Who invests in Research and Development?: A Model of Endogenous Sectoral Choice with Costly External Finance

Rasim Burak Uras

June 12, 2009

Abstract

In this paper a dynamic general equilibrium framework is analyzed in which limited contract enforcement not only affects a firm’s investment size but also its sectoral choice (R&D intensive and productivity enhancing sector (Pharmaceutical) versus sales oriented sector (Retail)). In the model firms are heterogeneous. Heterogeneity is introduced in terms of the initial collateralizable endowment positions firms are born with and the productive volatility. Endogenizing sectoral selection helps us to capture the implications of financial frictions extensively. The key result of the paper shows that, due to the limited contract enforcement, firms who have high collateral initially are larger ex-post and choose to operate in the R&D intensive sector. Another interesting result is that sectoral choice is not always monotonic in initial endowment. For a range of parameter values, due to the ease of collateral building, firms with medium levels of initial collateral tend to operate in the sales oriented sector, whereas firms who are poor in initial endowment select the R&D intensive sector. The policy analysis aims to identify the effects of R&D subsidies on aggregate welfare. The quantitative experiments show that although for some parameter values subsidization of R&D might be beneficial from an aggregate point of view, under other parameter conditions they become distortionary. The reason for the latter outcome is the high concentration of financially constrained medium size firms in the sales oriented sector. Firms allocated in the sales oriented sector end up carrying the burden of the public finance mechanism. I conclude that the effectiveness of the public R&D policies depend on the severity of financial market frictions in the economy as well as on the strength of the relationship between the R&D and the aggregate productivity.

Keywords: Sectoral Choice, Costly External Finance and Public R&D Policies.

JEL Classification Numbers: E44, E62, G11.

*Washington University in St. Louis, rburas@wustl.edu
†I am indebted to Ping Wang for many valuable discussions on this topic. All errors are mine.
1 Introduction

Greenwood and Jovanovic (1990), Bencivenga and Smith (1991), Banerjee and Newman (1993) and Aghion and Bolton (1997) were among many who mathematically formalized the influence of strong financial institutions on the well-being of the aggregate economy. These papers have shown that investment analysis at the entrepreneurial level is important to take into account in order to understand the link between the level of financial development and the macroeconomic performance. However, this strand of literature did not pay enough attention to the sectoral composition of entrepreneurs. The purpose of my analysis is to show that sectoral composition of entrepreneurs is relevant for the relationship between the level of financial development and macroeconomic performance.

In order to validate my claim, I develop a 3-period overlapping generations general equilibrium model with endogenous sectoral choice. Firms get started by young entrepreneurs who are heterogeneous in terms of their initial asset size -which can be used as collateral- and productive volatility. Depending on the initial asset size, in equilibrium, some firms get financed externally whereas others internally. I also assume that small scale firms have higher productive volatility compared to their larger counterparts. At the beginning of their lifetime entrepreneurs have to choose one of the following production options. A firm owner can choose to operate in the R&D intensive sector which generates technological spillovers -and enhances aggregate productivity- for the rest of the economy or she can choose to produce in the sales oriented sector. The R&D intensive sector can be exemplified with the pharmaceutical industry whereas the sales oriented sector can be exemplified with the retail industry. Following Griliches (1998), I assume that the output of the R&D intensive sector needs a longer adjustment period (long term investment) compared to the output of the sales oriented sector (short term investment). In an incomplete markets environment, this assumption makes the R&D intensive firms financially more vulnerable relative to sales oriented firms. As in Hart and Moore (1988) limited commitment to loan repayment is the source of the market incompleteness. Atkeson, Khan and Ohanian (1996), Basu and Fernald (1997), Basu, Fernald and Shapiro (2001) and Guner, Ventura and Xu (2006) show empirical evidence for the existence of diminishing returns to scale at the establishment level. Following the findings of this literature I assume that both short term and long term production technologies have diminishing returns to scale. Diminishing returns to scale assumption creates a non-degenerate firm size distribution in equilibrium. Due to the presence of limited commitment problem, ex-ante small firms are under-sized whereas ex-ante large firms are over-sized. My purpose is to study how limited commitment affect the concentration of firms in each sector.
The main result of the paper shows that the lack of a well established contract enforcement scheme distorts the distribution of the demand for short term loans less than it distorts the distribution of the demand for long term loans. Consequently, the equilibrium analysis reveals that, credit constrained small and medium size firms tend to over-emphasize sales oriented investment; whereas, unconstrained large firms over-emphasize R&D intensive investment. The reason why small and large firms concentrate in the sales oriented sector is the ease of collateral building associated with this sector. This collateral effect is similar to that of in Kiyotaki and Moore (1997) and Kiyotaki (1998). Therefore, in a financially under-developed economy the distribution of long term funds among borrowers is relatively more skewed towards ex-ante large firms compared to the distribution of short term funds. If an entrepreneur’s initial collateral is below a certain level, sales oriented production may be beneficial for her relative to operating in the R&D intensive sector. However this relationship is not always monotonic. Under certain parameter conditions collateral poor small scale entrepreneurs can end up in the R&D intensive sector as well. This result can hold if there are increasing returns to scale associated with collateral building in the sales oriented sector. The interaction between productive volatility and limited commitment to debt repayment is important for this outcome.

In section 8, I study the aggregate implications of R&D subsidies. In this section a government is introduced into the benchmark model, whose objective is to maximize the social output by following a balanced budget rule in every period. This analysis shows that the self-selection of some small scale firms into the R&D intensive sector does not make the use of R&D subsidization necessarily beneficial from an aggregate point of view. The reason for this result is the presence of financially vulnerable other small and medium size firms in the sales oriented sector, who get taxed to finance R&D subsidies. This result shows that an extensive analysis of macroeconomic implications of R&D public policies require the incorporation of firm size heterogeneity and sectoral composition of firms in the analyzed framework. In this type of a framework the strength of the relationship between the R&D and productivity, the severity of financial market frictions in the economy and the interactive effect of these two determine the effectiveness of public R&D policies.

The rest of the paper is organized as follows. Section 2 describes stylized differences between small and large firms. Section 3 develops the economic environment for the model. Sections 4, 5 and 6 provide solutions to three different versions of the model. Section 7 presents the benchmark calibration and section 8 the policy analysis. Section 9 concludes the paper.
2 Stylized Differences Between Small and Large Firms

There are stylized differences between small and large firms. Large firms are less volatile in profitability and have higher average long term credit rating, which play crucial role for investment finance behavior in an imperfect credit markets economy.

**Time Series Volatility in Firm Profits:** I extracted two samples of panel data (small firms (Employees < 500) and large firms (Employees > 5000)) and computed the average time-series variance in firm profits for each group. Firm profits are computed as (Total Sales - Cost of Goods Sold). Time series volatility in profits for each firm $i$ is computed as $^1$:

$$\left( \frac{\text{Standard Deviation}(1997:2006)}{\text{Mean}(1997-2006)} \right) = \sigma_i.$$

The results are as follows:

<table>
<thead>
<tr>
<th>Firm Size Class</th>
<th>median</th>
<th>mean</th>
<th>st.dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>1.2366</td>
<td>2.2344</td>
<td>9.2343</td>
</tr>
<tr>
<td>Large</td>
<td>0.2948</td>
<td>0.6554</td>
<td>6.4111</td>
</tr>
</tbody>
</table>

Source: Compustat. Small firms #Employees < 500, Large firms #Employees > 5000.

