Enforcement policies and innovation in the presence of commercial piracy*

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Abstract

In this paper we analyze the impact of regulatory enforcement policy on a firm’s incentive to innovate, which is measured by the quality of the product that the firm produces, in the presence of commercial policy. We also determine the socially optimal enforcement policy. An increase in the enforcement policy unambiguously increases the firm’s incentive to innovate. Even, the inclusion of innovation may result in no enforcement policy as socially optimal. If enforcement policy is socially optimal then the prevention of piracy is not guaranteed.

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1. **Introduction**

Commercial piracy has emerged as one of the leading global challenges faced by software businesses, entertainment industry, law enforcement agencies, and international trade partners.\(^1\) This issue assumes importance not only because of the high magnitude of the immediate loss in retail sale but also of its possible detrimental effects on the incentive to innovate. In terms of the reduced incentive to innovate, Business Software Alliance believes that “local software industries crippled from competition with high-quality pirated software” and International Federation of the Phonographic Industry (IFPI) in its 2005 Commercial Piracy Reports argues, “The illegal music trade is destroying creativity and innovation, …”\(^2\)

As a consequence, the government is often called upon to perform the duty of strengthening and enforcing copyright. The US Trade Representative in its recent 2004 Special 301 Report, a trade sanction tailored for intellectual property trade concerns, posited that “ineffective enforcement of intellectual property rights, commercial piracy - in particular the growing problem of pirate production of optical media such as CDs, DVDs and CD-ROMs,…continue to be a global threat.” The threat is so real as to force the Bush Administration to launch an inter-agency initiative called the *Strategy Targeting Organized Piracy* (STOP) in October 2004.

The common rationale for the existence of enforcement policies to combat piracy of digital products like software is to provide incentive to create and innovate. If the regulatory authority ignores innovation it implies that this authority places greater value on the short run benefits associated with piracy. Consequently, in the context of commercial piracy, allowing piracy and hence, no enforcement is the

\(^{1}\) Commercial piracy refers to a situation where a firm/s illegally reproduces and sells copies of legitimate products thereby competing with the original producer.

\(^{2}\) BSA in their 2005 Piracy Study claims US$34 billion in worldwide losses. BSA further projects that in the next five years almost US$200 billion worth of software will be pirated globally.
optimal policy in the short run because it results in larger market output and lower price thus leading to a higher consumer surplus and social welfare. Hence, the producers of digital products may lobby for the protection of their products. However, even lobbying may not be sufficient to generate enforcement as the optimal policy that may prevent piracy with certainty. In the case of end-user piracy, Chen and Png (1999), Cheng, Sims, and Teegen (1997) and Noyelle (1990 show that lower price rather than regulatory enforcement is a better strategy to combat this type of piracy in the short run.

Given the ineffectiveness of enforcement policies in preventing piracy in the short run where innovation is treated as a sunk cost it is important to analyze such policies in the long run that take into consideration innovation. Such an analysis will provide an insight into whether the regulatory authority’s stance towards piracy when innovation is taken into consideration differs from the short run situation. This is the main focus of this paper. For an accurate assessment of the efficacy of copyright protection we take into consideration the costs and benefits of developing a new product as well as the costs associated with administering copyright protection.

In this paper we introduce the regulatory authority’s (hereafter, referred to as the government) choice of monitoring, which is costly, as a measure of enforcement policy for copyright protection. We consider a sequential entry-deterrence framework where we introduce a firm’s (hereafter, referred to as the monopolist) choice of quality as a measure of the incentive to innovate. The purpose of our paper is two-fold. First, we analyze the effects of increased copyright protection on the social welfare loss due to underproduction in the shadow of commercial piracy. Second, we

3 See Banerjee 2006a and 2006b.
4 End-user piracy is defined as copying for personal consumption rather than for commercial purposes. Shy and Thisse (1999) show that in the presence of network externalities non-protection against piracy is an equilibrium. Takeyama (1994), Conner and Rumelt (1991), and Nascimento and Vanhonacker (1988) also discuss the role of network externalities on the marketing of software.
investigate whether the socially optimal copyright protection policy results in piracy or in its deterrence as the subgame perfect equilibrium in a full-scale strategic entry-deterrence framework.  

We find that increased copyright protection unambiguously improves the incentive to innovate in the equilibrium. However, the impact of the government’s optimal copyright policy on piracy remains ambiguous even in the long run scenario where innovation is included in the analysis. Specifically we show that monitoring may or may not be the socially optimal outcome. Even if monitoring is socially outcome it may not result in the deterrence of piracy. We specify the conditions under which all these different equilibria hold.

Novos and Waldman (1984) and Qiu (2006) study the impact of end-user piracy on the incentive to innovate. However, this literature assumes copyright protection to be costless and exogenously given. Hence, the optimal copyright policy is not endogenously determined and consequently whether the equilibrium regime allows or prevents copying cannot be identified.  

This paper is arranged as follows. In sections 2 and 3 we discuss the model and analyze the monopolist’s equilibrium strategies. Sections 4 and 5 contain the welfare analysis and the concluding remarks.

2. The Model 

Let us consider the market for a product like software that requires copyright protection. We begin our analysis by describing the monopoly situation in the absence

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5 This model encompasses the basic elements emphasized by Landes and Posner (1989: page 326). Furthermore, the social welfare maximizing equilibrium strategy is fully characterized in our model while previous papers have mostly performed comparative statics.

6 Novos and Waldman (1984) only considers the price-quality combination that allows copying and show that a sufficient condition is needed to sustain the common claim that increases in copyright protection decreases the social welfare loss due to underproduction. Qiu (2006) considers the impact of legal and copyright protection policies on software development and shows that only “customized software” is developed under weak copyright protection. Both “customized” and “packaged” software are developed under strong copyright protection.
of piracy. There is a continuum of consumers indexed by $\theta$, $\theta \in [0,1]$. $\theta$ is assumed to follow a uniform distribution. We assume there is no resale market for used software. Each consumer is assumed to purchase only one unit of the software. The utility of a type $\theta$ consumer is,

$$U(\theta) = \begin{cases} \theta Q - p_m & \text{if the consumer buys the software,} \\ 0 & \text{if the consumer does not buy.} \end{cases} \quad (1)$$

$\theta$ is the consumer’s valuation of the software, $Q$ is the quality of the software which is a measure of the incentive to innovate, and $p_m$ is the price of one unit of the software charged by the monopolist. Thus, in the model, consumers differ from one another on the basis of their valuation of the software.

