Exploring Innovation with Market Concentration and Consumer Income in Dynamic Markets

Abstract

Our study examines the disruptive innovations that arrive from outside of the industry through computational analyses that connect innovation rate to market concentration and consumer income in a dynamic setting. The findings show that a market of fewer incumbents (higher concentration) generates less innovations due to the reduced innovation premium and shortened length of incumbency; however, the equilibrium innovation rate increases in the market share of the top firm, due to enhanced innovation premium. We also find that more high-income consumers stimulate innovation. Lastly, higher consumer welfare and aggregate welfare are found in a more innovative market in the long term.

Keywords: Innovation, market structure, income, vertical differentiation, computation.

1. Introduction

Successful innovation that transforms the market landscape is increasingly exemplified in the rapidly evolving technology economy. From the 1980s to the early 1990s, the entrant firms producing smaller-size hard drives penetrated the market of large-size hard drives, which were widely used in the mainframe computers; in effect, the market of computers underwent repeated transformations from giant machines to minicomputers, and eventually to personal computers (Christensen 1997). In the recent shift to Cloud computing at the expansion of Internet bandwidth, Google pioneered an array of online services based on its innovative business model with a highly lucrative advertising mechanism. Its web-based office applications are leading an expedited market transformation from offline desktop software computing to software as a service. Microsoft is striving to follow the trend in this abrupt shift away from the offline software market, in which the company’s core business resides. A recent article from the Wall Street Journal reveals that this software giant has in several occasions missed the innovative
opportunity of integrating Internet advertising into their business model, which was eventually undertaken by Google (Guth 2009).

The current paper takes interest in the disruptive innovations described by the examples above. As Christensen puts it, “generally disruptive innovations were… often simpler than prior approaches. They offered less of what customers in established markets wanted so could rarely be initially employed there.” Deploying a disruptive innovation often requires the incumbent firms to re-engineer their business models and take the risks of cannibalizing their existing product market. “Thus, managerial decisions that make sense for companies outside a value network may make no sense at all for those within it” (Christensen 1997). These arguments reassert the idea that the existing firms are at a significant disadvantage in introducing a groundbreaking innovation that would shift the industry in unanticipated manners. In this paper, we focus on the power of strategic innovation to displace prominent market leaders by examining the impact of economic attributes, such as consumer income and market structure, on the occurrence of innovation from outside of the market.

For decades, scholars have studied the impact of economic factors on innovation. As Schumpeter has pointed out, “[profit] is the premium which capitalism attaches to innovation” (Schumpeter 1918), the connection between market concentration, which is linked to industry profits, and innovation has been a topic of continuing interest. Empirical evidence has suggested that the optimal market structure that maximizes innovation is an intermediate structure between a large number of small- to moderate-sized firms and a large monopoly (Scherer 1967; Mansfield 1963). Mansfield (1963) obtained divergent results showing that firms’ innovation depends on time varying factors such as innovation cost, and that the optimal firm size for innovation varies across industries. Scherer (1967) observed that innovation increases within low range of market concentration; the approximate optimal market concentration is a moderate four-firm concentration ratio. Through an analytical approach, Kamien and Schwartz (1976) also showed that an intermediate rivalry leads to the fastest pace in innovation speed.

The majority of these studies draw the connection between the introduction of innovation and the rivalry among the innovating firms. Thus, they have generalized incremental innovations through
which incumbent firms sustain their market positions and a sharply distinct concept, which is the disruptive innovations that occur from outside of the market and fundamentally refine the market landscape. We differentiate these two phenomena and focus our research attention on the latter innovation, which is often accomplished by the potential entrants. Instead of examining the competition among the innovating firms, we focus our attention on the competitiveness in the market that the innovating firms aspire to enter, because it has a direct impact on the market structure which determines the incentive for innovation. In a dynamic setting, we continue the investigation of the driving forces of innovation, and treat innovation as a continual process. By endogenizing market concentration with a market share restriction, our model offers direct insights for the degree of market competition on the innovation efforts. We further endogenize market structure with consumer income levels to establish linkages between this economic factor and the innovation rate.

We depart from the studies that have focused on innovation in a monopoly context (Segal and Whinston 2007, Aghion et al. 2001) with the view that a single-incumbent market is the outcome of competition among successful innovators and incumbent firms. While monopoly may be a special case of single-firm markets, the competition does not necessarily reward the single incumbent with monopoly rents. Rather than taking as given a monopolist market structure and firms’ profits, we take into account such an endogenous nature of firms’ profits in our formulation of a vertically differentiated market, where innovation produces the latest generation product, and consumers put a certain level of premium on the newer good. In this way, the market concentration and firms’ profits are the result of the competition between the successful innovator and the producers of the goods of previous generations.

As supported by views of classic economics, innovation orchestrates the cycles of the industries. Encapsulated in Schumpeter’s idea of “creative destruction,” while innovation stimulates the rise of new firms, new markets, and economic growth, it also has the destructive power of rendering the prior technologies and business models obsolete and discontinuing the market positions of the incumbents. While some analytical works have shown that a highly intensive rivalry in an industry leads to lower profits (Loury 1979; Lee and Wilde 1980), the static or finite models only capture a myopic problem considered by the competing firms. In our infinite-horizon setup,
firms’ equilibrium innovation decision is based on discounted future values of becoming an incumbent, which bears the risk of being displaced by future innovators. Thus, an increasing innovation rate shortens the length of equilibrium incumbency, reducing the potential entrant’s expected value of the innovation premium.