As Table 1 shows, on average a small firm has higher time-series variance in firm profits relative to a large firm. Large firms are known to be better in diversifying business risk. The undiversifiable business risk associated with small firm production have been studied by other authors before (see Herranz, Krasa and Villamil (2007)); however, the time series dimension of this risk has not.

**Long Term Credit Rating:** The Compustat data shows that the average institutional credit rating (S&P Long Term Domestic Credit Issuer Rating) is increasing monotonically with the firm size bracket with variance of the credit rating staying more or less the same for each bracket. The following table summarizes the findings:

--

$^1$I excluded firms which have negative profits from the samples.
Table 2. S&P Long Term Credit Rating (1997-2006)

<table>
<thead>
<tr>
<th>Firm Size Class</th>
<th>median</th>
<th>mean</th>
<th>st.dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; N &lt; 500</td>
<td>14=BB</td>
<td>14=BB</td>
<td>4.9</td>
</tr>
<tr>
<td>500 &lt; N &lt; 2000</td>
<td>13=BB+</td>
<td>13=BB+</td>
<td>4.4</td>
</tr>
<tr>
<td>2000 &lt; N &lt; 5000</td>
<td>12=BBB-</td>
<td>12=BBB-</td>
<td>3.9</td>
</tr>
<tr>
<td>5000 &lt; N &lt; 30000</td>
<td>11=BBB</td>
<td>11=BBB</td>
<td>4.0</td>
</tr>
<tr>
<td>N &gt; 30000</td>
<td>9=A-</td>
<td>9=A-</td>
<td>5.0</td>
</tr>
</tbody>
</table>


This piece of evidence shows that large firms tend to receive higher institutional credit ratings compared to their smaller counterparts, which is expected to make them relatively more pledgeable in financial markets.

3 The Economic Environment

Consider a three period overlapping generations model with constant population. Every period a continuum of young agents with unit measure enter the economy, who live for three periods (young, middle age, old). Time is discrete and denoted as $t = 0, 1, 2, \ldots$. There is a single good which can be either invested or consumed.

An agent who is born in period $t$ becomes a financier with probability $\theta$ at the beginning of his lifetime. By being a financier he inherits a special type of "Lucas tree". The tree generates a consumption good flow for the young financier, at the beginning of period $t$, denoted as $e$. The consumption good can be stored for multiple periods.

Financiers who are born at time $t$ have preferences which are similar to Diamond-Dybvig (1983),

$$ u(c_t^y, c_{t+1}^m, c_{t+2}^o) = -\frac{(c_{t+1}^m + \phi c_{t+2}^o)^{-\gamma}}{\gamma} $$

(1)

For each financier:

$$ \phi = \begin{cases} 
0 & \text{with probability } \pi \\
1 & \text{with probability } (1 - \pi) 
\end{cases} $$

Unlike in Diamond-Dybvig (1983), I take the realization of $\pi$ as publicly observable, because it is not the purpose of this paper to study bank runs. This setting creates two types
of risk-neutral financiers: ”Impatient” and ”Patient”. \( \pi \) gets realized at the beginning of the life-time of an entrepreneur. Financiers participate in the credit market when young\(^2\). At each period, a fraction \( \pi \) of the new born financiers experience the utility shock.

Given the preference shock, our setting yields a simple \textbf{financier’s problem}: Impatient types lend all their initial endowment short term; and the patient types lend funds long term if long term interest rate \( (R_2) \) are higher than the short term interest rate \( (R_1) \).

Similarly, with probability \( (1 - \theta) \) an agent becomes an \textit{entrepreneur} (a firm owner) when he is young. Unlike financiers, entrepreneurs do not receive any consumption good flows during any period of their lifetime, therefore they borrow from financiers to be able to invest. Entrepreneurs are risk neutral and care about the third period consumption only. Each entrepreneur randomly inherits a firm at the beginning of his lifetime. The entrepreneur cannot trade his firm in a secondary market.

A young (firm owner) entrepreneur have access to a \textit{Short Term} and a \textit{Long Term} investment project. For now, the long term investment project does not generate enhancing effects (technological spillovers) on the aggregate productivity. Technological spillovers will be incorporated into the model in section 8 while studying the effects of public R&D policies.

Consider a firm owner \( j \) born in period \( t - 1 \). He can choose to operate a short term investment project when he is young, whose proceedings get realized in period \( t \) when he is middle aged. If he chooses the short term investment opportunity he will have access to a second short term investment opportunity when he is middle aged, whose proceedings get realized in period \( t + 1 \) when he is old.

The short term investment opportunity in period \( t - 1 \) is:

\[
 s_{tj}(x_{t-1}) = z_{tj}Bx_{t-1}^\alpha
\]

The entrepreneur’s first period maximization problem is given as:

\[
 \max_{x_{t-1}}\{s_{tj}(x_{t-1}) - R^j_1(t-1)x_{t-1}\}
\]

\( x_{t-1} \) is the total borrowing of the entrepreneur in period \( t - 1 \), \( R^j_1(t-1) \) is the short term interest rate.
interest rate charged. $B$ is the project specific (Short Term) productivity level, which is fixed among different types and across time. $z_{tj}$ is a parameter measuring the state of the firm specific productivity level in period $t$. $z_{tj}$ is drawn from a two-point distribution, namely $z_{tj} \in \{z_g, z_b\}$ with $z_g > z_b$. In each period a fraction $\gamma$ of the young entrepreneurial population (both small and large) draw the firm specific productivity level $z_g$. When an entrepreneur borrows in period $t-1$ (short term or long term), $z_{tj}$ is known by the entrepreneur and by the lender, however $z_{(t+1)j}$ is unobserved until the beginning of period $t$.

The proceedings of the first period short term investment project get re-invested into the second short term investment opportunity when the entrepreneur is middle aged yielding the following lifetime profit function:

$$V_{j}^{ST} \equiv E[\Pi_{t+1}^{ST}(j)] = E_j[\Pi_{t+1}^{ST}(j)]B(z_{tj}Bx_{t-1}^\alpha - R_{j}^{t}(t-1)x_{t-1})^\alpha$$  \hspace{1cm} \text{(2)}$$

**Assumption 1:** Only young entrepreneurs can engage in the borrowing activity. The proceedings of the first period short term investment project get re-invested in a second short term investment project.

Similarly, a young (firm owner) entrepreneur $j$ may undertake a long term investment project when young. The long term investment opportunity in period $t-1$ is given as:

$$l(k_{(t-1)j}) = E_j[\Pi_{t+1}^{LT}(j)]Ak_{t-1}^\alpha$$

The first period maximization problem is as follows:

$$\max_{k_{t-1}}\{l(k_{(t-1)j}) - R_{j}^{t}(t-1)k_{t-1}\}$$

where $k_{t-1}$ is the long term borrowing of the young entrepreneur in period $t - 1$. $R_{j}^{t}(t-1)$ is the long term interest rate charged in period $t - 1$. The proceedings of the long term investment opportunity initiated in period $t - 1$ get realized in period $t + 1$. $A$ is the project specific productivity level associated with long term investment opportunity, which is fixed for all types and across time. Note that $x$ and $k$ denote the same consumption good, but for notational convenience different letters are assigned to different funds investment.
Lifetime profits from investing in the Long Term investment opportunity is:

\[ V^\text{LT}_j \equiv E_j[\Pi^\text{LT}_{t+1}(j)] = E_j[z_{t+1}|z_{tj}]Ak_{t-1}^\alpha - R^1_{j}(t-1)k_{t-1} \]  \hspace{1cm} (3)

**Assumption 2:** \( A > B \).

Following Romer (1990), Aghion and Howitt (1992 and 1998), Aghion, Angeletos, Benarjee and Manova (2005) I assume that long term projects are more productive than short term projects.

