Outsourcing with Heterogeneous Firms

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Very Preliminary, Comments Welcome

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Abstract

A dynamic model of outsourcing is introduced that incorporates heterogeneity among both upstream and downstream producers to explain the distribution of outsourcing firms observed in data. The environment is one of search frictions and incomplete contracts where final-good producers require a specialized input and, upon matching with a supplier, can only contract the quantity of input. The main predictions of this model are: 1) two integrated firms with the same productivity level might end up on different paths, so that one seeks outsourcing and the other stays integrated, 2) under certain conditions, high-productivity integrated firms have incentive to outsource, and 3) more productive producers have long lasting vertical relations with more productive suppliers, whereas lower productivity producers stay matched to lower productivity suppliers.

Keywords: Outsourcing, Productivity, Incomplete Contracts, Search.


1 Introduction

Overwhelming evidence from micro-level data shows that firms are vastly dispersed in their productivity and size, no matter how narrowly the industry is defined\(^1\). Outsourcing is one aspect of the firm behavior not immune to this critic: the sheer level of heterogeneity among

\(^1\)See Bartelsman & Dom (2000) for an extensive survey.
firms inevitably spills into their outsourcing patterns. In this paper, I setup a model that addresses this issue by incorporating heterogeneity among both specialized producers of a final good and middle suppliers of a required intermediate input. In particular, presence of heterogeneity has important implications on what kind of vertical relations can be formed and sustained in the long run.

My framework is a modification of the classic model introduced by Grossman & Helpman (2002). Specialized producers compete monopolistically, and each producer needs a specific intermediate input that can be obtained either by internalizing its production (vertical integration) or by forming a one-to-one relation with a specialized middle supplier (outsourcing). I assume that the production of final good uses two processes: (i) a core process, “final production” or assembly, that has to be internalized at all times or the producer ceases to exist, and (ii) an intermediate process, “middle production” or parts manufacturing, that can be either internalized or outsourced to a middle supplier\(^2\). Producers are differentiated in each process by making idiosyncratic draws of their labor productivities, independently of each other and independently for each process within firm. The total labor productivity of an integrated producer is then an aggregation of productivities for the two processes.

Vertical integration distracts focus and requires a specialized producer to use extra management resources to coordinate and plan the whole production process, hence, internalizing is costly. Specialized producers can alternatively order the required input from a middle supplier. There are two sources of transaction costs from outsourcing: search friction and uncertainty about the productivity of middle suppliers. Contractual incompleteness described by Grossman & Hart (1986) is also present, so that producers can only contract the quantity of input they need, but the quality of input cannot be verified by a third party. As a result, middle suppliers have incentive to offer low quality input, with zero value to production, and increase their profit margin. To avoid this situation, as in Grossman & Helpman (2002), both parties renegotiate and share the total revenue through a Nash bargaining.

Middle suppliers are also assumed, on average, more efficient in the production of intermediate input than producers, to reflect their higher degree of sophistication in their field. This assumption, however, is not the most crucial element in this model, and most results

\(^2\)Antras & Helpman (2004) use “manufacturing” and “headquarter services” to label upstream and downstream processes in an international setting.
do not depend merely on that. To specialized producers, outsourcing is beneficial because it boosts their productivity on final process by cutting managerial costs and focusing production. This boost is also used to symbolize the fact that outsourcing gives producers the chance to reinvestment in and improve their technology on the final process.

Presence of productivity heterogeneity has the important implication that not all the matches formed between producers and suppliers are desirable and lasting. Since future search is possible, middle suppliers are especially sensitive to the productivity of their matched producers. If the matched producer is too productive, compared to middle supplier, middle supplier has difficulty keeping up with the demanded quantity of input and staying profitable. Under such condition, middle supplier is better off by breaking the match and going on search for a producer with more "desirable" productivity level. Conversely, if the matched producer is too unproductive, compared to middle supplier, middle supplier can do better by, again, breaking the match and continuing search. However, middle suppliers get less selective as their productivities get higher, mainly because the density of high productivity producers diminishes.

The model suggests that specialized producers also select their "desirable" middle suppliers. In particular, high-productivity integrated producers outsource if there is a good chance of finding a high-productivity middle suppliers. The incentive comes from the fact that high-productivity integrated producers are very efficient in the final process as a whole and get the largest boost in productivity by outsourcing. However, if finding a high-productivity middle supplier is improbable, these producers integrate to avoid search risks.

The model also raises the possibility of two integrated producers with identical productivities to behave differently, so that one of them integrates and the other one goes on search. The key to this difference in behavior is the fact that these two producers have the same total productivity prior to outsourcing, but the one with higher productivity in its final process benefits greatly from outsourcing and goes on search. This behavior is, nevertheless, sensitive to the probability of finding a high-productivity supplier. If finding a high-productivity supplier is somewhat improbable, then no high-productivity integrated producer will attempt to outsource.

Most of the implications in this paper are in line with the existing empirical evidence.
For one thing, the empirical evidence on the role of firm heterogeneity on outsourcing behavior is still developing. Heterogeneity in outsourcing behaviors, the diversity of jobs being outsourced, and the intensity by which jobs get outsourced are demonstrated by Abraham & Taylor (1996) and Benson & Ieronimo (1996) using data on manufacturing in the US and Australia, respectively. In a more detailed study, Bakhtiari (2009) uses firm-level data on Australian manufacturing and points out that outsourcing firms, despite being mostly low productivity, are significantly dispersed in their productivity. Especially, there are many high-productivity firms who outsource. Productivity improvement turns out to be the main objective for outsourcing among Australian firms, a feeling shared among all firms of low to high productivity. As this paper also concludes, data shows that two firms with the same level of productivity can have different attitudes towards outsourcing, with one of them staying integrated and the other one outsourcing. My model also overcomes some other shortcomings of Antras & Helpman (2004). Particularly, Antras & Helpman (2004) suggests that there should be a gap in the productivity distribution of outsourcing firms because some of them outsource domestically and some others outsource internationally, a feature missing in the results of Bakhtiari (2009) and not present in my theoretical implications.