To gain insights from a public policy perspective, where incumbents’ market power can be directly constrained, we first use an exogenous variable that puts a restriction on the leading firm’s market share in Section 2. The computational analysis performed under this setup with market structure resulted from such a restriction and consumer preference introduces a clear understanding on the connection between degrees of competition and innovation. The market structure stimulus for innovation is found to reside in two dimensions: 1) the innovation premium, which is mainly characterized by the payoff of the top firm; and 2) the expected length of incumbency, which includes the probability of staying as the leading firm as well as remaining in the market after being pushed out of the top position. It can also be understood as the product life cycle from its successful appearance in the market to becoming obsolete.

The single-firm market is the outcome of aggressive price competition when no constraint is put on the competing firms’ market shares; as a consequence, the successful innovator producing the latest generation product lowers its price until it is able to obtain the entire market. When the top firm is has a constrained market share, a two-firm market is formed. Compared with the single-firm structure, the two-firm market—which has a lower concentration—offers a higher innovation premium and induces higher R&D efforts when considered with other market forces interplayed in this dynamic problem. The idea is that a less concentrated market tends to provide more market positions and promote a longer product life cycle. This insight is sharpened in the dynamic setting, as the innovators’ decisions take into account values of all future periods of potentially multiple incumbencies. At the same time, however, the two-firm scenario also shows that innovation rate increases as the market becomes more concentrated at the top firm. In other words, a weaker restriction results in a higher innovation rate as long as such a control is in place. This is due to that restrictive policy actually mitigates the price competition allowing more firms to survive and higher profits to be obtained.
While the topic on concentration and innovation has been one of the main foci, scant research attention is devoted to analyzing other economic forces behind the incentive for innovation. We build on the current analysis on market structure and establish the linkage to consumer income in the economy. Based on Shaked and Sutton’s work (1982, 1987), market concentration and equilibrium prices are largely dependent on the distribution of consumer income. Wealthy consumers’ purchasing ability helps sustain the profitability of the firms offering higher quality products, as poor consumers have less disposable income for newer and fancier goods. In Section 4, instead of imposing a market share constraint as in Section 2, we derive market concentration according to the premiums consumers of low and high income levels place on the latest generation product, and compute the equilibrium innovation rate in terms of proportion of high-income consumers. In effect, our work extends Shaked and Sutton’s work (1982) to a dynamic setting with the computation of equilibrium innovation rate in terms of consumer income distribution. We found that in an economy with more high-income consumers, or a higher ratio between high- and low-income consumers’ premiums for newer goods, a higher innovation rate is stimulated. The implications are that the wider spread of income heterogeneity as well as a larger segment of high-income consumers will create a more profitable market position for a potential innovator as well as longer market incumbency. Thus, due to the same effect as that of a market share constraint, higher innovation is induced under these income conditions.

In Section 2, we introduce the model, and derive equilibrium prices and profits, and potential entrants' maximization problem. We introduce a market share constraint under the “restricted prize” setting, where the leading firm that produces the latest generation product cannot obtain the entire market. In Section 3, we discuss the computation method and present our results on the comparison of the innovation rates, consumer welfare and aggregate welfare. And then in Section 4, we introduce income heterogeneity among consumers and derive the condition that results in a two-firm market structure similar to that in Section 3. We also present the computation results in terms of innovation rates and welfare in this setup. And finally, we conclude.

2. Model

2.1 Model Setup
The economy is populated with two kinds of agents: consumers and firms. There is a unit measure of consumers in the market, with homogeneous preferences over products. In every period, each consumer buys one unit of the vertically differentiated good from one of the incumbent firms. In every period \( t \) there are \( N > 0 \) incumbent firms in the market. Denote the number of potential entrants by \( M \), where \( M > 0 \). The ranking of the firms that have successfully innovated is based on the quality of their innovations, described in generations. Let product \( j_t \) denote the latest generation (highest quality) product in period \( t \), product \( (j_t - 1) \) be the previous generation (second highest quality) and so on. The incumbent firms in every period \( t \) are the ones that have successfully innovated and received a patent.

Assume for simplicity that only potential entrants can innovate, while the incumbent firms cannot. Thus, \( M \) firms compete in an R&D race in every period. If their efforts lead to success, one of the innovators replaces the highest generation product in the market, assuming consumers value the newest product more than the existing products. Also assume that the marginal cost of producing any quality \( \xi \) is constant. Denote by \( \beta \) the time discount factor.

We start by introducing exogenous restrictions on the maximum market share that the latest generation producer can occupy. Denote this market share restriction parameter by \( \alpha \), where \( \alpha \leq 1 \). We later show that market structure can be endogenized by allowing heterogeneity on the demand side of the market.

**Consumer Valuation**

We assume homogeneous consumer preferences over products. Let the consumers' utility increment between any two consecutive generations \( (j_t - x) \) and \( (j_t - x - 1) \), \( x \geq 0 \), be equal to \( \delta \), that

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1. We later introduce heterogeneous consumer income, which is reflected in differentiated consumer taste for product quality and determine market concentration.
2. As we have pointed out in the introduction, this study is interested in the disruptive innovation occurring from outside of the existing industry. This is consistent with the Schumpeterian view that innovation often occurs outside of the market and accomplished by a new entrant. Also discussed by Christensen in “The Innovator’s Dilemma,” while incumbents are commonly focused on improving their products based on consumers’ demand, they often have the disincentive to innovate in the sense potential entrants are capable of in order to evade the market. Arguably, a more realistic setting would allow the incumbents to perform incremental innovations as well. Our main focus, however, is on innovation incentive defined by the market structure. For this purpose, this simpler framework allows for a more clear presentation and interpretation of the results.
is \( u_{j_t-x} - u_{j_t-x-1} = \delta \). Thus, consumers are indifferent between products of generations \((j_t - x)\) and \((j_t - x - 1)\), if \( p_{j_t-x} = p_{j_t-x-1} + \delta \) for all \( x \geq 0 \). We resolve the indifference case by assuming that the consumer buys \((j_t - x)\) if she derives the same utility from \((j_t - x)\) and \((j_t - x - 1)\).

