Private Equity Acquisitions and Strategic Buyers: Information Discounts versus Synergies

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Abstract We characterize when private equity funds have a competitive advantage over strategic buyers in acquiring a target firm. Using their skill to mitigate informational frictions, private equity funds cut loss-making projects, potentially merge the target with similarly restructured firms, and exercise growth options. Instead, a strategic buyer integrates with the target to obtain a better competitive position. The private equity fund is more likely to win the takeover competition when its information skill is greater, its required return premium is smaller, and the strategic buyer's synergy gains are smaller. Such takeovers by private equity funds improve economic welfare.

Keywords: Information Frictions, Private Equity, Posterior Beliefs, Takeover Competition, Welfare

JEL subject codes: G24, G32, G34

1 Introduction

The amount of capital allocated to private equity funds is substantial and the resulting activities by such funds are impressive (e.g., Strömberg, 2008; Sørensen et al., 2013; Norbäck et al., 2013). For example, U.S. private equity funds raised about \$188 billion and European funds raised about €48 billion in 2015.¹ The persistent and substantial amount of capital allocated to private equity funds indicates that such funds are perceived to generate value to investors. Still, the economic value to society is debated among academics as well as in the media.² While there is an increasing amount of empirical literature analyzing the role of private equity funds, the theoretical literature on the subject remains thin, although theory is needed to address the question of social value.

The present paper provides a theoretical framework useful for analyzing the role played by private equity funds and how that role creates value. Our model considers a setup in which private equity funds compete with strategic buyers. We use their differences in both skills and objective functions to derive a realistic characterization of the key elements determining whether a private equity fund is successful in its acquisition effort. Competition with the strategic buyer raises the bar for the funds, and also gives the funds a role in preventing anti-competitive conduct by the strategic buyer. The theory allows us to address how private equity acquisitions may generate value to society and not only to investors.

Some stylized facts on private equity funds must be incorporated in our model. First, they typically undertake major changes to their acquired firms, delisting them from the stock exchange while assets are restructured. This can also involve buying additional firms and combining them with the initial portfolio firm (e.g. Hammer et al., 2017). Second, funds have restrictive requirements for generating high returns to the partners. Indeed, private equity funds seem to perform well relative to benchmarks (e.g. Harris et al., 2014; Ang et al., 2010; Axelson et al., 2013). Third, their investments must pay off on a shorter horizon, typically within six years (e.g., Kaplan and Strömberg, 2009). We take these fea-

¹Statistics from the American Investment Council 2016 Q1 Private Equity Trends Report and the European Venture Capital Association pan European report for 2015, respectively. Available at <http://www.investmentcouncil.org/> and <http://evca.eu/>, respectively. Accessed June 2016.

²See, e.g., “Amid Attacks on Private Equity, Efforts to Study Its Value”, The New York Times (January 24, 2012), “Private equity adds value to investors as well as businesses”, Financial Times (June 14, 2013), and “Private Equity – Adding Value?”, The Economist (February 8, 2013).

tures for granted, as exogenously given arrangements among limited and general partners.

Our core idea to explain a private equity fund's role is that the wider financial market's valuation of the target firm is distorted due to information frictions. For example, the target firm can have inside information about a profitable growth option, but a self-interested firm cannot credibly signal this information to outside investors on the financial market: all types of firms would pretend to be profitable. Such information frictions prevent the firm from following a value improving investment policy. Our model relies on the special skills used by private equity funds to mitigate information frictions of this kind.³ As Acharya et al. (2013) show, private equity investors managing the funds are more successful when they can use their specific background to the growth improving strategy implemented in the target firm. These skills create value to investors and society as soon as the growth option pays off in the restructured target firm. In successful outcomes, the private equity fund can sell again within its limited time horizon.

We pay attention to the fact that private equity funds, target firms, and other operating firms (strategic buyers) considering a takeover are pitted in a contest against each other, and therefore need to outbid each other. This provides an analysis of bidding incentives that is richer than in frictionless financial markets, but our focus is precisely on the role of private equity funds in markets with frictions. Strategic buyers desire to take over the target company in order to obtain a competitive advantage over other operating firms. However, like the target firm, a strategic buyer can be subject to an information friction which limits its ability to bid for the target.

A somewhat similar game between private equity funds and strategic buyers is considered in a few recent papers, e.g., Hege et al. (2013) and Martos-Vila et al. (2013). The latter rests on the assumption that the financial market evaluates debt incorrectly because uninformed investors have a wrong perception of the success probabilities. Their paper is related to ours in the sense that valuation and decision making is distorted by a lack of information. However, in contrast to the more behavioral approach in Martos-Vila et al. (2013), we assume that a private equity fund's information advantage is due to its skills to obtain information and to its credible commitment to a loss-cutting strategy. Indeed, we emphasize the private equity fund's willingness to repackage assets in acquired target

³As we explain later, there is a need for some investors to acquire reliable information, and reliability is highest when the investor conducts his own investigation.

firms, sometimes unsettling for the work force involved. This may reconcile the view that funds sell loss-making parts of firms with the evidence that average portfolio firms exhibit growth in economic activity and employment.

Our framework involves four players: the manager (target), the financial market (outside investors), a strategic buyer, and a private equity fund. Therefore, we need to be very specific about these players and to make assumptions which yield a parsimonious and tractable, yet insightful, model. The outside investors have no special information. Their only role is to provide a reservation value for the target firm when considering bids from the strategic buyer and the private equity fund. If the strategic buyer or private equity fund will pay more than this reservation value for the target firm, the firm is sold to the party with the highest value.

The strategic buyer's incentive to take over the manager's firm depends on the synergy gains which can be created by combining the strategic buyer and the target. Instead, the private equity fund exploits information frictions in financial markets by buying firms and subsequently restructuring the acquired firms in order to eliminate these frictions.⁴ Parallel to this, the private equity fund also aims to realize any embedded growth option such that the restructured firms can be sold back (i.e., evaluated without an informational friction discount) to the financial market with a higher return.

Our analysis shows under which conditions private equity funds have a competitive advantage in acquiring financially constrained firms with a potential. The private equity fund wins the takeover competition, unless the strategic buyer can avoid information costs and obtain a gain of similar size as the private equity fund. Since the activities of private equity funds are highly debated, we also consider welfare effects in terms of society. We show that when takeovers by private equity funds takes place, then they improve society's welfare. We also show that high return requirements within the private equity fund can crowd out takeover opportunities pointing towards a potential loss for society. Thus, private equity funds serve as a mean to mitigate information frictions.

The remainder of the paper is organized as follows. Section 2 sets up the model and we solve it in Section 3. Section 4 analyses welfare implications and discussion, and we consider various extensions in Section 5. Finally, we conclude in Section 6. Proofs are

⁴As part of this, we analyze a private equity fund's incentives to acquire several firms and restructuring them into one merged firm either sequentially or simultaneously.

postponed to the appendix.

2 Model

We model the potential sale of a target firm to a private equity (PE) fund. This section introduces the players and our assumptions in depth. To simplify the exposition, we set the risk-free interest rate to zero, and suppose that all parties are risk neutral.

