p} \) become weaker as \( p \) increases. The first boundary gives \( b_1' = \frac{4b_2-1}{(2+p)^2} < 0 \), the second boundary gives \( b_1' = \frac{-1-4b_2}{2p^2} < 0 \). In area 5 communication dominates delegation, see Result 12. In area 4, the area has the same property that when \( p \) increases, eventually communication will dominate delegation; see Result □

4.3 Subcase 3 — Both types extreme

For all \( b_1 \) and \( b_2 \) where both types are extreme, \( p|b_1| + (1-p)|b_2| > \frac{1}{4} \). From Result 7, communication is always optimal in this case.

Result 10. Communication is always optimal whenever both types are extreme.

4.4 Subcase 4 — \( b_1 \) uninformative, \( b_2 \) moderate

When \( b_1 \) is uninformative and \( b_2 \) is moderate, the receiver’s expected loss \( U^C \) is the sum of the losses in (22) and (23):

\[
U^C = -\frac{p}{2} \left[ X_H^2 + (1 - X_H)^2 \right] - \frac{1-p}{2} \left[ X_L^2 + (T_2 - X_L)^2 + (X_H - T_2)^2 + (1 - X_H)^2 \right].
\]

(37)
Substitute values of \( X_L \) and \( X_H \) from equation (24) and value of \( T_2 \) from (25) yields

\[
U^C = -\frac{2p + 1}{4(2 + p)} - \frac{4(1 - p)b_2^2}{2 + p}. \tag{38}
\]

The receiver’s expected utility from communication is greater than her expected utility from delegation if:

\[
-\frac{2p + 1}{4(2 + p)} - \frac{4(1 - p)b_2^2}{2 + p} > -p|b_1| - (1 - p)|b_2|.
\]

**Result 11.** For any \( b_1 \) and \( b_2 \) where Type 1 is uninformative and Type 2 moderate, there exists a \( p^* \) where communication is optimal for all \( p^* < p < 1 \).

**Proof.** The proof is similar to the proof of Result 4.2, part (1). Type 1 is uninformative if \( b_1 > \frac{1}{4} \), implying that for sufficiently high \( p \) the payoff from delegation is lower than \(-\frac{1}{4}\). \( \square \)

### 4.5 Subcase 5 — \( b_1 \) uninformative, \( b_2 \) extreme

For all \( b_1 \) and \( b_2 \) where \( b_1 \) is uninformative and \( b_2 \) is extreme, \( p|b_1| + (1 - p)|b_2| > \frac{1}{4} \).

From Result 7, communication is always optimal in this case.

**Result 12.** Communication is always optimal whenever one type is uninformative and the other extreme.

### 4.6 Summary

The working from the previous subsections shows that communication can be optimal in every communicative subcase. Moreover, when there is uncertainty about the sender’s bias, communication can be optimal even when both types are informative, and when both types biases are less than \( \frac{1}{4} \). Figure 7 presents values of \( b_1 \) and \( b_2 \) under which communication dominates delegation, for \( p = 0.5 \).

As noted before, in areas 6 and 7 communication is not possible. The shaded areas 1-5 show the region for which communication is possible and preferred to dele-
Figure 7: Regions where communication dominates delegation

- gation. Only in the blank area in between the two diagonals does delegation dominate – comparing Figure 7 and Figure 6, it is clear that communication is optimal for a significant proportion of the values of $b_1$ and $b_2$ that support communication. Moreover, when allowing for more messages would improve the information transmission, making it less likely that delegation is preferred to communication. The possibility of more than 2 messages being sent is considered in the next Section.

