Abstract

The purpose of this paper is to present a model of the endogenous evolution of a society’s property rights regime. We use an overlapping-generations framework in which capital accumulation takes place. Property rights enforcement is costly. Individuals decide collectively in each period the appropriate level of enforcement. They pay taxes to finance enforcement. Poor households have less interest in enforcement of property rights. As they become richer through the process of capital accumulation, they change.

JEL Classification: H10

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1 Introduction

The purpose of this paper is to present a model of the endogenous evolution of a society’s property rights regime. We use an overlapping-generations framework in which capital accumulation takes place. In the simplest version of the model, agents are homogeneous. They predate on each other, and yet realize that a perfect property rights regime, if it could be costlessly enforced, would benefit everyone. In our model, property rights enforcement is costly, and the cost of enforcement depends on the level of enforcement. Individuals must decide collectively in each period the appropriate level of enforcement for that period, knowing that they must pay taxes to finance the enforcement activities. We show that there are two interior steady-state equilibriums. Along the path approaching the stable steady state, the degree of enforcement increases as the stock of capital is accumulated. The intuition is simple. When households are poor, they do not have great interest in a regime with a high degree of enforcement of property rights. As they become richer through the process of capital accumulation, they become more committed to the idea of property rights protection.

We then extend the model to the case where agents are heterogeneous in terms of preferences. We take the case where there are two types of agents: those with strong preferences for present consumption (the “greedy type”) and those with weaker preferences for present consumption. We show that political power may shift from one group to another, with the possibility of reswitching at a later date. A discrete shift in power is called a “revolution”. It results in a new regime with a stronger (or weaker) commitment to property rights protection. Our numerical simulations show the possibility of two revolutions.

Our model borrows some features from Gradstein (2004, 2007, 2008). However, our first model differs from those of Gradstein in a number of
aspects. First, the degree of property-rights enforcement is modelled as a continuous variable which can move gradually from zero to 1. Second, the cost of enforcement is a function of two variables: the degree of enforcement, and the size of the economy (as measured by its capital stock). Third, the effectiveness of an individual’s rent-seeking activity is a function of three variables: the amount of resource he devotes to rent-seeking, the size of his investment, and society’s degree of enforcement of property rights. Fourth, there is a “subsistence level” of consumption, below which there will be no capital accumulation. Fifth, the model shows that there can be several steady-state aggregate capital stock levels, some of which are stable and others are unstable. They are associated with different endogenously chosen degrees of property-right enforcement. In our second model, we consider groups with different “subsistence levels” of consumption, and we allow population growth rates to depend on income level, a feature well documented by Schultz (2005), and model political power of a group in terms of its aggregate wealth.

There is a significant literature on the endogenous determination of property rights regime. Tornell (1997) presented a model of economic growth and decline with endogenous property rights. In his model, infinitely-lived agents solve a dynamic game over the choice of property-rights regime. He showed that a possible equilibrium of the game involves multiple switching of regimes. Dixit (2004) analyzed the benefits and costs of private enforcement of property rights. Guriev and Sonin (2009) considered a repeated game between two oligarchs and a dictator. The oligarchs cooperate when choosing a new dictator at the start of a new period, but do not cooperate within the period. Dictators come in two types: a strong dictator can only be removed when the two oligarchs join forces, while a weak dictator can be removed by either oligarch. A strong dictator can expropriate one oligarch as long as he keeps the other oligarch happy. The dictator asks the oligarchs to contribute
funds to him, and can choose “strong protection” (thus banning rent-seeking by the oligarchs) at a cost, or “no protection” at zero cost. Guriev and Sonin (2009) characterized the equilibrium of the game, and found conditions under which a dictator would choose strong protection or no protection, and conditions under which a dictator will be removed. Acemoglu and Robinson (2001, 2006) built dynamic models where coups and revolutions can occur when there are exogenous economic shocks.

