A Small Open Economy DSGE Model with a Housing Sector

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Abstract

A DSGE model with a housing sector for a small open economy is constructed and estimated using Australian data and Bayesian methods. The model is simulated to obtain insights about the effects of a number of shocks on the importance of housing. A comparison of the impulse responses for the model with and without the housing sector shows the role played by the relative flexibility of housing and goods prices in determining the dynamics of housing and consumption expenditure.

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Key words: DSGE, Small Open Economy, Housing market, Bayesian Estimation
1 Introduction

The demand for housing services is an important component of economic activity. This paper aims to shed some light on the dynamic interactions of consumption and housing using a DSGE model for a small open economy with a housing sector. The basic small open economy DSGE framework is based on Gali and Monacelli (2005), Lubik and Schorfheide (2007) and Christiano, Trabandt and Walentin (2007); while the introduction of credit constraints follows ideas expressed in Gammoudi and Mendes (2005), Campbell and Hercowitz (2005), Iacoviello (2005), Iacoviello and Minetti (2008) and Aoki, Proudman and Vlieghe (2004).

The DSGE model constructed has two novel features. The first lies with the treatment of the household sector. Unlike typical DSGE models, our household sector is split into two types - consumers and entrepreneurs. The consumers provide labour services, earn wages, and consume goods and housing services; while the entrepreneurs provide housing services, earn rents and consume goods. In other words, the consumers rent housing services from the entrepreneurs. The second novel feature lies with the specification of the risk premium which links the policy interest rate and the mortgage rate. In this model risk depends on the degree of borrowing by entrepreneurs to finance new investment - the higher the borrowing, the higher the risk margin.

The other aspects of the model are standard. The financial sector accepts deposits from the consumers, borrows from foreigners and lends to entrepreneurs for building up stock of houses. Firms import intermediate goods as capital inputs to produce final goods which may be consumed or turned into houses. The price of goods is sticky but the rental price is flexible. The model also includes a Taylor-type monetary policy rule.

This paper is organized as follows. Section 2 describes the small open economy DSGE model with a housing sector. Section 3 presents the estimated results for the Australian economy. An analysis of impulse responses to several orthogonal shocks is given in Section 4. Some conclusions are presented in Section 5.
2 A DSGE Model with a Housing Sector

The model contains six decision-making agents. They are consumers, bankers, entrepreneurs, firms, the government, and foreigners.

2.1 Consumers

The model economy is populated with infinitely lived consumers who supply labor services \( N \) at the nominal wage rate \( W \), consume non-housing goods \( C^c \), rent housing services \( H \) from the entrepreneurs, deposit savings \( B \) in banks at the risk free return \( R_f \), pay taxes \( T \) to the government, and receive real dividends \( D_f \) and \( D_b \) from firms and banks, respectively. Subject to budget constraints in every period, consumers aim to maximize their expected discounted lifetime utility, which is given by

\[
E_T \sum_{t=0}^{\infty} \beta^{t-r} U(C_t, N_t).
\]

Here, \( U(C_t, N_t) \) is the period utility function given by

\[
U(C_t, N_t) = \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\chi}}{1+\chi},
\]

where \( \beta \) is the discount factor, \( 1/\sigma \) denotes the consumers' intertemporal elasticity of substitution, \( \chi \) is the inverse of the Frisch labor supply elasticity, \( C_t \) is an index of composite consumption goods and \( N_t \) is the labor supply. The utility function captures the consumers' preference for more consumption goods and less working hours. The consumption index \( C_t \) composed of consumption goods, \( C^c_t \), and housing services, \( H_t \), is defined as

\[
C_t = g(C^c_t, H_t) = \left[ \gamma \left( (C^c_t)^{\frac{\eta-1}{\eta}} + (1-\gamma) \right)^{\frac{1}{\eta}} (H_t)^{\frac{\eta-1}{\eta}} \right]^{\frac{1}{\eta-1}},
\]

where \( \eta \) represents the intratemporal elasticity of substitution between consumption goods and housing services, and \( \gamma \in [0, 1] \) represents the share of consumption goods in the total composite consumption, reflecting the preference of non-housing goods in total consumption.\(^1\) In addition, \( C^c_t \) is a Dixit-Stiglitz aggregate of differentiated consumption goods and housing services become perfect substitutes as \( \eta \to \infty \) and perfect complements as \( \eta \to 0 \). If \( \eta = 1 \), the functional form of consumption bundle \( g(.) \) takes the Cobb-Douglas form: \( C_t = (C^c_t)^{\gamma} (H_t)^{1-\gamma} \).

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\(^1\)Consumption goods and housing services become perfect substitutes as \( \eta \to \infty \) and perfect complements as \( \eta \to 0 \). If \( \eta = 1 \), the functional form of consumption bundle \( g(.) \) takes the Cobb-Douglas form: \( C_t = (C^c_t)^{\gamma} (H_t)^{1-\gamma} \).
goods, defined as

\[ C_t^c = \left( \int_0^1 C_t^c(j)^{\theta-1} \frac{\theta}{\sigma} dj \right)^{-\frac{1}{\theta-1}}, \]

where \( j \in [0,1] \) is the index of differentiated consumption goods, \( \theta > 1 \) is the elasticity of substitution between differentiated goods. The corresponding price index for the consumption goods, \( P_{c,t} \), is given by

\[ P_{c,t} = \left( \int_0^1 P_{c,t}(j)^{1-\theta} \frac{1}{\theta-1} dj \right)^{-\frac{1}{\theta-1}}. \]

The period-t budget constraint is then given by²

\[ W_t N_t + R_{f,t-1} B_{t-1} + P_{c,t} D_{f,t} + P_{c,t} D_{h,t} = P_{c,t} C_t^c + P_{h,t} H_t + B_t + T_t. \] (4)

The consumers’ problem is characterized by the following equations, where Eq.(6) captures the optimal supply of labor and Eq.(7) captures the optimal deposits by consumers.

\[ \frac{g_2(C_t^c, H_t)}{g_1(C_t^c, H_t)} = \frac{P_{h,t}}{P_{c,t}} = \left( \frac{\gamma}{1 - \gamma C_t^c} \right)^{-\frac{1}{\eta}}, \] (5)

\[ \frac{W_t}{P_{c,t}} = \frac{U_2(C_t, N_t)}{U_1(C_t, N_t)}, \] (6)

\[ \beta E_t \left\{ \frac{U_1(C_{t+1}, N_{t+1}) g_1(C_{t+1}, H_{t+1})}{U_1(C_t, N_t) g_1(C_t^c, H_t)} \frac{P_{c,t}}{P_{c,t+1}} \right\} = \frac{1}{R_{f,t}}. \] (7)

Notice that \( U_1(C_t, N_t) \) and \( g_1(C_t^c, H_t) \) are the first order derivative of utility function and consumption budle with respect to the first argument, respectively. Similarly \( U_2(C_t, N_t) \) and \( g_2(C_t^c, H_t) \) are respect to the second argument.