The rest of the paper proceeds as such: Next section reviews the general literature on outsourcing and multi-nationalism. Section 3 sets up the theoretical model and describes the mechanism of outsourcing. Section 4 discusses the implied theoretical predictions. The paper is then concluded.

2 Related Literature

Study of outsourcing has been mostly popular in the context of trade and cross-regional or cross-country differences. Extensions of Heckscher-Ohlin model by Helpman (1984) and Helpman (1985) describe conditions under which inter- and intra-firm trade are possible across countries. Grossman & Helpman (2005) present a north-south model to explain the geographical aspects of domestic versus international outsourcing.

Grossman & Hart (1986) and Hart & Moore (1990) show how contractual incompleteness leads to certain ownership issues within both integrated and outsourcing firms. Based on these issues, Grossman & Helpman (2002) relate outsourcing behavior to the costs borne
out of incomplete contracts and search frictions. Briefly, if benefits from outsourcing surpass its implied costs, the whole industry outsources, and stays integrated otherwise. Antras (2003) and Antras & Helpman (2004) use similar approaches to explain different patterns of integration versus outsourcing. The effect of trade openness on outsourcing behavior is described by (McLaren 2000). He argues that, in a closed economy, integration is an exclusion mechanism to curb competition, while trade openness takes that advantage away from the integrated firms and pushes them towards outsourcing. Finally, Naghavi & Ottaviano (2009) study the effect of outsourcing on the rate of innovation using a dynamic model.

Antras & Helpman (2004) have one of the very few papers that incorporates firm heterogeneity into outsourcing. However, their model does not tackle many issues raised by data observations, which form the basis for this paper.

3 Theoretical Setup

3.1 Consumers

There is a representative consumer that gains utility over a continuum of consumption goods varieties indexed by $j$. The utility function of this consumer is

$$U = \left( \int_{j \in J} y_j^\alpha \, dj \right)^{1/\alpha},$$

where $J$ is the set of varieties being produced, and $\alpha < 1$. As a result, the elasticity of substitution among varieties is constant and equal to $\alpha$. Dixit & Stiglitz (1977) show that such economy can be represented by an aggregate output index $Y = U$ and the associated aggregate price index, $P$, defined as

$$P = \left( \int_{j \in J} p_j^{\alpha - 1} \, dj \right)^{\alpha - 1}.$$

The consumer's optimal demand for each variety can then be found as a function of $P$ and $Y$, which is

$$p_j = Ay_j^{-(1-\alpha)},$$
where \( A = PY^{1-\alpha} \) is an aggregate index. This demand function is the one with constant price-elasticity equal to \( 1/(1 - \alpha) \).

### 3.2 Integrated Firm

There is one sector, and each firm in this sector produces a distinct variety \( j \in J \) of final good. Since the space of varieties is continuous, the probability of two firms producing the same variety is zero, hence, \( j \) indexes both good variety and the corresponding firm.

To produce variety \( j \), a firm has to first produce a specialized intermediate input and then adapt it to the distinct variety. The production of intermediate input and final good are both constant returns processes with labor as the only input. Specialized producers are differentiated in their efficiency in producing the intermediate input and their efficiency in producing the final good. Specifically, producer \( j \) needs \( 1/\phi_j \) labor to produce one unit of input. \( \phi_j \) is randomly drawn from a known cumulative distribution \( F(\phi) \) with support \( \phi \geq 0 \) and is observed after sinking all entry costs. Every unit of input is then transformed into one unit of a specialized final good using \( 1/\lambda_j \) units of labor. \( \lambda_j \) is also randomly drawn from a known cumulative distribution \( G(\lambda) \) with support \( \lambda \geq 0 \) and observed after entry. The pair \( (\lambda_j, \phi_j) \) defines the overall production efficiency of firm \( j \).

The total labor required by the integrated producer \( j \) to produce one unit of final good can be conveniently described in terms of a total labor productivity, defined as:

\[
\chi_j = \left( \frac{1}{\lambda_j} + \frac{1}{\phi_j} \right)^{-1}.
\]

\( \chi \) is basically what is measured in the data as productivity, while \( \lambda \) and \( \phi \) are implied but not directly measured in general. In the remaining, I drop index \( j \) where it does not cause confusion.

For the moment, I focus on the one-period behavior of a specialized producer in equilibrium. Wage rate for the employed labor is fixed in equilibrium and, without loss of generality, normalized to one henceforth. Studying industry evolution is not the main objective of this paper, therefore, I am abstracting from including fixed costs and time variations in produc-

\( ^3 \chi_j \) is practically the harmonic mean of \( \lambda_j \) and \( \phi_j \).
tivities to keep the model tractable and also help focus on more important features\textsuperscript{4}. The cost of entry and dynamics of operation for producers are discussed later in Section 3.4.

Producers decide their production level by maximizing the profit function \( \pi_V(\chi) = Ay_j^\alpha - \frac{y}{\chi} \); subscript \( V \) referring to vertical integration. Solving the first-order condition gives a producer’s optimal output \( x_V \), price \( p_V \), and profit \( \pi_V \) as

\[
x_V(\chi) = (\alpha \chi A)^{\frac{1}{1+\alpha}},
\]

\[
p_V(\chi) = 1/(\alpha \chi),
\]

\[
\pi_V(\chi) = \left( \frac{1}{\alpha} - 1 \right) (\alpha A)^{\frac{1}{1+\alpha}} \chi^{\frac{\alpha}{1+\alpha}},
\]

Note that, because of the constant-elasticity demand, producers charge a constant \( \frac{1}{\alpha} - 1 \) markup over their production costs.

