When Expansionary Fiscal Policy is Contractionary: A Neoklassikal Scenario

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ABSTRACT

The paper presents a simple theoretical account of how an increase in government purchases may reduce total employment. It is shown that in a 'neoklassikal' model - in which utility maximisation saving choices are combined with a fixed-coefficient technology – a permanent increase in government purchases will reduce the demand for labour at the given wage rate. It is argued further that the endogenisation of the wage rate by way of wage bill maximisation will only exacerbate the reduction in employment.

1 Titles are not subject to copyright, which is all the more reason to acknowledge my evident debt to Makin (1998).
Introduction

The Keynesian doctrine of a government spending multiplier has been subject to an manifold critique, both empirically and theoretically. While much of this criticism is underpinned by a confidence in the ‘self-correcting’ properties of the economy, well known criticisms of the effectiveness of government spending in stimulating the economy have long been pursued within a framework of involuntary unemployment. Some of these critiques invoke the implications of monetary equilibrium; (interest rate crowding out; or, in the open economy, exchange rate crowding); others invoke capital market equilibrium (‘Treasury View’ of the 1930s); while another influential path of criticism has invoked the ‘rational consumer’ (the permanent income hypothesis and Ricardian equivalence).²

This paper articulates another criticism Keynesian doctrine of fiscal policy. The criticism goes further than that reached by allowance for monetary equilibrium or the rational consumer. For the present paper’s critique contends that government spending will be actually counter productive; it will reduce employment.

The paper’s critique of the government spending multiplier is fashioned in large part out of standard, neoclassical materials: perfect foresight, utility maximisation, perfect competition amongst firms, the absence of any money illusion or rigidities. But it is all done in the context of unemployment, and will allow, as we shall see, for employment to respond positively in some measure to ‘demand’.

To the extent that the paper has novel ingredient that accounts for its results it is a refusal of a standard neoclassical assumption. For it will be supposed that the demand for labour is not a manifestation of the diminishing marginal productivity of an aggregate production function. Instead of the ‘neoclassical’ twice differentiable aggregate production function, the model assumes a ‘klassikal’ fixed coefficient

² The critique by way of the rational consumer is not restricted to the efficacy of tax cuts. The permanent income hypothesis suggests a high marginal propensity to save, and so low multiplier. Ricardian Equivalence treats every increase in G as if accompanied by an equal increase in T, leaving the impact of G only a matter of the balanced budget multiplier.
technology operating in two sectors: consumption goods and capital goods. In consequence of this technology the wage is driven by the valuation on future consumption, measured in terms of current consumption. It is demonstrated that government expenditure reduces that valuation, so that the demand for labour falls.

The last section of the paper reinforces the conclusions of the earlier section by endogenising the wage rate. That section assumes that the wage rate is set so as to maximise the wage bill. As the wage bill can be interpreted as the expected income of a worker when all share the chance of unemployment equally, the wage rate is being set in some measure rationally (although not socially efficiently). This endogenisation of the wage rate implies that a permanent fiscal expansion will raise the wage rate, and thereby exaggerate the negative impact of fiscal expansion. Thus allowing for the wage setting process to have some ‘intelligence’ about it only increases the severity of the model’s critique of the government spending multiplier.

The elements of the model

The economy has two outputs; consumption and capital. There are two inputs, labour and capital.

The production of $\alpha$ units of consumption requires 1 unit of capital and $\beta$ units of labour. There is no possibility of substituting labour for capital in the production of consumption: the isoquants are L shaped, and there is no ‘marginal product of labour’. Thus as long as capital is not a free good,

$$C_t = \alpha K_t$$

(1)

The production of one unit of capital requires one unit of labour, and no other input (so we could imagine a labourer fashioning with their hands ‘a machine’). A unit of
capital ‘evaporates’ at rate $d$. Thus the quantity of capital in the next period equals $1-d$ of capital in the current period plus the production of capital this period. Since the production of capital equals total employment minus employment in the consumption sector we have,

$$K_{t+1} = K_t [1 - d] + L_t - \beta K_t$$

(2)