$\theta_m$ is the marginal consumer who is indifferent between buying and not buying:

$$U(\theta_m) = Q\theta_m - p_m = 0 \Rightarrow \theta_m = \frac{p_m}{Q}. \quad (2)$$

In the absence of piracy, the monopolist faces the demand function,

$$D_m(p_m) = \frac{1}{\theta_m} \int_{\theta_m}^1 d\theta = 1 - \frac{p_m}{Q}. \quad (3)$$

Let $F(Q)$ be the fixed cost of developing a software of quality $Q$. We assume $F'(Q) > 0$ and $F''(Q) > 0$ for $Q > 0$, and $F'(Q) = 0$ for $Q = 0$. For tractability we assume $F(Q)$ to take the form $F(Q) = \frac{Q^2}{2}$. The cost of replicating the software after it has been developed is assumed to be zero. So the monopolist’s profit is;

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7 In the copyright literature, it is also called the “cost of expression” which does not depend on the amount of goods produced. See Landes and Posner (1989).
\[
\pi_m = p_m D_m - \frac{Q^2}{2}.
\]
The monopolist chooses a price and quality that maximizes its profit. The equilibrium monopoly results are,
\[
Q^* = \frac{1}{4}, \quad p_m^* = \frac{Q^*}{2} = \frac{1}{8}, \quad \text{and} \quad \pi_m^* = \frac{1}{32}. \tag{4}
\]

Let us now introduce piracy in our model. We consider four types of agents: the consumers, the monopolist, a pirate who illegally reproduces and sells licensed software, and the government which is responsible for monitoring and penalizing the pirate. The game played between the government, the monopolist, the pirate, and the consumers is specified in extensive form as follows.

**Stage 1:** The government chooses a monitoring rate \( \alpha \) and a penalty \( G \).

**Stage 2:** The monopolist chooses a quality \( Q \) and a price \( p_m \).

**Stage 3:** The makes its entry decision. If the pirate enters then he chooses a price \( p_c \).

**Stage 4:** The consumers make their purchase decision.

Let us discuss the behavior of each of the agents in the model. The government only works through the supply side in controlling piracy. Users do not face the risk of prosecution from the use of pirated software. The government is responsible for monitoring and penalizing the pirate. Let \( \alpha \) be the monitoring rate which is the probability of detection and \( G \) the penalty. Let \( c(\alpha) \) be the cost of monitoring. We assume \( c(0) = 0, c'(\alpha) > 0, c'(0) = 0, c''(\alpha) > 0 \). For tractability we assume \( c(\alpha) = \frac{k\alpha^2}{2}, \quad k > 0 \). The pirate pays the penalty \( G \) to the government if his illegal operation is detected to compensate for incurring the monitoring cost.  

\footnote{Qiu (2006) also assumes that the government is responsible for collecting the penalty which is given exogenously because he does not consider costly monitoring.}
The government chooses $\alpha$ to maximize domestic social-welfare and the fine $G$ is given institutionally. Let $R$ be the net expected revenue of the government from its anti-piracy policy.

$$R = \alpha G - c(\alpha).$$

(5)

The pirated software is assumed to be an inferior substitute of the original software. Let $qQ$ be the quality of the pirated software, $q \in (0,1)$, $q$ is given exogenously and is common knowledge. The qualitative difference between the original and the pirated software arises because the support benefits and the full warranty that are included with the purchase of the original software do not come with the purchase of the pirated software. We also assume that the pirate’s marginal cost of duplicating is zero. The pirated software is not available if the pirate do not enter or if the pirate enters and is detected. The rationale behind this assumption is that the store selling pirated products is raided prior to selling.

The behavior of a type-$\theta$ consumer is as follows. One, he only buys the original software. Two, he buys the pirated software if it is available. Three, he buys nothing. The utility of a type $\theta$ consumer if the pirated software is available is, 

9 Banerjee (2006a) mention that the inferior quality of the illegal software can be viewed as the present discounted value of future updates that are available at a lower price and only come with the purchase of the legitimate software. The qualitative difference is intended to capture these aspects and is assumed to be common knowledge, which allows the monopolist to interact strategically with the pirate. Banerjee (2003), Besen and Kirby (1989), Takeyama (1994) also assume that originals and copies are imperfect substitutes.

10 We set this bound to ensure that the profits are not indeterminate.

11 According to the UK police, a factory raid has netted pirated video games, DVDs and CDs with a street value estimated at £1 million. 30,000 of the counterfeit discs were seized from the factory in West Midlands, and there was an estimated 10,000 each of DVDs, video games and audio CDs. Along with the discs, seven PCs were seized with 35 DVD re-writers, 19 HDDs, 15 Xbox 360 consoles and two Wii consoles. "Multiple" modchips were also found. This was reported on 9.03.2009 in http://www.afterdawn.com/news/archive/16302.cfm. Thousands of illegal CDs have been seized following police raids in Sydney to break up an international music piracy ring. The report titled, “Pacific Music Piracy Ring Broken”, published on 22.04.2008 in tvnz.co.nz/content/1734328, mentions, “A Sydney man has been charged after raids carried out by the Australian Federal Police on homes and businesses in NSW today and yesterday, including on a manufacturing plant and on retail outlets. It's alleged an organised gang in Australia has been manufacturing tens of thousands of re-mixed compilation albums and distributing them throughout Australia, New Zealand and the Pacific.
\[ U(\theta) = \begin{cases} 
\theta Q - p_m & \text{if the consumer buys the original software,} \\
q\theta Q - p_c & \text{if the consumer buys the pirated software,} \\
0 & \text{if the consumer does not buy.} 
\end{cases} \] (6)

If the pirated software is not available then the utility of a type \( \theta \) consumer is,

\[ U(\theta) = \begin{cases} 
\theta Q - p_m & \text{if the consumer buys the software,} \\
0 & \text{if the consumer does not buy.} 
\end{cases} \] (7)

If the pirated software is available then there are two marginal consumers, \( \theta_1^a \) and \( \theta_2 \). The marginal buyer \( \theta_1^a \) is indifferent between buying the original and the pirated software:

\[ \theta_1^a Q - p_m = q\theta_1^a Q - p_c \Rightarrow \theta_1^a = \frac{p_m - p_c}{Q(1-q)}. \] (8)

The marginal buyer \( \theta_2 \) is indifferent between buying the pirated software and not buying at all:

\[ q\theta_2 Q - p_c = 0 \Rightarrow \theta_2 = \frac{p_c}{qQ}. \] (9)

If the pirated software is not available then there is only one marginal consumer, \( \theta_1^b \), who is indifferent between buying the original software or not. So,

\[ \theta_1^b = \frac{p_m}{Q}. \] (10)

We derive the demand faced by the monopolist and the pirate from equations (9), (10), and (11) and is shown in equation (12).

\[ D_m(p_m, p_c) = \begin{cases} 
1 - \theta_1^a = 1 - \frac{p_m - p_c}{Q(1-q)} & \text{if the pirated software is available,} \\
1 - \theta_1^b = 1 - \frac{p_m}{Q} & \text{if the pirated software is not available.} 
\end{cases} \] (11)

\[ D_r(p_m, p_c) = \begin{cases} 
\theta_1^a - \theta_2 = \frac{(q p_m - p_c)}{qQ(1-q)} & \text{if the pirated software is available,} \\
0 & \text{if the pirated software is not available.} 
\end{cases} \] (12)
Table 1 summarizes the events and the corresponding demand, profit functions and consumer surplus and social welfare. The events are pirate do not enter, pirate enters and is detected, and pirate enters and is not detected.