### 3.3 Outsourcing Firm

A specialized producer can also order the required input from a middle supplier. Upon finding a supplier, the producer can contract the required input and its quantity. The middle supplier has the option to accept the production plan or break the match. If plan is accepted, production goes ahead with returns described below. The input produced in this stage is assumed useless outside the specific relation. Also, a high quality input is required for production; low quality input has zero value to production. If the match fails, both parties make zero profit in that period and continue search in the next period. For the moment, focus on the one-period operation of producers and suppliers.

Following Grossman & Hart (1986), I consider the relation between a specialized producer and its middle supplier to be governed by contractual incompleteness. Specialized producers can contract the required quantity of input, but if they make an ex ante commitment to price, since the quality of the intermediate good is not verifiable by a third party, middle supplier has every incentive to offer a low quality input and increase its own margin of profit. Ex post, however, both parties can renegotiate their claims to final revenue through a Nash
bargaining. But the middle supplier produces a specific input that is useless outside the relation, hence, it is left with little bargaining power at that point. Let the middle supplier’s bargaining power be $\omega \in (0, 0.5)$, reflecting this weak position. Middle supplier’s nonzero claims to the final revenue provides it with the incentive to produce high-quality input.

In addition to the above issues, the middle suppliers are also heterogeneous in their productivities, denoted as $\phi_o$. $\phi_o$ is drawn from a cumulative distribution $H(\phi_o)$ with support $\phi_o \geq 0$ and $E[\phi_o] > E[\phi]$; the intermediate industry is specialized in the production of the middle input and can offer higher productivity on average. The productivity of an individual middle supplier is unobserved by final producers until after a match is formed and the possibility of further search in that period is foregone. This uncertainty about middle supplier’s productivity is an additional risk factor from outsourcing.

The benefit from outsourcing is that producers require less overhead labor to manage and coordinate the whole production process. This cost saving is modeled here as a proportional increase in $\lambda_j$ by a factor $\mu(>1)$. $\mu$ is also used to account for improvement in final productivity as a result of the producer getting focused in its production and reinvesting some of the extra revenue to improve its final productivity.

An outsourcing decision starts by a specialized producer specifying the quantity of input $x (= y)$ it needs. The profit function for specialized producer, given a match is formed, is $\pi_S(\lambda) = (1 - \omega)Ay^\alpha - \frac{xO}{\mu A}$; subscript $S$ referring to specialized producer. Maximizing profit gives the optimal price ($p_O$) and demand for input ($x_O$) as

$$x_O(\lambda) = \left((1 - \omega)\alpha A\mu \lambda\right)^\frac{1}{1-\alpha}, \quad (8)$$

$$P_O(\lambda) = \frac{1}{((1 - \omega)\alpha A \mu \lambda)}. \quad (9)$$

The optimal profit for specialized producer is

$$\pi_S(\lambda) = \left(\frac{1}{\alpha} - 1\right) \left((1 - \omega)A\alpha\right)^\frac{1}{1-\alpha} (\mu \lambda)^\frac{1}{1-\alpha}. \quad (10)$$

The profit function for a middle supplier with productivity $\phi_o$ delivering quantity $x_O$ is $\pi_M(\lambda, \phi_o) = \omega Ax_O^\alpha - \frac{xO}{\phi_o}$; subscript $M$ referring to middle supplier. By delivering $x_O$, middle
supplier makes a profit of

$$\pi_M(\lambda, \phi_o) = \left( \frac{\omega}{\alpha(1 - \omega)} - \frac{\mu\lambda}{\phi_o} \right) \left[ (1 - \omega)\alpha A \right]^{\frac{1}{1 - \omega}} \left( \mu\lambda \right)^{\frac{\alpha}{1 - \omega}}. \quad (11)$$

### 3.4 Dynamics

The operation of integrated and outsourcing producers specified above repeats in every period, and future values are discounted for both producers and suppliers by a factor $\delta$, where $\delta \in (0, 1)$. In every period, there is a failure rate $\xi$ for a specialized producer which forces the producer, along with the matched supplier if outsourcing, to exit. $\xi$ is assumed non-degenerate and exogenously given. In what follows, $\xi$ mostly behaves like another discount factor, in addition to $\delta$. Therefore, to simplify matters, I define and use $\hat{\delta} = \xi\delta$ where appropriate.

Specialized producers and middle-suppliers find each other through a search process. Let $v$ be the number of producers that enter as vertically integrated, $s$ be the number of producers that enter seeking to outsource, and $m$ be the number of middle suppliers that enter. Number of matches formed in the period is $\eta(s, m)$, which is constant returns in $s$ and $m$. All producers seeking a supplier are equally likely to find one, hence, the probability of a match is $\eta(s, m)/s$. With constant returns matching, this probability can be stated as $\eta(r) = \eta(1, r)$, where $r = m/s$. On the other hand, the probability of a middle supplier finding a match is $\eta(s, m)/m$ or $\eta(r)/r$. Once a match is formed, middle suppliers can decide if they want to keep the match or break it and go on search again. If the match is kept, then it is permanent, unless both producer and supplier exit.

I assume a long-run situation throughout, where the dynamics of the industry has settled on a steady state path. As a result, $A, r, v, s,$ and $m$ are not time-varying. Specialized producers enter freely into market, paying a fixed entry cost $c_e > 0$ to cover their start-up costs, such as setting up the physical plant and penetrating market. Middle suppliers also enter market freely, paying a fixed entry cost $c_m > 0$. Entry costs for producers and suppliers need not be the same.
3.5 Middle Supplier’s Decision

The profit function (11) is not necessarily positive or, with the possibility of further search, in the long-run interest of the middle supplier. In particular, if middle supplier expects higher profits from future search, it will break the match. To characterize the nature of this decision, note that, in keeping a match, the present value of middle supplier is $\pi_M(\lambda, \phi_o)/(1 - \delta)$, given the match can be permanently kept. Figure 1 shows middle supplier’s profit as a function of $\lambda$ and for two different values of $\phi_o = \phi_1, \phi_2$ ($\phi_2 > \phi_1$). Clearly when the specialized producer is too unproductive, relative to middle supplier, the middle supplier is better off breaking the match and going on search again for a higher-productivity producer. On the other hand, if the specialized producer is too productive, relative to the middle supplier, the middle supplier is unable to keep up with the demanded quantity of input with enough profitability and can even make negative profits. Again, middle supplier has incentive to break the match. Let the lower and upper cutoff productivity of $\lambda$ corresponding to these cases be denoted as $\bar{\lambda}(\phi_o)$ and $\hat{\lambda}(\phi_o)$, respectively, so that a $\phi_o$-type middle supplier is willing to match to producers with $\lambda \in [\bar{\lambda}(\phi_o), \hat{\lambda}(\phi_o)]$.