Following Piazzesi, Schneider and Vuzel (2007), the expenditure ratio is defined as

\[ \omega_t = \frac{P_{c,t} C_t^c}{P_{c,t} C_t^c + P_{h,t} H_t}, \] (8)

where \( \omega_t \) denotes the proportion of expenditure on non-housing goods to total consumption expenditure. Then Eq.(6) and (7) can be rewritten as

\[ (C_t^c)^{-\sigma} \gamma^{1/\eta} \left( 1 - \frac{1 - \omega_t}{\eta-1} \right) \frac{W_t}{P_{c,t}} = N_t^X, \] (9)

²Eq.(4) utilises the result that the demand function for each differentiated consumption goods is given by \( C_t^c(j) = \left( \frac{P_{c,t}(j)}{P_{c,t}} \right)^{-\eta} C_t^c \) such that the expenditure on consumption goods \( \int_0^1 P_{c,t}(j) C_t^c(j) dj \) can be written as \( P_{c,t} C_t^c \).
\[
\frac{1}{R_{f,t}} = \beta E_t \left\{ \left( \frac{C_{t+1}^c}{C_t^c} \right)^{-\sigma} \left( \frac{\omega_{t+1}}{\omega_t} \right)^{\frac{n-1}{\eta-1}} \frac{P_{c,t}}{P_{c,t+1}} \right\}.
\] (10)

2.2 Entrepreneurs

Following Aoki, Proudman and Vlieghe (2004), the entrepreneurs are assumed to be real estate specialists and to be risk neutral. They receive rental income from consumers, own the entire housing stock, borrow in the form of mortgage \( M \) from banks, purchase new houses to expand their housing stock, pay back the previous mortgage loan, and consume non-housing consumption goods. The budget constraint for the entrepreneurs in period \( t \) is given by

\[
P_{h,t}H_t + (1 - \delta)P_{c,t}H_t + M_t = P_{c,t}H_{t+1} + R_{m,t-1}M_{t-1} + P_{c,t}C_t^e,
\] (11)

where \((R_{m,t} - 1)\) is the mortgage rate, \( C_t^e \) is the non-housing consumption for entrepreneurs, and \( \delta \) is the constant depreciation rate of housing stock. For simplicity, we consider a one-good model which can be transformed into consumption-type goods or shelter-type goods; in other words there is only one price of goods \( P_{c,t} \) in the model. Dividing \( P_{c,t} \) on both sides of the above equation gives the real budget constraint as:

\[
Q_tH_t + (1 - \delta)H_t + M_t/P_{c,t} = H_{t+1} + R_{m,t-1}M_{t-1}/P_{c,t} + C_t^e,
\] (12)

where \( Q_t = \frac{P_{h,t}}{P_{c,t}} \) is the relative rental price. The rental market is competitive, so that the rental price is endogenously determined to equate the supply of housing stock by entrepreneurs and the consumers’ demand for housing rentals. The stock of housing evolves as follows:

\[
H_{t+1} = I_t + (1 - \delta)H_t,
\] (13)

where \( I_t \) is the entrepreneurs’ investment in housing.

The entrepreneurs’ problem is to maximize their expected discounted consumption

\[
\sum_{t=\tau}^{\infty} (\beta^e)^{t-\tau} E_\tau [C_t^e],
\] (14)

subject to Eq.(12) for all \( t \), where \( \beta^e \) is the discount rate for entrepreneurs. It is assumed that \( \beta^e < \beta \), i.e., entrepreneurs are more impatient than consumers, such
that in equilibrium entrepreneurs are borrowers and consumers are lenders (through the intermediation of banks). The first order condition with respect to mortgages $M_t$ is given by
\[
\frac{1}{P_{c,t}} = \beta \frac{E_t}{P_{c,t+1}} R_{m,t},
\]
(15)
And the first order condition with respect to the housing stock $H_{t+1}$ is given by
\[
1 = \beta \frac{E_t}{P_{c,t}} ((Q_{t+1} + (1 - \delta)).
\]
(16)
Combining Eq.(15) and Eq.(16) yields
\[
R_{m,t} E_t \left( \frac{P_{c,t}}{P_{c,t+1}} \right) = E_t (Q_{t+1} + (1 - \delta)),
\]
(17)
where the left hand side is the real mortgage rate and the right hand side is the real gross return of housing.

An exogenous shock to the mortgage rate is introduced here:
\[
R_{m,t} \exp(\epsilon_{z,t}) E_t \left( \frac{P_{c,t}}{P_{c,t+1}} \right) = E_t (Q_{t+1} + (1 - \delta)),
\]
(18)
The shock is assumed to obey a first order autoregressive (AR(1)) process
\[
\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \epsilon_{z,t}, \quad \epsilon_{z,t} \sim N(0, \sigma^2_z),
\]
(19)
where $\rho_z$ is the smoothing parameter and the error term $\epsilon_{z,t}$ follows a normal distribution with zero mean and finite standard deviation $\sigma_z$. The shock $\tilde{z}_t$ is introduced to allow for government intervention in the mortgage maker.

Let $NT_{t+1}$ be the net worth of entrepreneurs defined as $NT_{t+1} = H_{t+1} - M_t / P_{c,t}$, where $M_t / P_{c,t}$ is the real amount of mortgage loans. $NT_{t+1}$ can be understood as the down payment of entrepreneurs for housing. Let $V_t$ be the entrepreneurs’ equity defined as
\[
V_t = Q_t H_t + (1 - \delta) H_t - R_{m,t-1} M_{t-1} / P_{c,t}.
\]
(20)
Then Eq.(12) implies that
\[
V_t = NT_{t+1} + C^e_t,
\]
(21)
i.e., entrepreneurs’ equity can be either paid to banks as the down payment for next period housing purchase or spent on consumption goods. For simplicity, we don’t model
the endogenous determination of the division of \( V_t \) between \( NT_{t+1} \) and \( C_t^e \), instead, we assume a simple accounting rule:

\[
NT_{t+1} = \chi_v V_t, \quad C_t^e = (1 - \chi_v) V_t. \tag{22}
\]

Here, \( 0 \leq \chi_v \leq 1 \) represents the fraction of equity retained, i.e., the retention rate of equity for reinvest in the housing market. If the retention rate is equal to 1, the entrepreneurs will put all their own resources in accumulating stock of housing and consume nothing.