The growth of consumption is determined by the maximisation of identical homothetic utility function

$$U = C_t^\gamma + \frac{C_{t+1}^\gamma}{1 + \delta} + \frac{C_{t+2}^\gamma}{[1 + \delta]^2} + ... \quad \gamma < 1$$

(3)

by a given cohort of infinitely lived persons, yielding.$^4$

$$\frac{C_{t+1}^1}{C_t^1} = \left[\frac{1 + \rho}{1 + \delta}\right]^{\sigma} \quad \sigma = \frac{1}{1 - \gamma}$$

(4)

This equality expresses the paradigmatic neoclassical explanation of the valuation of current consumption in terms of future consumption –i.e. the rate of profit, $1+\rho$. This valuation is explained in terms of the relative abundance of current and future consumption. The value of current consumption in terms of future consumption,

$^3$ We assume $\beta + d < 1$. Technologically, there is no requirement that $\beta + d < 1$. But $\beta + d > 1$ would imply that investment is irrational. It means that machines are being built that could not be operated; or only operated by abandoning the replacement of older machines. Suppose that 1000 of labour is available to build 1000 machines, after existing machinery has absorbed what labour it requires, and another portion of labour is allocated to construct replacement machines for the existing capital stock. If $\beta + d > 1$ then more than 1000 of labour would be required to operate (and replace) the constructed machines in the next period. But more than 1000 is not available.

$^4$ ‘Perfect capital markets’ are assumed to exist, by which we mean that capital can be sold short without restriction. Equivalently, we can suppose that there exist credit markets, in which borrowing and lending occurs at the same rate and without restriction.
$1 + \rho$, is increased the more plentiful future consumption, and reduced the more plentiful current consumption. The same result can be expressed in terms of the valuation of future consumption in terms of current consumption, $1/(1 + \rho)$. The value of future consumption in terms of current consumption is reduced the more plentiful future consumption, and increased the more plentiful current consumption. These propositions are the critical to the conclusions of the paper.

But what is the rate of profit? The rate of profit is derived from the relativities of costs and payoff from producing a machine,

$$1 + \rho_t = \frac{\alpha - \beta w_{t+1} + [1 - d]P_{k,t+1}}{w_t}$$

(5)

$P_{k,t} \equiv$ price of unit of capital in terms of consumption in $t$

But as long as some capital is produced in $t+1$ then,

$$P_{k,t+1} = w_{t+1}$$

thus

$$1 + \rho_t = \frac{\alpha + [1 - \beta - d]w_{t+1}}{w_t}$$

The model is completed by invoking a wage setting process.

Competitive Wage setting: a Full-employment Model

Suppose the wage adjusts to secure full-employment. Then we may write,

$$L_t = \Sigma$$
where \( \Sigma \) is the supply of labour, and so

\[
K_{t+1} = \Sigma + (1 - \beta - d)K_t
\]

This is the ‘equation of motion’ in the capital stock. It is a first order difference equation with the solution,

\[
K_t = [K(0) - \frac{\Sigma}{1 - \beta - d}] + \frac{\Sigma}{\beta + d}
\]

Evidently, the profile of capital over time is entirely determined by \( K(0), \Sigma, \beta \) and \( d \), and is completely independent of preference parameters, \( \delta \) and \( \sigma \).\(^5\) The equation of motion yields steady state quantity of capital.

\[
K_{ss} = \frac{\Sigma}{\beta + d}
\]

Unlike the neoclassical Ramsey-Solow model the steady state is not a matter of an equality between the rate of time preference and the marginal product of capital; there is no marginal product of capital. Unlike the classical Ricardian model the steady state is not a result of an equality between the marginal product of labour and the wage at which the net reproduction of the labour force is zero: there is no marginal product of labour. Rather, the steady state occurs because so much capital has been produced that the entire workforce is absorbed in either operating the existing capital, or in building machines to replace those that wear out. So there is no labour left over to add to the capital stock.