2.1 The Transaction

The owner-manager (henceforth, manager) of the target firm has three outside opportunities which influence the willingness to sell to the fund. Opportunity 1 is keeping the firm as it is. Opportunity 2 involves keeping ownership of the firm, but raising external finance from outside investors in order to exploit a growth option — in this connection, the manager may also consider to restructure the firm before raising finance. Opportunity 3 is selling the firm to a strategic buyer — in the main version of our analysis, we will focus on the case where the manager has all the bargaining power vis-a-vis competitive strategic buyers.⁵

The manager can rely on the most efficient of the three outside opportunities to define the reservation value of the firm in negotiations with the fund. We assume that the PE fund takes over the target when this is the efficient allocation among the investors, i.e., the fund is willing to pay more than this reservation value. If the PE fund thus can create most value for investors, the resulting transaction price is an outcome of bilateral bargaining between the fund and the manager.

The values of the manager's three outside opportunities, as well as the value created by the PE fund, depend on some frictions which we further specify below. The relative strengths of these frictions determine the outcome of the model, whether the fund takes over the target, or which outside opportunity is used by the manager. The frictions also have implications for the efficiency of this outcome.

⁵This assumption is relaxed in an extension. Full strength of the manager isolates the hardest case for the private equity fund to be able to beat its competition for a takeover of the target.

2.2 The Target

The target firm has productive assets in place which can be viewed as the union of A_1 and A_2 . As will become clear, A_1 represents the target's valuable opportunities, while A_2 represents a bad asset that is a source of frictions. The target firm has no liquid funds. The manager has limited liability, so there is no stage where the manager earns less than zero.

Asset A_1 provides the manager a private benefit of U_{A_1} . It also enjoys a value increasing potential. If no investment is made, A_1 returns zero cash-flow (beyond the private benefit). Utilizing the growth option requires an initial investment in the project, in order to generate an informative signal that can tell whether or not to terminate early.

The market value of the bad asset A_2 is normalized to 0, and it has no true potential. If kept to stage 2, the manager enjoys private benefits from A_2 , given by $U_{A_2} > 0$. Also, keeping asset A_2 unlocks further private benefits to a strategic buyer. The PE fund has no private benefits from A_2 .

The most important friction stemming from A_2 is that it complicates inference regarding A_1 . Initial divestment of A_2 will improve transparency in the firm, and simplify information generation regarding the potential in A_1 .

Figure 1 illustrates the timing of the model. An initial investment of $I_1 > 0$ is required to launch the first information gathering stage. If the required investment has been made in the first stage, the firm's owner observes the non-verifiable signal $s_A \in \{0, 1\}$. The ex ante probability that the signal realization is $s_A = 1$ depends on whether the bad asset A_2 is still part of the firm. If A_2 is divested, there is an ex ante probability $p \in (0, 1)$ that the signal realization is $s_A = 1$. If A_2 is kept, the inference regarding A_1 is less precise. We introduce the parameter $\eta \geq 1$ as a measure of the information friction—the informational cost of keeping A_2 —implying that the ex ante probability that the signal realization is $s_A = 1$ decreases to p/η . Essentially, the PE fund manager can increase the value of A_1 through better management of the requisite information, because the PE fund manager is not tempted to keep the confounding assets A_2 .⁶ We let η capture the friction in a reduced form model which potentially could be more complex and have different interpretations. We defer extensions in this direction until Section 5. At the end of the first stage, after

⁶Note that the skill of the PE fund manager cannot just be sold as a service to another owner who did not divest A_2 .

observing the signal, there is an option to terminate the project. If continuing, a further investment of $I_2 > 0$ is needed. A continued project returns a pledgable cash-flow $s_A V_{A_1}$, thus dependent on the signal. A terminated project returns zero, and also reduces the manager's private benefit from asset A_1 to zero.⁷

Figure 1: **Timeline of the model.**

To make the problem interesting, we impose the following two assumptions. Note that assumption 1 has two parts, to cover both the case where asset A_2 is retained and the case where it is sold off. Assumption 2 is stated for the case where A_2 has been sold off, and so the presence of A_2 may or may not render the project unprofitable depending on parameters.

Assumption 1 *Termination at $t = 2$ is efficient if and only if $s = 0$: $U_{A_1} + V_{A_1} > I_2 > U_{A_1} > 0$ and $U_{A_1} + U_{A_2} + V_{A_1} > I_2 > U_{A_1} + U_{A_2} > 0$.*

Assumption 2 *An efficiently terminated project is ex ante profitable: $p(U_{A_1} + V_{A_1} - I_2) > I_1 > 0$.*

2.3 Outside Investors

We assume that the outside investors are willing to inject capital as long as their net present value of doing so is at least 0. The outside investors have no special information and they cannot monitor the manager. A contract offered by the manager can specify the payments made by the investors and the cash flow to return. The manager has limited liability. The only pledgable cash-flow is V_{A_1} which is positive if investments have been made in both stages, and $s_A = 1$.

⁷A continued project gives private benefit U_{A_1} to the manager, independent of s_A .

2.4 Strategic Buyer

The strategic buyer is a firm that looks somewhat like the target. It has some free funds, but may also need funding from outside investors before a takeover. To capture its agency cost of funding, we let this firm have assets-in-place consisting of two parts.

The value of asset Q_1 is known and constant, $V_{Q_1} > 0$. The asset Q_2 contains an investment opportunity. The value is unknown ex ante, but it is revealed through a signal s_Q at time $t = 1$. s_Q is perfectly correlated with the other signal s_A .⁸ Specifically, we assume for tractability that the ex ante probability of $s_Q = 1$ is the same as for $s_A = 1$, thus depending on whether the bad asset A_2 is still in place in the acquired firm. The return from the investment is $s_Q V_{Q_2}$. To keep the model focused we assume that the strategic buyer does not need to make an initial investment (or that it has already been done) to obtain the signal s_Q . However, to exploit the investment opportunity the strategic buyer needs to pay the investment cost $I_2^{Q_2}$ at time $t = 2$.

We make the normalizing assumption that the strategic buyer has no private benefits of Q_1 nor Q_2 at this stage. We also suppose that efficient investment depends on the signal:

Assumption 3 $0 < I_2^{Q_2} < V_{Q_2}$.

The strategic buyer has sufficient cash (we think of the value from Q_1 as similar to cash) to fund the investment. The stand-alone ex ante expected utility of the strategic buyer is

$$\mathbb{E}U^{SB} = V_{Q_1} + p(V_{Q_2} - I_2^{Q_2}). \quad (1)$$

Most importantly, acquisition of the target unleashes synergy gains in two forms that we will more carefully distinguish in Section 4. The welfare-enhancing value is $G_{SB} \geq 0$, and private benefits (additional to the target manager's) by the amount $U_Q \geq 0$.

Assumption 4 *If the strategic buyer takes over the manager's firm and invests $I_1 + I_2 + I_2^{Q_2}$, then the strategic buyer has private benefits $U_{A_1} + U_{A_2} + U_Q < I_2 + I_2^{Q_2}$.*

In general, there is imperfect competition among strategic buyers to take over the firm. Bargaining with the manager over a transaction price, the strategic buyer then has

⁸This assumption allows us to disregard considerations of risk diversification.

bargaining power φ_T^{SB} . Until an extension in Section 5, we consider the simpler case where strategic buyers are perfectly competitive, $\varphi_T^{SB} = 0$. This case renders it least likely that the private equity fund will prevail in a takeover battle.