\section*{2.3 Banks}

Banks accept deposits \( B \) from consumers at a cost \((R_f - 1)\), borrow in the form of foreign debt from overseas at unit cost \((R_w - 1)\), and lend mortgages \( M \) to entrepreneurs at a mortgage rate \((R_m - 1)\). In addition, the banks pay audit fees to the government to hedge against the risk of mortgage defaults by entrepreneurs. The audit fee is defined as an increasing function of the amount of mortgage loan, \( \Phi(M_t) \), with \( \Phi(0) = 0 \).

Banks maximizes their expected discounted profits given by

\[
\sum_{t=\tau}^{\infty} E_t \{ \Lambda_{b,\tau,t} D_{b,t} \}, \tag{23}
\]

subject to the balanced sheet given by

\[
B_t + R_{m,t-1} M_{t-1} + \mathcal{E}_t F_t^* = M_t + R_{f,t-1} B_{t-1} + R_{w,t-1} \mathcal{E}_t F_{t-1}^* + \Phi(M_t) + P_{c,t} D_{b,t}, \tag{24}
\]

where \( \Lambda_{b,\tau,t} \) is the discount factor of the banks between period \( \tau \) and \( t \) which reflects the discounting of the consumers who own the banks, \( \mathcal{E}_t \) is the nominal exchange rate and \( F_t^* \) denotes the amount of foreign debt banks owe.

The unit borrowing cost of foreign debt is assumed to be a function of the world interest rate \( R_{w,t}^* \) and the amount of foreign debt:

\[
R_{w,t} = R_{w,t}^* \left( \frac{F_t^*}{F_t^*} \right) \xi_2, \quad \xi_2 > 0, \tag{25}
\]

where \( F_t^* \) denotes the steady state level of foreign debt. The parameter \( \xi_2 \) captures the degree of risk premium to compensate the default risk of foreign debt. With \( \xi_2 > 0 \),
$R_{w,t}$ increases with the amount of foreign debt. In particular, if $F_t^* > F_t^*$, $R_{w,t}$ is greater than the world interest rate to cover a possible default risk.

The world interest rate is exogenous and its law of motion is given by

$$R_{w,t} = (R_{w,t-1}^*)^{\rho_{rw}} \frac{R_{w,t}}{R_{w,t-1}} e^{\epsilon_{rw,t}}, \quad \epsilon_{rw,t} \sim N(0, \sigma_{rw}^2),$$

where $R_{w,t}^*$ denotes the steady state value of world interest rate, $\rho_{rw}$ is the smoothing parameter, and the disturbance term $\epsilon_{rw,t}$ is normally distributed with zero mean and standard deviation $\sigma_{rw}$. It’s clear that the logarithm of $R_{w,t}$ follows an AR(1) process.

The first order conditions with respect to $B_t$ and $F_t^*$ are given by

$$R_{f,t} = E_t \left\{ \frac{1}{\Lambda_{b,t+1}} \frac{P_{c,t+1}}{P_{c,t}} \right\}, \quad (27)$$

$$R_{w,t} + R_{w,t}^* = E_t \left\{ \frac{1}{\Lambda_{b,t+1}} \frac{S_{t}}{S_{t+1}} \right\}, \quad (28)$$

where $R_{w,t}^* = \xi_2 R_{w,t}^* \left( \frac{F_t^*}{F_t^*} \right)^{\xi_2-1} \left( \frac{1}{F_t^*} \right)^{\xi_2}$, and $S_t$ is the nominal exchange rate deflated by the consumption goods price index $P_{c,t}$, i.e. $S_t = \frac{S_t}{P_{c,t}}$. Combining Eq.(27) and Eq.(28) and ignoring the covariance terms yields the uncovered interest parity:

$$(1 + \xi_2) R_{w,t}^* \left( \frac{F_t^*}{F_t^*} \right)^{\xi_2} = R_{f,t} E_t \left\{ \frac{S_t}{S_{t+1}} \right\} E_t \left\{ \frac{P_{c,t}}{P_{c,t+1}} \right\}. \quad (29)$$

The first order condition with respect to $M_t$ is given by

$$1 = E_t \left\{ \frac{1}{\Lambda_{b,t+1}} \frac{P_{c,t+1}}{P_{c,t}} (R_{m,t} - \Phi_{m,t}) \right\}. \quad (30)$$

Combining Eq.(27) and Eq.(30) and also ignoring the covariance terms yields

$$R_{m,t} = R_{f,t} + \Phi_{m,t} \Rightarrow R_{m,t} = \left( 1 + \frac{\Phi_{m,t}}{R_{f,t}} \right) R_{f,t}. \quad (31)$$

Notice that there is a ‘wedge’ between the equilibrium mortgage rate and the risk-free interest rate, which captures the risk premium of mortgage loans. The question of how to model the ‘wedge’ is raised. Following Aoki, Proudman and Vlieghe (2004), who adopt the mechanism of Bernanke, Gertler and Gilchrist (1999) to model credit market

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3Other economists, such as Fernandez-Villaverde (2005), suggest thick tail processes rather than normal processes for the error term.
frictions and the so-called ‘financial accelerator’ effect, we assume that the mortgage rate takes the form

\[ R_{m,t} = \left( \frac{NTH_{t+1}}{H_{t+1}} \right)^{-\xi_1} R_{f,t}, \]  

where \( 0 < \xi_1 < 1 \). That is, the risk premium of mortgage loans is modelled as an increasing function of the leverage ratio, given as \( 1 - \frac{NTH_{t+1}}{H_{t+1}} = 1 - \phi_t \) (\( \phi_t \) is the ratio of the entrepreneurs’ internal funds to their desired level of housing stock). Since \( \phi_t \leq 1 \), \( \frac{R_{m,t}}{R_{f,t}} \geq 1 \), i.e., entrepreneurs have to pay higher interest rates on mortgage loans than on risk-free assets. When entrepreneurs only use their own resources to acquire the housing stock, i.e. \( \phi_t = 1 \), the mortgage rate is equal to the risk-free rate.

### 2.4 Firms

The fourth group of agents are the monopolistic competitive firms who employ labor services \( N \) and import intermediate goods \( K \) to produce output \( Y \).

Each firm is assumed to produce a differentiated product under monopolistic competition. The production function for firm \( i \) is given by the Cobb-Douglas form:

\[ Y_t(i) = A_t N_t(i)^{\alpha} K_t(i)^{1-\alpha}, \]

where \( i \in [0, 1] \) denotes the firm-specific index, \( A_t \) represents the common technology level across all firms, \( N_t(i) \) is firm \( i \)'s demand for labor services, and \( K_t(i) \) is firm \( i \)'s demand for imported intermediate goods. Notice that all imported goods are intermediate goods not consumption goods.\(^4\) The productivity or technology shift is assumed to follow a Markov process:

\[ A_t = (A_{t-1})^\rho_a \overline{A}^{1-\rho_a} e^{\epsilon_a,s}, \quad \epsilon_a,s \sim N\left(0, \sigma_a^2\right), \]

where \( \overline{A} \) denotes the steady state level of technology, \( \rho_a \) is the smoothing parameter, and the error term follows a normal distribution with zero mean and finite standard deviation \( \sigma_a \).