2 The Model

We consider an overlapping generation model. Time is discrete, $t = 0, 1, 2, 3\ldots$ and the economy goes on forever. In each period, there is a continuum of households, each indexed by $i$, where $i$ is a number in the unit interval $[0, 1]$. The head of household $i$ in period $t$ receives at the beginning of period $t$ a gross income $b_{it} > 0$. A tax rate $\tau_t$ is imposed on this income. He allocates the net income $(1 - \tau_t)b_{it}$ among three activities: consumption, denoted by $c_{it}$, investment, denoted by $k_{it+1}$, and rent-seeking, denoted by $r_{it+1}$. The total amount of capital in the economy at time $t + 1$ is then

$$K_{t+1} = \int_0^1 k_{jt+1}dj$$

Let $z_{it+1}$ denote the effective influence of agent $i$. The actual amount of capital that becomes available to household $i$ is denoted by $\kappa_{it+1}$. We take it that the size of $\kappa_{it+1}$ depends on the effective influence of agent $i$ relative to the average influence:

$$\kappa_{it+1} = \frac{z_{it+1}K_{t+1}}{\int_0^1 z_{jt+1}dj} \quad (1)$$

We assume that

$$z_{it+1} = (k_{it+1})^{\lambda_t} (r_{it+1})^{1-\lambda_t}$$

Gradstein (2007) assumed that $z_{it+1} = k_{it+1}r_{it+1}$.
where \( \lambda_t \) is the degree of property-rights enforcement in period \( t \). Notice that \( \lambda_t = 1 \) means perfect-perfect property rights protection: as \( \lambda_t \to 1 \), \( (r_{it+1})^{1-\lambda_t} \to 1 \) and hence \( z_{it+1} \to k_{it+1} \), which in turns implies that \( \kappa_{it+1} = k_{it+1} \).

At the beginning of period \( t+1 \), the (new) head of household \( i \) (whose father was head of household in period \( t \)) receives the gross income

\[
b_{it+1} = A (\kappa_{it+1})^\alpha (n_{it})^{1-\alpha}
\]

where \( n_{it} \) is the number of workers in the household (in this section, we assume \( n_{it} \) to be a constant, normalized at unity).

We assume that the objective function of the head of household \( i \) in period \( t \) is to maximize

\[
V_{it} = \ln(c_{it} - x_i) + \beta \ln(b_{it+1})
\]

subject to the budget constraint that

\[
(1 - \tau_t)b_{it} = c_{it} + r_{it+1} + k_{it+1}
\]

The parameter \( x_i > 0 \) is the (exogenous) perceived minimum subsistence level of consumption and we assume that \( (1 - \tau_t)b_{it} > x_i \). There is another interpretation of \( x_i \) in this model. Since a larger \( x_i \) will induce a lower bequest \( b_{it+1} \), we interpret \( x_i \) as a measure of selfishness or greed.

Substitution gives the objective function

\[
V_{it} = \ln \left[ (1 - \tau_t)b_{it} - r_{it+1} - k_{it+1} - x_i \right] + \beta \ln \left\{ A \left[ \left( \frac{k_{it+1}}{r_{it+1}} \right)^{\lambda_t} \frac{K_{t+1}}{z_{jt+1} dj} \right]^{\alpha} \right\}
\]

So

\[
V_{it} = \ln \left[ (1 - \tau_t)b_{it} - r_{it+1} - k_{it+1} - x_i \right] + \\
\alpha \beta \lambda_t \ln(k_{it+1}) + \alpha \beta (1 - \lambda_t) \ln(r_{it+1}) + \beta \ln \left\{ \frac{AK_{t+1}^{\alpha}}{\int_{0}^{1} z_{jt+1} dj} \right\}
\]