A middle supplier that breaks a match receives zero profit in current period and finds a match in next period with $\eta(r)/r$ probability. The probability that this new match is
desirable is $G(\lambda(\phi_o)) - G(\lambda(\phi_o))$. In this case, the present value of the middle supplier is

$$\frac{\hat{\delta}}{1-\delta} \frac{n(r)}{r} (G(\lambda(\phi_o)) - G(\lambda(\phi_o))) E[\pi_M(\lambda, \phi_o) | \lambda(\phi_o) \leq \lambda(\phi_o)].$$

But, with $1 - \frac{n(r)}{r} (G(\lambda(\phi_o)) - G(\lambda(\phi_o)))$ probability there is no new match or the match fails and the game repeats. Denote the value of going on search after breaking a match by $V_M(\lambda, \phi_o)$, then

$$V_M(\lambda, \phi_o) = \frac{\hat{\delta} \frac{n(r)}{r} (G(\lambda(\phi_o)) - G(\lambda(\phi_o)))}{1 - \delta + \hat{\delta} \frac{n(r)}{r} (G(\lambda(\phi_o)) - G(\lambda(\phi_o)))} E[\pi_M(\lambda, \phi_o) | \lambda(\phi_o) \leq \lambda(\phi_o)].$$

A $\phi_o$-type middle supplier breaks a match if the following holds

$$\pi_M(\lambda, \phi_o) < (1 - \delta)V_M(\lambda, \phi_o), \quad (12)$$

The right-hand side above is constant with respect to $\lambda$ and is represented by horizontal lines in Figure 1, whose intersections with profit functions identify the threshold values $\lambda$ and $\lambda$. Alternatively, these threshold values can be obtained by solving (12) with equality.

The behavior of $\lambda$ and $\lambda$ with $\phi_o$ is of particular interest and is described by the following series of propositions.

**Proposition 1** For any $\phi_o > 0$, $\lambda(\phi_o)$ and $\lambda(\phi_o)$ exist such that $\lambda(\phi_o) > 0$ and $\lambda(\phi_o) > \lambda(\phi_o)$.

**Proposition 2** If $\hat{\delta}$ is large enough, then $d\lambda(\phi_o)/d\phi_o > 0$ and $d\lambda(\phi_o)/d\phi_o > 0$.

**Proposition 3** As $\phi_o \to \infty$, $\lambda(\phi_o) \to \Lambda$ and $\lambda(\phi_o) \to \infty$, where $0 < \Lambda < \infty$.

Proposition 1 says that each middle supplier with nonzero productivity has incentive to match with a nonempty set of producers. Proposition 2 shows that as middle suppliers get more productive, they seek more productive producers. But, from Proposition 3 it is clear that as $\phi_o$ becomes very large, middle suppliers become less selective about the producers they want to match with, partly because the probability of finding a high-$\lambda$ producer diminishes.

Note that the condition to have large enough $\hat{\delta}$ in Proposition 2 is rather conservative: with many sets of parameters even $\hat{\delta} \approx 0$ is large enough for this result to hold.
3.6 Specialized Producer’s Decision

The implications of Proposition 2 is far reaching in terms of what kinds of matches between middle suppliers and specialized producers are successful. The following looks at this issue from the specialized producer’s point of view:

**Proposition 4** For a specialized producer with productivity pair $(\lambda, \phi)$ and for $\hat{\delta}$ large enough, there exist $\phi_o(\lambda)$ and $\bar{\phi}_o(\lambda)$, so that the specialized producer can build a lasting match with any middle supplier of productivity $\phi_o \in [\phi_o(\lambda), \bar{\phi}_o(\lambda)]$.

**Proposition 5** If $\hat{\delta}$ is large enough, then $\frac{d\phi_o(\lambda)}{d\lambda} > 0$ and $\frac{d\bar{\phi}_o(\lambda)}{d\lambda} > 0$.

**Proposition 6** As $\lambda \to \infty$, $\phi_o(\lambda) \to \infty$ and $\bar{\phi}_o(\lambda) \to \infty$.

Proposition 5 says that high-$\lambda$ producers seek high-productivity suppliers. Proposition 6 shows that, contrary to middle suppliers, producers stay selective about their middle suppliers as $\lambda$ gets very large.

If a specialized producer decides to outsource in the current period, a successful match happens with $\eta(r)(H(\bar{\phi}_o(\lambda)) - H(\phi_o(\lambda)))$ probability and the return is $\pi_S(\lambda)$ per each period. With probability $1 - \eta(r)(H(\bar{\phi}_o(\lambda)) - H(\phi_o(\lambda)))$ there is no match or the match fails. Since nothing has changed for the specialized producer, it continues to search in the next period and the game repeats. In total, the value of outsourcing is

$$V_S(\lambda) = \frac{\eta(r)(H(\bar{\phi}_o(\lambda)) - H(\phi_o(\lambda))) \pi_S(\lambda)}{1 - \hat{\delta} + \hat{\delta}\eta(r)(H(\bar{\phi}_o(\lambda)) - H(\phi_o(\lambda)))}.$$