**The R&D Race**

In period \( t \), potential entrants innovate for the possibility to overtake the latest generation in the previous period \( j_{t-1} \). Assume that the innovation process is binary: either “beat” \( j_{t-1} \) or not. If the innovation in period \( t \) is successful, \((j_t - 1) = j_{t-1}\), otherwise, \((j_t - 1) = (j_{t-1} - 1)\). In other words, successful innovation replaces the latest generation and pushes it down to the second latest generation; and if no innovation succeeds, the generations of products remain unchanged in this period.

We follow Segal and Whinston’s setup in the following notation for the probability of innovation and receiving the patent (Segal and Whinston 2007). Denote the potential entrant firms by \( i=1,...,M \). Each firm in period \( t \) chooses the innovation rate \( \phi_i \in [0,1] \), at cost \( c(\phi_i) \), where the cost function is strictly convex reflecting the diminishing returns to the innovation effort. The probability that firm \( i \) succeeds in developing the next generation product is \( \phi_i \). Since all potential entrants are identical, we consider only symmetric equilibria, where all firms choose the same innovation rate \( \phi \). Then, the probability that a new generation product is developed is \( \pi_M(\phi) = [1 - (1-\phi)^M] \). Note that \( \pi_M(\phi) \) is increasing in \( \phi \) and concave. Only one of the entrants who make a successful discovery (conditional on the discovery being made) will be awarded a patent. In a symmetric equilibrium, let that conditional probability be \( \gamma_M(\phi) \). If the patent holder is chosen randomly among all innovators, \( \gamma_M(\phi) = [1-(1-\phi)^M] / \phi M \). This function is decreasing and convex in \( \phi \). Denote the probability for firm \( i \) of being a successful innovator and winning a patent by \( \lambda_M(\phi_i, \phi_{-i}) \), where \( \phi_i \) is the innovation rate chosen by firm \( i \), and \( \phi_{-i} \) is the innovation rate chosen by each of the other competing entrants. Then, \( \lambda_M(\phi_i, \phi_{-i}) = \phi_i \gamma_M(\phi_{-i}) \).

**Timeline**

The timing of actions in each period is as follows:

1. Potential entrants choose respective innovation rates;
2. Success or failure of innovation occurs; and one of the successful firms is awarded a patent and becomes an incumbent;
3. Prices are set in the output market.

We now analyze the winner-take-all and the restricted-prize market structures. In each period, we solve the problem by backward induction: 1) Find the equilibrium prices and profits; 2) Derive the potential entrants' utility maximization problem.

2.2 Equilibrium Prices and Innovation Problems

For each of the winner-take-all and the restricted-prize cases, we derive the equilibrium prices and profits and then the optimization problem for the potential entrants' choice of innovation rate. Given the complexity of the problem, obtaining an analytical solution is infeasible. Thus, we use the numerical methods to compute the optimal innovation rates. We will present our analysis and compare the results in the Section 3.

2.2.1 Winner-Take-All: $\alpha=1$

In the winner-take-all case, the winning firm can take at the maximum the entire market (i.e., $\alpha=1$). The setup here is similar to Segal and Whinston's work (2007), which only consider the monopoly case. However, instead of assuming a particular profit function, the price and profit are derived from the consumer demand for the differentiated products.

Prices and Profits

**Lemma 1.** When $\alpha=1$, the firm producing generation $j_i$ sets $p_{j_i} = \delta + \xi$, and its profit is $\Pi_{j_i} = \delta$, while the profits of all other firms are zero.

**Proof.** We can argue this by contradiction. Suppose not, and $p_{j_i} > \delta + \xi$, say $p_{j_i} = \delta + \xi + \varepsilon$.

Then, firm $(j_i - 1)$ can charge $p_{j_i - 1} = \xi + \frac{\varepsilon}{2}$ and get all of the market, making $\frac{\varepsilon}{2}$. Firm $j_i$ would end up making 0. Thus, $p_{j_i} = \delta + \xi$. \qed
Given the maximum allowed market share being 1, the top firm only has two options—charging $p_{j_1} = \delta + \xi$ and getting the entire market, or charging any higher price and being undercut by the firm with the second highest generation product, in which case the top firm will get nothing.

**Innovation Decision**

In the beginning of period $t$, the value of the incumbent firm from period $(t-1)$ is

$$V^I = (1 - \pi_M(\phi))[(\delta + \beta V^I) + \pi_M(\phi)\beta V^E].$$

(1)

The entrant’s problem is to maximize

$$V^E = \max_{\phi}\{\lambda_M(\phi,\phi_-)[\delta + \beta V^I] + (1 - \lambda_M(\phi,\phi_-))\beta V^E - c(\phi)\}.$$ 

(2)

From (1) and (2), we obtain

$$V^I = \frac{\delta(1 - \pi_M(\phi)) + \pi_M(\phi)\beta V^E}{1 - \beta(1 - \pi_M(\phi))},$$

and

$$V^E = \max_{\phi}\{\frac{\lambda_M(\phi,\phi_-)}{1 - \beta(1 - \pi_M(\phi))}[\delta + \beta^2 \pi_M(\phi)V^E] + (1 - \lambda_M(\phi,\phi_-))\beta V^E - c(\phi)\}.$$ 

(3)

We denote the optimal innovation rate chosen by the entrant by $\phi^W$. Due to the symmetry between the entrants, in equilibrium, they choose the same innovation rate $\phi = \phi_- = \phi^W$.