2.5 Private Equity Fund

The private equity (PE) fund has the ability to observe the signal s_A — the manager may choose to let the PE investor control the termination decision, as part of the financial contract. The PE fund has promised partners a higher risk-neutral return $r > 0$ on its capital. The higher return is effectively the rent earned by the ability to take part in the firm's control. This ability is not priced in the market except through this very channel that higher returns can be earned on projects where the PE fund provides financing.⁹

Due to the construction of the PE fund we assume that it has no private benefits. Therefore, the PE fund will get rid of A_2 after a takeover. Thus, it has an ex ante probability p of $s_A = 1$, it pays I_1 to make the first stage investment, and it invests I_2 in stage 2 when $s_A = 1$.

More explicitly, we suppose that the private equity fund has assets-in-place F from a previous (first) acquisition. The private equity fund has already eliminated private benefits associated with F . The stand-alone value of the assets is V_F and it can be realized at time $t = 2$. However, there can be synergies from merging with our target firm, which thus plays the role of the second acquisition opportunity. A successful merger results in the gain G_{PE} with probability p .

3 Analysis of the Outcome

To analyze the manager's choice between the PE fund and the three outside opportunities, we consider each opportunity in turn.

⁹This required return represents a friction in our model. It naturally stems from the agency problem between the general and limited partners of the fund, which puts a constraint on the available capital.

3.1 Stand Alone

The manager does not use the growth option, but only derives private benefits from the assets. The manager's value from this opportunity is

$$U_{A_1} + U_{A_2}. \quad (2)$$

3.2 Outside Investors

3.2.1 Both Assets Kept

Assume that the manager keeps A_2 and then contracts with outside investors. The manager's signal is not contractible, but both the initial investment in the growth option and the decision to continue are contractible. Value can be generated only with successful investment, so a contract requires the investors to provide the initial I_1 as well as the additional I_2 if the manager decides to continue.

Since the manager has private benefits from continuation, but continuation is efficient only when the signal realization is high, the contract must incentivize the manager to honestly follow the signal. The contract therefore specifies that investors provide compensation $C \geq 0$ to the manager in case the project is terminated.

The only cash flow going back to investors is a fraction $\alpha \in [0, 1]$ of the final cash-flow $s_A V_{A_1}$. The outside investors accept any contract where their present expected net cash flow is non-negative. The manager designs contract parameters α and C to maximize own expected utility, subject to investor participation.

Lemma 5 *It is possible to obtain financing from outside investors if and only if*

$$\frac{p}{\eta} (V_{A_1} - I_2 + U_{A_1} + U_{A_2}) \geq I_1 + U_{A_1} + U_{A_2}. \quad (3)$$

One optimal contract then has incentive compensation $C = U_{A_1} + U_{A_2}$, with efficient termination.

Since outside investors obtain zero expected profit, it is easy to calculate the manager's expected utility. When outside financing is impossible, it remains at the level of (2). When possible, the manager's expected utility is

$$\frac{p}{\eta} (V_{A_1} - I_2 + U_{A_1} + U_{A_2}) - I_1. \quad (4)$$

Note that the necessary condition (3) directly implies that the manager prefers OI funding with both assets, whenever it is feasible.

3.2.2 Initial Asset Sale

We also consider the possibility that the manager initially sells A_2 for the value of 0. The manager loses the benefit U_{A_2} , but gains that inference regarding A_1 is easier. Lemma 5 carries over, by setting $U_{A_2} = 0$ and $\eta = 1$ in the Lemma. The exact condition for the possibility of OI funding with one asset is thus

$$p(V_{A_1} - I_2 + U_{A_1}) \geq I_1 + U_{A_1}. \quad (5)$$

Since (3) implies (5), this financing condition is easier to satisfy.

If OI funding is possible in this case, the manager's utility becomes

$$p(V_{A_1} - I_2 + U_{A_1}) - I_1. \quad (6)$$

3.2.3 Comparison

To conclude, outside funding is feasible when (5) holds. If also (3) holds, the manager can choose between selling or keeping the additional asset A_2 . Comparing (4) to (6), the manager prefers to sell the additional asset A_2 precisely when

$$\frac{U_{A_2}}{\eta - 1} \leq V_{A_1} - I_2 + U_{A_1}. \quad (7)$$

Intuitively, if the agency rent of the additional asset is small relative to the higher investment cost, then the manager prefers to sell the additional asset.

3.3 Strategic Buyer

This analysis focuses on the case where A_2 is kept, for two reasons. First, we take the strategic buyer to be less dependent on outside investors, and hence less in need to bring down private benefits. Second, we find it natural that the strategic buyer is interested in harvesting the private benefits from A_2 , while agency concerns in the PE fund renders it relatively more eager to remove sources of private benefits. We will end this section with a discussion of the possibility to sell A_2 .

After a takeover, the strategic buyer invests I_1 to receive a signal about the new assets (A_1, A_2) . If the signals are good, $s_A = s_Q = 1$, the strategic buyer invests $I_2 + I_2^{Q_2}$ and obtains the value of the assets and the synergy gain, $G_{SB} > 0$. Thus, the strategic buyer's (ex post) utility is

$$V_{Q_1} + (V_{Q_2} - I_2^{Q_2}) + (V_{A_1} - I_2) + (U_{A_1} + U_{A_2} + U_Q) + G_{SB} - I_1. \quad (8)$$

If the signals are bad, assumption 4 implies that the strategic buyer abandons the investment.¹⁰ The ex post utility is simply the value of the strategic buyer's initial asset $V_{Q_1} - I_1$.

3.3.1 Strategic Buyer's Alternative

Without the takeover, the strategic buyer continues with a stand-alone firm. If the signal is good, $s_Q = 1$, the strategic buyer invests $I_2^{Q_2}$ and obtains utility

$$V_{Q_1} + (V_{Q_2} - I_2^{Q_2}). \quad (9)$$

If the signal is bad, the utility is just V_{Q_1} .

3.3.2 Manager's Utility

We have assumed that the manager has all the bargaining power against the strategic buyer. The strategic buyer's gain at a good signal is the difference between (8) and (9). The loss at a bad signal is the additional investment I_1 . Hence, the SB is willing to offer:

$$\frac{p}{\eta} (V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_Q + G_{SB}) - I_1. \quad (10)$$

3.3.3 Comparison

Since a strategic buyer can get synergy effects by combining the assets from the two firms it is normally better for the manager to make a deal with a strategic buyer than to turn to outside investors.

Suppose that the manager could obtain OI funding with A_2 . The manager's utility was then given by (4). It is immediate to see that the SB's offer exceeds this by $\frac{p}{\eta}(U_Q + G_{SB})$. Essentially, if the synergy gains are positive, the SB can beat the market.¹¹ This condition implies that OI funding is entirely irrelevant, for feasibility of outside funding (3) implies positivity of the SB's offer (10).

¹⁰We discuss an alternative version of this at the end. The private benefits play a more interesting role if the strategic buyer needs outside funding for the takeover. The case in which (29) holds is in the appendix.

¹¹The appendix considers the more complicated case where the SB needs to obtain OI funding.