I consider a simple debt contract as the only way of financing in this setting. \( R^1_j \) and \( R^2_j \) are respectively short term and long term interest rates charged to type \( j \) entrepreneurs. However, there’s no probability of failure in this setup\(^3\). Since there’s no risk associated with firm lending and debt is the only financing option, all borrowers face the same interest rates. Therefore for notational convenience we drop the firm specific superscript \( j \) from interest rates.

Entrepreneurs own two types of firms in the model, Small and Large. Firm ownership is determined randomly at the beginning of an entrepreneur’s lifetime. Firms cannot be traded out across agents. We introduce firm size heterogeneity with the following two assumptions:

**Assumption 3:** For a large firm owner \( j \), which has \( z_{tj} = z_k \) realized where \( z_k \in \{z_g, z_b\} \), the conditional expectation of \( E_{\text{Large}}[z_{t+1}|z_{tj}] \):

\[
E_{\text{Large}}[z_{t+1}|z_{tj}] = \bar{z}_k = \left( \frac{1}{2} + \mu \right) z_k + \left( \frac{1}{2} - \mu \right) z_n
\]

\[0 < \mu < 1/2\]

with \( k \neq n \). For a small firm owner \( i \) on the other hand, given \( z_{ti} = z_k \):

\[
E_{\text{Small}}[z_{t+1}|z_{ti}] = \bar{z} = \frac{1}{2} z_k + \frac{1}{2} z_n.
\]

Following the observation in the data, I assume, that large firms are more persistent in their productivity levels compared to their smaller counterparts; or in other words the prob-

\(^3\)I consider equilibria at which productivity levels are always high enough to cover the cost of capital borrowed.
ability of drawing $z_{g}$ for the last period productivity level is path dependent for large firms.

To distinguish large firms from small firms I make the following assumption as well:

**Assumption 4:** All large firms’ output are fully pledgeable, whereas only $\lambda_{i}$ portion of a small firm $i$’s output is pledgeable (for both short term production and long term production)

This assumption creates the following financial constraints. If a small firm $i$ borrows $m$ units in the credit market in period $t - 1$, then the total repayment cannot exceed:

$$\lambda_{i} z_{t} B m^\alpha \geq R_{1} m \quad (4)$$
$$\lambda_{i} \bar{z} A m^\alpha \geq R_{2} m \quad (5)$$

If constraints (4) and (5) are violated then the entrepreneur will not commit to repay his debt, and he will choose to default. With this I assume that there’s limited commitment to debt repayment in this economy. (4) is the borrowing constraint associated with short term investment and (5) is the borrowing constraint associated with long term investment. I assume that large firms are fully pledgeable; $\lambda_{l} = 1$ for all large firm $l$. $\lambda_{i}$ is distributed uniformly for small firms with the support $[0, \bar{\lambda}]$. $\lambda_{i}$ is perfectly observable.

Due to the lack of adequate level of pledgeability some small firms will be financially constrained in equilibrium.

**Lemma 3.1** (4) and (5) are binding whenever $\lambda_{i} < \alpha$.

**Proof** The unconstrained optimum short term and long term funds demanded can be derived as:

$$x = \left( \frac{z_{t} B \alpha}{R_{1}^{S}} \right)^{\frac{1}{1 - \alpha}} \quad (6)$$
$$k = \left( \frac{\bar{z} A \alpha}{R_{1}^{L}} \right)^{\frac{1}{1 - \alpha}} \quad (7)$$

Using (6) and (7) in (4) and (5) we can find that (4) and (5) are binding if $\lambda_{i} < \alpha$. Q.E.D.

An entrepreneur becomes a Large (Small) firm owner with probability $\eta ((1 - \eta))$ at the beginning of his lifetime. I do not incorporate firm dynamics into the model. A young small
firm owner continues to be a small firm owner when he is middle aged and old. I assume that small and large firms do differ from each other in the two dimensions listed above. An arbitrarily chosen large firm is more persistent and more pledgeable compared to a small firm.

In this setting an entrepreneur born in period \((t-1)\) is a triple as \(i \equiv \{\text{Firm Size}_i, z_{ti}, \lambda_i\}\) with:

\[
\text{Firm Size}_i \in \{\text{Small, Large}\}
\]

\[
z_{ti} \in \{z_g, z_b\}
\]

\[
\lambda_i \in [0, \bar{\lambda}].
\]

### 3.1 Firm’s Problem

Each entrepreneur \(i\) has to choose to operate a single investment project (short term or long term) when young, yielding a simple discrete choice problem as the following:

\[
\max \{ V_i^{ST}(x^*) , V_i^{LT}(k^*) \}
\]

If \(i\) is a small firm, then the maximization problem associated with short term investment yields:

\[
x_{\text{Small},i}^* \equiv \arg \max_x z_t B x^\alpha - R_1 x
\]

\[
\text{s.t. } \lambda_i z_t B x^\alpha \geq R_1 x.
\]

If \(i\) is a large firm, then \(x_{\text{Large},i}^*\) is the simple unconstrained optimum short term funds demand.

Similarly, for a small firm \(i\), the maximization problem associated with long term investment is:

\[
k_{\text{Large},i}^* \equiv \arg \max_k \bar{z} A k^\alpha - R_2 k
\]

\[
\text{s.t. } \lambda_i \bar{z} A k^\alpha \geq R_2 k
\]

, whereas for a large firm \(i\) \(k_{\text{Large},i}^*\) is the simple unconstrained optimum long term funds demand.
3.2 The Credit Market

Financiers and firm owners interact in the credit market. In equilibrium all arbitrage opportunities will be wiped out for lenders. The equilibrium in the credit market will determine the equilibrium for this setting. In equilibrium aggregate loan demand equals to aggregate loan supply for both short term and long term loans. Throughout the paper I will be interested in equilibria in which both short term and long term lending exists, which will be satisfied as long as the equilibrium long term interest rate $R^*_2$ exceeds the equilibrium short term interest rate $R^*_1$.

The time-line of events in the model is as the following:

1. Publicly observable preference shocks of financiers, and productivity and firm ownership shocks of entrepreneurs get realized.
2. Young entrepreneurs borrow short term or long term funds from young financiers.
3. Young entrepreneurs invest short term funds in short term investment projects or long term funds in long term investment projects.
4. Project returns get realized.
5. Repayment of debt takes place.
6. Consumption takes place.

I will provide three versions of the model to grasp intuition regarding the channels which will potentially affect the results. The first model (section 4) analyzes an economy in which small firms are more volatile than large firms; however for all small firms the level of pledgeability is fixed at $\lambda$. This model shows that, for low levels of $\lambda$, small firms are more keen in investing short term compared to large firms. Section 5 provides a model in which small firms vary across each other in both volatility and the level pledgeability, however investment size is fixed and given exogenously for all small firms. In this model, fixed investment size creates credit rationing in equilibrium. The size of the investment determines which small firms can borrow in the credit market. Section 6 provides a solution to the full model with varying volatility and pledgeability across firms. In this final version of the model investment size and portfolio choice are determined endogenously.
4 Fixed level of Pledgeability ($\lambda$)

Suppose for small firms the level of contract enforcement is constant at $\lambda$ and for large firms it is constant at $\lambda_l = 1$. Then (4) and (5) become:

$$\lambda z_t B_m^\alpha \geq R_1 m$$  \hspace{1cm} (8)
$$\lambda E[z_{t+1}] A_m^\alpha \geq R_2 m$$  \hspace{1cm} (9)

I also assume:

**Assumption 5:** $\lambda < \alpha$.

Following Lemma 2.1, Assumption 4 makes all small firms financially constrained.