On the other hand, if this producer stays fully integrated, its present value would be $V_V(\lambda, \phi) = \pi_V(\chi)/ (1 - \hat{\delta})$. The specialized producer outsources if

$$V_V(\lambda, \phi) < V_S(\lambda). \quad (13)$$

The right-hand side is a constant for any given $\lambda$, while the left-hand side is a monotonically increasing function of $\chi(\lambda, \phi)$, starting from zero and increasing to infinity as $\chi \to \infty$. As a result, there exists a $\chi^*(\lambda)$ (and a corresponding $\phi^*(\lambda)$) where a $\lambda$-type specialized producer outsources if $\chi(\lambda, \phi) < \chi^*(\lambda)$, and stays integrated if $\chi(\lambda, \phi) > \chi^*(\lambda)$. Firms with $\chi(\lambda, \phi) = \chi^*(\lambda)$ are indifferent between outsourcing or staying integrated.
3.7 Closing Model

Free Entry

Specialized producers enter freely, therefore they should expect to make zero profits in steady state to curb excessive entry. Zero expected profit condition requires that

$$
\int_0^\infty \left( \int_{\phi^*(\lambda)}^\infty V_T(\lambda, \phi) dF(\phi) + F(\phi^*(\lambda))V_S(\lambda) \right) dG(\lambda) = c_e. \tag{14}
$$

The left-hand side above is the expected profit in case a producer ends up as integrated or outsourcing. This profit is offset by the cost of entry to generate the required condition.

Middle suppliers also enter freely and their zero profit condition can be formulated as

$$
\int_0^\infty \int_{\Delta(\phi_o)}^{\bar{\lambda}(\phi_o)} V_M(\lambda, \phi_o) dG(\lambda) dH(\phi_o) = c_m. \tag{15}
$$

Solving equations (14) and (15) together provides the two unknowns $A$ and $r$.

Labor Clearing

Assume the industry is endowed with a fixed amount of labor $L$. In steady state the labor demand is equal to labor supply. If the mass of integrated producers is $N$ and the mass of outsourcing producers is $M$ then the condition is

$$
\int_0^\infty \left( \int_{\phi^*(\lambda)}^\infty \frac{xV(\chi)}{\chi} NdF(\phi) + F(\phi^*(\lambda)) \int_0^\infty \frac{xO(\lambda)}{\chi_o} MdH(\phi) \right) dG(\lambda) = L, \tag{16}
$$

where $\chi_o = \left( \frac{1}{\chi_N} + \frac{1}{\chi_M} \right)^{-1}$. The first term on the right-hand side is the labor used by all integrated producers, and the second term is the labor used by the matched outsourcing producers and their suppliers.

Entry and Exit

In steady state, the rate of entry and exit are equal so that

$$
v + s = \xi N, \quad m = \xi M. \tag{17}
$$

The fraction of outsourcing versus integrated producers is the last equation to solve for.
unknowns and formulates to
\[
\frac{M}{N} = \frac{m}{v} = \frac{\int_0^\infty F(\phi^*(\lambda))dG(\lambda)}{1 - \int_0^\infty F(\phi^*(\lambda))dG(\lambda)}.
\] (18)

**Definition 1** A steady state equilibrium for the outsourcing problem with heterogeneous producers and heterogeneous suppliers is the tuplet \((\lambda(\phi_o), \bar{\lambda}(\phi_o), \phi_o(\lambda), \bar{\phi}_o(\lambda), \phi^*(\lambda), \Lambda, N, M, v, s, m)\) such that (i) \(\lambda(\phi_o)\) and \(\bar{\lambda}(\phi_o)\) satisfy (12) with equality, (ii) \(\phi_o(\lambda)\) and \(\bar{\phi}_o(\lambda)\) satisfy (13) with equality, and (iii) (14), (15), (16), (17), and (18) are satisfied, given values of \(\alpha, L, c_e, c_{e_m}, \omega, \mu, \delta, \xi\) and the distributions \(F(\phi), G(\lambda)\) and \(H(\phi_o)\).

### 4 Theoretical Implications of Outsourcing

Outsourcing benefits specialized producers two-folds: first, by not having to produce the intermediate input, their productivity is now defined by \(\lambda > \chi(\lambda, \phi)\). Second, productivity level \(\lambda\) is even magnified by a factor \(\mu\). Clearly, if \(\mu\) is much larger than one, then all producers find it optimal to outsource as the benefits overshadow any cost. To have an industry that is a mix of both vertically integrated and outsourcing producers, the cost and benefit of outsourcing should be in some balance as described below:

**Theorem 1** If \((1 - \omega)\mu^\alpha < 1\), then the equilibrium is a mix of vertically integrated and outsourcing producers.

**Proposition 7** In a mixed equilibrium, \(\phi^*(0) = 0\). Also, there exists a \(\bar{\lambda} > 0\) (with possibility of \(\bar{\lambda} \to \infty\)) such that \(d\phi^*(\lambda)/d\lambda > 0\) for \(\lambda \in [0, \bar{\lambda}]\).

Producers with very low values of \(\lambda\) are the ones that benefit the least from outsourcing, and, by proposition 7, outsourcing is more scarce among those firms. An interesting implication of Proposition 7 is the following:

**Theorem 2** In a mixed equilibrium, two integrated producers with the same total productivity can end up in different paths, with one of them outsourcing and the other staying integrated.

As shown in Proposition 2, not every specialized producer can form a lasting match with a certain middle supplier. The range of productivities that would stay matched is a window
of productivities $[\bar{\lambda}(\phi_0), \bar{\lambda} (\phi_0)]$, that moves towards higher productivities as $\phi_o$ gets larger (Figure 2). This effect leads to the following result:

**Theorem 3** On average, specialized producers with higher productivity on their final process form successful and lasting matches with more productive middle suppliers. Conversely, on average, specialized producers with lower productivity on their final process form successful and lasting matches with less productive middle suppliers.