3.3.4 Discussion

Let us return to the question whether it's even better for the manager to sell off A_2 before approaching the SB. Assumption 4 defined the private benefits to the SB as $U_{A_1} + U_{A_2} + U_Q$, under the implicit assumption that both assets are present. If A_2 is not there, let us suppose that SB obtains private benefits $U_{A_1} + U_{Q \cup A_1}$. Extending the spirit of assumption 4, suppose that $U_{A_1} + U_{Q \cup A_1} < I_2^{Q_2} + I_2$. The SB then continues to invest precisely when $s_A = s_Q = 1$. The ex ante probability of a good signal increases from $\frac{p}{\eta}$ to p .

It follows that the SB is willing to offer

$$p(V_{A_1} - I_2 + U_{A_1} + U_{Q \cup A_1} + G_{SB}) - I_1. \quad (11)$$

This is no better than (10), provided $U_{A_2} + U_Q > (\eta - 1)(V_{A_1} - I_2 + U_{A_1} + G_{SB}) + \eta U_{Q \cup A_1}$. In words, the SB's private benefits from keeping A_2 outweigh the informational benefits from selling A_2 . Since $U_{Q \cup A_1}$ is specific to this corner of the analysis, we can impose that this condition is true without interfering with any other assumption.¹²

3.4 Private Equity Fund

As mentioned before, we consider an acquisition by a private equity fund conditional on the fund having already acquired another firm. An extension in Section 5 addresses the issue of acquiring the initial firm in the private equity fund's portfolio.

The private equity fund has assets-in-place F from a previous (first) acquisition. The private equity fund has already eliminated private benefits associated with F . The stand-alone value of the assets is V_F and it can be realized at time $t = 2$.

The private equity fund can acquire the manager's firm with assets $A_1 \cup A_2$ and merge it with the first acquisition. A successful merger results in the gain G_{PE} . However, as argued above the private equity fund has no private benefits of its assets, and hence it sells the asset A_2 while restructuring its assets. Therefore, a successful merger occurs with probability p .

The PE fund takes over the manager's firm by entering a contract similar to the one possible for the outside investors. Since the PE fund effectively controls the investments in the two stages, it sets $C = 0$, and requires a share α of the manager's firm. Following

¹²Again, feasibility of outside funding (5) implies positivity of the SB's offer.

a takeover, before any new investments have been materialized, the PE fund's expected utility is

$$-I_1 + \frac{1}{1+r} \left(p \left[-I_2 + \frac{1}{1+r} \alpha (V_{A_1} + G_{PE} + V_F) \right] + (1-p) \alpha \frac{V_F}{1+r} \right) \quad (12)$$

$$= \frac{p [\alpha (V_{A_1} + G_{PE}) - (1+r)I_2] - (1+r)^2 I_1 + \alpha V_F}{(1+r)^2}. \quad (13)$$

Lemma 6 *The PE fund requires at least an ownership share of $\hat{\alpha}_{min}$, before it is willing to takeover the manager's firm, where*

$$\hat{\alpha}_{min} = \frac{p(1+r)I_2 + (1+r)^2 I_1 + V_F}{p(V_{A_1} + G_{PE}) + V_F}. \quad (14)$$

The ownership share increases in the private equity fund's return r , and it is less than one if r is not too large.

Lemma 6 shows that the minimum share required by the private equity fund is increasing in the size of the initial assets-in-place.

Corollary 7 *Assume that the private equity fund's minimum ownership is less than one. Then the ownership in the new acquisition increases at a decreasing rate in its earlier acquired assets-in-place:*

$$\frac{d\hat{\alpha}_{min}}{dV_F} > 0, \quad (15)$$

$$\frac{d^2\hat{\alpha}_{min}}{dV_F^2} < 0, \quad (16)$$

$$\hat{\alpha}_{min} \longrightarrow 1, V_F \rightarrow \infty. \quad (17)$$

On the other hand, the manager of the target firm has a minimum requirement on the ownership share, because the manager can turn to the strategic buyer (or the outside investors), as an alternative to the PE fund. A contract with the PE fund gives the manager utility

$$(1-\alpha)(p(V_{A_1} + G_{PE}) + V_F). \quad (18)$$

Let us maintain the assumption from the end of Section 3.3 that the manager's best outside opportunity is the takeover by an SB, keeping assets A_2 . According to (10), the manager requires at least

$$\frac{p}{\eta} (V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_Q + G_{SB}) - I_1. \quad (19)$$

By comparing (18) to (10) we obtain the minimum ownership share required by the manager. We denote the maximum share to the private equity fund as $\bar{\alpha}_*$, which is increasing in the private equity fund's assets-in-place and the potential gain obtained by the private equity fund.

Lemma 8 *The manager can allow that the private equity fund at most gets the share*

$$\bar{\alpha}_* \stackrel{def}{=} 1 - \frac{\frac{p}{\eta}(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_Q + G_{SB}) - I_1}{p(V_{A_1} + G_{PE}) + V_F}. \quad (20)$$

Moreover, $\frac{\partial \bar{\alpha}_*}{\partial V_F} > 0$, $\frac{\partial \bar{\alpha}_*}{\partial G_{PE}} > 0$, $\frac{\partial \bar{\alpha}_*}{\partial G_{SB}} < 0$, and $\bar{\alpha}_* \leq 1$ iff

$$\frac{p}{\eta}(V_{A_1} + U_{A_1} + U_{A_2} + U_Q + G_{SB}) \geq I_1 + \frac{p}{\eta}I_2. \quad (21)$$

By assumption, a takeover by the PE fund occurs whenever these requirements are compatible, $\hat{\alpha}_{\min} < \bar{\alpha}_*$. A contract is feasible, when the private equity fund's ownership is sufficiently beneficial. This will be the outcome unless the investment costs or the information signaling costs due to private benefits are too large. The following proposition also shows that the private equity fund's assets-in-place actually do not play any role in this respect.

Proposition 9 *Assume that $\hat{\alpha}_{\min} < 1$. A takeover by the PE fund occurs if the return premium r is not too big, the synergy gain G_{PE} from a PE-takeover is sufficiently larger than G_{SB} from a SB-takeover, the private benefits $U_{A_1} + U_{A_2} + U_Q$ are not too big, and information signaling costs η are large enough. Specifically, $\hat{\alpha}_{\min} \leq \bar{\alpha}_*$ iff*

$$\begin{aligned} & ((1+r)^2 - 1) I_1 + p \left((1+r) - \frac{1}{\eta} \right) I_2 \\ & \leq p \left((V_{A_1} + G_{PE}) - \frac{1}{\eta}(V_{A_1} + G_{SB}) \right) - \frac{p}{\eta}(U_{A_1} + U_{A_2} + U_Q). \end{aligned} \quad (22)$$

It may seem counterintuitive that the private equity fund's assets-in-place do not play a role in terms of feasibility of a contract between the manager and the private equity fund. The reason that this result holds is due to the fact the higher share required by the private equity fund is exactly off set by the willingness from the manager to give up a share in the firm.

3.5 Comparison

Our analysis is useful for showing under which conditions private equity funds have a competitive advantage over strategic buyers in acquiring constrained firms with a potential.