For small firms the lifetime value functions associated with short term ($V_{ST}$) and long term ($V_{LT}$) investment given the realization of $z_j$ in period $(t - 1)$, can be derived as the following:

$$V_{ST}^{Small}(z_j) = \bar{z} B_{1-\alpha}^{1-\alpha} (z_j)^{\alpha\alpha} (1 - \lambda)^{\alpha} (R_1^l)^{\frac{\alpha}{1-\alpha}}$$  \hspace{1cm} (10)
$$V_{LT}^{Small}(z_j) = (\bar{z} A)^{1-\alpha} \lambda^{1-\alpha} (1 - \lambda) (R_2^l)^{\frac{\alpha}{1-\alpha}}$$  \hspace{1cm} (11)

Similarly for large firms:

$$V_{ST}^{Large}(z_j) = \bar{z}_j B_{1-\alpha}^{1-\alpha} (z_j)^{\alpha\alpha} (1 - \alpha)^{\alpha} (R_1^l)^{\frac{\alpha}{1-\alpha}}$$  \hspace{1cm} (12)
$$V_{LT}^{Large}(z_j) = (\bar{z}_j A)^{1-\alpha} \alpha^{1-\alpha} (1 - \alpha) (R_2^l)^{\frac{\alpha}{1-\alpha}}$$  \hspace{1cm} (13)

From (10) and (11) with the Assumption 1 we can capture the following:

$$V_{ST}^{Small}(z_g) > V_{ST}^{Small}(z_b)$$
$$V_{LT}^{Small}(z_g) = V_{LT}^{Small}(z_b)$$

Also, from (12) and (13):

$$V_{ST}^{Large}(z_g) > V_{ST}^{Large}(z_b)$$
\[ V^{LT}_{\text{Large}}(z_g) > V^{LT}_{\text{Large}}(z_b) \]

We can also observe that:

\[ V^{ST}_{\text{Large}}(z_g) > V^{ST}_{\text{Small}}(z_g) > V^{ST}_{\text{Small}}(z_b) \]
\[ V^{LT}_{\text{Large}}(z_g) > V^{LT}_{\text{Small}}(z_g) = V^{LT}_{\text{Small}}(z_b) \]

However, we cannot rank \( V^{LT}_{\text{Large}}(z_b) \) with \( V^{LT}_{\text{Small}}(z_g) = V^{LT}_{\text{Small}}(z_b) \); and \( V^{ST}_{\text{Large}}(z_g) \) with \( V^{ST}_{\text{Small}}(z_g) \) and \( V^{ST}_{\text{Small}}(z_b) \).

### 4.1 Equilibria

Consider the following type of groups and the corresponding ratios:

1. \((\text{large} - z_b) : r_1 = V^{LT}_{\text{Large}}(z_{t+1} = z_b)/V^{ST}_{\text{Large}}(z_{t+1} = z_b)\)
2. \((\text{large} - z_g) : r_2 = V^{LT}_{\text{Large}}(z_{t+1} = z_g)/V^{ST}_{\text{Large}}(z_{t+1} = z_g)\)
3. \((\text{small} - z_b) : r_3 = V^{LT}_{\text{Small}}(z_{t+1} = z_b)/V^{ST}_{\text{Small}}(z_{t+1} = z_b)\)
4. \((\text{small} - z_g) : r_4 = V^{LT}_{\text{Small}}(z_{t+1} = z_g)/V^{ST}_{\text{Small}}(z_{t+1} = z_g)\)

The first group (\((\text{large} - z_g) \) (a young large firm owner who is born in period \((t - 1)\) with \(z_t = z_g \) realized)) is relatively more keen to take long term investment (less keen in short term investment), whereas the fourth group is relatively less keen in long term investment (more keen in short term investment). However, as noted in the previous subsection that we cannot get the ranking \( r_1 > r_2 > r_3 > r_4 \) for all parameter values.

**Proposition 4.1** The “preference ranking”, \( r_1 > r_2 > r_3 > r_4 \) holds iff

\[ \frac{V^{LT}_{\text{Large}}(z_g)}{V^{ST}_{\text{Large}}(z_g)} > \frac{V^{LT}_{\text{Small}}(z_b)}{V^{ST}_{\text{Small}}(z_b)} \tag{14} \]

and \((14)\) holds only if:

\[ \left( \frac{\alpha}{\lambda} \right)^\alpha \left( \frac{1 - \alpha}{1 - \lambda} \right)^{(1-\alpha)} > \left( \frac{\bar{z}/z_g}{z_b/z_g} \right) \]

**Lemma 4.2** The LHS of the inequality \((14)\) increases monotonically as \( \lambda \) decreases. The RHS decreases monotonically as the ratio \( z^b/\bar{z} \) increases.
This lemma shows that, small firm owners born in period \((t - 1)\) who draw a \(z_b\) type firm productivity in the same period are keen in investing short term at the beginning of period \((t - 1)\) if either the level of contract enforcement is weak or the volatility in firm specific productivity is small. The reason why high volatility makes some small firms willing to invest long term is the following: If the realization of \(z_t\) is low for a young agent, he still has a chance to have a relatively high state realization in the next period if the volatility in \(z\) is high enough. Diamond (1991) has also pointed out that firms with low reputation would choose to issue long term debt. In our setting different from Diamond (1991), the borrowing behavior affects the investment choice and the productivity level of the firm as well.

Suppose (14) holds. Then an equilibrium can occur with

1. (Case 1) Only \((Large - z_b)\) types investing in long term projects,
2. (Case 2) Only \((Large - z_b)\) and \((large - z_g)\) investing in long term projects,
3. (Case 3) \((Large - z_b), (Large - z_g)\) and \((Small - z_b)\) investing in long term projects.
4. (Case 4) All firms investing in long term projects.

4.2 Equilibrium at ”Case 3”

Suppose all the top 3 groups \(((Large - z_b), (Large - z_g)\) and \((Small - z_b))\) invest in long term investment projects, whereas the bottom in short term investment projects. And suppose the equilibrium occurs such that, one type group of entrepreneurs are indifferent between investing in short term and long term projects in equilibrium. Parameter conditions which support this equilibrium will be derived at the end.