Proposition 2 especially shows that middle suppliers become less selective as they become increasingly productive. This is expected as the density of high-\(\lambda\) producers diminishes. On the other hand, high-\(\lambda\) producers, would outsource if they are likely to find a suitable match. However, this is not an absolutely pressing matter for producers as they can always internalize. As a result, and also shown by Proposition 2, high-\(\lambda\) producers stay selective in their choice of middle suppliers and outsource only if the likelihood of finding a high-productivity supplier is large enough. In other words,

**Theorem 4** If finding a high-productivity middle-supplier is probable enough, a large fraction of high-\(\lambda\) producers outsource. If finding a high-productivity middle-supplier is improbable enough, then all high-\(\lambda\) producers stay integrated.

In particular, this model predicts that some high-productivity integrated producers (with high values of $\chi$) should be outsourcing.

**Theorem 5** If finding a high-productivity middle-supplier is probable enough, the set of high-productivity integrated producers that go on search for a middle supplier is nonempty.
This theorem comes as a corollary to Theorem 4 because very high-productivity integrated producers are those with high λ’s. With the probability of finding a high-productivity supplier large enough, Theorem 4 predicts that a proportion of these producers should be outsourcing (Figure 3). Theorems 1, 2, and 5, in particular, conform with the empirical evidence presented by Bakhtiari (2009).

5 Conclusion

Empirical evidence emphasizes that industries are a mix of integrated and outsourcing firms. Firms with the same level of productivity might have different attitudes towards outsourcing, with one of them outsourcing and the other staying vertically integrated. High productivity firms are also engaged in outsourcing. The model presented in this paper is capable of explaining these observed facts using a two stage production process and heterogeneous productivities for each stage of production among firms and their suppliers. In addition, this model suggests an ordering in the productivities of upstream suppliers and downstream producers, where, through a selection mechanism, producers and suppliers roughly match their productivities to build vertical relations.
References


A Technical Appendix

Proof of Proposition 1: Since \( \pi_M(0, \phi_o) = 0 \) for any \( \phi_o > 0 \), clearly \( \lambda(\phi_o) > 0 \). The unimodal form of \( \pi_M(\lambda, \phi_o) \) shows that any possible set of solutions will form one connected interval \( [\Delta(\phi_o), \bar{\lambda}(\phi_o)] \). Let \( \Delta(\phi_o) = \bar{\lambda}(\phi_o) \) for any \( \phi_o \); i.e., the set of producers that can have lasting match with this supplier is measure zero. Then the right-hand side of (12) is zero. But, (11) clearly shows that \( \pi_M(\lambda, \phi_o) > 0 \) for all \( \lambda < \frac{\phi_o}{\alpha(1 - \omega)}, \) meaning that the set \( [\lambda, \bar{\lambda}] \) must have nonzero measure. This is a contradiction to the initial assumption. Therefore, it must be that \( \bar{\lambda}(\phi_o) > \lambda(\phi_o) \). ■

Proof of Proposition 2: First note the following preliminary results used in the course of proof:

\[
\frac{\partial \pi_M(\lambda)}{\partial \phi_o} > 0, \quad \frac{\partial^2 \pi_M(\lambda, \phi_o)}{\partial \lambda \partial \phi_o} > 0, \quad \frac{\partial \pi_M(\lambda)}{\partial \lambda} > 0, \quad \frac{\partial \pi_M(\bar{\lambda})}{\bar{\lambda}} < 0.
\]

I am dropping \( \phi_o \) as argument where obvious to save space. Define

\[
B \equiv \frac{r(1 - \hat{\delta})}{\delta \eta(r)}, \quad G \equiv G(\Delta), \quad \bar{G} \equiv G(\bar{\lambda}), \quad g = \frac{dG(\lambda)}{d\lambda} \bigg|_{\Delta}, \quad \bar{g} = \frac{dG(\lambda)}{d\lambda} \bigg|_{\bar{\lambda}},
\]

and

\[
\bar{\pi}_M \equiv \pi_M(\bar{\lambda}), \quad \lambda \equiv \bar{\pi}_M(\bar{\lambda}).
\]

Also let \( \hat{E}(\cdot) \equiv \int^{\bar{\lambda}}_{\Delta} (\cdot) dG(\lambda) \).

To start with proof, at \( \Delta \) and \( \bar{\lambda} \) we have

\[
\bar{\pi}_M = \hat{\pi}_M = \frac{1}{B + G - G} \hat{E}[\pi_M(\lambda)].
\]

Taking derivatives with respect to \( \phi_o \) and rearranging terms gives

\[
\left( \frac{\partial \pi_M}{\partial \lambda} + \frac{\hat{g} \hat{E}[\pi_M(\lambda)]}{(B + G - G)^2} + \frac{\pi_M}{B + G - G} \right) \frac{d\lambda}{d\phi_o} = \left( \frac{\hat{g} \hat{E}[\pi_M(\lambda)]}{(B + G - G)^2} + \frac{\pi_M}{B + G - G} \right) \frac{d\bar{\lambda}}{d\phi_o} = \frac{\partial \pi_M}{\partial \phi_o} + \frac{1}{B + G - G} \hat{E} \left[ \frac{\partial \pi_M(\lambda)}{\partial \phi_o} \right].
\]

(19)
and
\[
\left( \frac{g\hat{E}\left[\pi_M(\lambda)\right]}{(B + G - G)^2} + \frac{\pi_M}{B + G - G} \right) \frac{d\lambda}{d\phi_o} - \left( \frac{\partial\pi_M}{\partial\lambda} + \frac{\partial\hat{E}\left[\pi_M(\lambda)\right]}{(B + G - G)^2} + \frac{\pi_M}{B + G - G} \right) \frac{d\lambda}{d\phi_o} = \]
\[
- \frac{\partial\pi_M}{\partial\phi_o} + \frac{1}{B + G - G} \hat{E} \left[ \frac{\partial\pi_M}{\partial\phi_o} \right].
\]