This can be seen from Proposition 9 and, in particular, condition (22). We consider the trade-offs between the key elements further in the numerical analysis to follow. For this analysis we use a base case with an ex ante success probability $p = 0.5$, a successful investment returns $V_{A_1} = 30$. The initial investment cost is $I_1 = 4$ and the second stage investment cost is $I_2 = 14$. Hence, the project yields a net present value of 4, if there are no frictions. The informational friction induced by keeping the bad asset A_2 is $\eta = 5/3$, and hence the posterior belief of a successful outcome is $p\eta = 0.3$. The strategic buyer's stand alone project returns $V_{Q_2} = 30$ in the next period with an investment cost of $I_2^{Q_2} = 14$. We assume the private benefits are equally spread out in the various stages so that $U_{A_1} = U_{A_2} = U_Q = 3$.¹³ Furthermore, the direct gain of a takeover by the strategic buyer is $G_{SB} = 10$. Similarly, we let the direct gain of a takeover by the private equity fund be $G_{PE} = 10$, thus ensuring that there is no higher gain in terms of return realization for the private equity fund compared to the strategic buyer. Finally, the private equity fund's return requirement is $r = 0.1$.

Panel (a) in Figure 2 shows the trade-off between the required return premium r and the information signaling costs η . The purple line corresponds to a case with low information costs $\eta = 1.5$. For a given level of information signaling costs, a higher return premium makes a private equity acquisition more difficult and, eventually, not possible. As the purple line illustrates, when the information signaling costs are lower, the slack for a return premium is smaller. This makes good sense. Lower information signaling costs make an acquisition by SB more attractive (and it is easier to obtain OI financing), and a higher return premium puts pressure on the private equity fund's minimum share in the acquired firm. As a result, a private equity takeover is less likely. Similarly, a takeover by the private equity fund is less likely to occur when the private benefits from a strategic buyer's takeover, U_Q , increases because the strategic buyer is willing to pay a higher price for the manager's firm. As seen in panel (b), a higher return requirement to the private equity fund amplifies this effect. The direct effect of the key friction—the information signaling cost— η is illustrated in panel (c). If the information friction is sufficiently low, the features of a private equity fund are not valuable enough to compensate for the higher return requirement. Hence, the higher the return requirement, the larger must the information friction be, before the private equity fund plays a role. Noteworthy,

¹³We ensure that Assumption 1–4 are satisfied in the parameterizations we consider.

the private equity fund plays a role in mitigating the information friction in our base case ($\eta = 5/3$) albeit the direct gain of a takeover is the same whether the takeover is done by the private equity fund or the strategic buyer ($G_{PE} = G_{SB}$). Panel (d) considers how the gain by a private equity takeover influences the outcome. Naturally, G_{PE} and G_{SB} are important determinants for whom takes over and for efficiency for society. The outcome of a takeover game is highly sensitive in G_{PE} and G_{SB} , and these gains are difficult to estimate in practice. However, the important issue is how uncertain the gain difference $G_{PE} - G_{SB}$ is. If the difficulty in estimating G_{PE} is similar to those in estimating G_{SB} , and if both components are affected similarly by estimation difficulties, then $G_{PE} - G_{SB}$ can be relatively robust. Therefore, we only consider variations in G_{PE} . Intuitively, a higher gain makes it easier for the private equity fund to undertake a takeover, and it also provides more leeway for a high return premium. However, the private equity fund can play a role even if the direct gain is lower than for the strategic buyer. In our base case this occurs as long as G_{PE} is up to 20% lower than G_{SB} . This observation is important when evaluating the benefits of private equity funds. Although the private equity fund may apparently yield a lower gain than the strategic buyer, and in addition require a return premium, the private equity fund can actually be the preferred one to takeover the manager's firm.

The size of the investment cost matters for the role played by a private equity fund. Panel (a) in Figure 3 considers a higher investment cost in the first stage and panel (b) considers a higher investment cost in the second stage. A higher investment costs in either stage decreases the possibility of a private equity fund takeover. We also note the intuitive effect that if information signaling cost is low, the private equity fund loses its competitive advantage in which case higher investment costs makes a private equity takeover less likely. Interestingly, increasing the investment cost in one of the two stages does not imply the same outcome. In an information frictionless world, the net present value of asset A_1 is positive as long as $I_1 < 8$ or $I_2 < 22$ (keeping the other investment cost fixed at its base case level). For the highest stage one investment cost, $I_1 = 8$, the private equity fund will take over the manager's firm. In contrast, the private equity fund will not undertake a takeover if the stage two investment costs is above 18. This difference is due to the fact that I_1 in our base case is low compared to I_2 and that I_1 is paid before the signal is revealed so the information friction impacts the effect of the two types of investment

costs differently.¹⁴ To understand the implications of investment costs better, we perform an analysis in which we keep the net present value of A_2 constant and vary the first stage investment cost. Thus, as I_1 increases, I_2 decreases just enough to maintain the net present value. Panel (c) depicts the effect of this exercise. We now see that a higher stage one investment cost makes it more likely that the private equity fund undertakes the takeover. This is due to the fact that as I_2 decreases the present value after making the first investment increases. Since the private equity fund has an informational advantage in the second stage, this advantage thus increases as the initial investment cost accounts for a larger share of the total investment costs. Of course, the private equity fund has less to say if the information friction is lower. Indeed, if $\eta = 1.5$ the private equity fund will not take over the manager's firm unless the initial investment cost is at least 5.

4 Welfare Implications and Discussions

Our analysis sheds light on the circumstances under which private equity funds are beneficial to society. A central element is how society evaluates private benefits. This depends on how private benefits are interpreted. One extreme is that private benefits are essentially saved efforts which are costly to provide for the manager (or the strategic buyer). Another extreme case is that private benefits are perks spent by the manager, and these perks are costly for the firm — in this vein, also the synergy benefits enjoyed by the strategic buyer may arise from anti-competitive conduct with gain to investors but actual welfare loss. In the former case, private benefits should be included in society's assessment of the ownership structure, in the latter case they should not. To handle this, we introduce a parameter $u \in [0, 1]$. u is the weight society puts on private benefits.

We can now analyze the effect of different forms of takeovers on society's valuation, i.e., the total value for society of the assets in the manager's firm, the strategic buyer's firm, and the private equity fund.