**Table 1: Events, Demand, Profits, Consumers Surplus and Government Revenue**

<table>
<thead>
<tr>
<th>Events</th>
<th>Pirate does not enter</th>
<th>Pirate enters and is not detected with probability ((1 - \alpha))</th>
<th>Pirate enters and is detected with probability (\alpha)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Outcomes</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Market Demand</strong></td>
<td>(D_m = 1 - \frac{p_m}{Q})</td>
<td>(D_m = 1 - \frac{p_m - p_c}{Q(1 - q)})</td>
<td>(D_m = 1 - \frac{p_m}{Q})</td>
</tr>
<tr>
<td></td>
<td>(D_c = 0)</td>
<td>(D_c = \frac{(q p_m - p_c)}{q Q(1 - q)})</td>
<td>(D_c = 0)</td>
</tr>
<tr>
<td><strong>Monopolist’s Profit</strong></td>
<td>(p_m - \frac{p_m^2}{Q} - F(Q))</td>
<td>(p_m - \frac{p_m^2 - p_m p_c}{Q(1 - q)} - F(Q))</td>
<td>(p_m - \frac{p_m^2}{Q} - F(Q))</td>
</tr>
<tr>
<td><strong>Pirate’s Profit</strong></td>
<td>0</td>
<td>(\frac{(q p_m - p_c)p_c}{q Q(1 - q)})</td>
<td>(\frac{(q p_m - p_c)p_c}{q Q(1 - q)} - G)</td>
</tr>
<tr>
<td><strong>Consumer Surplus</strong></td>
<td>(\int_{\theta_1}^{\theta_2} (\theta Q - p_m) , d\theta)</td>
<td>(\frac{\int_{\theta_1}^{\theta_2} (\theta Q - p_m) , d\theta + \int_{\theta_2}^{\theta_3} (q \theta Q - p_c) , d\theta}{1})</td>
<td>(\frac{\int_{\theta_1}^{\theta_2} (\theta Q - p_m) , d\theta}{1})</td>
</tr>
<tr>
<td><strong>Net Government Revenue</strong></td>
<td>(-c(\alpha))</td>
<td>(-c(\alpha))</td>
<td>(G - c(\alpha))</td>
</tr>
</tbody>
</table>

Using Table 1 we get the consumer surplus as,

\[
CS = \begin{cases} 
\alpha \int_{\theta_1}^{\theta_2} (\theta Q - p_m) d\theta + (1 - \alpha) \left( \int_{\theta_1}^{\theta_2} (\theta Q - p_m) d\theta + \int_{\theta_2}^{\theta_3} (q \theta Q - p_c) d\theta \right) & \text{if pirate enters,} \\
\int_{\theta_1}^{\theta_2} (\theta Q - p_m) d\theta & \text{if pirate do not enter.}
\end{cases}
\]  

(12)

Let us discuss the behavior of the firms. We assume that a firm remains in the market only if it is making nonzero profit. From Table 1 we get the expected profits
of the monopolist and the pirate as follows.

\[
\pi_m(p_m, p_c, \alpha) = \begin{cases} 
\alpha(p_m - \frac{p_m^2}{Q}) + (1 - \alpha)(p_m - \frac{p_m^2 - P_m^c}{Q(1 - q)}) - F(Q), & \text{if pirate enters,} \\
qP_m - F(Q), & \text{if pirate do not enter.} 
\end{cases}
\]

(13)

\[
\pi_c(p_m, p_c, \alpha) = \begin{cases} 
(1 - \alpha)(\frac{qp_m - P_c}{qQ(1 - q)}) - \alpha G, & \text{if pirate enters,} \\
0, & \text{if pirate do not enter.} 
\end{cases}
\]

The monopolist chooses either an *accommodating (ac)* strategy or an *aggressive (ag)* strategy. In the case of the *ac-strategy* the monopolist behaves as a leader and chooses a combination of price and quality that maximizes profit assuming that the pirate may enter the market. So the *ac-strategy* plays no role in eliminating the possibility of the pirate’s entry. It is only the monitoring rate that can prevent the pirate’s entry. The *ag-strategy* is a limit price strategy such that it is not profitable for the pirate to enter the market. In this case, the monopolist plays a strategic role by eliminating the possibility of a pirate’s entry.

Social-welfare (SW) is the sum of the monopolist’s and pirate’s profits, the consumer surplus and the government’s net revenue.

\[
SW = \begin{cases} 
\pi_m + \pi_c + CS + \alpha G - c(\alpha), & \text{if the pirate enters,} \\
\pi_m + CS - c(\alpha), & \text{if the pirate do not enter.} 
\end{cases}
\]

(14)

3. Equilibrium Accommodating And Aggressive Strategies

In this section we discuss the equilibrium accommodating and aggressive strategies. In view of equation (14), the pirate’s reaction function, if he enters, is,

\[
p_c = \frac{qp_m}{2}.
\]

(15)

3.1. Equilibrium Accommodating Strategy

Suppose the pirate decides to enter the market. In the *ac-subgame* the monopolist’s price and the quality strategy allows the pirate’s entry. Substituting the
pirate’s reaction function into the monopolist’s profit function in equation (13), and equating its first derivatives with respect to \( p_m \) and \( Q \) to zero gives us the equilibrium ac-strategy. The results are summarized in Proposition 1.

**Proposition 1.** (i) The monopolist’s equilibrium ac-strategy is the pair \((p_m^{ac^*}, Q^{ac^*})\)

where \( p_m^{ac^*} = \frac{(1-q)^2}{2(2-q-aq)^2} \) and \( Q^{ac^*} = \frac{1-q}{2(2-q-aq)} \).

(ii) The pirate’s equilibrium profit is \( \pi_c^{ac^*} = \frac{q(1-\alpha)(1-q)^2}{8(2-q-aq)^3} - \alpha G \) and he does not enter if \( \bar{\alpha} \leq \alpha \leq 1 \), where \( \bar{\alpha} \) satisfies \( \pi_c^{ac^*} (\bar{\alpha}) = 0 \).