(2)
Since the individual takes the last term in equation (2) as given, the FOCs are
\[
\frac{\partial V_{it}}{\partial k_{it+1}} = -\frac{1}{(1-\tau_t)b_{it} - x_i - r_{it+1} - k_{it+1}} + \frac{\alpha\beta\lambda_t}{k_{it+1}} = 0 \quad (3)
\]
\[
\frac{\partial V_{it}}{\partial r_{it+1}} = -\frac{1}{(1-\tau_t)b_{it} - x_i - r_{it+1} - k_{it+1}} + \frac{\alpha\beta(1-\lambda_t)}{r_{it+1}} = 0 \quad (4)
\]
These two equations imply
\[
r_{it+1} = \frac{(1-\lambda_t)}{\lambda_t} k_{it+1} \quad (5)
\]
Equation (5) indicates that when the property rights regime is perfect ($\lambda_t = 1$), no individual would choose a positive level of rent seeking. Substitute equation (5) into (3) to get
\[
(1-\tau_t)b_{it} - x_i = k_{it+1} \left[ \frac{1}{\lambda_t} + \frac{1}{\alpha\beta\lambda_t} \right] \equiv k_{it+1} \frac{1}{\mu_t}
\]
where
\[
\mu_t = \frac{\lambda_t\alpha\beta}{1 + \alpha\beta}
\]
Hence the household’s investment rule is:
\[
k_{it+1} = \mu_t [(1-\tau_t)b_{it} - x_i] \quad (6)
\]
and its non-consumption expenditure rule is:
\[
r_{it+1} + k_{it+1} = \frac{\alpha\beta}{1 + \alpha\beta} [(1-\tau_t)b_{it} - x_i] \quad (7)
\]
Thus its influence is
\[
z_{it+1} = (k_{it+1})^{\lambda_t} (r_{it+1})^{1-\lambda_t} = (k_{it+1})^{\lambda_t} \left[ \frac{1 - \lambda_t}{\lambda_t} k_{it+1} \right]^{1-\lambda_t}
\]
\[
= k_{it+1} \left[ \frac{1 - \lambda_t}{\lambda_t} \right]^{1-\lambda_t} = \mu_t [(1-\tau_t)b_{it} - x_i] \left[ \frac{1 - \lambda_t}{\lambda_t} \right]^{1-\lambda_t}
\]
Next, define aggregate bequest at $t$ by

$$B_t \equiv \int_0^1 b_{jt} dj$$

and let the aggregate minimum consumption expenditure be

$$X \equiv \int_0^1 x_{jd}$$

Then

$$Z_{t+1} \equiv \int_0^1 z_{jt+1} dj = \mu_t \left[ (1 - \tau_t) B_t - X \right] \left[ \frac{1 - \lambda_t}{\lambda_t} \right]^{1 - \lambda_t}$$

and

$$K_{t+1} \equiv \int_0^1 k_{jt+1} dj = \mu_t \left[ (1 - \tau_t) B_t - X \right]$$

For household $i$, its Nash equilibrium effective capital is, from (1)

$$\kappa_{it+1} = \mu_t \left[ (1 - \tau_t) b_{it} - x_i \right] \left[ \frac{1 - \lambda_t}{\lambda_t} \right]^{1 - \lambda_t} \frac{K_{t+1}}{Z_{t+1}} = \mu_t \left[ (1 - \tau_t) b_{it} - x_i \right]$$

and its equilibrium utility from consumption in period $t$ is

$$\ln \left[ (1 - \tau_t) b_{it} - x_i - r_{it+1} - k_{it+1} \right] = \ln \left\{ \left( \frac{1}{1 + \alpha \beta} \right) \left[ (1 - \tau_t) b_{it} - x_i \right] \right\}$$

Its second period wealth is then a function of its first period wealth and $\mu_t$ and $\tau_t$

$$b_{it+1} = A (\kappa_{it+1})^\alpha (n_{it})^{1 - \alpha} = A \left[ \frac{\alpha \beta \lambda_t}{1 + \alpha \beta} \right]^\alpha [(1 - \tau_t) b_{it} - x_i]^\alpha (n_{it})^{1 - \alpha}$$

Thus the evolution of $b_{it+1}$ depend only on $b_{it}$, $\tau_t$ and $\lambda_t$. Here $n_{it} = 1$ for all $t$ (in this section).