5 Extension

In this section we generalize the results of Ottaviani (2000), who analyzes the case where $b_1 = -b_2$ and $p = 0.5$. First, let $b_1 = b$ for simplicity. Second, suppose there are $N$ actions, $n = 1, ..., N$. 
5.1 Sender behaviour

With $N$ actions, there are $N - 1$ indifference points. The $n$th indifference point of Type 1 is given by:

$$T_n^1 = \frac{X_n + X_{n+1}}{2} - b.$$  \hfill (39)

The $n$th indifference point of Type 2 is:

$$T_n^2 = \frac{X_n + X_{n+1}}{2} + b = T_n^1 + 2b.$$  \hfill (40)

5.2 Receiver’s actions

5.2.1 Interior actions

If both types report messages $n - 1$ and $n + 1$, as $p = 0.5$, by symmetry equilibrium actions must satisfy (see Figure 8):

$$X_n = \frac{T_n^2 + T_{n-1}^1}{2} = \frac{T_n^2 + T_{n-1}^1}{2}.$$  

Substituting for indifference points 9:

$$X_n = \frac{X_{n-1} + 2X_n + X_{n+1}}{4} = \frac{X_{n-1} + X_{n+1}}{2}.$$  \hfill (41)

![Figure 8: Optimal ‘interior’ actions](image)

9Note that this result is independent on whether sender types are moderate or extreme.
5.2.2 Actions on the ‘edges’

Action 1

There are 3 subcases depending on the type of the first sender.

Moderate

Given the receiver’s \( N \) actions, suppose that \( b \) is sufficiently small that both types report \( X_1 \), and do so as moderates. The receiver’s problem is:

\[
\min_{X_i} \frac{1}{4} \left[ 2X_i^2 + (T_1^1 - X_i)^2 + (T_1^2 - X_i)^2 \right].
\] (42)

Taking the FOC:

\[
X_i = \frac{T_1^1 + T_1^2}{4} = \frac{X_1 + X_2}{4} = \frac{X_2}{3}.
\] (43)

Extreme

Continue to suppose that both types report \( X_1 \), but Type 1 is extreme. Then, the receiver solves:

\[
\min_{X_i} \frac{1}{4} \left[ 2X_i^2 + (T_1^2 - X_i)^2 - (X_1 - T_1^1)^2 \right].
\] (44)

The FOC is:

\[
X_i = \frac{T_1^2 - T_1^1}{2} = \frac{X_1 + X_2}{2} + b - \frac{X_1 + X_2}{2} + b = b.
\] (45)

Uninformative

Suppose that Type 1 does not report \( X_1 \). The receiver solves:

\[
\min_{X_i} \frac{1}{4} \left[ X_i^2 + (T_1^2 - X_i)^2 \right].
\]

The FOC is:

\[
X_i = \frac{T_1^2}{2} = \frac{X_1 + X_2}{4} + \frac{b}{2} = \frac{2b}{3} + \frac{X_2}{3}.
\] (46)
However, if Type 1 is uninformative

\[ T_1^1 < 0. \quad (47) \]

Equations (46) and (47) together imply that \( X_1 > X_2 \), which is not permissable. There is no equilibrium, for finite \( N \), where either type does not report all messages.

**Action N**

There are 3 subcases depending on the type of the second sender.

**Moderate**

In this case, both types report action \( N \) as moderates:

\[
\min_{X_N} \frac{1}{4} \left[ (X_N - T_{N-1}^1)^2 + (X_N - T_{N-1}^2)^2 + 2 (1 - X_N)^2 \right] 
\]

resulting in a first-order condition of:

\[
(X_N - T_{N-1}^1) + (X_N - T_{N-1}^2) + 2 (X_N - 1) = 0
\]

or

\[
X_N = \frac{1}{2} + \frac{T_{N-1}^1 + T_{N-1}^2}{4} = \frac{1}{2} + \frac{X_{N-1} + X_N}{4} = \frac{2}{3} + \frac{X_{N-1}}{3}. \quad (49)
\]

**Extreme**

If Type 2 reports action \( N \) as an extreme type, the receiver’s problem is:

\[
\min_{X_N} \frac{1}{4} \left[ (X_N - T_{N-1}^1)^2 - (T_{N-1}^2 - X_N)^2 + 2 (1 - X_N)^2 \right] 
\]

with a first-order condition

\[
T_{N-1}^2 - T_{N-1}^1 + 2 (X_N - 1) = 0
\]

\[
X_N = 1 + \frac{T_{N-1}^1 - T_{N-1}^2}{2} = 1 - \frac{T_{N-1}^2 - T_{N-1}^1}{2} = 1 - b. \quad (51)
\]
5.2.3 Equilibrium actions

Recall that by symmetry, if both types report action 1 as moderate types, then they will report action $N$ as moderate types.