(iii) The monopolist’s equilibrium profit is

\[
\pi_m^{ac^*} = \begin{cases} 
\pi_m^{ac_{m1}} = \frac{(1-q)^2}{8(2-q-aq)^3}, & \text{if } \alpha < \bar{\alpha}, \\
\pi_m^{ac_{m2}} = \frac{(1-q)^2(2+q-3aq)}{8(2-q-aq)^3}, & \text{if } \alpha \geq \bar{\alpha}.
\end{cases}
\]

(iv) \( p_m^{ac^*}, Q^{ac^*}, \pi_{m1}^{ac^*}, \pi_{m2}^{ac^*} \) are increasing in the monitoring rate, and \( \pi_{m1}^{ac^*} \) and \( \pi_{m2}^{ac^*} \) are convex in the monitoring rate.

(v) \( Q^{ac^*} \leq Q^*, \pi_m^{ac^*} \leq \pi_m^*, \) and the monopoly outcome is restored at \( \alpha = 1 \).

We discuss the proof of part (ii) of Proposition 1 in the main text as it is instructive. The rest of the proof is in the Appendix. Substituting \( p_m^{ac^*} \) and \( p_c^{ac^*} \) in the pirate’s profit function yields \( \pi_c^{ac^*} = \frac{q(1-\alpha)(1-q)^2}{8(2-q-aq)^3} - \alpha G \). Let us study the properties of \( TR_c^{ac^*} = \frac{q(1-\alpha)(1-q)^2}{8(2-q-aq)^3} \) which is the pirate’s expected revenue.

\[
dTR_c^{ac^*} = \frac{q(1-q)^2(2q-1-\alpha q)}{4(2-q-aq)^4}, \quad \frac{dTR_c^{ac^*}}{d\alpha} = \frac{q(1-q)^2(2q-1-\alpha q)}{4(2-q-aq)^4} \leq 0 \text{ for } 0 < q \leq 0.5.
\]

Hence in this range of \( q \) \( \pi_c^{ac^*}(\alpha) \) is monotonically decreasing in \( \alpha \). For \( 1 > q > 0.5 \)
the sign of \( \frac{d TR_{ac}^*}{d \alpha} \) depends on the sign of \( (2q - 1 - \alpha q) \). For any given \( q \) in the range 1 > \( q > 0.5 \), 2q - 1 - \( \alpha q \geq 0 \) if \( \frac{2q - 1}{q} \geq \alpha \). This is shown in Figure 1.

![Graph of (-1+2q-aq) for given values of q, against \( \alpha \)](image)

**Figure 1: Graph of (-1+2q-aq) for given values of q, against \( \alpha \)**

From Figure 1 we observe that for each \( q \) in the range 1 > \( q > 0.5 \),

\[ 2q - 1 - \alpha q \geq 0 \]

for a monitoring rate below a certain critical level. For example, say \( q = 0.9 \). Then for \( \frac{8}{9} \geq \alpha \), 2q - 1 - \( \alpha q \geq 0 \). This means for each \( q \) in the range 1 > \( q > 0.5 \), \( TR_{ac}^* \) has an inverted U-shape. Thus for 1 > \( q > 0.5 \), \( \pi_{ac}^* \) is either monotonically decreasing or also has an inverted U-shape with respect to the monitoring rate depending on the value of \( G \). Hence, for all \( q \in (0,1) \) there exists an \( \alpha \), say \( \alpha^* \), at which \( \pi_{ac}^{\alpha^*} = 0 \) and \( \pi_{ac}^{\alpha^*} \) is monotonically decreasing for \( \alpha^* \leq \alpha \leq 1 \). Hence, \( \pi_{ac}^{\alpha^*}(\alpha) \leq 0 \) for \( \leq \alpha \leq 1 \).
The above analysis has the following implications. An increase in the monitoring rate has two opposing effect on the pirate’s profit. An increase in the monitoring rate raises $Q^{ac*}$ which in turn raises the pirate’s profit because the pirated product’s price is directly related to $Q^{ac*}$ via $p_m$ as seen from the reaction function. However, an increase in the monitoring rate increases the probability of detection which in turn reduces the pirate’s profit. These two combined effects determine the shapes of $TR^c_m(\alpha)$ and $\pi^c_m(\alpha)$ with respect to the monitoring rate. The shape of the latter is also determined by the strength of the penalty $G$.

Parts (ii) and (iii) of Proposition 1 implies that the pirate’s entry solely depends up on the monitoring rate. Specifically if the monitoring rate is at least as much as $\alpha = \bar{\alpha}$ the pirate’s entry is prevented and monopolist’s equilibrium profit jumps from $\pi^m_1$ in the interval $\alpha \in [0, \bar{\alpha})$ to $\pi^m_2$ in the interval $\alpha \in [\bar{\alpha}, 1]$. The monopoly outcome is restored when detection of piracy takes place with certainty, that is, at $\alpha = 1$, $\pi^m_2(\alpha = 1) = \pi^+_m$, $Q^{ac*}(\alpha = 1) = Q^*$ and $p^m_2(\alpha = 1) = p^*_m$.

Part (iv) of Proposition 1 implies that an increase in the monitoring rate increases the product quality because an increase in the monitoring rate decreases the likelihood of the pirate’s entry and hence the incentive to invest more in innovation increases which is captured by the improvement in product quality. Correspondingly, there is an increase in the equilibrium price.

From part (ii) of Proposition 1 it follows that there are two consumer surplus and social welfare functions for $\alpha \in [0, \bar{\alpha})$ (when the pirate enters) and $\alpha \in [\bar{\alpha}, 1]$ (when he does not enter). These functions expressed in terms of $Q^{ac*}$ are,
The monopolist’s ag-strategy is the pair \( (p_m, Q) \) which is as follows. The monopolist chooses a limit price strategy such that it is not profitable for the pirate to enter the market and correspondingly chooses the profit maximizing quality. Alternatively, the monopolist can also choose a quality level such that it is not profitable for the pirate to enter the market and correspondingly chooses the profit maximizing price. The result is the same in both cases. We use the first approach in this paper. Substitution of the pirate’s reaction function in its profit function yields

\[
\pi_c = \frac{(1 - \alpha)q p_m^2}{4(1 - q)Q} - \alpha G.
\]

The pirate does not enter if

\[
\pi_c = \frac{(1 - \alpha)q p_m^2}{4(1 - q)Q} - c(\alpha) \leq 0,
\]

which is

\[
p_m^2 \leq \frac{4(1 - q)c(\alpha)}{q(1 - \alpha)}.
\]

The results for the equilibrium ag-strategy are summarized in Proposition 2 and the proof is given in the Appendix.