Hence the (optimized) welfare of household $i$ is, using (9),

$$V_{it}^*(b_{it}, \lambda_t, \tau_t) = \ln \left\{ \left( \frac{1}{1 + \alpha \beta} \right) [(1 - \tau_t) b_{it} - x_i] \right\} + \beta \ln A (\kappa_{it+1})^\alpha =$$
\[
\ln \left( \frac{1}{1 + \alpha \beta} \right) + \beta \ln A + (1 + \alpha \beta) \ln [(1 - \tau_t)b_{it} - x] + \alpha \beta \ln \lambda_t + \alpha \beta \ln \left( \frac{\alpha \beta}{1 + \alpha \beta} \right)
\]

(11)

Now assume that the degree of enforcement \( \lambda_t \) in an economy with size \( B_t \) is possible if and only if the government spends an amount \( G(\lambda_t, B_t) = \gamma \lambda_t B_t \) units of resources (where \( \gamma \) is a positive constant, assumed to be smaller than 1). Assume this expenditure is financed by a tax \( \tau_t \) on \( b_{it} \) for all households. Then the requirement of balanced budget implies \( \tau_t B_t = \gamma \lambda_t B_t \). This implies that \( \tau_t = \gamma \lambda_t \). Then

\[
V_{it}^*(b_{it}, \lambda_t) = \ln \left( \frac{1}{1 + \alpha \beta} \right) + \beta \ln A + (1 + \alpha \beta) \ln [(1 - \gamma \lambda_t)b_{it} - x_{it}] + \alpha \beta \ln \lambda_t + \alpha \beta \ln \left( \frac{\alpha \beta}{1 + \alpha \beta} \right)
\]

What is individual \( i \)'s most preferred \( \tau_t \)? This is found by maximizing \( V_{it}^*(b_{it}, \lambda_t) \) with respect to \( \lambda_t \) subject to \( 0 \leq \lambda_t \leq 1 \). The solution is denoted by \( \lambda_{it}^* \). The first-order condition for an interior maximum is

\[
\frac{-(1 + \alpha \beta)\gamma b_{it}}{(1 - \gamma \lambda_{it}^*)b_{it} - x_{it}} + \frac{\alpha \beta}{\lambda_{it}^*} = 0
\]

(12)

This shows that the individual's most preferred \( \tau_t \) depends on his wealth \( b_{it} \).

Hence

\[
\lambda_{it}^* = \frac{\alpha \beta}{\gamma(1 + 2\alpha \beta)} \left[ \frac{b_{it} - x_{it}}{b_{it}} \right] = \frac{\alpha \beta}{\gamma(1 + 2\alpha \beta)} \left[ 1 - \frac{x_{it}}{b_{it}} \right]
\]

(13)

Thus,

\[
\frac{\partial \lambda_{it}^*}{\partial b_{it}} > 0
\]

and

\[
\frac{\partial \lambda_{it}^*}{\partial x_{it}} < 0
\]

The wealthier a household is, the more law enforcement they want and the greedier a person is, the less law enforcement they want. The latter is explained by the fact that they will bequeath a smaller amount, hence have less interest in it being protected.
3 Homogeneous households

Now consider the special case where all households are identical, \( x_i = x \). At each time period \( t \), given their wealth \( b_{it} = b_t \) (the same for all households) they vote for the level of enforcement \( \lambda^*_t = \lambda^*(b_t) \). From equation (13)

\[
\lambda^*_t = \frac{\alpha \beta}{\gamma(1 + 2\alpha \beta)} \left[ 1 - \frac{x}{b_{it}} \right] \equiv \omega \left[ 1 - \frac{x}{b_t} \right] \equiv \lambda^*(b_t) \tag{14}
\]

where we assume \( \omega \leq 1 \) to ensure that \( \lambda^*_t < 1 \). As \( b_t \to \infty \), \( \lambda^*_t \to \omega \). (This means \( \gamma \geq \frac{1}{2+1/\alpha \beta} \), so \( \gamma \) cannot be too small.)

The dynamic evolution of the system is described by the following difference equation which is obtained from (10) and (14)

\[
b_{t+1} = Q b_t^{-\alpha} (b_t - x)^{2\alpha} \equiv \psi(b_t) \tag{15}
\]

where

\[
Q \equiv A \left[ \frac{\alpha \beta}{1 + \alpha \beta} \right]^\alpha \omega^\alpha [1 - \gamma \omega]^{-\alpha} = A \gamma^{-\alpha} \left( \frac{\alpha \beta}{1 + 2\alpha \beta} \right)^{2\alpha} > 0
\]

Notice that \( \psi(b) \) is an increasing function for \( b > x \), and

\[
\frac{\psi(b)}{b} \to 0 \text{ as } b \to \infty
\]

indicating that, as \( b \to \infty \), the curve \( \psi(b) \) will lie below the 45 degree line.