**Moderates**

Combining the FOCs from sections 5.2.1 and 5.2.2, if $b_1$ and $b_2$ are moderate, then:

$$X_1 = \frac{X_2}{3}, \quad X_n = \frac{X_{n-1} + X_{n+1}}{2}, \quad X_N = \frac{2}{3} + \frac{X_{N-1}}{3}. \quad (52)$$

The equation for $X_n$ implies that the distance between each action is equal. The equation for $X_1$ ($X_N$) implies the distance between 0 (1) and $X_1$ ($X_N$) is half the distance between each action. That is:

$$X_n = \frac{2n - 1}{2N}, \quad n = 1, ..., N. \quad (53)$$

The equilibrium with moderates holds if $T_1 < X_1$, which results in the following condition

$$b < \frac{1}{2N}. \quad (54)$$

That is, if $b$ is sufficiently small, all actions are evenly spaced apart.

**Extreme**

From sections 5.2.1 and 5.2.2, if both types are extreme:

$$X_1 = b, \quad X_n = \frac{X_{n-1} + X_{n+1}}{2}, \quad X_N = 1 - b. \quad (55)$$

All actions are still evenly spaced apart, but the distance between 0 and $X_1$ (as well as 1 and $X_N$) is larger. The distance between $X_1$ and $X_N$, $1 - 2b$, determines $X_n$:

$$X_1 = b, \quad X_n = b + \left(\frac{n - 1}{N - 1}\right)(1 - 2b), \quad X_N = 1 - b. \quad (56)$$
This equilibrium holds if \( X_1 > T_1 > 0 \), which results in the following condition

\[
\frac{1}{2N} < b < \frac{1}{2}.
\]  

(57)

That is, if \( b \) is too large to support completely evenly spread apart actions, communication still possible and the first and last actions are closer to \( \frac{1}{2} \).

5.3 Welfare

In this subsection the receiver’s utility under delegation, constrained delegation is combined to his utility from communication with any finite number of actions.

5.3.1 Delegation

The receiver’s utility from delegation is:

\[
U^R = -b.
\]  

(58)

5.3.2 Constrained Delegation

As shown by Ottaviani (2000), the constrained delegation solution, for Type 1, is:

\[
m_b(x) = \begin{cases} 
  x + b & \text{if } x \in [0, 1 - 2b], \\
  1 - b & \text{if } x \in [1 - 2b, 1]. 
\end{cases}
\]

where \( m_b(x) \) is the message sent by Type 1. The solution for Type 2 is symmetric. Thus, the receiver’s utility is:

\[
U^R = -(1 - 2b)b - b^2 = -b + b^2.
\]  

(59)

The term \( b^2 \) is the receiver’s utility gain from constrained delegation over delegation.
5.3.3 Communication

Moderates

Given all actions are evenly spaced $\frac{1}{N}$ apart, as the sender’s bias is a constant her indifference points are spaced apart by the same distance as the actions. That is, partitions $T_1$ to $T_{N-1}$ are $\frac{1}{N}$ apart. As $b < \frac{1}{2N}$, this implies that $X_{n+1} > T_n > X_n$. In words, this implies that in this case sender’s are moderate for all actions.