Eliminating \(d\lambda/d\phi_o\) between (19) and (20) results in \(T d\lambda/d\phi_o = \tilde{S}\), where
\[
T = \frac{\partial\pi_M}{\partial\lambda} \left( \frac{\partial\pi_M}{\partial\lambda} + \frac{\hat{E}\left[\pi_M(\lambda)\right]}{(B + G - G)^2} + \frac{\pi_M}{B + G - G} \right) + \frac{\partial\pi_M}{\partial\lambda} \left( \frac{\hat{E}\left[\pi_M(\lambda)\right]}{(B + G - G)^2} + \frac{\pi_M}{B + G - G} \right)
\]
\[
\tilde{S} = \left( \frac{\partial\pi_M}{\partial\phi_o} - \frac{\partial\pi_M}{\partial\phi_o} \right) \left( \frac{\hat{E}\left[\pi_M(\lambda)\right]}{(B + G - G)^2} + \frac{\pi_M}{B + G - G} \right) + \frac{\partial\pi_M}{\partial\lambda} \left( \frac{1}{B + G - G} \hat{E} \left[ \frac{\partial\pi_M}{\partial\phi_o} \right] - \frac{\partial\pi_M}{\partial\phi_o} \right).
\]

Based on the preliminary results above, \(T\) is negative. Also, based on the preliminary results, \(\partial\pi_M/\partial\phi_o < \partial\pi_M/\partial\phi_o\). Besides, \(\hat{E}[\partial\pi_M/\partial\phi_o] > \partial\pi_M/\partial\phi_o\), and the last term in \(\tilde{S}\) will be negative if \(\delta\) is large enough (making \(B\) small enough). In this case, \(\tilde{S}\) is negative, making \(d\lambda/d\phi_o > 0\).

Similarly, eliminating \(d\lambda/d\phi_o\) between (19) and (20) results in \(T d\lambda/d\phi_o = \tilde{S}\), where
\[
\tilde{S} = \left( \frac{\partial\pi_M}{\partial\phi_o} - \frac{\partial\pi_M}{\partial\phi_o} \right) \left( \frac{\hat{E}\left[\pi_M(\lambda)\right]}{(B + G - G)^2} + \frac{\pi_M}{B + G - G} \right) + \frac{\partial\pi_M}{\partial\lambda} \left( \frac{1}{B + G - G} \hat{E} \left[ \frac{\partial\pi_M}{\partial\phi_o} \right] - \frac{\partial\pi_M}{\partial\phi_o} \right).
\]

From before, it is given that \(T < 0\). Also, given the preliminary results and noting that now \(\hat{E}[\partial\pi_M/\partial\phi_o] > \partial\pi_M/\partial\phi_o\), for large enough values of \(\delta\) we have \(\tilde{S} < 0\). As a result, \(d\lambda/d\phi_o > 0\).

**Proof of Proposition 3:** Let \(\lambda(\phi_o) \to \infty\) as \(\phi_o \to \infty\). If \(\hat{\lambda}(\phi_o)\) stays finite, then it is an immediate contradiction. Hence, assume that \(\hat{\lambda}(\phi_o) \to \infty\). Then, the right-hand side in (12) goes to zero. But, \(\pi_M(\lambda, \infty)\) is positive for any value of \(\lambda > 0\), i.e. middle supplier is willing to match with any specialized producer, which is a contradiction. Therefore, there should exist a \(0 < \lambda < \infty\), such that \(\lambda(\phi_o) \to \lambda\) as \(\phi_o \to \infty\).

Now assume that \(\hat{\lambda}(\phi_o) \to \hat{\lambda}, \lambda < \hat{\lambda} < \infty\), as \(\phi_o \to \infty\). As a result, the right-hand side in (12) converges to a constant \(K(\hat{\lambda}) < \infty\). But, the middle supplier is obviously willing to match with any specialized producer with\(^5\)
\[
\lambda > \frac{1}{\alpha(1 - \omega)\mu_{\hat{\lambda}}^{1/\omega}} \left( \frac{K(\hat{\lambda})}{\omega} \right)^{\omega},
\]
which includes any \(\lambda > \hat{\lambda}\), a contradiction to \(\hat{\lambda}\) being finite. Thus \(\lambda(\phi_o) \to \infty\) as \(\phi_o \to \infty\).

\(^5\)just set \(\pi_M(\lambda, \infty) > K(\hat{\lambda})\)
Proof of Proposition 4: $\bar{\phi}_o$ is the solution to the following

$$\bar{\phi}_o(\lambda) = \min \left\{ \phi_o|\bar{\lambda}(\phi_o) \geq \lambda \right\}.$$  \hspace{1cm} (21)

Since $\bar{\lambda}(\phi_o)$ is continuous, increasing in $\phi_o$, and $\bar{\lambda}(\phi_o) \to \infty$ as $\phi_o \to \infty$, the above solution exists.

$\bar{\phi}_o$ is the solution to the following

$$\bar{\phi}_o(\lambda) = \max \left\{ \phi_o|\bar{\lambda}(\phi_o) \leq \lambda \right\} \quad \text{if } \lambda < \Lambda,$$

$$\infty \quad \text{if } \lambda \geq \Lambda.$$  \hspace{1cm} (22)

Again, since $\bar{\lambda}(\phi_o)$ is continuous and increasing in $\phi_o$, the above solution exists. \hfill \blacksquare

Proof of Proposition 5: Based on (21) and (22), it is immediately clear that $\phi_o(\lambda)$ and $\bar{\phi}_o(\lambda)$ should both be increasing in $\lambda$. \hfill \blacksquare

Proof of Proposition 6: Letting $\lambda > \Lambda$, it is clear that $\bar{\phi}_o(\lambda) \to \infty$. Also, since $\bar{\lambda}(\phi_o)$ is an increasing and one-to-one mapping, from (21) it is clear that $\lambda \to \infty$ implies that $\phi_o(\lambda) \to \infty$. \hfill \blacksquare