### 2.2.2 Restricted Prize: $\alpha<1$

In the restricted-prize setting, we limit the share of the latest generation producer to less than the entirety of the market, such that the firm producing $(j_t)$ obtains $\alpha<1$. As shown in Lemma 2 below, this restriction results in two incumbents charging different prices.

**Prices and Profits**

Using the same argument as in Lemma 1, Lemma 2 shows that the latest generation producer is able to charge a higher price than the winner-take-all case. And both firms make positive profits.

**Lemma 2.** The firm producing generation $j_t$ sets $p_{j_t} = 2\delta + \xi$, and earns $\Pi_{j_t} = \alpha 2\delta$; The firm producing generation $(j_t-1)$ sets $p_{j_t-1} = \delta + \xi$ and earns $\Pi_{j_t-1} = (1 - \alpha)\delta$.
Proof. Suppose firm \((j_t - 1)\) charges \(p_{j_t-1} = \delta + \xi + \epsilon\), by the same argument as the Lemma 1 proof, firm \((j_t - 1)\) will be undercut by generation \((j_t - 2)\) producer and make 0 profit. Since charging a price lower than \(p_{j_t-1} = \delta + \xi\) cannot make firm \((j_t - 1)\) better off, \(p_{j_t-1} = \delta + \xi\) is the best response to \(p_{j_t} = 2\delta + \xi\). Similarly, given \(p_{j_t-1} = \delta + \xi\), firm \(j_t\) is not better off by charging any higher or lower than \(p_{j_t} = 2\delta + \xi\) due to the restricted market share at \(\alpha\). Therefore, the equilibrium prices are \(p_{j_t} = 2\delta + \xi\) and \(p_{j_t-1} = \delta + \xi\).

By restricting the top firm's market share, the market becomes that of two differentiated goods allowing the highest generation to charge a higher price than in the winner-take-all case.

Innovation Decision

In the beginning of period \(t\), the latest generation producer's expected utility is,

\[
V^I (j) = (1 - \pi_{M} (\phi))[\alpha 2\delta + \beta V^I (j)] + \pi_{M} (\phi)[(1 - \alpha)\delta + \beta V^I (j - 1)] ,
\]

and the firm producing the previous generation product has

\[
V^I (j - 1) = (1 - \pi_{M} (\phi))[\alpha \delta + \beta V^I (j - 1)] + \pi_{M} (\phi)\beta V^E .
\]

The entrants solve

\[
V^E = \max_{\phi} \{\lambda_M (\phi, \phi_{-})[\alpha 2\delta + \beta V^I (j)] + (1 - \lambda_M (\phi, \phi_{-}))\beta V^E - c(\phi)\} .
\]

From (4) and (5), we rewrite (6) as

\[
V^E = \max_{\phi} \left\{ \frac{\lambda_M (\phi, \phi_{-})}{1 - \beta(1 - \pi_{M} (\phi))} \left[ \frac{\alpha 2\delta + \beta(\pi_{M} (\phi)[(1 - \alpha)\delta + \beta^2 \pi_{M} (\phi)V^E]\phi]}{(1 - \beta(1 - \pi_{M} (\phi)))(1 - \alpha)\delta + \beta^2 \pi_{M} (\phi)V^E} \right] \right\} .
\]

Denote the optimal innovation rate \(\phi\) that solves (7) by \(\phi^M\). In equilibrium, all entrants choose the same innovation rate \(\phi = \phi_{-} = \phi^M\).

3. Analysis and Results

In this section we discuss the computational findings and their implications\(^3\). We use value function iteration to numerically estimate the equilibrium innovation rates for equations (3) and

\(^3\) The complete set of code for the computation analysis is available online. Please contact the authors for access.
and analyze the relative performance of the two market structures and contrast the difference. The convex innovation cost function is computed with the form $c(\phi) = A\phi^\eta$, where $A>0$ and $\eta>1$. Note that the cost of innovation function is quadratic, since the issues with the existence of the equilibrium solution arise for a non-convex cost function. We have performed the robustness testing under a large set of parameter values and will qualify our findings in the discussion.

3.1 Analysis of Restricted-Prize Case

While $\alpha$ is fixed at 1 for the winner-take-all case, the analysis of the restricted-prize case is conditional on the $\alpha$ value. Thus, we obtain results for increasing market concentration below 1, and examine separately the one-firm case, where the market concentration is the largest (at 1).