Actions 1 and $N$

First, the receiver’s expected utility from actions 1 and $N$ is:

$$-2 \left[ \frac{1}{2} \left( \frac{X_1^2}{2} + \frac{(T_1 - X_1)^2}{2} \right) + \frac{1}{2} \left( \frac{X_1^2}{2} + \frac{(T_1 - X_1)^2}{2} \right) \right] = -2 \left( \frac{1}{4N^2} \right) - b^2.$$

Actions 2 to $N - 1$

The receiver’s expected utility from actions 2 to $N - 1$ is:

$$-(N - 2) \left[ \frac{(X_n - T_{n-1})^2}{2} + \frac{(X_n - X_n)^2}{2} \right] = -(N - 2) \left[ \frac{1}{4N^2} + b^2 \right].$$

Combining, the receiver’s utility from communication is:

$$U^R = -\frac{1}{4N} - (N - 1) b^2. \quad (60)$$

Extreme

Similarly, as indifference points are the same distance apart as actions, if $T_1 < X_1$, then $T_n < X_n$.

Actions 1 and $N$

As actions are symmetric, the receiver’s expected utility from actions 1 and $N$ is:

$$- \left[ \frac{X_1^2}{2} - \frac{(X_1 - T_1)^2}{2} + \frac{X_1^2}{2} + \frac{(T_1 - X_1)^2}{2} \right] = -b^2 - (X_2 - X_1) b.$$

Actions 2 to $N - 1$

The receiver’s expected utility from actions 2 to $N - 1$ is:
\[- \frac{N-2}{2} \left[ (X_n - T_{n-1})^2 - (X_n - T_n)^2 \right] = -(N-2) (X_n - X_{n-1}) b.\]

Combining, the receiver’s utility is:

\[ U^R = -b + b^2. \]  

(61)

To summarize:

\[ U^R = \begin{cases} 
\frac{-1}{4N} - (N-1) b^2 & \text{if } N < \frac{1}{2b} \\
-b + b^2 & \text{if } N > \frac{1}{2b}.
\end{cases} \]

That is, for any finite \( N > \frac{1}{2b} \) the receiver’s utility from communication is exactly equal to his utility from constrained delegation. Note that the communicative equilibrium in Ottaviani (2000) requires an infinite number of potential messages, while the equilibrium described here achieves the same result with a finite number of messages.

6 Discussion

In this paper, following Crawford and Sobel (1982), we develop a framework in which a principal chooses whether to: seek the advice of an expert with unknown bias in order to implement a decision; or, alternatively, delegate the decision-making rights to the expert. The choice for the principal is a familiar one of retaining the decision-making powers but having to endure strategic communication versus the loss of control associated with delegation.

In this framework we are able to derive several interesting results. First, we show that communication is possible with a larger range of biases. Even when there is a single sender who may be of multiple types, there is no longer a unique upper bound on the level of bias required for communicative signalling. Indeed, sender types can be informative when their bias is greater than \( \frac{1}{4} \). Second, we are able to generalize the result that delegation dominates communication whenever communication is possible of Dessein (2002) from a situation in which the expert’s bias is systematic (or when
$b_1 = b_2$ in our model) to one in which the sender’s bias is uncertain, provided the direction of the possible biases are the same. Third, we show that when the possible biases are of opposite signs that it is possible that the receiver’s welfare under communication can now be greater than under delegation, providing a counterexample to the delegation dominates communication result in Dessein (2002).

Underlying these results is that, in equilibrium, uncertainty regarding the bias of the expert affects the sender’s incentives to engage in strategic communication. As the receiver chooses actions based on the reporting strategies of all types, uncertainty regarding sender’s bias and having possible sender types with opposing biases can allow the receiver to take more moderate actions. In effect, the opposing biases allow the receiver to take a higher Low action and a lower High action. In response, this shifts both types’ indifference points towards $\frac{1}{2}$ that: (a) expands the set of biases that support communication; and (b) improves welfare.

The second effect of communication under uncertainty is that it allows for informative signalling from informative types, while minimizing the receiver’s welfare loss from uninformative types. By retaining control over the action, an acutely biased type is prevented from taking actions that are not in the receiver’s interest. At the same time, the receiver can still maintain incentives for relatively unbiased sender types to be informative. Further to that, when the proportion of uninformative types is high, the receiver’s loss of utility from communication will be small compared to his loss of utility from relinquishing decision rights. Communication can be optimal when the receiver believes that a sender’s bias might be high.

References