Proof of Theorem 1: Writing a simplified version of (13) shows that outsourcing producers are those for whom

$$\chi < \left( \frac{\eta(r)(H(\bar{\phi}_o(\lambda)) - H(\phi_o(\lambda)))}{1 - \delta + \delta\eta(r)(H(\phi_o(\lambda)) - H(\bar{\phi}_o(\lambda)))} \right)^{\frac{1-\alpha}{\alpha}} \left( (1 - \omega)\mu^\alpha \right)^{1/\alpha} \lambda.$$  

It can be shown that the term in the first parentheses in the right-hand side is less than one. Consequently, if $(1 - \omega)\mu^\alpha < 1$, then as $\phi \to \infty$, the inequality is violated, which means that there are producers that stay integrated. Obviously producers with $\phi_o = 0$ outsource. So there are both types of producers present in equilibrium. \hfill \blacksquare

Proof of Proposition 7: from (13), $\chi^*(\lambda)$ can be written as $\chi^*(\lambda) = P(\lambda)\lambda$, where

$$P(\lambda) = \mu(1 - \omega)^{\frac{1}{2}} \left( \frac{\eta(r)(H(\bar{\phi}_o(\lambda)) - H(\phi_o(\lambda)))}{1 - \delta + \delta\eta(r)(H(\phi_o(\lambda)) - H(\bar{\phi}_o(\lambda)))} \right)^{\frac{1-\alpha}{\alpha}} \lambda.$$  

Since $P(\lambda) \to 0$ as $\lambda \to 0$, then $\chi^*(\lambda)$ goes to zero at a faster rate than $\lambda$. But $\phi^*(\lambda) = \frac{\lambda\chi^*(\lambda)}{\lambda \chi^*(\lambda)}$. Hence, as $\lambda \to 0$, $\phi^*(\lambda) \to 0$ from above.

In a mixed equilibrium, $\phi^*(\lambda) > 0$ for some $\lambda > 0$, which means that, by continuity of $\phi^*(\lambda)$, there must exist a $\check{\lambda} > 0$, such that $d\phi^*(\lambda)/d\lambda > 0$ for $\lambda \in [0, \check{\lambda}]$. \hfill \blacksquare
Proof of Theorem 2: I just need to show that there is a nonempty set of producers with the same total productivity but different paths. Let $\lambda_1 = r\lambda$, where $r < 1$ and $\lambda$ comes from Proposition 7. Define the following integrated producers:

**Producer 1:** has productivity pair $\left(\lambda_1 - \epsilon, \frac{\lambda_1\phi^*(\lambda_1)(\lambda_1 - \epsilon)}{\lambda_1(\lambda_1 - \epsilon) - \phi^*(\lambda_1)}\right)$. 

**Producer 2:** has productivity pair $\left(\lambda_1 + \epsilon, \frac{\lambda_1\phi^*(\lambda_1)(\lambda_1 + \epsilon)}{\lambda_1(\lambda_1 + \epsilon) - \phi^*(\lambda_1)}\right)$. 

Both producers 1 and 2 have the same total productivity equal to $\lambda_1\phi^*(\lambda_1)/(\lambda_1 + \phi^*(\lambda_1))$. Note that, $\frac{\lambda_1\phi^*(\lambda_1)(\lambda_1 - \epsilon)}{\lambda_1(\lambda_1 - \epsilon) - \phi^*(\lambda_1)} > \phi^*(\lambda_1) > \phi^*(\lambda_1 - \epsilon)$, for small enough $\epsilon$. The last result comes from Proposition 7 and the fact that $\phi^*(\lambda)$ is increasing at $\lambda_1$. Similar reasoning shows that $\frac{\lambda_1\phi^*(\lambda_1)(\lambda_1 + \epsilon)}{\lambda_1(\lambda_1 + \epsilon) - \phi^*(\lambda_1)} < \phi^*(\lambda_1) < \phi^*(\lambda_1 + \epsilon)$, for small enough $\epsilon$. Therefore, for small enough $\epsilon$, producer 1 stays integrated, whereas producer 2 goes on search for a middle supplier, proving the result. ■

Proof of Theorem 4: Focus on the right-hand side in (13). As $\lambda \to \infty$, $\tilde{\phi}_o(\lambda), \check{\phi}_o(\lambda) \to \infty$ which lets

\[
R = \frac{\eta(r)(H(\tilde{\phi}_o(\lambda)) - H(\check{\phi}_o(\lambda)))}{1 - \delta + \delta\eta(r)(H(\tilde{\phi}_o(\lambda)) - H(\check{\phi}_o(\lambda)))} \to 0.
\]

But, at the same time, $\pi_S(\lambda) \to \infty$. There are two cases:

**Case 1:** $H(\tilde{\phi}_o(\lambda)) - H(\check{\phi}_o(\lambda)) = O(\lambda^n)$, where $n < -\alpha/(1 - \alpha)$. In this case, for large enough $\lambda$, $R$ goes to zero faster than $\pi_S(.)$ goes to infinity, and the right-hand side in (13) goes to zero. As a result, the fraction of $\lambda$-type producers that outsource converges to zero as $\lambda \to \infty$. Notice that $H(\tilde{\phi}_o(\lambda)) - H(\check{\phi}_o(\lambda))$, when $\lambda \to \infty$, directly relates to the density of high-productivity suppliers.

**Case 2:** $H(\tilde{\phi}_o(\lambda)) - H(\check{\phi}_o(\lambda)) = O(\lambda^n)$, where $n > -\alpha/(1 - \alpha)$. In this case, $H(.)$ goes to zero more slowly than $\pi_S(.)$ goes to infinity or stays positive, and the right-hand side in (13) goes to infinity. As a result, the fraction of $\lambda$-type producers that outsource converges to one as $\lambda \to \infty$. ■