The characterization of the equilibrium will be given by:

\[
V^{ST}_{small}(z_b) = V^{LT}_{small}(z_b) \tag{15}
\]

which implies

\[
\bar{z}B^{\frac{1}{1-\alpha}}(z_b)^{\frac{\alpha}{1-\alpha}}\lambda^{\frac{\alpha^2}{1-\alpha}}(1 - \lambda)^{\alpha}(R_1)^{\frac{-\alpha^2}{1-\alpha}} = (\bar{z}A)^{\frac{1}{1-\alpha}}\lambda^{\frac{\alpha}{1-\alpha}}(1 - \lambda)(R_2)^{\frac{-\alpha}{1-\alpha}} \tag{16}
\]

(16) can be re-written as:

\[
\left(\frac{\bar{z}}{z_b}\right)^{\frac{\alpha}{1-\alpha}}\left(\frac{A}{B}\right)^{\frac{1}{1-\alpha}}\lambda^{\alpha}(1 - \lambda)^{1-\alpha} = \left(\frac{R_2}{R_1^\alpha}\right)^{\frac{-\alpha}{1-\alpha}} \tag{17}
\]
The Intuition is straightforward. In order to be able to characterize the equilibrium, we need to figure out the identity of the marginal loan demander, the entrepreneur who is indifferent between investing in short term and long term. Until this entrepreneur is indifferent between investing short term and long term, patient (long term lenders) and impatient (short term lenders) types will raise the short term and long term interest rates respectively.

In order (17) to characterize the equilibrium, the equilibrium interest rates should meet some conditions:

\[
\begin{align*}
R_1 &\geq 1 \\
R_2 &\geq 1 \\
R_2 &\geq R_1
\end{align*}
\]

(18) \hspace{1cm} (19) \hspace{1cm} (20)

Definition The dynamic competitive interior\(^4\) equilibrium of the economy is a stream of short term and long term interest rates, \{\(R_1(t)^*, R_2(t)^*\)\}_{t=0}^{\infty}\), at which firms maximize profits subject to (8) and (9) and financiers maximize utility subject to their lifetime income, markets clear and conditions (18), (19) and (20) hold.

The following parameter conditions will ensure that there exists an equilibrium at ”Case 3” with \((small - z_b)\) types being indifferent between short term and long term investment, satisfying (18), (19) and (20):

First, we will derive parameter conditions such that (20) holds. We will ensure that the same parameters will rule out equilibria at Case 1 and at Case 2.

Define the following interest rates using the market ”clearance conditions”:

\[
\begin{align*}
(R_2)_{\text{min}} &= \left(\frac{1 - \theta}{\eta e(1 - \pi)\theta}\right)^{1-\alpha} A(\gamma(\bar{z}_g)^{\frac{1}{1-\alpha}} + (1 - \gamma)(\bar{z}_b)^{\frac{1}{1-\alpha}})^{1-\alpha} \\
(R_1)_{\text{max}} &= \left(\frac{1 - \theta}{\eta e\pi\theta}\right)^{1-\alpha} \left(\gamma(B\lambda\bar{z}_g)^{\frac{1}{1-\alpha}} + (1 - \gamma)(B\lambda\bar{z}_b)^{\frac{1}{1-\alpha}}\right)^{1-\alpha}
\end{align*}
\]

(21) \hspace{1cm} (22)

\(^4\)Both short term and long term lending/borrowing exists.
Define:

\[ \bar{Z}_1 \equiv \left( \gamma \left( \bar{z}_{g} \right) \frac{1}{1-\alpha} + (1 - \gamma) \left( \bar{z}_{b} \right) \frac{1}{1-\alpha} \right)^{1-\alpha} \]

\[ \bar{Z}_2 \equiv \left( \gamma \frac{\bar{z}_{g}}{1-\alpha} + (1 - \gamma) \frac{\bar{z}_{b}}{1-\alpha} \right)^{1-\alpha} \]

\((R_2)_{\text{min}}\) and \((R_1)_{\text{max}}\) are lower and upper bounds for \( R_2 \) and \( R_2 \) respectively. For these values, there is "relatively" more abundant demand for short term loanable funds, and less abundant demand for long term loanable funds. Also consider:

\[(R_2)_{\text{max}} = \left( \eta \frac{1 - \theta}{e(1 - \pi)\theta} \left( (A\alpha \bar{Z}_1) + (1 - \eta)(1 - \gamma) \frac{1 - \theta}{e(1 - \pi)\theta} (A\lambda \bar{Z}) \right)^{1-\alpha} \right) \]  

(23)

\[(R_1)_{\text{min}} = \left( (1 - \eta) \frac{1 - \theta}{e\pi\theta} \gamma (B\lambda z_g) \right)^{1-\alpha} \]  

(24)

If the parameters, defining \((R_2)_{\text{min}}\) and \((R_1)_{\text{max}}\), \((R_2)_{\text{min}}\) and \((R_1)_{\text{max}}\) satisfy:

\[(R_2)_{\text{min}} < (R_1)_{\text{max}} \]  

(25)

\[(R_2)_{\text{max}} > (R_1)_{\text{min}} \]  

(26)

then we can have an equilibrium at "Case 3".

These parameter conditions can be summarized as the following:

(25) holds if:

\[ \eta < \frac{\lambda B \bar{Z}_2}{\alpha A \bar{Z}_1 + \lambda B \bar{Z}_2} \equiv \eta^{**} \]

And (26) holds if:

\[ \eta > \frac{\lambda B \bar{Z}_2 - 2(1 - \gamma)(B\lambda z_b) \frac{1}{1-\alpha}}{\alpha A \bar{Z}_1 + \lambda B \bar{Z}_2 - 2(1 - \gamma)(B\lambda z_b) \frac{1}{1-\alpha}} \equiv \eta^* \]

If (25) and (26) hold, we can have an equilibrium at Case 3. Notice, since \((R_2)_{\text{min}} < (R_1)_{\text{max}}\), under these parameter conditions we cannot have equilibria at Case 1 and Case 2, which would violate (20).

We should also determine how to rule out equilibria at Case 4, given the parameter conditions determined above.
The equilibrium interest rates should be such that, (18) and (19) hold. Consider:

\[
\left( \frac{\bar{z}}{z_g} \right)^{\frac{\alpha}{1-\alpha}} a^{\frac{1}{1-\alpha}} \lambda^\alpha (1 - \lambda)^{1-\alpha} = \left( \frac{R_1}{R_1^\alpha} \right)^{\frac{\alpha}{1-\alpha}} < \left( \frac{R_2}{R_1^\alpha} \right)^{\frac{\alpha}{1-\alpha}}
\] (27)

Suppose, (20) is binding with the abuse of language. Then an equilibrium cannot occur at Case 4 if

\[
\left[ \left( \frac{\bar{z}}{z_g} \right)^{\frac{\alpha}{1-\alpha}} a^{\frac{1}{1-\alpha}} \lambda^\alpha (1 - \lambda)^{1-\alpha} \right]^{\frac{1}{\alpha}} < 1
\] (28)

To ensure the existence of an equilibrium at "Case 3" satisfying (18) and (19), we should assume:

\[
\left[ \left( \frac{\bar{z}}{z_b} \right)^{\frac{\alpha}{1-\alpha}} a^{\frac{1}{1-\alpha}} \lambda^\alpha (1 - \lambda)^{1-\alpha} \right]^{\frac{1}{\alpha}} > 1
\] (29)

Note that, (28) and (29) are sufficient but not necessary.