Lemma 10 *Society's valuation depends on the form of ownership:*

1. *If there is no takeover and the manager cannot fund using outside investors, then*

¹⁴The effect depends on the fact that the private equity fund on the one hand benefits from its skill in saving information signaling costs but, on the other hand, requires a higher return. That is, the net return effect on the initial investment cost is $((1+r)^2 - 1)I_1 = r(2+r)I_1$, but the net return on the stage two investment cost is $((1+r) - \frac{1}{\eta})I_2 < (1+r)I_2$.

society's value is

$$V_{OI} + V_{Q_1} + p(V_{Q_2} - I_2^{Q_2}) + V_F, \quad (23)$$

if the manager can fund using outside investors, the value is

$$V_{OI} + \frac{p}{\eta}(V_{A_1} - I_2 + u(U_{A_1} + U_{A_2})) - I_1 + V_{Q_1} + p(V_{Q_2} - I_2^{Q_2}) + V_F, \quad (24)$$

2. If the strategic buyer takes over, society's value is

$$V_{OI} + \frac{p}{\eta}(V_{A_1} - I_2 + V_{Q_2} - I_2^{Q_2} + G_{SB}) - I_1 + V_{Q_1} + V_F + u\frac{p}{\eta}(U_{A_1} + U_{A_2} + U_Q), \quad (25)$$

3. If the strategic buyer takes over, the gain to society (compared with no takeover) is

$$\frac{p}{\eta} \left(G_{SB} + uU_Q - (\eta - 1)(V_{Q_2} - I_2^{Q_2}) \right). \quad (26)$$

Lemma 10 shows that society's value of a takeover by the strategic buyer (25) is independent of whether the strategic buyer needs to raise capital on the financial market or not. That the value does not depend on the strategic buyer's source of funding is due to the fact that the outside investors enter a contract with the strategic buyer which has a zero net present value. Thus, there is only a distributional effect between the strategic buyer and the outside investors. However, if the strategic buyer needs to make a contract with the outside investors, this can indirectly impact the value to society, since the size of the private benefits can be large enough so that a contract is not possible. Furthermore, the gain to society of a strategic buyer is intuitive from (26). A takeover by the strategic buyer is beneficial, if it adds expected synergy effects or if it provides valuable private benefits.

Given the value of the various strategies we can address which strategy is efficient from society's point of view.

Proposition 11 *If the private equity fund takes over, society's value is*

$$V_{OI} + V_{Q_1} + p(V_{A_1} + G_{PE} + V_{Q_2} - I_2^{Q_2}) + V_F - I_1 - pI_2. \quad (27)$$

If the private equity fund takes over, the gain to society (compared to a takeover by the strategic buyer) is

$$p \left(\left(1 - \frac{1}{\eta} \right) (V_{A_1} - I_2 + V_{Q_2} - I_2^{Q_2}) + \left(G_{PE} - \frac{1}{\eta} G_{SB} \right) - \frac{u}{\eta} (U_{A_1} + U_{A_2} + U_Q) \right). \quad (28)$$

The special characteristic of private equity funding that private benefits play no direct role is highlighted in (27).

The trade-off in our model between a takeover by a private equity fund or by a strategic buyer depends on four central elements. A direct channel is the difference in the takeover gain, $G_{PE} - G_{SB}$. Intuitively, if the gain of a takeover by the private equity fund is larger than the gain of a takeover by a strategic buyer, then society prefers that the private equity fund takes over. However, even if the gain of a takeover is largest if the private equity fund takes over, this outcome can be obstructed by the (excess) return requirement r . Thus, the internal contractual arrangement in the private equity fund leading to the return requirement, can result in a friction for society. Another friction is the information cost η . This measures the competitive advantage the private equity fund has over the strategic buyer, so the gain to society of a takeover by the PE fund is larger, the larger is the information cost. Finally, the weight society puts on the private benefits also play a central role in evaluating the efficiency of a takeover by the private equity fund. When u is close to one, private benefits are assessed as being beneficial for society as well. Since private benefits are distorted by a private equity fund's takeover, a higher u makes such a takeover less efficient. We collect the analysis of the various trade-offs below.

Corollary 12 *From Proposition 11 we get*

1. *If the private equity fund requires no excess return, $r = 0$, then the private equity fund's takeover is always beneficial for society.*
2. *If society has no value of private benefits, $u = 0$, and if the excess takeover gain $G_{PE} - G_{SB}$ and information cost η are only moderate, then society incurs a loss due to an inefficient takeover by a strategic planner.*
3. *If a takeover by the private equity fund is possible, i.e., (22) holds, then such a takeover is efficient from society's point of view.*

Figure 4 illustrates a number of trade-offs. The effects that a higher signal information costs and a lower value for society of private benefits increases society's gain of a private equity takeover are intuitive (panel (a) and (b)). This also holds if the direct gain G_{PE} is high enough and the probability of a good signal — and hence implementation of the gain — increases (panel (c)). Panel (d) shows that this is reversed, if the signal information cost and gain are low enough. However, the figures also illustrate that there is a loss to society in many cases in which the PE fund does not succeed in acquiring the target. This is most clearly illustrated in panel (e). That panel corresponds to panel (a) in Figure 2. Recall from the latter figure that a private equity fund cannot undertake a takeover, if the signal information cost is low ($\eta = 1.5$) and the return premium high enough ($r \geq 0.085$). In this case the takeover is done by the strategic buyer which in turn implies a loss for society (a value loss of about 5.5).¹⁵

5 Extensions

To make the analysis as simple as possible, we have made a number of assumptions. This section explores the robustness of our results under relevant alterations to the model.

5.1 Information Production

We have emphasized the special skill of the PE fund in acquiring information about the uncertain outcome s_A , i.e., whether the growth option pays off or not. Let us highlight this fundamental ingredient of the model through extension to a more traditional comparison of informative signals.

¹⁵For our base case the loss is about 7 if $r > 0.16$.

Our model lets any decision maker observe a payoff-relevant signal s_A . We have directly posited that keeping A_2 reduces the chance that s_A has the high value. A more traditional approach to the comparison of information separates this into both an unobserved payoff-relevant state $\theta_A \in \{0, 1\}$ and an observable signal s_A . For a comparison of signal informativeness, the prior belief $\Pr(\theta_A = 1)$ is held fixed, while the joint distribution of (θ_A, s_A) is changed.

To accomplish this separation in our model, we can let $s_A = \theta_A$ in the case where A_2 is divested. When A_2 is kept, we instead let $s_A \in \{0, 1\}$ satisfy $\Pr(s_A = 1|\theta_A = 1) = 1/\eta$ and $\Pr(s_A = 1|\theta_A = 0) = 0$. With this definition, our previous analysis goes through without modification. First, $\Pr(s_A = 1) = p/\eta$ when A_2 is kept. Second, upon observation of s_A , Bayes' rule provides posterior $\Pr(\theta_A = 1|s_A = 1) = 1$ and $\Pr(\theta_A = 1|s_A = 0) = (p\eta - p)/(\eta - p)$. Here is one difference from our analysis so far—the manager who keeps A_2 and observes $s_A = 0$ still expects some positive cash-flow from the investment opportunity. However, the posterior belief is lower than after $s_A = 1$, so it does not affect the implication that such a manager prefers the private benefits from A_2 over taking the option to invest.

Returning to general specifications of the signal, Bayes' rule allows computation of the posterior belief π . This is a random variable that satisfies $\mathbb{E}[\pi] = \Pr(\theta_A = 1)$. A better informed decision maker obtains a distribution of π that is more variable in the sense of a mean-preserving spread, or equivalently in the sense of second-order stochastic dominance, or the convex order: for any convex function h , $\mathbb{E}[h(\pi)]$ is greater.

When deciding on the initial investment at stage 1, the decision maker has two possible choices: invest or not. For any fixed choice, the expected payoff is a linear function of the belief π . The optimal choice allows the decision maker to obtain the upper envelope of these two linear functions, thus a convex function of π .

Due to private benefits, for any given belief π , we have assumed that the PE fund has lower value of either choice, since the owner and strategic buyers can enjoy private benefits on top of the firm's cash flow. We illustrate the situation in Figure 5.