As \( C_t = \alpha K_t \), the path of consumption exactly tracks the quantity of capital\(^6\)

\(^5\) Socially speaking, there is no ‘choice’ with respect to investment at all, at least given our assumption that all capital is utilised.

\(^6\) \[
\alpha K_{t+1} = \alpha \Sigma + (1 - \beta - d)\alpha K_t
\]

\[
C_{t+1} = \alpha \Sigma + (1 - \beta - d)C_t
\]
Using the equation of motion for capital we may infer,

\[
\frac{C_{t+1}}{C_t} = \frac{K_{t+1}}{K_t}
\]

This equality evidently determines the rate of profit in any period, by reference to the supply of labour per unit of capital in \( t \), \( \Sigma/K_t \), and parameters. As the \( \Sigma/K \) falls over time the profit rate falls over time until the steady state quantity of capital is reached yielding a steady rate of profit,

\[ \rho_{ss} = \delta \]

The expression for the rate of profit rate can be rewritten to determine the wage rate.

\[
w_t = \frac{\alpha}{1 + \rho_t} + \frac{1}{1 + \rho_t} \left[ 1 - \beta - d \right] w_{t+1}
\]

Repeated leading and substitution yields,

\[
C_t = \alpha \left[ K(0) - \frac{\Sigma}{\beta + d} \right] [1 - \beta - d]' + \frac{\alpha \Sigma}{\beta + d}
\]

7 The declining profit rate of implies a declining rate of consumption growth. That implies a declining rate of capital growth. The growth rate of capital (evaluated at constant prices) declines as the steady state is approached.

\[
\frac{K_{t+1} - K_t}{K_t} = \frac{\beta \left[ \frac{1}{\beta} - K(0) \right] [1 - \beta]'}{[K(0) - \frac{1}{\beta}][1 - \beta]'} + \frac{1}{\beta}
\]
The wage rate equals sort of ‘discounted’ product of capital…Given the profit rates

\[ 1 + \rho_{t+k} = \left[ \sum_{K_{t+k}} + 1 - \beta - d \right]^{1/\sigma} [1 + \delta], \]

the wage in \( t \) can be written as purely a function of parameters and the quantity of capital in \( t \). And given the monotonic fall in the rate of profit over time as capital rises over time to its steady state, the wage raises over time. It ultimately assumes its own steady state value,

\[ w_{SS} = \frac{\alpha}{\delta + \beta + d} \]

In summary terms, the competitive equilibrium of this neoklassikal model mimics the standard neoclassical model: a falling profile of profit, a rising profile of the wage, a rising capital per head; all flattening out in steady state values.

UnCompetitive Wage setting: an Unemployment Model

Suppose now that the wage is not set competitively. Employment is now an endogenous variable dependent on the wage rate. What is the character of that dependency?

The demand labour is the sum of ‘consumption sector labour’,

\[ L_{c,j} = \beta K_j \]

and ‘investment sector labour’

\[ L_{i,j} = K_{t+1} - [1 - d] K_t \]
Consumption sector labour is completely inelastic to the wage rate, at least until the wage is so high that profit on producing consumer goods has shrunk to zero.

But investment sector labour is negatively related to the wage rate, as a higher wage means a lower profit rate, and so a lower growth rate in consumption, and so a lower growth rate in capital. And so less investment... and so less investment labour. Algebraically,

$$1 + \rho_t = \frac{\alpha + [1 - \beta - d]w_{t+1}}{1 + \delta}$$

but

$$\left[\frac{1 + \rho_t}{1 + \delta}\right]^\sigma = \frac{C_{t+1}}{C_t}$$

and

$$\frac{C_{t+1}}{C_t} = \frac{K_{t+1}}{K_t}$$

and

$$\frac{K_{t+1}}{K_t} = \frac{L_t}{K_t} + 1 - \beta - d$$

yielding

$$L_t = \left[\frac{\alpha + [1 - \beta - d]w_{t+1}}{w_t[1 + \delta]}\right]^\sigma - 1 + \beta + d]K_t$$

where the negative impact of w reflects the reduction of investment labour as w rises.
The Demand for Labour in $t$ is a Negative Function of the Wage Rate in $t$.