\(^4\) According to "Composition of Trade" (2008), Department of Foreign Affairs and Trade (available at http://www.dfat.gov.au/publications), around half of the imported goods were utilized as input to produce domestic final goods in Australia.
The demand function for $Y_t(i)$, as a result of the consumers’ problem, is given by

$$Y_t(i) = \left( \frac{P_{c,t}(i)}{P_{c,t}} \right)^{-\theta} Y_t,$$

(35)

where $P_{c,t}(i)$ is the price set by firm $i$ and $Y_t$ is aggregate output. Given the Calvo (1983) model of price setting, the firm’s price-setting problem in period $\tau$ is to maximize the expected discounted nominal profits:

$$E_{\tau} \sum_{t=\tau}^{\infty} \vartheta^{t-\tau} \Lambda_{\tau,t} \left[ (Y_t(i)P_{c,t}(i) - Y_t(i)MC_t(i)) \right],$$

(36)

subject to the production function, Eq.(33); where $\vartheta$ is the probability of not setting a new price. The discount factor is given by

$$\Lambda_{\tau,t} = \beta^{t-\tau} \left( \frac{C_t}{C_{\tau}} \right)^{-\sigma} \left( \frac{P_{c,\tau}}{P_{c,t}} \right).$$

(37)

The first-order condition with respect to optimal price $P_{c,t}(i)$ is given by

$$E_{\tau} \sum_{t=\tau}^{\infty} \vartheta^{t-\tau} \Lambda_{\tau,t} Y_t \left( \frac{P_{c,\tau}(i)}{P_{c,t}} \right)^{-\theta} \left[ (1 - \theta) + \theta \left( \frac{P_{c,\tau}(i)}{P_{c,t}} \right)^{-1} MC_t(i) \right] = 0,$$

where $MC_t(i)$ denotes the nominal marginal cost, which is the same as the average cost given the Cobb-Douglas production function. In a symmetric equilibrium, all firms will choose the same optimal price

$$MC_t(i) = MC_t, \quad P^o_{c,t}(i) = P^o_{c,t},$$

and hence the optimal pricing decision can be rewritten as

$$P^o_{c,t} = \frac{\theta}{\theta - 1} \frac{E_{\tau} \sum_{t=\tau}^{\infty} \vartheta^{t-\tau} \Lambda_{\tau,t} Y_t \left( (P_{c,t})^\theta MC_t \right)}{E_{\tau} \sum_{t=\tau}^{\infty} \vartheta^{t-\tau} \Lambda_{\tau,t} Y_t (P_{c,t})^\theta}.$$

(38)

In addition, the dynamics for aggregate consumer price index is given by

$$P_{c,t} = \left[ (1 - \vartheta) (P^o_{c,t})^{1-\theta} + \vartheta (P_{c,t-1})^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

(39)

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5The aggregate of differentiated output and the corresponding price index are given by

$$Y_t = \left[ \int_0^1 Y_t(i) \vartheta^{i-1} di \right]^{\frac{1}{1-\theta}}, \quad P_{c,t} = \left[ \int_0^1 P_{c,t}(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}.$$

with $\theta > 1$. 

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If there is no price stickiness, i.e. $\theta = 0$, $P_{c,t}^0 = P_{c,t}$ and the flexible price is given by

$$P_{c,t} = \frac{\theta}{\theta - 1} MC_t,$$

(40)

and the real marginal cost, $\frac{MC_t}{P_{c,t}}$, is a constant over time. However, in sticky price framework it varies.

2.5 Government

The government sector is a combination of the Treasury and the Central Bank. The Treasury collects taxes $T$ from consumers, purchases public goods $G$, and earns audit fees $\Phi$ by providing audit services to banks. The budget constraint is given by

$$P_{c,t} G_t = T_t + \Phi(M_t).$$

(41)

The government expenditure is assumed to evolve exogenously according to

$$G_t = (G_{t-1})^{\rho_g} \overline{G}^{1-\rho_g} e^{g,t}, \quad e_{g,t} \sim N(0, \sigma^2_g),$$

(42)

where $\overline{G}$ is the steady state value of government spending, the $\rho_g$ is the smoothing parameter, and the disturbance term follows a normal distribution with zero mean and standard deviation $\sigma_g$. The Central Bank sets the risk-free rate $R_f$ according to a Taylor-type rule:

$$R_{f,t} = \left[ R_f \pi R_w \left( \frac{\pi_t}{\pi} \right)^{\psi} \left( \frac{t_w}{t_w} \right)^{1-\rho_R} R_{f,t-1}^{\rho_R} \exp(\epsilon_{R,t}) \right] \epsilon_{R,t} \sim N(0, \sigma^2_R),$$

(43)

where $R_f$, $\pi$, and $R_w$ are the steady state values of domestic interest rate, the target inflation rate and the world interest rate, respectively. The smoothing parameter $\rho_R$ captures the feature of inertia in interest rate setting, and $\psi$ is the elasticity of desired interest rate with respect to deviations of inflation from its target $\pi$. The existence of a unique stationary rational expectation equilibrium is ensured by the “Taylor principle”, which requires $\psi > 1$. The disturbance term $\epsilon_{R,t}$ follows a normal distribution with zero mean and finite standard deviation $\sigma_R$.

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6The proposition states that central banks can stabilize the macroeconomy by setting a greater than one-for-one response of the interest rate with respect to inflation rate.
The government has three policy tools to influence the domestic economy in this model. In addition to determining government spending and adjusting the domestic interest rate in response to changes in the price level, the government can affect the housing market by influencing the mortgage rate.

2.6 Foreigners

The last group of agents are the foreigners who lend in the form of foreign debt $F^*$ to the domestic banks at interest rate $(R_w - 1)$, demand goods in the form of exports $X$ and sell intermediate goods $K$ to the domestic firms. The balance of payments equation is given by

$$P_{c,t}X_t + E_t F_t^* = P_{t}^e K_t + R_{w,t-1} E_{t-1} F_{t-1}^*.$$  \hspace{1cm} (44)

For simplicity, we assume that the exports and the foreign price level are constant over time at its steady state value, such that

$$X_t = X, \quad P_t^* = P_t^e.$$  \hspace{1cm} (45)

In the model, the current account is in deficit, that is consistent with the reality in Australia. Thus the cumulated current account is defined as foreign debt rather than foreign asset.