**Proposition 4.3** There exists an equilibrium at "Case 3" characterized as

\[
\left( \frac{\bar{z}}{z_b} \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{A}{B} \right)^{\frac{1}{1-\alpha}} \lambda^\alpha (1 - \lambda)^{1-\alpha} = \left( \frac{R_2}{R_1^\alpha} \right)^{\frac{\alpha}{1-\alpha}}
\]

If

\((R_2)_{\text{min}} < (R_1)_{\text{max}}\)

\((R_2)_{\text{max}} > (R_1)_{\text{min}}\)

or

\(\eta^* < \eta < \eta^{**}\)

and

\[
\left[ \left( \frac{\bar{z}}{z_g} \right)^{\frac{\alpha}{1-\alpha}} a^{\frac{1}{1-\alpha}} \lambda^\alpha (1 - \lambda)^{1-\alpha} \right]^{\frac{1}{\alpha}} < 1
\]

\[
\left[ \left( \frac{\bar{z}}{z_b} \right)^{\frac{\alpha}{1-\alpha}} a^{\frac{1}{1-\alpha}} \lambda^\alpha (1 - \lambda)^{1-\alpha} \right]^{\frac{1}{\alpha}} > 1.
\]

When the equilibrium is characterized as in (17), then the level of contract enforcement has an effect on both interest rates and firm level investment composition. From (17) we can see that as \(\lambda\) rises, then holding the short term interest rate \(R_1\) constant, the long term
interest rate $R_2$ increases. The reason for this rise is that the increase in the level of contract enforcement makes small firms less keen to take short term investment projects to build up assets. If the rise in $\lambda$ is big enough, then the economy might experience an equilibrium switch, where in the new equilibrium ($small - z_g$) types invest in long term investment projects as well.

As a thought experiment let’s consider the following: Suppose instead of a uniform level of pledgeability $\lambda$ across investment projects, for all small firms short term invested funds can be seized at a higher rate than long term invested funds. That means, the longer the maturity of the investment project the harder it becomes to separate the capital from the entrepreneur. Formally, suppose $\lambda_1$ is the level of pledgeability associated with short term investment projects and $\lambda_2$ is the level of pledgeability associated with long term investment projects.

**Assumption 5’:** $\alpha > \lambda_1 > \lambda_2$.

If this assumption holds, a Case 3 equilibrium can be characterized with:

$$\bar{z}B^{1-\alpha}(z)\frac{\alpha}{\alpha-\mu}(1-\lambda_1)^{\alpha}(R_1)^{\alpha-\mu} = (\bar{z}A)^{1-\alpha}\lambda_2^{\alpha}(1-\lambda_2)(R_2)^{-\alpha}$$

(30)

Since $\lambda_1 > \lambda_2$, and both the LHS and the RHS of (30) are increasing in $\lambda_1$ and $\lambda_2$ respectively, $R_2/R_1$ ratio would be smaller than if $\lambda_1$ were equal to $\lambda_2$. The intuition is simple, $\lambda$ constrains the demand for loanable funds. The wedge between $\lambda_1$ and $\lambda_2$ creates a wedge between the short term and long term loan demands compared to the case where $\lambda_1 = \lambda_2$. Since, the equality (30) determines the equilibrium, the rise in $R_1/R_2$ corrects the drop in relative loan demands.

## 5 Fixed Investment Size For Small Firms, Varying $\lambda_i$’s

In this section I consider a model in which small firms have fixed investment production technologies, such as:

$$s(m) = z_tBm$$

$$l(m) = E[z_{t+1}]Am$$

where $m$ is the fixed investment size for both short term ($s(m)$) and long term ($l(m)$) production technologies and it is exogenously given.
Large firms on the other hand have concave production technologies.

**Assumption 5**: For small firms $\lambda_i$ is distributed uniformly with the support $[0, \bar{\lambda}]$, with $\bar{\lambda} < \alpha$; for large firms, $\lambda_l = 1$.

Finally, suppose that the parameter conditions hold, such that $(z_g - large)$ and $(z_g - small)$ types invest short term and $(z_b - large)$ and $(z_b - small)$ invest long term.

In this case, the equilibrium interest rates are pinned down by the marginal person, who is being able to borrow in the credit market:

$$ R_1 = z_g B \lambda_1^c $$
$$ R_2 = \bar{z} A \lambda_2^c $$

where $\lambda_2^c$ is the critical (threshold) firm in the distribution of small firms borrowing short term, where by the construction of the equilibrium all types are $z_g$. If a $(small - z_g)$ type has a pledgeability smaller than $\lambda_2^c$, he can not borrow short term, and gets credit rationed. Similarly, $\lambda_1^c$ is the critical (threshold) firm in the distribution of small firms, where all types are $z_b$. Below $\lambda_1^c$ non of the $z_b$ types can borrow long term. The fixed investment size creates credit rationing in equilibrium. The equilibrium can be characterized as the following:

The long term loanable funds market clearence:

$$ \eta (1 - \gamma) \left( \frac{\alpha z_b}{\bar{z} \lambda_1^c} \right)^{\frac{1}{1-\alpha}} + (1 - \eta) (1 - \gamma) (\bar{\lambda} - \lambda_1^c) m = \frac{e(1 - \pi) \theta}{1 - \theta} \quad (31) $$

The short term loanable funds market clearence:

$$ \eta \gamma \left( \frac{\alpha}{\lambda_2^c} \right)^{\frac{1}{1-\alpha}} + (1 - \eta) \gamma (\bar{\lambda} - \lambda_2^c) m = \frac{e \pi \theta}{1 - \theta} \quad (32) $$

Following the characterization of the equilibrium we can have the following two results

**Proposition 5.1** From (31) and (32) we can observe that if $\pi = \gamma = \eta = 1/2$, then $\lambda_2^c > \lambda_1^c$.

**Proposition 5.2** The higher the fixed investment size $m$, the smaller is the measure of small firms (both $\lambda_1^c$ and $\lambda_2^c$) who can participate in the credit market.

Without knowledge of $\alpha$ we cannot solve for $\lambda_i^c$ explicitly.
In order the characterized solution to have as an equilibrium, interest rates should satisfy:

\[ R_2 > R_1 \rightarrow \frac{\lambda c_1}{\lambda c_2} < \frac{z^gB}{\bar{z}A} \]

\[ R_2 > R_1 > 1 \rightarrow \lambda c_1 > \frac{1}{z^gB} \]

\( z^g - small \) is willing to invest in S.T. if:

\[ A(1 - \lambda c_2) < z_gB^2(1 - \lambda c_1) \quad (33) \]

\( z^b - small \) is willing to invest in L.T. if:

\[ A(1 - \lambda c_2) > z_bB^2(1 - \lambda c_1) \quad (34) \]

\( z^g - large \) is willing to invest in S.T. if:

\[ (1 - \alpha)A\bar{z}^g\left(\frac{\alpha\bar{z}_g}{\lambda c_2\bar{z}}\right)^{\frac{\alpha}{1-\alpha}} < \bar{z}_gB \left[ (1 - \alpha)z_gB \left( \frac{\alpha}{\lambda c_2} \right)^{\frac{\alpha}{1-\alpha}} \right]^\alpha \quad (35) \]

\( z^b - large \) is willing to invest in S.T. if:

\[ (1 - \alpha)A\bar{z}_b\left(\frac{\alpha\bar{z}_b}{\lambda c_2\bar{z}}\right)^{\frac{\alpha}{1-\alpha}} > \bar{z}_bB \left[ (1 - \alpha)z_bB \left( \frac{\alpha}{\lambda c_2} \right)^{\frac{\alpha}{1-\alpha}} \right]^\alpha \quad (36) \]

6 The Solution to the Full Model

In this section I let firm level investment size and investment portfolio determined endogenously. Assumption 5" holds. I am interested in the characterization of an equilibrium where all the large firms and at least some fraction of small firms borrow and invest long term.