A steady-state is a fixed point \( b^* \) such that

\[
b^* = \psi(b^*)
\]

Then \( b^* \) is a solution of

\[
b^* = Q(b^*)^{-\alpha} (b^* - x)^{2\alpha} \tag{16}
\]

or

\[
b^* = Q^{1/(1+\alpha)} [b^* - x]^{2\alpha/(1+\alpha)} \tag{17}
\]
Now $\alpha < 1$ implies $2\alpha/(1 + \alpha) < 1$. Hence the left hand side of (17) is a linear function of $b^*$ and the right-hand side is a concave curve for $b^* > x$. So there are two intersections, $b^*_L$ and $b^*_H$ where $x < b^*_L < b^*_H$. It easy to see that $b^*_H$ is a stable equilibrium, and $b^*_L$ is unstable.

The evolution of the property rights regime is as follows. If $b_0 \in (b^*_L, b^*_H)$, both $b_t$ and $\lambda_t$ will be increasing with time, converging to $b^*_H$ and $\lambda^*_H$ where

$$\lambda^*_H \equiv \omega \left(1 - \frac{x}{b^*_H}\right) < 1$$

If $b_0 \in (x, b^*_L)$, then $b_t$ falls steadily toward $x$ and $\lambda_t$ falls steadily to zero.

4 **Heterogeneous Households**

In the preceding section, we assumed that individuals were identical, therefore the legal system (i.e., the property rights and their enforcement) was optimally chosen by the representative individual. We now turn our attention to an heterogeneous society. The key state variable in our model is the wealth bequeathed to individuals in the next generation. One of the main determinants of this process is the parameter $x_i$ which is usually considered as the minimum level of consumption. However what a household deems a bare minimum depends on their tastes. It is not a biological necessity in modern society. (Plasma TV’s, mobile phones and brand-name jeans are examples of what some households consider essential.) The larger $x_i$, the smaller will $b_{it+1}$ be, and we have interpreted $x_i$ as selfishness or greed parameter. Therefore the intergenerational transmission of wealth depends on $x_i$. We now assume that there are two types of households with $x_1 > x_2$. As a working hypothesis, we take it that children inherit the characteristics of their parents. With two distinct types of agents, the population share of each type may be changing over time.

It is an empirically accepted fact that fertility decreases with income. (See for instance Schultz, 2005.) For simplicity we do not model this explicitly
here but instead assume that the number of children a household has is a function of its initial wealth. Specifically it takes the form

\[ n_{it} = M b_{it}^{-\sigma}, \sigma > 0, M > 0 \]  

(18)

We now assume that the political system is such that the group that decides on the legal system for the next generation is the group with the largest total wealth, \( N_{i,t} b_{it} \), where

\[ N_{it} = N_{i,t-1} n_{i,t-1} \]  

(19)

and \( N_{i,0} \) is the number of households of type \( i \) at the beginning of period 1. The fact that the number of people has an influence represents some ideal of democracy. The inclusion of individuals’ wealth reflects a fact of life in all democracies.

Therefore the evolution of bequests can be determined by two distinct mechanisms. One mechanism uses \( \lambda_{1,t}^* \) which is preferred by type 1 and the other \( \lambda_{2,t}^* \) which is the choice of type 2. At each generation, we calculate \( N_{i,t} b_{i,t} \) (\( i = 1, 2 \)) and whichever one is greater will determine the property rights system for the next generation. Given \( N_{1,1}, N_{2,1}, b_{1,1} \) and \( b_{2,1} \), we can compute \( N_{1,1} b_{1,1} \) and \( N_{2,1} b_{2,1} \) and know which group is in power in period 1. At each generation we recalculate total wealth for each group and let the political process choose the next legal system. If there is a switch we say that a “revolution” has taken place.