**Result 1.** In the restricted-prize case, the optimal innovation rate $\phi^M$ is increasing in $\alpha$. (See Figure. 1)

Result 1 states that holding the number of incumbents fixed, higher concentration at the top firm induces innovation. In other words, when the market is shared between two firms, the innovation rate is increasing in the top firm’s market share size. This idea can be seen clearly from a static standpoint, which specifies that the innovator is only maximizing the payoff in the next period of potentially becoming an incumbent. A higher market share is equivalent to a higher innovation premium; the only constraint on innovation is the cost of innovating. The dynamic model presents a more challenging problem. The innovator’s problem takes into account events in all future periods. Therefore, it is not straightforward that the innovator necessarily increases its R&D effort in equilibrium given a higher incumbent profit, because a higher innovation rate itself is a constraint by reducing the innovation premium—the profitability of incumbency is undermined by a higher likelihood of being displaced by aggressive innovators like itself. However, as Result 1 indicates, the observation of a simplified static case extends, not in a trivial manner, to the dynamic view. We also note in advance that the results obtained for very large values of $M$ demonstrate that as the innovation race becomes more competitive (i.e., the probability of obtaining a patent becomes minuscule), the optimal innovation rate tends to zero for any value of $\alpha$. This relationship between the innovation rate and the number of potential
entrants is indicated in Result 4. In the next result, we discuss the entrant’s value at the equilibrium innovation rate.

![Figure 1](image-url)

**Figure 1.** Equilibrium Innovation Rates ($\beta=0.98$, $M=100$, $\delta=0.01$, cost is quadratic with $A=10$)

**Result 2.** In the restricted-prize case, the value of the potential entrant is increasing in $\alpha$. (See Figure. 2).
The similar tension as that discussed under Result 1 exists in evaluating the equilibrium value of the potential entrants. The increase in innovation premium brought by a higher $\alpha$ outweighs the reduction in other aspects such as innovation cost, probability of success, and probably of sustaining the incumbency. Thus, the above result follows.

Results 1 and 2 are both robust to changes in all parameter values.

3.2 Comparing Optimal Innovation Rates

Result 3. For sufficiently high values of $\alpha$, $\phi^M > \phi^W$ (see Figure 1).

The result that the innovation rate is higher when firms split the market is due to both greater profits for the latest generation producer, and the positive market share and profits for the
previous generation producer. In case of a two-firm market, the successful entrant enjoys two profitable periods before being finally pushed out of the market. Since the leading firm obtains higher profit, a higher market share for the leading firm is valued more than that of the second-generation producer. Therefore, when such a market share is insignificant, the innovators are inclined to innovate more in the single-firm case, where the successful entrant reaps the entire innovation reward upon entering the market.

This is one of the key results in this paper addressing the question of whether higher market concentration induces innovation. As analyzed above, the answer lies in two dimensions, the profitability upon entry which is the innovation premium, and the length of incumbency which includes the likelihood of remaining the leading firm and the length of continued market position after further occurrence of other entrants’ innovation. In a vertically differentiated competition, both of these factors are strengthened with more incumbents. In other words, more firms in the market allows for higher prices as well as more periods of incumbency after a successful innovation. Therefore, Result 3 is obtained with a two-firm market inducing a higher innovation rate than a single-firm market structure. However, it is crucial to note that we do not conclude that lower concentration induces innovation; in fact, from Result 1 clearly indicates that given the number of incumbents, market concentration at the top firm does induce innovation, which interestingly does not contradict the findings in Result 3. While a static interpretation of the current problem helps simplify the main intuition behind Result 1 (although leaving out important factors), in contrast, Result 3 cannot be captured in a static setup and is sharpened in a dynamic setting that we follow.

Result 3 is robust to changes in all parameters: $\beta, A, M$, and $\delta$. And the cut-off value for $\alpha$ has never exceeded 1/5 in any of the robustness tests. In Figure 1, the single-firm case generates higher innovation rates for $\alpha < 0.03$. The dependence of the exact cutoff value for $\alpha$ and the relative distance between $\phi^M$ and $\phi^W$ on other parameters are illustrated in the next two results.

Result 4. *In both cases, the optimal innovation rate decreases in $M$: both $\phi^W$ and $\phi^M$ approach zero as $M \to \infty$. The cutoff value for $\alpha \to 0$ as $M \to \infty$. *
When a large number of firms compete for a patent, the probability of becoming the successful entrant is small, which diminishes the innovation premium and discourages R&D investment. In both winner-take-all and restricted-prize cases, the optimal innovation rate approaches zero as $M$ gets infinitely large. Also, the dependency on $\alpha$ weakens in the number of potential entrants—as the possibility of winning the patent fades, the prize becomes increasingly irrelevant. 

**Result 5.** Both $\phi^W$ and $\phi^M$ increase in the ratio $\delta/A$. The difference between the innovation rates decreases in $\delta/A$, while the range of values for $\alpha$ such that $\phi^M > \phi^W$ stays the same. 

$\delta/A$ roughly reflects the profitability of innovation in terms of its cost. When this ratio is high, firms are more inclined to invest in R&D in both winner-take-all and restricted-prize cases. As such profitability increases, the dependency of innovation rate on the particular market structure becomes less relevant, thus the innovation rates get closer.

### 3.3 Consumer Welfare

Thus far, we have compared the optimal innovation rates in the two market structures and found that the two-firm market induces higher innovation compared to the single-firm market and the increase in innovation premium, or the market profitability, results in a higher equilibrium innovation rate in this dynamic problem. The fact that consumers pay higher prices in equilibrium in the more rapidly-innovating market structure raises the question of whether higher innovation is necessarily a desirable outcome from the welfare perspective. In this section, we explore the impact of higher innovation rate on consumer welfare.