As our analysis has shown, once the PE fund's information advantage is sufficiently great, its expected value at stage 0 exceeds that of the other parties. With reference to Figure 5, this is possible when the distribution of the PE fund's posterior belief is sufficiently more variable than those of the other parties. The PE fund better exploits the convexity in the payoff function, and can therefore have a higher expected value despite

the belief-by-belief dominated payoff. Indeed, convexity of the payoff function is due to optionality.

Similarly, we have also assumed that there is perfect information regarding the gains G_{PE} and G_{SB} . A classical issue in takeovers is how to convince the incumbent shareholders to sell their shares to a raider who can improve the value (e.g., Grossman and Hart, 1980; Shleifer and Vishny, 1986). Clearly, we could consider a more complex model in which the bargaining game between the manager and the private equity fund is more elaborate, or we could introduce asymmetric information regarding the size of the gains. In the latter case, the expected gains would be the central ingredient (perhaps conditional on the takeover offer being profitable to the private equity fund), and we would get some further conditions for when a private equity takeover is possible. However, we do not see that these conditions change the first-order effects seen in our model, and thus we disregard this more complex setting to have a tractable model.

5.2 Strategic Buyer

Our parsimonious modeling of the strategic buyer can be extended in several directions. We consider the possibility that the strategic buyer is capable of obtaining some value of its pre-acquisition asset Q_2 . Apparently, this puts the strategic buyer in a better position, however, lowering the ex post cost of a bad signal makes it more difficult to commit to a termination strategy ex ante. Thus, a higher return from existing assets in the bad state increases the strategic buyer's incentive to invest. That is, if society evaluates private benefits lower than the strategic buyer, then a higher return in the bad state can induce inefficient overinvestment.

This can be seen as follows. Consider an extension of the model in which the signal s_Q returns 1 with probability p and with probability $1 - p$ it returns $s_Q^{\min} \in [0, 1)$. The return from the investment is $s_Q V_{Q_2}$.

Lemma 13 *Suppose signals are bad, $s_A = 0$, $s_Q = s_Q^{\min}$. The strategic buyer prefers to invest if*

$$s_Q^{\min} V_{Q_2} + (U_{A_1} + U_{A_2} + U_Q) \geq I_2 + I_2^{Q_2}. \quad (29)$$

Proof. Suppose $s_A = 0$, $s_Q = s_Q^{\min}$. The strategic buyer prefers to invest at time $t = 2$ iff

$$V_{Q_1} + (s_Q^{\min} V_{Q_2} - I_2^{Q_2}) - I_2 + (U_{A_1} + U_{A_2} + U_Q) \geq V_{Q_1}, \quad (30)$$

iff

$$s_Q^{\min} V_{Q_2} + (U_{A_1} + U_{A_2} + U_Q) \geq I_2 + I_2^{Q_2}. \quad (31)$$

■

5.2.1 Funding Using Outside Investors

The strategic buyer may not be able to fund the takeover and the subsequent investments internally. As long as the manager is paid with a share in the new firm, the issue is whether outside investor are willing to fund the subsequent investments. Assume the strategic buyer has enough cash to invest in its existing options, $V_{Q_1} > I_2^{Q_2}$, but the strategic buyer needs to make a contract with outside investors to finance the investments in the acquired assets (albeit it could internally have excess cash). Thus, an OI-contract must provide $I_1 + I_2$ to the strategic buyer.

Lemma 14 *If $V_{Q_1} + \frac{p}{\eta}(U_{A_1} + U_{A_2} + U_Q + V_{A_1} - I_2 + V_{Q_2} - I_2^{Q_2} + G_{SB}) \geq (U_{A_1} + U_{A_2} + U_Q) + I_1$, outside financing is possible, otherwise it is not. An optimal contract has incentive compensation $D = U_{A_1} + U_{A_2} + U_Q$ and efficient termination, while the outside investor share β solves $\frac{p}{\eta}\beta(V_{Q_1} - I_2^{Q_2} + V_{Q_2} + V_{A_1} + G_{SB}) = I_1 + \frac{p}{\eta}I_2 + (1 - \frac{p}{\eta})(U_{A_1} + U_{A_2} + U_Q - V_{Q_1})$.*

We observe from (71) that the possibility to make a contract with the outside investors depends on the private benefits which the strategic buyer can obtain after a takeover. The larger these private benefits are, the more difficult it is to get a contract.

Corollary 15 *Suppose the manager's private benefits are maximal with respect to getting a contract with outside investors, i.e., there is equality in (53). Then it follows from (71) that the strategic buyer can obtain a contract with the outside investors if $(1 - \frac{p}{\eta})U_Q \leq V_{Q_1} + \frac{p}{\eta}(V_{Q_2} - I_2^{Q_2} + G_{SB})$.*

Therefore, a contract can be made if the net present value of the strategic buyer's single-firm investment opportunity and the takeover gain are large enough.

As long as a contract with the outside investors can be made, the strategic buyer and the manager get a value similar to the case in which the strategic buyer can make the takeover alone. This is due to the fact that the net present value to the financial market is zero and, importantly, that there are no marginal friction coming into play when the investments cost must be externally funded. That is, the only new friction stems from the condition on the size of the private benefits U_Q .

5.3 Private Equity Fund's Acquisition of Two Firms

As discussed in the introduction, an important role of private equity funds is to mitigate information frictions in a portfolio of firms. Indeed, such a buy and build strategy has recently become very popular (e.g., Hammer et al., 2017). As we shall see, this role is difficult for strategic buyers to fill out, and thus private equity funds make a beneficial contribution to society. To analyze this, we consider two firms, A and B . To avoid unnecessary complexity in the analyzes, we let each firm be similar in construction to the manager's firm considered earlier. Thus, firm A has assets A_1 and A_2 , and firm B has assets B_1 and B_2 . Only asset A_1 (B_1) can eventually provide a profitable return. Besides an obvious extension of notation, we introduce the gain of a private equity fund takeover, when two firms are acquired. Let $G_{PE,1}$ be the gain if the private equity fund successfully takes over either firm A or firm B , and let $G_{PE,2}$ be the gain if the private equity fund successfully takes over A and B . We assume that

$$G_{PE,2} = 2(G_{PE,1} + \frac{1}{2}G_{PE,syn}). \quad (32)$$

That is, the gain of taking over two firms is twice the gain of taking over one firm added with a synergy gain, $G_{PE,syn}$. Finally, we assume that $d_S = 0$.

Lemma 16 *The PE fund requires at least an ownership share of $\hat{\alpha}_{min}$, before it is willing to takeover the manager's firm, where*

$$\hat{\alpha}_{min} = \frac{p(1+r)I_2 + (1+r)^2 I_1}{p(V_{A_1} + G_{PE,1} + \frac{1}{2}G_{PE,syn})}. \quad (33)$$

The ownership share increases in the private equity fund's return r , and it is less than one if r is not too large.

The payment to the manager of firm A and the manager of firm B , respectively, can take one of two forms. Either as a share of the restructured (merged) firm, or by receiving a cash payment at the time of the acquisition. We consider the former form first. In this case, the managers' share benefits from the synergy gain, $G_{PE,syn}$, if the acquisition goes through and the following signal is positive.