**Proposition 2.** (i) The equilibrium ag-strategy is the pair \( (p_m^{ag*}, Q^{ag*}) \) where

\[
p_m^{ag*} = \min \left( 2Q^{ag*}_{m}, \frac{Q^{ag*}}{2} \right) \quad \text{and} \quad Q^{ag*} = \left( \frac{(1 - q)\alpha G}{(1 - \alpha)q} \right)^{\frac{1}{3}}.
\]
profit is, \( \pi_m^{ag*}(\alpha) = \begin{cases} \frac{3Q^{ag*2}}{2} - 4Q^{ag*3}, & \text{for } 0 \leq \alpha \leq \alpha_{\text{max}}, \\ \frac{Q^{*} - Q^{*2}}{4}, & \text{for } \alpha_{\text{max}} \leq \alpha \leq 1, \end{cases} \)

where \( \alpha_{\text{max}} \) satisfies

\[
\frac{\alpha_{\text{max}} G}{1 - \alpha_{\text{max}}} = \frac{qQ^{ag*}}{16(1 - q)}.
\]

(ii) \( p_m^{ag*}, Q^{ag*} \) and \( \pi_m^{ag*} \) are monotonically increasing in \( \alpha \) in the range \( \alpha \in [0, \alpha_{\text{max}}] \). \( \pi_m^{ag*} \) is concave in \( \alpha \).

(iii) \( Q^{ag*} \leq Q^*, p_m^{ag*} \leq p_m^*, \) and \( \pi_m^{ag*}(\alpha) \leq \pi_m^* \). The monopoly outcome is restored at \( \alpha \geq \alpha_{\text{max}} \).

At, \( \alpha = \alpha_{\text{max}} \), the entry-deterring limit price is the monopoly price, which is \( \frac{Q^*}{2} \). For monitoring rates above the critical level, \( \alpha_{\text{max}} \), there is no reason to choose a price more than \( \frac{Q^*}{2} \), since that lowers profit and has no effect on entry. For monitoring rates below \( \alpha_{\text{max}} \) the entry-deterring limit price is less than the monopoly price. When there is no monitoring, the equilibrium limit price is zero which is obtained by substituting \( \alpha = 0 \) in \( p_m^{ag*} = 2Q^{ag*2} \). At \( \alpha = 0 \), \( Q^{ag*} = 0 \). Therefore, \( \pi_m^{ag*}(\alpha = 0) = 0 \).

The consumer surplus and the social-welfare functions for the equilibrium ag-strategy expressed in terms of \( Q^{ag*} \), are

\[
CS^{ag*} = \begin{cases} \frac{Q^{ag*2}}{2} - 2Q^{ag*3} + 2Q^{ag*3}, & \text{for } 0 \leq \alpha \leq \alpha_{\text{max}}, \\ \frac{Q}{8}, & \text{for } \alpha_{\text{max}} \leq \alpha \leq 1. \end{cases}
\] (20)
$SW_{ag}^* (\alpha) = \frac{Q_{ag}^*}{2} - \frac{Q_{ag}^{*2}}{2} - \frac{k\alpha^2}{2}, \text{ for } 0 \leq \alpha \leq \alpha_{max}$; \\
$SW_{ag}^* (\alpha) = \frac{3Q_{ag}^*}{8} - \frac{Q_{ag}^{*2}}{2} - \frac{k\alpha^2}{2}, \text{ for } \alpha_{max} \leq \alpha \leq 1.$

(21)

3.3. Comparative Static Analysis

Using the results from Propositions 1 and 2 we compare the comparative static analysis of the monopolist’s profits for the $ac$- and $ag$-strategies with respect to the monitoring rate. The results, which are summarized in Proposition 3, will help us to determine the subgame perfect equilibrium strategies and the relevant outcomes for the socially optimal monitoring rates that are discussed in the next section. The proof of Proposition 3 is given in the Appendix and is shown in Figure 2.

**Proposition 3.** There exists a unique monitoring rate $\alpha_1$, $\alpha_1 \in (0, \bar{\alpha})$, where $\bar{\alpha}$, satisfies $\pi_{ac}^m (\alpha) = 0$, at which $\pi_{ac}^m (\alpha_1) = \pi_{ac}^m (\alpha_1)$. At $\alpha = \bar{\alpha}$, $\pi_{ac}^m (\bar{\alpha}) = \pi_{ac}^m (\bar{\alpha})$.

The $ac$-strategy is strongly dominant and the $ag$-strategy is weakly dominant in the intervals $\alpha \in [0, \alpha_1)$ and $\alpha \in [\alpha_1, 1]$.

![Figure 2: Comparative static analysis of monopolist’s profits with respect to $\alpha$](image-url)
Figure 2 shows that the relevant range of monitoring rate that we need to consider for the rest of the analysis is $\alpha \in [0, \alpha_{\text{max}}]$. This is because, for $\alpha \geq \alpha_1$, the equilibrium *ag-strategy* is weakly dominant, and hence is credible. Increasing $\alpha$ beyond $\alpha_{\text{max}}$ does not change profit or consumer surplus, because the monopoly results are restored in the interval $\alpha \in [\alpha_{\text{max}}, 1]$, but the cost of monitoring, which is a deadweight loss, increases. Therefore, we need to consider the social welfare function corresponding to the equilibrium *ac-strategy* in the interval $\alpha \in [0, \alpha_1)$, which is $SW_{1}^{ac}$, and the social welfare function corresponding to the equilibrium *ag-strategy* in the interval $\alpha \in [\alpha_1, \alpha_{\text{max}}]$, which is $SW_{1}^{ag}$ as given in equations (17) and (21).

4. Social Welfare Analysis

The government seeks the monitoring rate that maximizes social welfare. Let $\alpha^{ac*}$ and $\alpha^{ag*}$ be the monitoring rates that maximizes

$$SW_{1}^{ac} = \frac{Q^{ac*}}{2} - \frac{3Q^{ac*2}}{4} - Q^{ac*3} - \frac{k\alpha^2}{2}$$ in the interval $\alpha \in [0, \alpha_1)$, and

$$SW_{1}^{ag} = \frac{Q^{ag*}}{2} - Q^{ag*3} - \frac{k\alpha^2}{2}$$ in the interval $\alpha \in [\alpha_1, \alpha_{\text{max}}]$. Let $\alpha^*$ be the socially optimal monitoring rate. The results are summarized in Proposition 4. The proof of Proposition 4 (i) is given in the Appendix. We discuss the proof of Proposition 4 in the main text because it is instructive.