Before we move to analysing employment of government spending it is worth noting that this employment equation has a flavour of Keynesian models, in that $L$ will increase in response to a rise in the ‘marginal efficiency of investment’. Let’s suppose there is an increase in the profitability of machinery because future capital will be more automated, and so $\beta_{t+1}$ falls below $\beta_t$.

$$L_t = \left[\frac{\alpha + [1 - \beta_{t+1} - d]w_{t+1}}{w_t[1 + \delta]}\right]^\sigma - 1 + \beta_t + d]K_t$$

If $\beta_{t+1}$ falls, or is expected to fall, the *ex ante* profit rate rises and employment rises, for a given wage.

The Impact of Government Spending on Unemployment.

We are now prepared to approach the analysis of the magnitude of the government spending multiplier.
A fiscal framework

A sound analysis of the government spending multiplier calls for some of characterisation of the spending. The simplest way to think about G is to suppose that the government purchases G of consumption output, and throws it away.

To conceive of G as the government purchasing of consumer goods, and throwing them away, may seem an assumption that is ‘biased’ against the Keynesian doctrine. But the Keynesian doctrine of the government spending multiplier should not be identified with the mere contention that more government spending might be a good thing. Everyone, presumably, will allow that, in certain circumstances, it might be a good thing for the government to build (say) more bridges. The Keynesian doctrine is much more surprising: it contends that in times of unemployment it will be good thing to spend money ‘uselessly’; to pay people dig holes, and fill them up again.\(^8\) To pay someone to do that is no different from paying someone to make a widget, and then smash it. So we will proceed in this section on the assumption that this is what government ‘demand’ amounts to, and later analyse the multiplier when G consisting the hiring, and payment, of persons onto the government payroll.

A sound analysis of the government spending multiplier also requires a characterisation of the financing of G. The simplest way of thinking about how that is to suppose a flat tax. But the analysis would be unaltered if we supposed G was financed by a consumption tax at such a rate as to yield revenues of G. And, if householders have foresight of their future tax liabilities, then the analysis would be also unaltered if we supposed G was financed by government borrowing, as long as the future tax burden is financed by a consumption tax.

\(^8\) Or, in Keynes’ comparison, to pay them to fill holes and dig them up again. ‘If the Treasury were to fill old bottles with bank-notes, bury them ant suitable depths in disused cool mines, which are then filled to the surface with town rubbish, and leave it private enterprise on well-tried principles of \textit{laissez faire} to dig the notes up again, there need be no more unemployment and. With the help of repercussions, the real income of the community …would probably become a good deal larger than it actually is. It would , indeed, be more sensible to build houses and the like; but if there political and practical difficulties in the way of this, the above would be better than nothing. (Keynes 1936, p129).
A temporary shock to $G$

Suppose now the government decides to spend $G$, in real terms, in the current period, but not other period. Under the fiscal framework assumed in the preceding subsection, the consumption growth rate equation is now altered to,

$$\frac{C_{t+1}}{C_t} = \frac{\alpha K_{t+1}}{\alpha K_t - G}$$

Thus the employment equation becomes,

$$L_t = \left[ K_t - G / \alpha \right] \left[ \frac{\alpha + [1 - \beta - d] w_{t+1}}{w_t [1 + \delta]} \right]^{\beta} - [1 - \beta - d] K_t$$

Evidently, employment for a given wage is reduced by $G$. Equivalently, the wage rate for a given level of employment is reduced by $G$. Why? Anyone who pays labour to build a machine is effectively sacrificing current consumption to future consumption. How much you are willing to pay that person is dictated by how much value future consumption, in terms of current consumption. The value of future consumption, in terms of current consumption, has fallen on account of the reduction in current consumption, or the increased ‘scarcity’ of current consumption.