Assumption 6:

\[ \left( \frac{\alpha}{\lambda} \right)^\alpha \left( \frac{1 - \alpha}{1 - \lambda} \right)^{(1-\alpha)} > \left( \frac{\bar{z}/\bar{z}^g}{z^b/z^g} \right) \]

The assumption 6 ensures that, "the best type" in the group (small-\( z_b \)) prefers short term investment whenever the entrepreneurs in (large-\( z_b \)) is indifferent between short term and long term investment. Also, assume that \( \bar{\lambda} < \alpha \). Following this, in equilibrium \( (1 - \lambda c_1) \) fraction of small firms which have \( z_b \) realized and \( (1 - \lambda c_2) \) fraction of small firms which have \( z_g \) realized will invest in long term projects, whereas the rest invests in short term investment.
projects.

**Definition** The dynamic competitive interior equilibrium of the economy is a stream of short term and long term interest rates, \( \{R_1(t^*), R_2(t^*)\}_{t=0}^{\infty} \), at which firms maximize profits subject to (4) and (5) and financiers maximize utility subject to their lifetime income, markets clear and conditions (18), (19) and (20) hold.

The general equilibrium conditions will be given as the following:

For the long term loanable funds market:

\[
\frac{e(1-\pi)\theta}{1-\theta} = \eta \left( \gamma \left( \frac{\alpha A\bar{z}_2}{R_2} \right)^{\frac{1}{1-\alpha}} + (1-\gamma) \left( \frac{\alpha A\bar{z}_b}{R_2} \right)^{\frac{1}{1-\alpha}} \right) + (1-\eta) \left( \gamma \int_{\lambda_2^l}^{\lambda_2^h} \left( \frac{\lambda_i A\bar{z}_2}{R_2} \right)^{\frac{1}{1-\alpha}} d\lambda_i + (1-\gamma) \int_{\lambda_1^l}^{\lambda_1^h} \left( \frac{\lambda_i A\bar{z}_b}{R_2} \right)^{\frac{1}{1-\alpha}} d\lambda_i \right)
\]

And for the short term loanable funds market we have:

\[
(1-\eta) \left( \gamma \int_{0}^{\lambda_2^l} \left( \frac{\lambda_i B\bar{z}_2}{R_2} \right)^{\frac{1}{1-\alpha}} d\lambda_i + (1-\gamma) \int_{0}^{\lambda_1^l} \left( \frac{\lambda_i B\bar{z}_b}{R_2} \right)^{\frac{1}{1-\alpha}} d\lambda_i \right) = \left( \frac{e\pi\theta}{1-\theta} \right)
\]

From (37) and (38) we can capture the following result:

**Lemma 6.1** There exist an upper bound \( \eta'' \) and a lower bound \( \eta' \) such that for

\[
\eta' \leq \eta \leq \eta''
\]

all large firms invest in long term investment projects, and at least a fraction of small firms invest in long term investment projects.

We can re-write (37) and (38) as:

\[
\frac{e(1-\pi)\theta}{1-\theta} = \eta \left( \gamma \left( \frac{\alpha A\bar{z}_2}{R_2} \right)^{\frac{1}{1-\alpha}} + (1-\gamma) \left( \frac{\alpha A\bar{z}_b}{R_2} \right)^{\frac{1}{1-\alpha}} \right) + (1-\eta) \left( \gamma \left( \frac{\lambda_i A\bar{z}_2}{2R_2} \right)^{\frac{1}{1-\alpha}} + (1-\gamma)(1-\lambda_2^l) \left( \frac{\lambda_i A\bar{z}_b}{2R_2} \right)^{\frac{1}{1-\alpha}} \right)
\]
and
\[
(1 - \eta) \left( \gamma (\lambda_2^c) \left( \frac{\lambda_2^c B z_g}{2 R_2} \right)^{\frac{1}{\alpha}} + (1 - \gamma) (\lambda_1^c) \left( \frac{\lambda_1^c B z_b}{2 R_2} \right)^{\frac{1}{\alpha}} \right) = \left( \frac{e \pi \theta}{1 - \theta} \right) \quad (40)
\]

As one can guess, there is a critical person \( \lambda_1^c \) in group \((small - z^b)\) and a critical person \( \lambda_2^c \) in group \((small - z^g)\), for whom the following holds:

\[
V_{ST_{Small}}(z_b, \lambda_1^c) = V_{LT_{Small}}(z_b, \lambda_1^c) \quad (41)
\]
\[
V_{ST_{Small}}(z_g, \lambda_2^c) = V_{LT_{Small}}(z_g, \lambda_2^c) \quad (42)
\]

(41) and (42) can be re-written as:

\[
\left( \frac{\tilde{z}}{z_b} \right)^{\frac{\alpha}{\alpha - 1}} \left( \frac{A}{B} \right) \left( \lambda_1^c \right)^{\alpha} (1 - \lambda_1^c)^{1 - \alpha} = \left( \frac{R_2}{R_1} \right)^{\frac{1}{\alpha}} \quad (43)
\]
\[
\left( \frac{\tilde{z}}{z_g} \right)^{\frac{\alpha}{\alpha - 1}} \left( \frac{A}{B} \right) \left( \lambda_2^c \right)^{\alpha} (1 - \lambda_2^c)^{1 - \alpha} = \left( \frac{R_2}{R_1} \right)^{\frac{1}{\alpha}} \quad (44)
\]

**Lemma 6.2 (Existence)** If

\[
\left( \frac{e \theta}{1 - \theta} \right) > 0 \quad (45)
\]

and

\[
0 < \pi < 1 \quad (46)
\]

there exists an equilibrium.

**Proof** (45) and (46) ensure a positive supply of funds, which is enough to satisfy the existence in the setting described above.

**Lemma 6.3 (Uniqueness)** The equilibrium of the system of equations governed by (38), (37), (43) and (44) is unique.

**Proof** RHSs of (38) and (37) are monotonically decreasing in \( R_1 \) and \( R_2 \), whereas LHSs are constant. Therefore (38) and (37) pin down the unique equilibrium interest rates. Given the equilibrium interest rates, (43) and (44) pin down \( \lambda_1^c \) and \( \lambda_2^c \). Q.E.D.

Since \( z_g > z_b \), from (43) and (44) we can derive the following result.
Proposition 6.4 In equilibrium, $\lambda_2 > \lambda_1$. That’s the fraction of $(small - z_g)$ types who invest long term is smaller than the fraction of $(small - z_b)$.

This result can be interpreted as the following: The level of pledgeability of a firm, determines its investment composition in an imperfect credit markets economy, which in return has a direct effect on its productivity level. Regardless of its draw from $z_t \in \{z_g, z_b\}$ in period $(t-1)$, a firm which is highly pledgeable chooses to borrow and invest long term. On the other hand, regardless of the realization of $z_t$, a firm which has a very low $\lambda_i$ invests short term to build up assets.

This result shows that, when there are credit market imperfections; for small firms the investment composition, hence the productivity level, is sensitive to the firm level pledgeability. Since for all large firms $\lambda_l > \alpha$, credit market imperfections do not have a direct consequence on their investment composition. However, since it is a general equilibrium model, there is a feedback effect on the allocation of capital through the equilibrium interest rates. Since both short term and long term interest rates are low in an imperfect credit market economy, large firms are over-sized compared to a frictionless economy.

From (43) and (44), we can see that the larger the ratio $z_b/z_g$, the closer are $\lambda_1$ and $\lambda_2$ to each other. That means, excluding large firms, the level of cross-sectional variance affects the distribution of types in loanable funds markets. If the volatility is high, $z_g$ type firms (firms owned by young entrepreneurs who draw $z_t = z_g$ in period $(t-1)$) self-select themselves to short term loanable funds market, whereas $z_b$ type firms to long term loanable funds market.