**Proposition 4.** (i) Any $\alpha$ in the interval $\alpha \in [0, \alpha_1)$ maximizes $SW_{1}^{ac} (\alpha)$, that is, $\alpha^{ac*} \in [0, \alpha_1)$ and any $\alpha$ in the interval $\alpha \in [\alpha_1, \alpha_{\text{max}}]$ maximizes $SW_{1}^{ag} (\alpha)$, that is, $\alpha^{ag*} \in [\alpha_1, \alpha_{\text{max}}]$.

(ii) The socially optimal monitoring rates and the monopolist’s subgame perfect strategies are: (1) $\alpha^* = \alpha^{ac*}$ if $SW_{1}^{ac} (\alpha^{ac*}) > SW_{1}^{ag} (\alpha^{ag*})$ and *ac-strategy* is the
subgame perfect equilibrium; (2) $\alpha^* = \alpha^{ag*}$ if $SW_1^{ag} (\alpha^{ag*}) > SW_1^{ac} (\alpha^{ac*})$ and ag-strategy is the subgame perfect equilibrium.

$$\frac{dSW_1^{ac}}{d\alpha} = \left( \frac{1}{2} - \frac{3Q^{ac*}}{8} - 3Q^{ac*+2} \right) \frac{dQ^{ac*}}{d\alpha} - k\alpha.$$ The first expression on the right side is positive because for the highest value of $Q^{ac*}$ which is $Q^{ac*} = \frac{1}{4}$ the term in parenthesis is positive, hence for lower values of $Q^{ac*}$ it must be positive, and

$$\frac{dQ^{ac*}}{d\alpha} > 0.$$ Hence depending on the strength of $k$, $\frac{dSW_1^{ac}}{d\alpha} > 0$. So $\alpha^{ac*} \in [0, \alpha_1)$.

$$\frac{dSW_1^{ag}}{d\alpha} = \left( \frac{1}{2} - 3Q^{ag*+2} - Q^{ag*} \right) \frac{dQ^{ag*}}{d\alpha} - k\alpha=0$$ by the same argument. So $\alpha^{ag*} \in [\alpha_1, \alpha_{\text{max}}]$.

Proposition 4(i) can be explained by decomposing the effect of an increase in the monitoring rate on the monopolist’s profit, consumer surplus and the pirate’s profit. An increase in the monitoring rate unambiguously increases the monopolist’s profit for both ac- and ag-strategies in the defined ranges as shown in Propositions 1 and 2. However, the effect on consumer surplus is ambiguous. An increase in the monitoring rate increases the product quality and price which has opposing effects on consumer surplus. The pirate’s profit is only relevant for $SW_1^{ac}$. As explained in the proof of part (ii) of Proposition 1, an increase in the monitoring rate starting from zero may initially increase the pirate’s profit because the effect on the pirated product’s price via the quality of the legitimate product may dominate the higher cost of entering the market. But then gradually the second effect dominates the first and the pirate’s profit starts decreasing for further increases in the monitoring rate. However,
the positive effect of increased monitoring on the monopolist’s profit dominates the ambiguous effects on consumer surplus and pirate’s profit thus resulting in
\[
\left( \frac{1}{2} - \frac{3Q^{ac}\alpha^*}{8} - 3Q^{ac}\alpha^* - 2 \right) \text{ and } \left( \frac{1}{2} - 3Q^{ag}\alpha^* - Q^{ag} \right)
\]
(which are the terms in parentheses in the expressions \(dSW_{1ac}^{ac} = \alpha\) and \(dSW_{1ag}^{ag} = \alpha\)) to be positive. So the ambiguity is due to the relative strengths of the first expression and the marginal cost of monitoring in the expressions for \(dSW_{1ac}^{ac} = \alpha\) and \(dSW_{1ag}^{ag} = \alpha\). 12

Hence, any monitoring rate in the interval \(\alpha \in [0, \alpha_1]\) maximizes \(SW_{1ac}^{ac} = \alpha\) and any monitoring rate in the interval \(\alpha \in [\alpha_1, \alpha_{\text{max}}]\) maximizes \(SW_{1ag}^{ag} = \alpha\). Enhanced copyright protection may be a welfare-worsening scenario when the heterogeneity of consumers is such that price-sensitive consumers opt out of the legal market more than those who are lured by higher quality.

Proposition 4(ii) follows from the fact that the government chooses the monitoring rate that yields the highest social welfare. If \(SW_{1ac}^{ac} = \alpha^{ac*} > SW_{1ag}^{ag} = \alpha^{ag*}\), then the socially optimal monitoring rate is \(\alpha^* = \alpha^{ac*} \in [0, \alpha_1]\). From Figure 2 we see that in this range of monitoring rate the accommodating strategy is dominant, and hence, is the subgame perfect equilibrium. So there is piracy in equilibrium.

Alternately, if \(SW_{1ag}^{ag} = \alpha^{ag*} > SW_{1ac}^{ac} = \alpha^{ac*}\), then the socially optimal monitoring rate is \(\alpha^* = \alpha^{ag*} \in [\alpha_1, \alpha_{\text{max}}]\). In this case the aggressive strategy is weakly dominant and hence is the subgame perfect equilibrium. Consequently, piracy is deterred in equilibrium.

12 The important implication of the above discussion is that removal of the pirate’s profit from \(SW_{1ac}^{ac}\) will reduce the ambiguity and hence the results stated in Proposition 4 will hold.
The implications of Proposition 4 are as follows. One, it is possible that even the inclusion of innovation may not result in monitoring by the government as the socially optimal policy. This will be the case if \( \alpha^* = \alpha^{ac^*} = 0 \). Two, monitoring may be the socially optimal policy but it may not be sufficient to prevent the pirate’s entry. This will be the case if \( \alpha^* = \alpha^{ac^*} \) when \textit{ac-strategy} is the subgame perfect equilibrium. Three, monitoring may be the socially optimal outcome and piracy is deterred if \( \alpha^* = \alpha^{ag^*} \). In this case the \textit{ag-strategy} is the subgame perfect equilibrium.

\textbf{Corollary 1.} If \( k = 0 \), that is monitoring is costless then \( \alpha^* = \alpha^{ag^*} = \alpha_{\text{max}} \) and the monopoly outcome is restored.