A future shock to $G$

Suppose now the government decides to spend $G$, in real terms, in the future period, but not other period. Under the fiscal framework assumed in the preceding subsection, the consumption growth rate equation is now altered to,

$$\frac{C_{t+1}}{C_t} = \frac{K_{t+1} - G / \alpha}{K_t}$$

Thus the employment equation becomes,
Evidently, employment for a given wage is increased by \( G \). Equivalently, the wage rate for a given level of employment is increased by \( G \). Why? Anyone who pays labour to build a machine is effectively sacrificing current consumption to future consumption. How much you are willing to pay that person is dictated by how much value future consumption, in terms of current consumption. The value of future consumption, in terms of current consumption, has been increased on account of the reduction in future consumption. Thus the amount you a willing to pay someone to build a machine has increased.

A permanent shock to \( G \)

Suppose now the government decides to spend \( G \), in real terms, in all periods. The consumption growth rate equation is now altered to,

\[
\frac{C_{t+1}}{C_t} = \frac{K_{t+1} - G / \alpha}{K_t - G / \alpha}
\]

and so

\[
L_t = [K_t - G / \alpha][\frac{\alpha + [1 - \beta - d]w_{t+1}}{w_t[1 + \delta]}]^{\sigma} - [1 - \beta - d]K_t + G / \alpha
\]

Evidently, the net effect of current and future \( G \) on labour demand is negative. Employment for a given wage is reduced by \( G \). This is because a given absolute reduction in \( C \) in both the present and in the future, increases the ratio of \( C_1 \) to \( C \), and so reduces the valuation of future consumption.

A shock to \( G \) in the form of increased government employment
We now turn to analyse the case where tax revenues are used by the government to hire people. So we move to the situation where the G consists of ‘public works’ and ‘job creation schemes’.

We could suppose that these employees are paid to construct machines. The income stream of these machines, we might suppose, is used to reduce tax liabilities. We can model this as the government giving away the machines.

What will be the impact on total employment of this sort of ‘nation building’ initiative? To the extent that this type of G shock increases total employment in t,

\[
\frac{K_{t+1}}{K_t}
\]

must rise. But, critically, the implication of this new characterisation of G has restored the equality of the growth in consumption to the growth in the capital stock, as G does not waste capital’s output of consumer goods. So it is once more true to say,

\[
\frac{C_{t+1}}{C_t} = \frac{K_{t+1}}{K_t}
\]

Thus to the extent that this type of G shock increases total employment in t it increases \(\frac{C_{t+1}}{C_t}\), and that will reduce the valuation placed on future consumption. And that will reduce the wage.

But the wage cannot be reduced, by assumption.

The conclusion is… this type of G shock does not increase total employment at all. Government investment employment exactly displaces an equal amount of private ‘investment employment’. In effect, households - seeing the government tax them (say) $1m to build machines that they will give back to them - just let the government do their investment for them to the extent of $1m, and scale back their own investment by $1m.
But what if government hires people not to add to the capital stock? Perhaps it employs people to do something useful (even if non-capital in nature), such as picking up litter. Or perhaps it employs people to do something useless; counting the stars, or digging holes and filling them up again, or building machines and throwing them into the sea. Whatever the case, this form of employment does not reduce the magnitude of consumption in any period, and so it will not affect the valuation of future consumption, and so - by previously rehearsed argument - will not effect investment employment.\(^9\) And so will not effect private sector employment.

Yet there will however be an increase in government employment; that will simply equal \(G/w\). And since private employment has not fallen, total employment will rise.

And there will be an increase in \(Y\), simply equal to \(G\). There is a multiplier it seems, of 1.

To what extent does this result vindicate Keynesian doctrine? Not very much. For we are not seeking the Keynesian multiplier at work here, for there may have been zero social benefit consequent upon \(G\). There will be no social benefit at all if persons are employed simply to count the stars etc. In that case there has been, in this model, no true increase in social income. In that case the increase in \(G\) is purely notional, a mismeasurement, in truth; for that government spending is no more than transfer. The measured increase in \(Y\) is the same spurious increase in \(Y\) as if all age pensioners were tomorrow declared government employees. And of course, as we have stressed, the Keynesian contention is that there will have been a social benefit even from hiring people even to watch the grass grow …

\(^9\) We are assuming here that if government hires people to do something useful, then its contribution to utility is separable from the contribution of consumption. 

\[ U = C_t^r + \frac{C_{t+1}^r}{1+\delta} + \frac{C_{t+2}^r}{[1+\delta]^2} + \ldots + \nu(G) \]. In other words, the marginal rate of substitution between future and current ‘private’ consumption is not affected. We are assuming, in other words, that picking up ice cream wrappers today does not make us more (or less) impatient for the consumption of ice cream tomorrow.
The Impact of Government Spending on Unemployment when the Wage Bill is Maximised.