Finally, the resource constraint of the economy is given by

$$Y_t = C_t^c + G_t + C_t^w + X_t + I_t.$$  \hspace{1cm} (46)

2.7 Log-linearized Model

The variables are of two types - state variables or observed variables. The definitions of the variables are summarized in Table 1. A term with a tilde denotes the log deviation from steady state, for instance, $\tilde{c}_t^c = \ln C_t^c - \ln C^c$. The parameters can be summarized as $\Theta = \{\vartheta, \alpha, \eta, \sigma, \delta, \chi, \phi, \psi, \rho_R, \rho_z, \rho_{rw}, \rho_g, \rho_a, \sigma_R, \sigma_z, \sigma_{rw}, \sigma_g, \rho_a\}$, and the steady state values are $\bar{L}, \bar{G}, \bar{C}, \bar{K}, R_f, R_w, \text{ and } R_m$. In the following section, the parameters in the set $\Theta$ are estimated by Bayesian methods, and steady state values are calibrated to match actual averages. The log-linearized equations are summarized below:

$$\tilde{c}_t^c = E_t(\tilde{c}_{t+1}^c) - \frac{\eta \sigma - 1}{\sigma (\eta - 1)} (E_t(\tilde{\omega}_{t+1}) - \tilde{\omega}_t) + \frac{1}{\sigma} [E_t(\tilde{\pi}_{c,t+1}) - \tilde{r}_{f,t}]$$  \hspace{1cm} (47)
\[ \tilde{\omega}_t = (1 - \tilde{\omega})(\tilde{c}^e_t - \tilde{q}_t - \tilde{h}_t) \quad (48) \]

\[ \tilde{\pi}_{c,t} = \left(\frac{1 - \eta \beta}{\eta}\right) \tilde{m}c_t + \beta E_t \left(\tilde{\pi}_{c,t+1}\right) \quad (49) \]

\[ \tilde{m}c_t = \chi \tilde{y}_t - (1 + \chi)\tilde{a}_t - (1 - \alpha) \chi \tilde{k}_t + (1 - \alpha) \tilde{s}_t + \sigma \alpha \tilde{c}^e_t + \frac{(1 - \eta \sigma) \alpha}{\eta - 1} \tilde{\omega}_t \quad (50) \]

\[ \tilde{y}_t = \frac{I}{Y} t + \frac{G}{Y} \tilde{g}_t + \frac{C^e}{Y} \tilde{c}^e_t + \frac{C^e}{Y} \tilde{c}^e_t \quad (51) \]

\[ \tilde{k}_t = \frac{\sigma \alpha \tilde{c}^e_t + (1 + \chi)(\tilde{y}_t - \tilde{a}_t) - \alpha \tilde{s}_t}{1 + \chi - \chi \alpha} + \frac{(1 - \eta \sigma) \alpha}{(\eta - 1)(1 + \chi - \chi \alpha)} \tilde{\omega}_t \quad (52) \]

\[ \tilde{s}_t = \tilde{r}_{w,t}^* + \xi_2 \cdot \tilde{f}_t - \tilde{r}_{f,t} + E_t(\tilde{\pi}_{c,t+1}) + E_t(s_{t+1}) \quad (53) \]

\[ \tilde{r}_{m,t} = \frac{R_m + \delta - 1}{R_m} E_t(\tilde{q}_{t+1}) + E_t(\tilde{\pi}_{c,t+1}) - \tilde{z}_t \quad (54) \]

\[ \tilde{q}_t = -\frac{1}{\eta} \left(\tilde{h}_t - \tilde{c}^e_t\right) \quad (55) \]

\[ \tilde{h}_{t+1} = \tilde{d} \tilde{t} + (1 - \delta) \tilde{h}_t \quad (56) \]

\[ \tilde{r}_{m,t} = \tilde{r}_{f,t} + \xi_1 \left(\tilde{h}_t - \tilde{n}_t\right) \quad (57) \]

\[ \tilde{f}_t = K \tilde{k}_t + \left(\frac{R_f}{1 + \xi_2} + \frac{K}{F} - 1\right) \tilde{s}_t + \frac{R_f}{1 + \xi_2} \left(\tilde{r}_{w,t}^* - 1\right) + R_f \cdot \tilde{f}_{t-1} \quad (58) \]

\[ \tilde{n}_t = \tilde{R}_m + \frac{\delta - 1}{\phi} \tilde{q}_t \left(1 - \frac{1}{\phi}\right) \tilde{r}_{m,t-1} + \tilde{n}_t \quad (59) \]

\[ \tilde{c}^e_t = \tilde{n}_{t+1} \quad (60) \]

\[ \tilde{r}_{f,t} = (1 - \rho_R) \left[\psi \tilde{\pi}_{c,t} + \psi(1 - \gamma)(\tilde{q}_t - \tilde{q}_{t-1}) + \tilde{r}_{w,t}^*\right] + \rho_R \tilde{r}_{f,t-1} + \epsilon_{R,t}, \quad \epsilon_{R,t} \sim N\left(0, \sigma^2_{\epsilon_R}\right) \quad (61) \]

\[ \tilde{a}_t = \rho_a \tilde{a}_{t-1} + \epsilon_a, \quad \epsilon_a \sim N\left(0, \sigma^2_{\epsilon_a}\right) \quad (62) \]

\[ \tilde{r}_{w,t} = \rho_{r_{w,t}} \tilde{r}_{w,t-1} + \epsilon_{r_{w,t}}, \quad \epsilon_{r_{w,t}} \sim N\left(0, \sigma^2_{r_{w}}\right) \quad (63) \]

\[ \tilde{g}_t = \rho_g \tilde{g}_{t-1} + \epsilon_g, \quad \epsilon_g \sim N\left(0, \sigma^2_{g}\right) \quad (64) \]

\[ \tilde{z}_t = \rho_z \tilde{z}_{t-1} + \epsilon_z, \quad \epsilon_z \sim N\left(0, \sigma^2_{z}\right) \quad (65) \]

Eq.(47) is derived from the consumption Euler equation. The conventional Phillips curve is expressed in Eq.(49). Eq.(51) is the log-linearized approximation of the demand function. Eq.(53) incorporates the international borrowing premium in the UIP. Eq.(54) embeds a shock term in the relationship linking the mortgage rate and the return on the
housing market. Eq.(57) shows the relationship between the mortgage rate and the risk-free interest rate. Eq.(55), (58), and (59) are the transition equations for the three state variables: investment in housing, foreign debt, and net worth of entrepreneurs. The net worth can be substituted out using Eq.(60). Eq.(61) is the Taylor-type monetary policy rule. Eq.(62) to (65) give the log-linearized law of motion for the exogenous shocks.