This follows from the fact that 
\[
\frac{dS_{1ac}}{d\alpha} = \left( \frac{1}{2} - \frac{3Q^{ac^*}}{8} - \frac{3Q^{ac^*^2}}{8} \right) \frac{dQ^{ac^*}}{d\alpha} > 0 \quad \text{and} \\
\frac{dS_{1ag}}{d\alpha} = \left( \frac{1}{2} - \frac{3Q^{ag^*}}{8} - \frac{3Q^{ag^*^2}}{8} \right) \frac{dQ^{ag^*}}{d\alpha} > 0.
\]

Since both \( S_{1ac} \) and \( S_{1ag} \) are monotonically increasing in \( \alpha \), hence, \( \alpha^{ac^*} \approx \alpha_{1} \) and \( \alpha^{ag^*} = \alpha_{\text{max}} \) maximizes \( S_{1ac} \) and \( S_{1ag} \). Since \( S_{1ac} (\alpha_{1}) = S_{1ag} (\alpha_{1}) \) and \( S_{1ag} \) is monotonically increasing in \( \alpha \) hence, \( \alpha^* = \alpha^{ag^*} = \alpha_{\text{max}} \) is the socially optimal monitoring rate. That is, when monitoring is costless the monopoly outcome is socially optimal and the highest possible quality is produced.\(^{13}\)

Let us discuss the comparative static analysis with respect to the penalty, \( G \).

We begin with the the \textit{ac-strategy}. The pirate’s profit, \( \pi_{ac}^{ac^*} = \frac{q(1-\alpha)(1-q)^2}{8(2-q-\alpha q)} - \alpha G \), is decreasing in \( G \). So an increase in \( G \) shifts down \( \pi_{ac}^{ac^*} \) measured against \( \alpha \) thereby, reducing \( \alpha \), at which \( \pi_{ac}^{ac^*} = 0 \). There is no change in \( p_{m}^{ac^*}, Q^{ac^*} \) and \( \pi_{m}^{ac^*} \), only \( \pi_{m1}^{ac^*} \)

\(^{13}\) Costless monitoring is only a sufficient condition for this result.
jumps to $\pi^{ac*}_{m2}$ at a lower $\alpha$. The properties of the social welfare functions are retained because the penalty is a transfer from the pirate to the government and hence do not appear in the social welfare function.

For the ag-strategy, $p^{ag*}_m$, $Q^{ag*}$ and $\pi^{ag*}_m$ are increasing in $G$. That is, for any given $\alpha$ an increase in $G$ pushes up $\pi^{ag*}_m$. Hence, $\alpha_{\max}$ decreases and $\alpha_1$ (at which $\pi^{ag*}_m = \pi^{ag*}_{m1}$) falls. All other properties of the social welfare function remain the same. Thus an increase in $G$ reduces $\bar{\alpha}$, $\alpha_i$ and $\alpha_{\max}$. This is shown in Figure 3 where we use the following notations: $G_2 > G_1$, $\alpha_i^1 = \alpha_i(G_i)$, $\bar{\alpha}_i = \bar{\alpha}(G_i)$, $\alpha_{\max i} = \alpha_{\max i}(G_i)$, $i = 1,2$.

![Figure 3: Comparative static analysis of monopolist’s profits with respect to G](image-url)
5. Conclusion

To achieve accurate assessments of the efficacy of copyright protection, we developed a framework that weaves together the strategic interaction among the copyright enforcing government, the innovating and price-setting monopolist, a potential entrant who practices commercial piracy, and a set of taste-heterogeneous consumers.

By incorporating the innovation phase, a costly enforcement scheme, and using an entry-deterring framework, this paper has demonstrated that increased copyright protection via increased government monitoring effort unambiguously improves the incentive to innovate. Nevertheless, except in the limiting case where the monopoly outcome is restored, the commercial piracy, deterred or otherwise, does dampen the incentive to innovate.

Regarding the socially optimal monitoring rates and the resultant equilibrium market structures, we find that even the inclusion of innovation may not result in monitoring by the government as the socially optimal policy. Consequently, there is piracy in equilibrium. Monitoring may be the socially optimal policy but its intensity may not be sufficient to prevent the pirate’s entry. Alternatively, monitoring may be the socially optimal outcome and piracy is deterred in equilibrium.

Appendix

Proof of Proposition 1: (i) Substituting \( p_r = \frac{qp_m}{2} \) in the monopolist’s profit function and applying the first order conditions yield;

\[
\frac{d\pi_m}{dp_m} = 1 - \frac{(2 - q - \alpha q)p_m}{(1 - q)Q} = 0, \text{ and } \frac{d\pi_m}{dQ} = \frac{(1 - q)}{2(2 - q - \alpha q)} - Q = 0.
\]
Solving the two we get $p_{m}^{ac^*} = \frac{(1-q)^2}{2(2-q-\alpha q)^2}$ and $Q_{m}^{ac^*} = \frac{1-q}{2(2-q-\alpha q)}$.

Consequently, $p_{c}^{ac^*} = \frac{q(1-q)^2}{4(2-q-\alpha q)^3}$. For the second order condition we construct the Hessian determinant which is

$$
H = \begin{vmatrix}
\frac{d^2 \pi_m}{dp_m^2} & \frac{d^2 \pi_m}{dQdp_m} & \frac{d^2 \pi_m}{d^2 Q} \\
\frac{d^2 \pi_m}{dp_m dQ} & (1-q)Q_{ac^*} & (1-q)Q_{ac^*}^2 \\
\frac{d^2 \pi_m}{d^2 Q} & (2-q-\alpha q) p_{m}^{ac^*} & (2-q-\alpha q) p_{m}^{ac^*^2} - 1.
\end{vmatrix}
$$

$H_1 = \frac{-(2-q-\alpha q)}{(1-q)Q_{ac^*}^2} < 0$ and $H_2 = \frac{(2-q-\alpha q)}{(1-q)Q_{ac^*}} > 0$. Hence the second order conditions of maximization are satisfied.

(iii) Substituting the expressions from (i) we get the monopolist’s equilibrium profit as,

$$
\pi_{m}^{ac^*}(\alpha) = \begin{cases}
(1-q)Q_{ac^*}^{m} - F(Q_{ac^*}^{m}), & \text{if } \alpha < \bar{\alpha}, \\
\frac{Q_{ac^*}^{m}(1-q)(1-\alpha q)}{(2-q-\alpha q)^2} - F(Q_{ac^*}^{m}), & \text{if } \alpha \geq \bar{\alpha}.
\end{cases}
$$

(iv) $\frac{dQ_{ac^*}}{d\alpha} = \frac{q(1-q)^2}{2(2-q-\alpha q)^2} > 0$, $\frac{dp_{m}^{ac^*}}{d\alpha} = \frac{q(1-q)}{(2-q-\alpha q)^2} > 0$.

$$
\frac{d\pi_{m1}^{ac^*}}{d\alpha} = \frac{q(1-q)^2}{4(2-q-\alpha q)^3} > 0 \text{ and } \frac{d^2 \pi_{m1}^{ac^*}}{d\alpha^2} = \frac{3q^2(1-q)^2}{4(2-q-\alpha q)^4} > 0.
$$

$$
\frac{d\pi_{m2}^{ac^*}}{d\alpha} = \frac{6q^2(1-q)^2}{8(2-q-\alpha q)^3} > 0 \text{ and } \frac{d^2 \pi_{m2}^{ac^*}}{d\alpha^2} = \frac{3q^3(1-q)^2}{2(2-q-\alpha q)^5} > 0
$$