7 Calibration

7.1 Calibration Strategy

We calibrate the model for target moments we observe in the U.S. data. We define a small firm as a firm which has less than 500 employees, a large firm as a firm which more than 5000 employees:

For the time period 1985-2005, we observe the following time-series averages in Compustat database.:

1. $(\text{Total Short Term}^5 \text{ Loans Outstanding} / \text{Total Loans Outstanding}) = 0.28$

2. $(\text{Total Short Term Loans Outstanding} / \text{Total Loans Outstanding})_{\text{Small Firms}} = 0.3$
3. \( \text{(Total Short Term Loans Outstanding / Total Loans Outstanding)}_{\text{Large Firms}} = 0.2 \)

4. Population Ratio of Large Firms = 0.15.

In the equilibrium we have presented for the full model, all large firms borrow long term only. However, as we observe in the data, aggregate short term borrowing of large firms is not zero. To be able to capture this empirical fact in our calibrated model, we extend the benchmark model in one further dimension. We assume that large firms is a heterogeneous group in terms of the productive volatility. That’s a fraction of young large firms have the persistence parameter \( \mu_h \), whereas the rest have \( \mu_l \), instead of all large firms having \( \mu \) as in the benchmark model. We assume the following:

\textbf{Assumption 7:} \( \mu_h > \mu_l \).

In this setting, we can choose \( \mu_h, \mu_l \) and \( s \) appropriately, such that 20% of large firms invest short term when they are young.

We do not face a similar problem for small firms. The heterogeneity in firm level pledge-ability \( \lambda_i \) helps us to calibrate the model for the target aggregate debt structure (short term/long term) of small firms.

### 7.2 Benchmark Calibration

For the same time period (1985-2005), using the average monthly fed funds rate we target:

\[
R_{S.T.} = 1.05,
\]

and using average 5 year T-Bill rates, we target:

\[
R_{L.T.} = 1.07,
\]

We set our period length in the model as 18 years to convert the annual short term and long term interest rates to the model short term \( (R_1) \) and long term \( (R_2) \) interest rates.

We choose \( \alpha \), the share of capital in production technologies, as the commonly used value 0.36; and the population share of financiers (non-entrepreneurs) as 0.9 which can be observed in the aggregate U.S. data.
Some model parameters are normalized. We normalize the population ratio of young-$z_g$ types ($\gamma$) to 0.5, and choose the ”persistence parameter” of large firms which are less volatile ($\mu_h$) as 0.25 and the persistence parameter of more volatile large firms ($\mu_l$) as 0. The population share of $\mu_h$ types is normalized to be $s = 0.6$. The initial endowment level of young financiers ($e$) is chosen as 0.05. The rest of the model parameters get calibrated to match the 6 moment conditions mentioned above. The calibration results are as the following:

The total fraction of outstanding short term loans helps us to calibrate, the population share of impatient types as 22 %. The productivity parameter of long term production technology gets calibrated as 3.3892, whereas that of short term production technology as 1.1876. $z_g$ the short term ”high productivity” shock is determined as 1.8946. In the calibrated model, all small-$z_g$ types choose to invest short term, which gives rise to $\lambda_g = 0.36$. A big fraction of small-$z_b$ types invest long term, which gives $\lambda_b = 0.001$. Also, in the large firms class; all $\mu_l-z_g$ (high volatility $z_g$ type large firms) choose to invest short term, whereas the rest of the large firms choose to invest long term. The population share of young entrepreneurs who draw the good productivity shock $z_g$ in the first period is calibrated as $\gamma = 0.3528$. The calibration results can be found in the table 2.
Table 2. Benchmark Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated-Normalized Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>Population Share of Financiers</td>
<td>0.9</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Population Share of Large Firms</td>
<td>0.15</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Population Share of Young Good Types</td>
<td>0.5 (Normalized)</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Population Share of Impatient Financiers</td>
<td>0.22 (Calibrated)</td>
</tr>
<tr>
<td>$s$</td>
<td>Population Share of Persistent Large Firms</td>
<td>0.6 (Normalized)</td>
</tr>
<tr>
<td>$\mu_h$</td>
<td>High Persistency Parameter</td>
<td>0.25 (Normalized)</td>
</tr>
<tr>
<td>$\mu_l$</td>
<td>Low Persistency Parameter</td>
<td>0 (Normalized)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Share of Capital</td>
<td>0.36 (Normalized)</td>
</tr>
<tr>
<td>$A$</td>
<td>Long Term Productivity</td>
<td>3.3892 (Calibrated)</td>
</tr>
<tr>
<td>$B$</td>
<td>Short Term Productivity</td>
<td>1.1876 (Calibrated)</td>
</tr>
<tr>
<td>$z_g$</td>
<td>Good Productivity Shock</td>
<td>1.8946 (Calibrated)</td>
</tr>
<tr>
<td>$z_b$</td>
<td>Bad Productivity Shock</td>
<td>0.1054 (Calibrated)</td>
</tr>
</tbody>
</table>

### 7.3 Comparative Statics

The following comparative statics results are based on the benchmark calibration of the model:

1. A 1% rise in the parameter $A$ leads to:
   - 0.0001 % decrease in $\lambda_b$.
   - 0.0001 % decrease in $R_1$.
   - 0.71 % increase in $R_2$.

2. A 1% rise in the parameter $B$ leads to:
   - 0.0004 % increase in $\lambda_b$.
   - 1 % increase in $R_1$.
   - 0.0009 % decrease in $R_2$.

3. A 1% rise in the parameter $z_g$ leads to:
   - 0.007 % decrease in $\lambda_b$.
   - 1 % increase in $R_1$.
   - 0.41 % decrease in $R_2$. 
These results are intuitive. The changes in productivity parameters directly affect the conditions in loanable funds markets with which they are associated. The reason why changes in $B$ are more effective on market conditions ($R_1$, $R_2$ and $\lambda_b$) compared to the effects of changes in $A$ is the re-investment opportunity of short term investment projects.

4. A 1% rise in the parameter $\mu_h$ leads to:
   - 0.0001 % decrease in $\lambda_b$.
   - 0.0001 % decrease in $R_s$.
   - 0.0029 % decrease in $R_l$.

The intuition for this effect is straightforward as well: The increase in $\mu_h$ decreases the long term funds demanded of large firms. (The decrease in $z_b$ borrowing is greater than the increase in $z_g$ borrowing.) Hence, $R_l$ decreases, whereas $R_s$ stays the same. Therefore, "some" small firms shift from short term to long term borrowing. Critical level $\lambda_b$ decreases, this shift makes $R_s$ decrease.

8 Public R&D Finance

9 Conclusion

Depending on the results derived from the three models analyzed, we can conclude that, the level of firm level pledgeability has important effects on the sectoral choice of an entrepreneur. The volatility plays a crucial role as well. The higher the volatility in the stochastic firm specific productivity, the measure of small firms which borrow and invest long term in period $(t - 1)$, consists mostly those ones with a "bad" realization of $z_t$, as the volatility starts to decrease small firms which have a "good" realization of $z_t$ start to enter the long term loanable funds market as well. These results show that volatility and pledgeability are important in the determination of firm level sectoral composition and productivity and should to be taken into account for aggregate policy analysis.

References