Suppose \( N_{1,0} = N_{2,0} = 1 \) and \( n_{1,0} = n_{2,0} = n \) where \( n \) is an arbitrary constant. Then \( N_{1,1} = N_{1,2} = n \). The initial numbers of children having a strong influence on the early evolution of the system, we do not want to influence things either way.

Given the values of \( N_{i,t-1}, n_{i,t-1} \) and \( b_{i,t} \) we can calculate \( N_{i,t} b_{i,t} \) and hence we know which tax regime will prevail in period \( t \). From this, we can calculate \( b_{i,t+1} \). Suppose the chosen value for generation \( t \) is \( \lambda_{i,t}^* \), then

\[
 b_{1,t+1} = A \left[ \lambda_{1,t}^* \frac{\alpha \beta}{1 + \alpha \beta} \right]^\alpha \left[ (1 - \tau_t) b_{1,t} - x_1 \right]^\alpha (b_{1,t})^{(\alpha - 1)\sigma}
\]
and
\begin{equation}
b_{2,t+1} = A \left[ \lambda_i \frac{\alpha \beta}{1 + \alpha \beta} \right]^\alpha \left[ (1 - \tau_t) b_{2,t} - x_2 \right]^{(\alpha - 1) \sigma} \tag{20}
\end{equation}

or, using (14),
\begin{equation}
b_{1,t+1} = A \left[ \left( \frac{\alpha \beta}{1 + \alpha \beta} \right)^{\frac{1}{\gamma}} \left( \frac{\alpha \beta}{1 + 2 \alpha \beta} \right) \left( 1 - \frac{x_i}{b_{i,t}} \right) \right]^\alpha \left[ (1 - \frac{\alpha \beta}{1 + 2 \alpha \beta} \left( 1 - \frac{x_i}{b_{i,t}} \right) ) b_{1,t} - x_1 \right]^{(b_{1,t})(\alpha - 1) \sigma} \tag{21}
\end{equation}

The political power in period \( t+1 \) is determined by the higher of the two \( P_{j,t}, j = 1, 2 \), where (when the ruling tax regime was \( i \) in period \( t \))

\begin{align*}
P_{j,t} & \equiv N_{j,t} b_{j,t} = N_{j,t-1} n_{j,t-1} b_{j,t} \\
P_{j,t} & = N_{j,t-1} n_{j,t-1} A \left[ \left( \frac{\alpha \beta}{1 + \alpha \beta} \right)^{\frac{1}{\gamma}} \left( \frac{\alpha \beta}{1 + 2 \alpha \beta} \right) \left( 1 - \frac{x_i}{b_{i,t-1}} \right) \right]^\alpha \times \\
& \left[ (1 - \frac{\alpha \beta}{1 + 2 \alpha \beta} \left( 1 - \frac{x_i}{b_{i,t-1}} \right) ) b_{j,t-1} - x_j \right]^{(b_{j,t-1})(\alpha - 1) \sigma}
\end{align*}

where \( i \) may or may not differ from \( j \).

The following examples illustrate such switches.

We have chosen the following parameter values: \( A = 12\sqrt{30}, \alpha = 0.5, \beta = 0.5, \gamma = 0.3, \sigma = 0.1, x_1 = 2, x_2 = 1.1, M = 1.55 \), and also \( N_{1,0} = N_{2,0} = 1, n_{1,0} = n_{2,0} = 0.645 \).

Therefore households of type 1 are 'greedier' than those of type 2 and will pass on less capital, everything else equal, but consequently will also have more children in future. This will turn out to be the decisive factor in gaining the upper hand in the long run.

The values of \( A, M, N_{i,0} \) and \( n_{i,0} \) are irrelevant scaling factors. We chose \( M \) and \( n_{i,0} \) so that the number of children per household would be approximately 1 at the steady states.
The value of $\sigma = 0.1$ yields a relationship between $n$ and $b$ which, once scaled, is almost undistinguishable from the one plotted by Schultz (2005, Fig. 1).

The first example, exhibiting one revolution (property rights regime change) has initial conditions: $b_{1,0} = 4, b_{2,0} = 7$.

The second example, exhibiting two revolutions has initial conditions: $b_{1,0} = 8, b_{2,0} = 5$.