$Q_{ac^*} \leq Q^*$ because $Q_{ac^*}$ is increasing in $\alpha$ and $Q_{ac^*} = Q^*$ at $\alpha = 1$. Substituting $\alpha = 1$ in $Q_{ac^*}$, $p_{m}^{ac^*}$ and $\pi_{m2}^{ac^*}$ we get the monopoly outcome. $Q.E.D.$

**Proof of Proposition 2:** (i) The monopolist faces the following constrained profit maximization problem.
max \( \pi = p_m - \frac{p_m^2}{Q} - \frac{Q^2}{2} \)

subject to \( g(p_m, Q, \alpha) = \frac{p_m^2}{Q} - \frac{4(1-q)\alpha G}{q(1-\alpha)} \leq 0. \)

Hence the Lagrangian function of this optimization is

\[
L(p_m, Q, \lambda) = p_m - \frac{p_m^2}{Q} - \frac{Q^2}{2} - \lambda \left( \frac{p_m^2}{Q} - \frac{4(1-q)\alpha G}{q(1-\alpha)} \right).
\]

\[
\frac{\partial L}{\partial p_m} = 1 - \frac{2p_m}{Q} - \frac{2\lambda p_m}{Q} = 0,
\]

The first order conditions are

\[
\frac{\partial L}{\partial Q} = \frac{p_m^2}{Q} - Q + \frac{\lambda p_m^2}{Q^2} = 0,
\]

\[
\frac{\partial L}{\partial \lambda} = \left( \frac{p_m^2}{Q} - \frac{4(1-q)\alpha G}{q(1-\alpha)} \right) = 0.
\]

It follows that \( p_m^{ag*}(\alpha) = 2Q^{ag*+2} \) where \( Q^{ag*} = \left( \frac{(1-q)\alpha G}{q(1-\alpha)} \right)^{\frac{1}{3}}. \) For the second order

condition we construct the bordered Hessian which is,

\[
H = \begin{pmatrix}
0 & g_{p_m} & g_Q \\
g_{p_m} & L_{p_m p_m} & L_{p_m Q} \\
g_Q & L_{Q p_m} & L_{Q Q}
\end{pmatrix} = \begin{pmatrix}
0 & 2p_m & -\frac{p_m^2}{Q} \\
\frac{2p_m}{Q} & -\frac{2(1+\lambda)}{Q} & \frac{2(1+\lambda)p_m}{Q^2} \\
\frac{p_m^2}{Q^2} & \frac{2(1+\lambda)p_m}{Q^2} & -\frac{2(1+\lambda)p_m^2}{Q^3} - 1
\end{pmatrix}.
\]

\[
H_1 = -\frac{4p_m^{ag*+2}}{Q^{ag*+2}} < 0 \text{ and } H_2 = \frac{2(1+\lambda)p_m^{ag*+2}}{Q^{ag*+2}} + \frac{4p_m^{ag*+2}}{Q^{ag*+2}} > 0. \text{ Hence the second order
}

conditions of maximization are satisfied.

(ii) \( \frac{dQ^{ag*}}{d\alpha} > 0, \text{ because } \frac{\alpha}{1-\alpha} \text{ is increasing in } \alpha; \text{ and } \frac{dp_m^{ag*}}{d\alpha} > 0; \text{ and }

\[
\frac{d\pi^{ag*}_m(\alpha)}{d\alpha} = (3Q^{ag*} - 12Q^{ag*+2}) \frac{dQ^{ag*}}{d\alpha} \geq 0 \text{ because } \frac{dQ^{ag*}}{d\alpha} > 0. \text{ Equating }
\]

\[
\frac{d\pi^{ag*}_m(\alpha)}{d\alpha} = (3Q^{ag*} - 12Q^{ag*+2}) \frac{dQ^{ag*}}{d\alpha} \text{ to zero yields } Q^{ag*} = Q^* = \frac{1}{4}. \text{ That is}
\]
Proof of Proposition 3. From Proposition 1 we know that \( \pi_m^{ag} \) is monotonically increasing and convex in \( \alpha \), \( \alpha \in [0, \overline{\alpha}] \). Further, \( \pi_m^{ag} (\alpha = 0) > 0 \) and \( \pi_m^{ag} (\alpha = 1) = \pi^* \). From Proposition 2 we know that \( \pi_m^{ag} (\alpha) \) is also monotonically increasing in \( \alpha \), \( \alpha \in [0, \alpha_{\text{max}}] \). Further, \( \pi_m^{ag} (\alpha = 0) = 0 \) and \( \pi_m^{ag} (\alpha = \alpha_{\text{max}}) = \pi^*_m \).

Therefore, \( \pi_m^{ag} \) is steeper than \( \pi_m^{ac} \) in the range, \( \alpha \in [0, \overline{\alpha}] \), hence the two profit functions intersect say at \( \alpha_1 \), where \( \alpha_1 \in [0, \overline{\alpha}] \). At \( \alpha = \overline{\alpha} \)

\[
\pi_m^{ag} (\overline{\alpha}) = \frac{q(1-\overline{\alpha})(1-q)^2}{8(2-q-\overline{\alpha}q)} - \overline{\alpha}G = 0 \text{ which on rearrangement yields}
\]

\[
\frac{(1-q)^3}{8(2-q-\overline{\alpha}q)} = \frac{(1-q)\overline{\alpha}G}{q(1-\overline{\alpha})} \Rightarrow Q^{ag+3} = Q^{ag+3}. \text{ That is, at } \alpha = \overline{\alpha}, \ Q^{ag+3}(\overline{\alpha}) = Q^{ag+3}(\overline{\alpha}),
\]

\[
p_m^{ag} (\overline{\alpha}) = p_m^{ag} (\overline{\alpha}) \text{ and } \pi_m^{ag} (\overline{\alpha}) = \pi_m^{ag} (\overline{\alpha}). \text{ Since } \pi_m^{ag} \text{ and } \pi_m^{ac} \text{ are monotonically increasing in } \alpha \text{ and } \pi_m^{ag} (\alpha_{\text{max}}) = \pi_m^{ag} (\alpha = 1) = \pi^*_m, \text{ hence, } \pi_m^{ag} (\alpha) \geq \pi_m^{ac} (\alpha) \text{ for } \alpha \in [\alpha, \alpha_{\text{max}}] \text{ and } \overline{\alpha} < \alpha_{\text{max}}.
\]

Q.E.D.
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