First, we compute utility of consumers in a given period $t$. In the winner-take-all case, all consumers purchase from the latest generation producer. Thus, their utility is $U^W_t = u_{j_t} - p_{j_t}$, or $U^W_t = u_{j_t} - \delta - \xi$. In the restricted-prize case, after plugging in equilibrium prices the consumer welfare in any period $t$ is $U^M_t = u_{j_t} - 2\delta - \xi$. Clearly, in any given period the utility of

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4 For clarity $M$ is modeled as an exogenous parameter in the current work. In practice, it is likely that a more profitable industry attracts more innovators. From this result, we can infer that in a setting where $M$ is an endogenized by the profitability of the incumbency, the effect of higher concentration in inducing higher equilibrium innovation rate as derived in Result 1, will be dampened. It is reasonable to conjecture that, if we allow the innovation premium to increase competition intensity among the potential entrants, the impact on innovation will vary in ways based on the interplay between these elements in the exact formulation.
consumers is greater in a lower-price, or less profitable, market. On the other hand, higher occurrence of innovations improves consumer utility. For both cases, if innovation occurs in period \( t+1 \), \( U_{t+1} = U_t + \delta \), otherwise \( U_{t+1} = U_t \). Thus, over time consumers may be better off in a more rapidly-innovating market even though they are paying more for the new products. To see this, we consider the cumulative consumer welfare over the extended time horizon. We compute consumer welfare over a given number of periods, from \( t=0 \) to 750. Since we are interested only in the welfare difference between the two setups, assume that \( u_{j0} = 0 \) and \( \xi = 0 \). The expected utility over \( T \) periods is obtained numerically. In both winner-take-all and the restricted-prize cases, the \( \pi_M(\phi^*) \) function is used for the probability of innovation occurring in any given period \( t \), where \( \phi^* \) is the optimal innovation rate in each case.

In the restricted-prize case, \( \phi^* \) also depends on \( \alpha \). Since welfare is cumulative, it is increasing in \( T \). It is also slightly increasing in \( \alpha \) at all values of \( T \), because the optimal innovation rate is non-decreasing in \( \alpha \). Next we compare consumer welfare in two cases (see Figure 3, 4 and 5).

**Result 6.** Consumer welfare is higher in the winner-take-all case for a short period of time, however, for sufficiently large values of \( \alpha \), consumer welfare is greater in the restricted-prize case through the extended horizon after the initial periods.
Figure 3. Consumer Welfare ($\beta=0.98$, $M=100$, $\delta=0.01$, cost is quadratic with $A=10$)
Figure 4. Consumer Welfare at T=100 ($\beta=0.98$, $M=100$, $\delta=0.01$, cost is quadratic with $A=10$)
Initially, consumer welfare is higher in the winner-take-all case for all values of $\alpha$. For a few number of periods, consumers have not experienced enough innovative products to justify the higher prices paid in a two-firm market. However, the consumption of innovative products accumulates over time to the point (with these parameter values, beginning at $t=20$ when $\alpha$ is close to 1) where higher innovation rate from the two-firm market results in greater consumer welfare than the single-firm market. This result highlights the merit of the dynamic view at the innovation problem, as the static setting would not lead to such an important insight. The intuition is that above a certain level of market concentration in the two-firm market, the relative difference between the rates of innovation in two market structures is sufficiently large. At this equilibrium innovation rate, over a length of periods consumers derive a higher utility from more new products introduced than the disutility due to the elevated price paid compared to the single-firm market. This is more clearly shown in the two-dimension cross sections of Figure 3, at $T=100$ and $T=700$, see Figures 4 and 5 above.
In Results 4 and 5, we showed the impact of other parameters on the main findings. In the following, we discuss the implications of changes in these parameters on consumer welfare.

**δ to A Ratio**

By Result 5, the difference between the innovation rates decreases in the ratio \( \frac{\delta}{A} \). Thus, a high \( \frac{\delta}{A} \) value leads to diminished consumer utility gain from more innovation in the two-firm market, which may not offset the higher prices paid. Consumer welfare is then higher in the single-firm case for a very large \( \frac{\delta}{A} \).

**Number of Potential Entrants, M**

Result 4 states that the optimal innovation rate goes to zero when \( M \) is very large for both market structures. In that case, the only relevant factor for consumer welfare is the equilibrium prices. Thus, Result 6 holds as long as \( M \) is not too large; otherwise, in the extended horizon the consumer welfare is higher for the single-firm case.

### 3.4 Aggregate Welfare

We now analyze the aggregate welfare, which combines firms' values and consumer welfare. This measure is calculated by adding incumbents' profits to the consumer welfare, and subtracting the innovation costs for the potential entrants. The aggregate welfare in each period for the two cases is expressed in the following equations:

\[
AW_t^W(\phi^*) = U_t^W + \delta - M \cdot c(\phi^*)
\]

\[
AW_t^M(\phi^*) = U_t^M + 2\alpha\delta + (1-\alpha)\delta - M \cdot c(\phi^*)
\]

Again, in both cases, \( \pi_M(\phi^*) \) determines the probability of an innovation occurrence in any given period \( t \), where \( \phi^* \) is the optimal innovation rate in each case.

**Result 7.** The aggregate welfare is higher in the restricted-prize case over time and when \( \alpha \) is sufficiently large. The difference between the two cases is larger in the aggregate welfare than in the consumer welfare. (See Figure 6, 7, and 8)
The result of higher welfare with restricted-prize over longer time horizons is even stronger for the aggregate welfare case because of the firm valuations added in the calculation. The increased profitability for the incumbents yields a higher welfare in the aggregate sense than the consumer welfare; the result that the two-firm market shows a higher welfare is more evident in the aggregate case. The only exception is when $M$ tends to infinity, the miniscule innovation (see Result 4) makes the comparison irrelevant. Otherwise, Result 7 is robust to changes in all other parameters.