3 Bayesian Estimation

The linear system given in Eq.(47)-(65) has a standard linear state-space representation. It is solved using the algorithm of Sims (2002) for solving linear rational expectation models and the parameters $\Theta$ are estimated by Bayesian methods. This approach for solving and estimating DSGE models has been widely applied to deal with DSGE models, see Ingram and Whiteman (1994), Schorfheide (2000), An and Schorfheide (2006), Justiniano and Preston (2006), Lubik and Schorfheide (2005, 2007), and Matheson (2006) for examples. Several of the priors for the parameters are obtained from studies using Australian data, namely Justiniano and Preston (2006), Lubik and Schorfheide (2007), and Nirmark (2007).

In summary, there are five independent structural shocks in the model:

$$\epsilon_t \equiv (\epsilon_{R,t}, \epsilon_{\omega,t}, \epsilon_{r_{w,t}}, \epsilon_{g,t}, \epsilon_{z,t}).$$

Define the state variables as:

$$x_t \equiv \left( c_t^c, \pi_{c,t}, \omega_t, r_f, s_t, q_t, i_t, r_{m,t}, f_t, r_t^c, k_t, a_t, r_{w,t}, g_t, z_t, E_t(c_{t+1}), E_t(\omega_{t+1}), E_t(\pi_{c,t+1}), E_t(s_{t+1}), E_t(q_{t+1}) \right)' .$$

The five observable variables are: the growth rate of non-housing consumption $c_t^c - c_{t-1}^c + \pi$, the inflation rate of the price level of non-housing consumption $\pi_{c,t}$, the growth rate in investment $i_t - i_{t-1} + \pi_t$, the domestic interest rate $r_{f,t}$, and the rate of change in the real exchange rate $\Delta s_t$.

3.1 Data

Our sample period covers the inflation targeting period 1993:2 to 2008:1. All variables are sourced from the Australian Bureau of Statistics at a quarterly frequency. Non-
housing consumption is the difference between the final consumption expenditure (series ID: A2303280V) and rent and other dwelling expenditures (series ID: A2303254R) while investment in housing is the value of approved total new residential building (series ID: A419851R). The inflation rate is calculated as the log difference of the CPI index for all groups excluding housing and financial services (series ID: A2332686K) and adjusted for the GST effect following Valadkhani and Layton (2004). The 90-day bank rate is used as the measure of the domestic risk free interest rate and the trade weighted real exchange rate is used as the measure of the nominal exchange rate.

In summary, there are 60 observations for each of the five observable variables. All variables are demeaned and tested to be stationary.

### 3.2 Estimation Results

The steady state values are calibrated to match the corresponding average values across the sample periods. The government expenditure ratio is \( G/Y = 0.18 \), household composite consumption ratio is \( C/Y = 0.55 \), and the investment ratio is \( I/Y = 0.09 \). The consumption for entrepreneurs is not observed but can be inferred using the steady state result,

\[
\frac{C^e}{Y} = \frac{1 - \chi_v \phi}{\chi_v} \frac{I}{\delta Y}. \tag{66}
\]

In addition, the consumption for consumers is given by

\[
\frac{C^c}{Y} = \frac{\gamma}{1 - \gamma} \frac{(R_m + \delta - 1)\eta}{\delta} I. \tag{67}
\]

Parameter \( \gamma \) and \( \overline{\sigma} \) can be determined by the other parameters, given by

\[
\gamma = \frac{C \delta - (R_m - 1) \phi \phi}{C \delta - (R_m - 1) \phi \phi + \frac{1}{1} (R_m + \delta - 1)\eta},
\]

\[
\overline{\sigma} = \left[ \left( \frac{1 - \gamma}{\gamma} \right) (R_m + \delta - 1)^{1-\eta} + 1 \right]^{-1}. \tag{68}
\]

In the data, the standard deviations are: for the domestic interest rate 0.0024, for GDP 0.1549 and for government expenditure 0.0279. We assume that the income share attributed to technology in output is 0.4, then \( \sigma_{\alpha} = 0.0619 \). The standard deviation of the world interest rate is calculated as the standard deviation of the interest rate
of the U.S. over the sample period 1993:3 to 2008:3, which is 0.0042. The standard deviation for the mortgage shock is based on the volatility of actual mortgage rates; namely $\sigma_z = 0.0024$.

<table>
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<tr>
<th>$R_f$</th>
<th>$R_m$</th>
<th>$R_w$</th>
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<th>$G/Y$</th>
<th>$I/Y$</th>
<th>$K/F$</th>
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<td>1.0125</td>
<td>1.02</td>
<td>1.01</td>
<td>0.55</td>
<td>0.18</td>
<td>0.09</td>
<td>0.17</td>
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The steady state values are summarized in Table 1, and the prior distribution for the parameters are shown in the left panel of Table 2. The first three columns indicate the mean, standard deviation and distribution for the priors. The fourth column is the starting values while conducting the Bayesian estimation through Markov Chain Monte Carlo methods. The results of the Bayesian estimation are given in the right panel of Table 2. The columns entitled ‘mean’ and ‘std’ in the right panel contain the mean and standard deviation for estimated parameters. $CI(low)$ and $CI(high)$ represent the lower bound and upper bound of the 90% confidence interval for the estimates, respectively. The estimated model is then simulated to obtain the impulse responses to the exogenous shocks.
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<tr>
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<th>Prior Distribution</th>
<th>Estimation Results</th>
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</thead>
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<td>$\phi$</td>
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<td>$\phi$ 0.4814 0.0435 0.4102 0.5529</td>
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<tr>
<td>$\alpha$</td>
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<td>$\alpha$ 0.7629 0.0261 0.7210 0.8058</td>
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<tr>
<td>$\eta$</td>
<td>0.9000 0.05 Beta</td>
<td>$\eta$ 0.9095 0.0451 0.8418 0.9796</td>
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<tr>
<td>$\sigma$</td>
<td>2.2000 0.20 Gamma</td>
<td>$\sigma$ 2.1306 0.1792 1.8388 2.4275</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.0200 0.005 Beta</td>
<td>$\delta$ 0.0196 0.0042 0.0126 0.0263</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.7000 0.10 Beta</td>
<td>$\phi$ 0.6647 0.0630 0.5615 0.7686</td>
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<tr>
<td>$\psi$</td>
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<td>$\chi$</td>
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<td>$\rho_R$</td>
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<tr>
<td>$\sigma_a$</td>
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<td>$\sigma_a$ 0.0116 0.0013 0.0096 0.0136</td>
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<tr>
<td>$\sigma_{rw}$</td>
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<td>$\sigma_{rw}$ 0.0027 0.0007 0.0017 0.0038</td>
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<tr>
<td>$\sigma_z$</td>
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<td>$\sigma_z$ 0.0015 0.0002 0.0011 0.0018</td>
</tr>
</tbody>
</table>

4 Simulation - importance of housing

In this section, impulse responses following a range of exogenous shocks are simulated. The simulated state variables are output, the expenditure ratio, real consumption goods for consumers and entrepreneurs, housing stock, investment in housing market, the inflation rate, relative rental price, the nominal domestic interest rate, the mortgage rate, the nominal exchange rate and foreign debt. All these variables are expressed as percentage deviations from their steady states. The magnitude of each shock is set at
This section investigates the influence of a housing sector on a small open economy by comparing the differences in the impulse responses for a simple small open economy with and without a housing sector.