We show the evolution of wealth for the two types, $b_{1,t}$ and $b_{2,t}$ on one graph and the evolution of political power $R_t \equiv P_{1,t} / P_{2,t}$ on another. The position of $R_t$ relative to 1 determines which tax regime is used for the next generation.

There are common features to these two (and all) examples: Type II households have a higher wealth in the long run - the steady-state values of $b_{1,t}$ and $b_{2,t}$ are reached relatively soon. Type I households have more children and consequently dominate the political process in the long run - this can take quite a long time as example two makes clear. The ratios $P_{1,t} / P_{2,t}$ and $N_{1,t} / N_{2,t}$ have very similar values after a while. (This is due to the similar steady state values for wealth in these examples.). The above ratios never converge however.

We observe one ‘revolution’ in the first example. This is because the ‘greedy’ type (type I) begins with less wealth, hence the political power rests with type II at first. Then the demographics overwhelm them and this never changes. In example 2 group I starts much wealthier, consequently they have less children; this allows type II to overtake them politically (the first revolution), and maintain their political superiority for over 80 generations. In generation 88 type I finally wins ascendency (the second revolution) and never relinquish it.

Therefore we have demonstrated that political change, defined as a change of property rights regime can be generated from within our model. We
nonetheless refrain from claiming this as a vindication of Marxist thought. The steady state values are independent of the initial conditions only because the type I regime always prevails in the end. However the nature of the property rights regime can be affected for a long time by initial conditions.

We now briefly examine the properties of the steady states. Because we know regime I will prevail, we can define the steady state for \( b^*_i \) using \( i = 1 \). We obtain

\[
 b^*_1 = T_1 [b^*_1 - x_1]^{2/\alpha/(1+\alpha+\sigma(1-\alpha))}
\]

where \( T_1 \) is a positive constant and the exponent is less than 1.

Once \( b^*_1 \) is defined we can use \( \text{(22)} \) to define \( b^*_2 \). We obtain

\[
 b^*_2 = T_2 G_1 [G_1 b^*_2 - x_2]^{2/\alpha/(1+\alpha+\sigma(1-\alpha))}
\]

where \( T_2 \) is a positive constant and \( 0 < G_1 = 1 - x_1/b^*_1 < 1 \). Note that \( G_1 \) decreases as \( x_1 \) increases and increases with \( b^*_1 \).

There are two steady states but only the larger one is stable. The curve begins at \( x_1 \) on the horizontal axis, therefore an increase in \( x_1 \) will decrease \( b^*_1 \). As expected, the ‘greedier’ are households, the less wealthy they will be in equilibrium. The same pattern is valid between \( b^*_2 \) and \( x_2 \). However, \( b^*_2 \) is also dependent on \( b^*_1 \) and \( x_1 \). To see this it is convenient to rewrite\( \text{(23)} \) as

\[
 T_3 [b^*_2]^{(1+\alpha+\sigma(1-\alpha))/2\alpha} = [G_1]^{(1+\alpha+\sigma(1-\alpha))/2\alpha} [G_1 b^*_2 - x_2], \text{where } G_1 = 1 - x_1/b^*_1
\]

The left-hand side of \( \text{(24)} \) is convex while the right-hand side is linear in \( b^*_2 \).

Recall that \( G_1 \) decreases as \( x_1 \) increases and increases with \( b^*_1 \).

Consider the effect of a larger \( x_1 \) on \( b^*_2 \). The left-hand side is unaffected. An increase in \( x_1 \) will decrease \( G_1 \) as well as decrease \( b^*_1 \), so the total effect on \( G_1 \) is unambiguous. The slope of the line on the right-hand side will also
decrease. The result of an increase in $x_1$ is to decrease $b_2$. The increased greed of type I will make type II less wealthy in the long run.

5 Concluding Remarks

We have shown how property rights regime may evolves along paths of capital accumulation. There are various extensions that may be worthwhile. First, technological shocks can result in a change in property rights regime. Second, the opening of an economy to international trade and investment can make households re-evaluate their opportunities and this may result in their collective decisions about property rights enforcement. Third, the propagation of ideas across national borders can be an important factor.

References