There is another query about all the earlier sections’ analysis of shocks to G. One way or another it would seem that the wage rate, after taxes, has been reduced by G. But the model is built on wage rigidity. Does it make sense to assume a rigid wage, and then give the government free movement to reduce the effective wage? And if Government has that power, why does it not just reduce wages as a policy measure?

We need to rationalise the wage rate setting process, and then explore the multiplier within that rationalisation.

We will rationalise the wage rate as that which maximises the wage bill.

We first consider wage bill maximisation on the absence of government spending.

*With G = 0*

A wage bill maximiser selects the wage rate so that the elasticity with respect to the wage rate is 1. Given,

\[ L_t = \left[ \frac{\alpha + [1 - \beta - d]w_{t+1}}{w_t[1 + \delta]} \right]^{\sigma} - 1 + \beta + d \]

we may infer,

\[ e_t = -\frac{w_t L_t}{L_t \frac{\partial L_t}{\partial w_t}} = \frac{\sigma}{1 - [1 - \beta - d][\frac{1 + \delta}{1 + \rho_t}]^\sigma} \]

If there was full employment we can further infer,
Thus under full-employment the elasticity of the demand for labour is a positive function of the quantity of capital in the economy, relative to the labour force.

The Elasticity of Labour Demand is a Positive Function of Capital per Unit of Labour Supply

\[ e_i = \frac{\sigma}{1-[1-\beta-d]\left[\frac{1}{\Sigma K_i}+1-\beta-d\right]} \]

Evidently there is some magnitude of the capital stock, relative to the labour supply, such that the elasticity of demand is equal to 1 at full-employment. This ‘critical’ magnitude of capital can from the equality above be solved out as,

\[ K_{CRITICAL} = \frac{\Sigma}{1-\beta-d} \frac{1-\sigma}{\sigma} \]
For any quantity of capital greater than $K_{\text{CRITICAL}}$, then, the elasticity of demand for labour exceeds 1 under full-employment. For any quantity of capital greater than $K_{\text{CRITICAL}}$, then, the wage bill maximising wage rate is simply that which secures full-employment; for any higher wage would only reduce the wage bill. But any quantity of capital less than $K_{\text{CRITICAL}}$, a wage bill maximiser would raise the wage in the current period above its full employment rate. It would set the wage rate to satisfy,

$$\frac{\sigma}{1 - [1 - \beta - d][\frac{w_t[1 + \delta]}{\alpha + [1 - \beta - d]w_{t+1}}]^\sigma} = 1$$

But given

$$\left[\frac{1 + \rho_{\text{CRITICAL}}}{1 + \delta}\right]^\sigma = \frac{1 - \beta - d}{1 - \sigma}$$

this ‘elasticity of labour demand equals one’ expression can be written as

$$w_t = \frac{\alpha + [1 - \beta - d]w_{t+1}}{[1 + \rho_{\text{CRITICAL}}]}$$

Evidently ‘the’ wage bill maximising wage rate actually grows over time according to the expression. This profile of raising wage rates will be “anchored” by $w_T = w_{\text{CRITICAL}}$, where $T = \text{that period when sufficient capital has accumulated so that } K_T = K_{\text{CRITICAL}}$, and $w_{\text{CRITICAL}}$ is the full-employment wage when $K = K_{\text{CRITICAL}}$.\(^{10}\)

\(^{10}\) Capital does grow. It grows at the rate of consumption, and consumption growth is dictated by the critical rate of profit.
To find employment we substitute back in to

\[ L_t = \left[ (\alpha + [1 - \beta - d] w_{t+1})/w_t [1 + \delta] \right]^\sigma - 1 + \beta + d]K_t \]

yielding

\[ L_t = \frac{\sigma}{1 - \sigma} [1 - \beta - d]K_t \]

This is the expression for employment under wage bill maximisation, but in the absence of government spending.