The log-linearized equations of a DSGE model without a housing sector is as follows:

\[
\begin{align*}
\tilde{c}_t &= E_t (\tilde{c}_{t+1} - \frac{1}{\sigma (\tilde{r}_{f,t} - E_t (\pi_{t+1})))} \\
\pi_t &= \frac{(1 - \vartheta \beta) (1 - \beta)}{\vartheta} \left[ (1 - \alpha) \tilde{s}_t - (1 + \chi) \tilde{a}_t + (\alpha \sigma + \chi \tilde{C}) \tilde{c}_t + \chi \tilde{C} \tilde{g}_t - (1 - \alpha) \chi \tilde{k}_t \right] + \beta E_t (\pi_{t+1}) \\
\tilde{k}_t &= \frac{1 + \chi}{1 + \chi - \chi \alpha} \left( \frac{G}{Y} \tilde{g}_t + \frac{C}{Y} \tilde{c}_t - \tilde{a}_t \right) + \frac{\alpha}{1 + \chi - \chi \alpha} (\sigma \tilde{c}_t - \tilde{s}_t) \\
\tilde{r}_{f,s} &= \tilde{r}_{w,t}^{*} + \xi_2 \cdot \tilde{f}_{s} + E_t (\pi_{t+1}) + E_t (\tilde{s}_{t+1}) - \tilde{s}_t \\
\tilde{f}_t &= \frac{K}{F} \tilde{k}_t + \left( \frac{R_f}{1 + \xi_2} + \frac{K}{F} - 1 \right) \tilde{s}_t + \frac{R_f}{1 + \xi_2} \left( \tilde{r}_{w,t-1}^{*} \right) + R_f \tilde{f}_{t-1} \\
\tilde{r}_{f,s} &= \rho_R \tilde{r}_{f,s-1} + (1 - \rho_R) \tilde{\psi}_t + (1 - \rho_R) \tilde{r}_{w,t}^{*} + \epsilon_{R,t}, \quad \epsilon_{R,t} \sim N \left( 0, \sigma_R^2 \right) \\
\tilde{a}_t &= \rho_g \tilde{a}_{t-1} + \epsilon_{a,t}, \quad \epsilon_{a,t} \sim N \left( 0, \sigma_a^2 \right) \\
\tilde{r}_{w,t}^{*} &= \rho\tilde{r}_{w,t-1}^{*} + \epsilon_{r_{w,t}}, \quad \epsilon_{r_{w,t}} \sim N \left( 0, \sigma_{r_{w}}^2 \right) \\
\tilde{g}_t &= \rho_g \tilde{g}_{t-1} + \epsilon_{g,t}, \quad \epsilon_{g,t} \sim N \left( 0, \sigma_g^2 \right)
\end{align*}
\]

The above equations are similar to those in the core model, except that the parameter \( \gamma \) is set to be zero. The variables pertinent to the housing sector are thus excluded in this model, i.e. \( \tilde{h} = \tilde{h} = \tilde{z} = \tilde{r}_m = \tilde{c}_c = \tilde{q} = 0, \tilde{c} = \tilde{c}_c \) and \( \tilde{\pi} = \tilde{\pi}_c \).

Eq.(69) is the conventional Euler equation, and Eq.(70) represents the Phillips curve. The uncovered interest rate parity (UIP) is described as Eq.(72). The UIP shows the expected depreciation(appreciation) of the domestic currency, to compensate for the difference in domestic and world interest rate\(^7\). Eq.(73) is the transition equation for foreign debt. The Taylor-type monetary rule is expressed in Eq.(74). This rule shows that the central bank adjusts the nominal interest rate in response to the lagged interest rate, the current inflation rate and the current world interest rate. Finally, Eq.(75),

\(^7\)The nominal exchange rate is the price of one unit of foreign currency, measured in units of domestic currency. Therefore an increase of nominal exchange rate implies a depreciation of domestic currency.
Eq.(76), and Eq.(77) describe the exogenous processes, for technology, the world interest rate, and government spending, as first order stationary autoregressive processes.

The calibrated parameters are the same as those estimated for the core model. In the following sub-sections, the impulse responses are explored for each orthogonal shock.

4.1 Productivity Shock

Figure 1 contains the impulse response functions for several key variables following a productivity shock or a technology shock. The impulse responses for the model with a housing sector (hereafter the "core" model) are represented by solid lines, while the impulses for the model without a housing sector (hereafter "basic" model) are represented by dashed lines.

Since this is a supply shock, output initially increases, which increases consumers’ income, and hence consumption. As supply initially exceeds demand, the price level and inflation decreases. According to the Taylor-type rule, the nominal domestic interest rate decreases with the fall in inflation. As the interest rate falls, foreigners are less inclined to invest in assets denominated in domestic currency, and the capital outflows cause a depreciation of the exchange rate. The foreign debt worsens with the increased importation of intermediate goods.

The impact of productivity shock on output for the "core" model is bigger than that for the model without a housing sector. First, as consumers’ disposable income rises, expenditure on both consumption goods and housing services increases (unlike in the base model where all the increase in expenditure falls on consumer consumption). Then, since the rental price is flexible while the price of the consumption good is sticky, the increase in rental prices will be larger\(^8\) than the increase in the prices for consumption goods. This stimulates investment in the housing market. As inflation falls by less in the core model compared to the basic model, it follows that the nominal domestic interest rate will decrease by a smaller extent, and the depreciation of the exchange rate would be less. However, the foreign debt worsens by more because the import of intermediate goods.

\[^8\]The relative rental price is defined as \(Q_t = \frac{P_{h,t}}{P_{c,t}}\). As the denominator decreases, without any other change in numerator, the price of rental property, the relative rental price increases.
goods for producing the output is higher. Thus, introducing a housing sector amplifies the effect of the shock through the investment in housing market channel.

4.2 Fiscal Shock

Figure 2 shows the impulse responses of state variables to a positive government expenditure shock. For the basic model, in the short run, a rise in government expenditure increases the demand for goods. As demand exceeds supply, the price level increases, and hence the rate of inflation increases. As the price level increases, the real wage rate decreases, and households’ income and consumption falls. Responding to the rise in inflation rates, the domestic interest rate increases according to the Taylor-type monetary rule. This attracts capital inflows, the nominal exchange rate appreciates. The foreign debt worsens as output and the importation of intermediate goods rise.