Figure 6. Aggregate Welfare ($\beta=0.98$, $M=100$, $\delta=0.01$, cost is quadratic with $A=10$)
Figure 7. Aggregate Welfare at T=100 (\(\beta=0.98, M=100, \delta=0.01\), cost is quadratic with \(A=10\))
Figure 8. Aggregate Welfare at T=700 ($\beta=0.98$, $M=100$, $\delta=0.01$, cost is quadratic with $A=10$)

4. Heterogeneous Consumers

The two-firm market structure analyzed in the previous section was obtained by a particular market policy imposed on the top firm to permit incumbency of the firm producing the second generation product. The existence of multiple incumbents can also result from other economic factors, such as heterogeneity among consumers. Shaked and Sutton’s seminal work (1982) on market structure under differentiated consumer income provides an elegant result indicating the entry of two firms in a vertically differentiated market given a range of consumer income, when wealthier consumers are more willing to pay for the later generation product. In this section, we introduce a similar heterogeneity among consumers, through which the two-firm market structure is endogenized. For simplicity, we consider two discrete types of consumers--which is
sufficient for eliciting insights in the current model--rather than a continuous distribution of income.

Let us consider two types of consumers in the market, high- and low-income consumers. There is a continuum of consumers of measure one, with fraction $\alpha \in (0,1)$ of high-income consumers putting a premium of $\delta_H$ on the newer generation product. That is, for these consumers, $u_{j,t-x} - u_{j,t-x-1} = \delta_H$. The rest $(1 - \alpha)$ of low-income consumers have a lower newer generation premium $\delta_L < \delta_H$. Then, the high-income consumers are indifferent between purchasing from firm $(j_t - x)$ and $(j_t - x - 1)$ if and only if $p_{j_t-x} = p_{j_t-x-1} + \delta_H$. Low-income consumers are indifferent if and only if $p_{j_t-x} = p_{j_t-x-1} + \delta_L$. The two incumbents can share the market if $p_{j_t-x-1} + \delta_L < p_{j_t-x} \leq p_{j_t-x-1} + \delta_H$, provided that they price out all of the other competing generations.

**Lemma 3.** When $\delta_H / \delta_L \geq (1 - \alpha)/\alpha$, in equilibrium the latest generation producer $j_t$ prices at $p_j^* = \delta_H + \delta_L + \xi$, $\pi_j^* = \alpha(\delta_H + \delta_L)$; firm $j_t-1$ prices at $p_{j_t-1}^* = \delta_L + \xi$, and $\pi_{j_t-1}^* = (1 - \alpha)\delta_L$.

**Proof.** The latest generation producer $j_t$ has two choices: 1) lower its price to obtain the entire market, or 2) share the market with the other firm(s). To achieve the latter, it must set its price at $p_{j_t} = \delta_L + \xi$, to push out all competitors, and will earn a profit of $\delta_L$. To achieve the former, it can focus on the high-income consumers, to whom it can sell at a higher price of $p_{j_t} = p_{j_t-1} + \delta_H$. In this case, the previous generation producer, firm $j_t-1$, competes with firm $j_t-2$ for the low valuation consumers. By Lemma 1, firm $j_t-1$ pushes firm $j_t-2$ out of the market at $p_{j_t-1} = \delta_L + \xi$. The best response of firm $j_t$ is then pricing at $p_{j_t} = \delta_H + \delta_L + \xi$, to get the higher valuation consumers; the profit is then $\alpha(\delta_H + \delta_L)$. Firm $j_t$ will find profitable to do so as long as it can make at least as much profit as it would have by targeting both type of consumers. Thus, we obtain the following condition:

$$\delta_H / \delta_L \geq (1 - \alpha)/\alpha. \quad (8)$$
Corollary 1. When $\alpha \geq 1/2$, for all values of $\delta_H$ and $\delta_L$, in equilibrium the latest generation producer $j$, prices at $p^*_j = \delta_H + \delta_L + \xi$, $\pi^*_j = \alpha(\delta_H + \delta_L)$; firm $j$, -1 prices at $p^*_{j-1} = \delta_L + \xi$, and $\pi^*_{j-1} = (1 - \alpha)\delta_L$.

Proof. Clearly, condition (8) will hold for any $\delta_H$ and $\delta_L$ if $\alpha \geq 1/2$.

Lemma 4. When $\delta_H / \delta_L < (1 - \alpha)/\alpha$, in equilibrium the latest generation producer $j$, prices at $p^*_j = \delta_L + \xi$, and obtains the entire market, $\pi^*_j = \delta_L$.

Proof. The result of Lemma 4 follows from the proof of Lemma 3.

When there exist a significant number of high-income consumers in the economy, or if the difference in the quality taste between the two types of consumers is sufficiently large, the economy opens a profitable niche for a second incumbent in the market. In this equilibrium, the top firm accumulates adequate profits targeting only the high-income consumers such that lowering its price to get the whole market is a suboptimal strategy.

In particular, for the values of $\delta_H$ and $\delta_L$ both close to $\delta$, firms’ profits are approximately identical to those in the two-firm case under the exogenous market share restriction (Lemma 2); thus, the relative innovation rates under Lemma 3 and 4 should resemble those of the two market structures examined in the previous section.