We are now ready to examine the impact of government spending.

*A temporary shock to \( G \)*

Since

\[ L_t = [K_t - G/\alpha] \left( \frac{\alpha + [1 - \beta - d] w_{t+1}}{w_t [1 + \delta]} \right)^\sigma - [1 - \beta - d]K_t \]

the wage bill maximising wage rate is now,

\[ K_{t+1}/K_t = C_{t+1}/C_t = \frac{1 - \beta - d}{1 - \sigma} \]

\[ L_t = \left[ \frac{1 + \rho_{t+1}}{1 + \delta} \right] - 1 + \beta + d]K_t, \text{ but } \left[ \frac{1 + \rho_{\text{critical}}}{1 + \delta} \right] = \frac{1 - \beta - d}{1 - \sigma} \text{ from wage bill maximisation.} \]
\[ w_t = [K \frac{[1 - \beta - d]}{[K - \frac{G}{\alpha}][1 - \sigma]}]^{1/\sigma} \left( \frac{\alpha + [1 - \beta - d]w_{t+1}}{1 + \delta} \right) \]

We see the wage rate is depressed by government spending.\(^\text{12}\) In fact the new wage rate falls just enough so that employment is exactly unchanged.

\[ L_t = \frac{\sigma}{1 - \sigma}[1 - \beta - d]K_t \]

The previously contractionary effect of government spending is now arrested by a fall in the wage rate sufficient to preserve employment.

The preservation of employment implies that the economy’s capital stock in the next period is unchanged by the G shock. (And so we have the same wage rate in the next period as we would have had). And that implies the economy’s consumption in the next period is unchanged by the G shock. The impact of this temporary shock is temporarily reduce consumption, this period. That reduces the valuation of next period consumption, in terms of current consumption. And so the wage falls. And profits rises.

In summary, the G shock does not do anything but waste consumption, and impose the waste entirely upon wage earning section of the population.

*permanent G*

The wage bill is

\[ w_tL_t = w_t[K_t - G / \alpha][\frac{\alpha + [1 - \beta - d]w_{t+1}}{w_t[1 + \delta]}]^{\sigma} - w_t[1 - \beta - d]K_t + w_tG / \alpha \]

\[^{12}\frac{\Delta w}{\Delta G} = -\frac{\Delta G / C}{\sigma}\]
Thus

\[
\frac{\partial W_t}{\partial w_t} = w_t^{-\sigma} [1 - \sigma][K_t - G / \alpha][\frac{\alpha + [1 - \beta - d]w_t^{\prime-1}}{1 + \delta}] - [1 - \beta - d]K_t + G / \alpha
\]

To discover the impact of G on w, we evaluate \( \frac{\partial W_t}{\partial w_t} \) at the wage bill maximising rate in the absence of G;

\[
w_t = \left[\frac{1 - \beta - d}{1 - \sigma}\right]^{1/\sigma} \left[\frac{\alpha + [1 - \beta - d]w_t^{\prime-1}}{1 + \delta}\right]
\]

Substituting this back into \( \frac{\partial W_t}{\partial w_t} \) implies

\[
\frac{\partial W_t}{\partial w_t} = \frac{G}{\alpha} (\beta + d) > 0
\]

Thus it is not wage bill maximising to keep w at its former level when G rises permanently. It is wage bill maximising to increases the wage rate when G rises permanently. Thus in the context of wage bill maximisation the response of w to a G shock is to exacerbate the negative impact on employment of G.

Conclusions

The paper has advanced a model of the macroeconomy which is simple, has intelligible microfoundations, allows for unemployment, and predicts a positive response in Y to an increased profitability of investment; but in which a permanent increase in government purchases will reduce employment.
References