In contrast, for the core model, the inflation rate decreases because demand falls by more in the core model compared with the basic model. When a housing sector is included, the fall in rental prices induces a fall in the investment in housing services. Note that when consumers are faced with a limited disposable income, they will adjust their expenditure on housing services to maintain, to some extent, their consumption.9 Also following the fall in the rental price, entrepreneurial net income falls and consumption by entrepreneurs also decreases in the short run. The fall in domestic interest rate now generates an outflow of capital and the exchange rate depreciates. Foreign debt worsens by less than in the basic case because the respond of output is less in the core model.

4.3 Monetary shock

Figure 3 illustrates the movement of the impulse responses following a tight monetary policy. The dashed lines represent the results from a simple open economy without a housing sector. The rise in the nominal domestic interest rate reflects the positive

9Notice that in this estimated model, the estimate of the intertemporal elasticity, i.e. \( \frac{1}{\rho} = 0.4694 \), is less than the estimate of intratemporal elasticity, i.e. \( \eta = 0.9095 \). As Piazzesi, Schneider, and Vuzel (2007) mentioned if the intertemporal elasticity is less than the intratemporal elasticity, the household is more willing to substitute housing services than he/she is to substitute the overall consumption bundle.
monetary shock. Intuitively, consumers have an incentive to save rather than to spend because of the higher interest rate. Consequently, consumption and output decrease. As the demand for goods falls, the price level and inflation rate decrease. The rise in the domestic interest rate induces capital inflows, and the nominal exchange rate appreciates. The foreign debt decreases or equivalently improves.

The solid lines show the results from the core model with a housing sector. In this case, the increase in the domestic interest rate also reduces investment in housing. As the fall in demand is greater in this scenario, the fall in the price level, and in the inflation rate is greater which then mitigates against the rise in the domestic interest rate and the appreciation of the exchange rate. While the fall in prices is large enough to induce a small increase in consumption goods by consumers, overall, tight monetary policy discourages the demand for goods, and the final output decreases by more than in the basic case due to the fall in investment in housing as well as the fall in consumption by entrepreneurs as their rental income falls. The contraction in the economic activity improves foreign debt by more than in the basic case.

4.4 World interest rate shock

Figure 4 shows the impulse responses to a world interest rate shock. The dashed lines represent the results from the simple open economy without a housing sector. As indicated by the Taylor-type monetary rule, the domestic interest rate increases corresponding to the world interest rate shock. Households are reluctant to spend, and hence the demand for goods decreases. Consequently, consumption and output falls. A positive shock to the world interest rate induces capital outflows and the exchange rate depreciates. The demand for imported intermediate goods falls and the foreign debt improves initially.

The solid lines represent the impulse responses to a world interest rate shock for the core model. Output falls by more compared with the simple basic open economy model again because the fall in housing investment exacerbates the fall in output. The fall in output decreases disposable income, and reduces expenditure on both consumption goods and housing services. As the demand for housing services falls, it induces a fall
in the rental price which in turn implies that entrepreneurs are reluctant to invest in housing. Given the flexibility of the rental price, it falls by more than the price of consumption goods and the rate of inflation decreases by more than in the basic model. The overall effect is to induce a fall in the domestic interest rate in the model with a housing sector in contrast to the rise in interest rate in the model without a housing sector.

4.5 Mortgage rate shock

The fifth impulse response examined here is for the case, when a shock is introduced to drive a wedge between the return to housing and the mortgage rate. Figure 5 shows the impulse responses following a positive mortgage rate shock. The shock may be interpreted as an effort by the government to cool the housing market by forcing the mortgage rate to rise above observed housing returns.

As a result of this shock, mortgage loan approvals falls both because banks raise the leverage ratio to reduce the probability of defaults and entrepreneurs desire for housing investment wanes in the face of increasing difficulties with borrowing. This shock diminishes the supply of housing services, and hence raises the relative rental price. To maintain the consumption of goods, the consumers adjust their expenditure on housing services. The fall in housing investment reduces aggregate demand for output, which generates a situation of excess supply. Thus the price level decreases and the inflation rate and hence domestic interest rate falls. In accordance with the interest parity condition, the fall in domestic interest rate induces a depreciation of the domestic currency. As output falls, the demand for imported intermediate goods also falls and the foreign debt improves in the short run.

5 Conclusion

The paper has proposed a small DSGE model with a housing sector for a small open economy. The model is estimated to fit the Australian economy. A version of the model with a housing sector and a version without a housing sector were shocked and
the impulses compared.

One insight is that the relative flexibility of housing and goods prices play an important role in determining the dynamics of housing and consumption expenditure. The effects of a supply shock are exaggerated whereas the effects of a demand shock are dampened. The effects of shocks to the interest rate on output and inflation are, not surprisingly, greater in the model with a housing sector.

The role of a government shock in the housing market was explored. This would be the subject of future research where the modelling of the production sector would be disaggregated to distinguish between the production of consumption goods from the production of shelter type goods. This would then clarify the relative role of government intervention in the demand and supply of housing.
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6 Appendix: Tables and Figures

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<th>Definition</th>
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<td>$A$</td>
<td>Productivity index</td>
<td>$\omega$</td>
<td>Expenditure ratio</td>
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<td>$B$</td>
<td>Deposits</td>
<td>$N$</td>
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<td>Government expenditures</td>
<td>$T$</td>
<td>Taxes</td>
</tr>
<tr>
<td>$H$</td>
<td>Housing stock</td>
<td>$V$</td>
<td>Equity</td>
</tr>
<tr>
<td>$I$</td>
<td>Investment in houses</td>
<td>$W$</td>
<td>Wage rate</td>
</tr>
<tr>
<td>$K$</td>
<td>Imported intermediate goods</td>
<td>$X$</td>
<td>Exports</td>
</tr>
<tr>
<td>$M$</td>
<td>Mortgage</td>
<td>$Y$</td>
<td>Output</td>
</tr>
<tr>
<td>$MC$</td>
<td>Marginal cost</td>
<td>$S$</td>
<td>CPI-deflated exchange rate</td>
</tr>
</tbody>
</table>
Figure 1: Impulse responses following a technology shock: with (solid line) and without (dashed line) a housing sector
Figure 2: Impulse responses following a fiscal shock: with (solid line) and without (dashed line) a housing sector
Figure 3: Impulse responses following a monetary shock: with (solid line) and without (dashed line) a housing sector
Figure 4: Impulse responses following a world interest rate shock: with (solid line) and without (dashed line) a housing sector
Figure 5: Impulse responses following a regulatory shock: with (solid line) and without (dashed line) a housing sector