Result 8. When the values of $\delta_H$ and $\delta_L$ are closer together, the equilibrium innovation rate is increasing in a given $\alpha \geq 1/2$. (See Figure. 9)
Figure 9. Equilibrium Innovation Rate with Consumer Heterogeneity ($\beta=0.98$, $M=100$, $\eta=2$, $\delta_H=0.01$, $\delta_L=\delta_H-\epsilon$, cost function is quadratic with $A=10$)

The implication in the above result is that depending on the taste difference between consumer of two income levels, an economy with more high-income consumers has a higher equilibrium innovation rate when the proportion of high-income consumers exceed a threshold; and the innovation rate is the same for the high-income proportion below such a threshold. Moreover, Lemma 3 and 4 suggest that with a larger taste difference, the equilibrium innovation rate starts increasing at a lower threshold of high-income consumers. Thus, both the heterogeneity in consumer quality taste induced by income, and an increased number of high-income (high-taste) consumers stimulate innovation. Parallel to the setup in Section 2, the entrant’s value is also increasing for sufficiently large proportion of high-income consumers.

**Result 9.** When the values of $\delta_H$ and $\delta_L$ are closer together, the entrant’s value is increasing in $\alpha$ given $\alpha \geq 1/2$. (See Figure. 10)
We compute consumer welfare over multiple periods by the method identical to that in the previous section. Since the innovation rate is constant for $\alpha < 1/2$, consumer welfare is constant in $\alpha$ within this range. We see an upward jump in welfare at $\alpha = 1/2$. For larger values of $\alpha$, consumer welfare is increasing due to the increasing innovation rate. And the rate of increase is magnified over higher number of periods. The intuition in the results of the previous section applies here. Although consumers are paying higher prices when two firms occupy the market instead of one firm, over longer periods of time the benefit from more innovative products outweighs the cost.

**Result 10.** Consumer welfare is increasing in $\alpha$ for $\alpha \geq 1/2$ for sufficiently large values of $T$. (See Figures 11, 12, and 13).
Figure 11. Consumer Welfare under Consumer Heterogeneity ($\beta=0.98, M=100, \eta=2, \delta_H=0.01, \delta_L=\delta_H-\epsilon$, cost function is quadratic with $A=10$)
Figure 12. Consumer Welfare under Consumer Heterogeneity at T=100 ($\beta=0.98$, $M=100$, $\eta=2$, $\delta_H=0.01$, $\delta_L=\delta_H-\epsilon$, cost function is quadratic with $A=10$)
Similarly, we compute the aggregate welfare by adding incumbents' profits to the consumer welfare, and subtracting the innovation costs for the potential entrants. The results show pronounced welfare increase compared to consumer welfare in the size of the high-income consumer segment.

**Result 11.** Aggregate welfare is increasing in $\alpha$ for $\alpha \geq 1/2$ for sufficiently large values of $T$. (See Figures 14).
Figure 14. Aggregate Welfare under Consumer Heterogeneity \( (\beta=0.98, M=100, \eta=2, \delta_H=0.01, \delta_L=\delta_H-\varepsilon, \text{cost function is quadratic with } A=10) \)

5. Conclusion

Observing the distinction between the incremental innovations within the industry and the disruptive innovations that arrive from outside of the industry and transform the market landscape, we analyze the connection of the disruptive innovation with market concentration as well as consumer income in a dynamic setting. Our study focuses on the competitiveness within the output market, where the successful innovators collect their innovation premium that fundamentally drives the developments of new products.

We first use an exogenous market share constraint to bring a clear understanding of the impact of different levels of market competition among incumbents on the equilibrium innovation rate of the entrants. Our results show that the highest possible concentration characterized by a single-firm market is the outcome of intense price competition, which does not reward the incumbent
with monopoly rents. Thus, a two-firm market induce a higher equilibrium innovation rate. However, the equilibrium innovation rate is stimulated by an increased concentration on the top firm. Furthermore, we endogenize market structure with consumers’ income levels to extend Shaked and Sutton’s seminal work (1982) to a dynamic analysis on innovation. By establishing the link between income levels and innovation, we find that income heterogeneity and sharper difference between the premiums that high- and low-income consumers’ place on a newer good create profitable market niche for successful innovators and encourage innovation.

The dynamic setting considered in our work is essential for understanding the economic forces that drive innovation. In particular, the innovators’ R&D investment is not solely based on the innovation premium within the profits of a successful innovation; the dynamic setting captures the influence of continuing market positions—which is a potential benefit of less concentrated market—on the innovators’ decision. Moreover, while markets of higher profitability attract higher innovation, the welfare effect is best studied in an infinite horizon. We compute both consumer welfare and aggregate welfare over large number of periods, and find that in the long term, the higher prices paid by consumers are justified by more innovative products in a two-firm market. Thus, both consumer welfare and aggregate welfare are higher and increasing in the more innovative market, for sufficiently large number of periods. Our analysis holds implications in terms of market regulation that restrictive public policies can mitigate competition and in effect boost firms’ profits and prolong incumbency. As a result, such policies stimulate innovation and improve welfare in the long run.

This work offers several directions of interesting future studies that will potentially have significant research and social values. The current setup of two income levels suggests the impact of income distribution on innovation due to the underlying market profitability connection the two elements. An extensive analysis of this linkage can be performed with the tool of computation that maps to realistic data of income distribution. A robust calibration of the model will provide important guidelines for designing public policies and market regulations with the goal of stimulating an innovative economy. Moreover, while empirical efforts have investigated the life cycles of products in the market, the research gap is still open on the theoretical explanation of market shares of newer and older goods. A generalization of the model
in the current study will generate insights for the effects of economic factors such as innovation on the lingering tail of outdated products.

References