Lemma 17 *The coalition of the A -manager and the B -manager can allow that the private equity fund at most gets the share*

$$\bar{\alpha}_* \stackrel{def}{=} 1 - \frac{\frac{p}{\eta}(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_{SB,A} + G_{SB}) - I_1}{p(V_{A_1} + G_{PE,1} + \frac{1}{2}G_{PE,syn})}, \quad (34)$$

and $\bar{\alpha}_* \leq 1$ iff

$$p(V_{A_1} + U_{A_1} + U_{A_2} + U_{SB,A} + G_{SB}) \geq \eta I_1 + pI_2. \quad (35)$$

Thus, a contract is feasible unless the investment costs or the information signaling costs due to private benefits are too large. Note that condition (35) corresponds to requiring that the offer from a strategic buyer to A and the offer from (perhaps) another strategic buyer to B has positive value.

It is not obvious that a contract can be made to satisfy the private equity fund's minimum share and the managers' minimum share, i.e., $\alpha \in [\hat{\alpha}_{\min}, \bar{\alpha}_*]$. The following proposition shows that this is possible when the private equity fund is sufficiently beneficial.

Proposition 18 *A takeover by the PE fund is possible, i.e., $\hat{\alpha}_{\min} < \bar{\alpha}_*$, if r is not too big, the gain from a PE-takeover is sufficiently larger than the one from a SB-takeover, the private benefits are not too big, and information signaling costs due to private benefits, η , are large enough. Specifically, $\hat{\alpha}_{\min} < \bar{\alpha}_*$ iff*

$$\begin{aligned} & (1+r)((1+r)I_1 + pI_2) \\ & \leq p \left(V_{A_1} \left(1 - \frac{1}{\eta}\right) + G_{PE,1} + \frac{1}{2}G_{PE,syn} - \frac{G_{SB}}{\eta} \right) - \frac{p}{\eta}(U_{A_1} + U_{A_2} + U_{SB,A}) + I_1 + \frac{p}{\eta}I_2, \end{aligned} \quad (36)$$

and the restriction that $\bar{\alpha}_* \leq 1$ implies

$$(1+r)((1+r)I_1 + pI_2) \leq p(G_{PE,1} + \frac{1}{2}G_{PE,syn} + V_{A_1}). \quad (37)$$

Consider now the other payment form, i.e., the private equity fund makes a cash payment to the A -manager and the B -manager, respectively. In this case the manager does not know whether the private equity fund will buy another firm as well, so the manager perceives the value of the acquired firm as $p(V_{A_1} + G_{PE,1})$. Therefore, the minimum payment to either manager is

$$\frac{p}{\eta}(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_{SB,A} + G_{SB}) - I_1, \quad (38)$$

and so the joint payment is

$$2\left(\frac{p}{\eta}(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_{SB,A} + G_{SB}) - I_1\right). \quad (39)$$

If instead the managers get a share as in the former case, the joint value of this is

$$\alpha_{A,\min}p(2V_{A_1} + G_{PE,2}) + \alpha_{B,\min}p(2V_{A_1} + G_{PE,2}) \quad (40)$$

i.e.

$$2\left(\frac{p}{\eta}(V_{A_1} - pI_2 + U_{A_1} + U_{A_2} + U_{SB,A} + G_{SB}) - I_1\right). \quad (41)$$

Thus, the value of the minimum payment to the managers is the same in the two cases.

The A -manager and B -manager can consider to merge their firm and get funding from outside investors. Following the steps from the case of one firm only reveals that the condition for obtaining funding is

$$\frac{p}{\eta}(\alpha V_{A_1} - I_2) \geq \left(1 - \frac{p}{\eta}\right)(U_{A_1} + U_{A_2}) + I_1. \quad (42)$$

This is precisely as if there is only one firm, and is due to our parsimonious modeling. It is easy to imagine that information costs or private benefits are higher in the combined AB -firm. This increases the right-hand side in (42), implying the intuitive result that the combined firm finds it more difficult to obtain funding from outside investors.

5.3.1 Acquisition of Two Firms – Welfare Implications

To further discuss welfare implications we assume that there are two similar strategic buyers who can acquire firm A and firm B , respectively. As before, the private equity fund has liquid funds V_F to acquire firms.

Lemma 19 *Society's valuation depends on the form of ownership:*

1. *If there is no takeover and the manager cannot fund using outside investors, then society's value is*

$$V_{OI} + 2(V_{Q_1} + p(V_{Q_2} - I_2^{Q_2})) + V_F, \quad (43)$$

if the manager can fund using outside investors, the value is

$$V_{OI} + 2\left(\frac{p}{\eta}(V_{A_1} - I_2 + u(U_{A_1} + U_{A_2})) - I_1 + V_{Q_1} + p(V_{Q_2} - I_2^{Q_2})\right) + V_F, \quad (44)$$

2. *If the strategic buyer takes over, society's value is*

$$V_{OI} + 2\left(V_{Q_1} + \frac{p}{\eta}(V_{Q_2} - I_2^{Q_2} + V_{A_1} - I_2 + u(U_{A_1} + U_{A_2} + U_Q) + G_{SB}) - I_1\right) + V_F, \quad (45)$$

3. *If the strategic buyer takes over, the gain to society (compared with no takeover) is*

$$2\frac{p}{\eta}\left(G_{SB} + uU_Q - (\eta - 1)(V_{Q_2} - I_2^{Q_2})\right). \quad (46)$$

Proposition 20 *If the private equity fund takes over, society's value is*

$$V_{OI} + 2(V_{Q_1} + p(V_{Q_2} - I_2^{Q_2} + V_{A_1} + G_{PE,1} + \frac{1}{2}G_{PE,syn}) - I_1 - pI_2) + V_F, \quad (47)$$

and the gain to society (compared to a takeover by the strategic buyer) is

$$2p \left(\left(1 - \frac{1}{\eta} \right) (V_{A_1} - I_2 + V_{Q_2} - I_2^{Q_2}) + \left(G_{PE} - \frac{1}{\eta} G_{SB} \right) - \frac{u}{\eta} (U_{A_1} + U_{A_2} + U_Q) \right). \quad (48)$$

Hence, we see that the intuition from the case with one target firm carries over to this case.

6 Conclusion

Firms with a potential for value creation are limited in exploiting their growth opportunities, if they are subject to information frictions. Private equity funds and strategic buyers compete about acquiring such firms. Following an acquisition, the two types of buyers vary in their ex post strategies. Due to the framework in which private equity funds are set up, they have a relatively short time horizon for getting verifiable signals supporting improvements. This leads to immediately cutting loss-making projects and potentially merge with other restructured constrained firms. This provides a specific role for private equity funds in realizing gains. In contrast, a strategic buyer focuses on integrating the acquired firm into the existing business plan. This difference leads to different incentives. We show under which conditions private equity funds have a competitive advantage in acquiring constrained firms with a potential. The PE fund wins the takeover competition, unless the strategic buyer can avoid information costs and obtain a gain of similar size as the private equity fund. Since the activities of private equity funds are highly debated, we also consider welfare effects in terms of society. We show that when takeovers by private equity funds takes place, then they improve society's welfare. We also show that high return requirements within the private equity fund can crowd out takeover opportunities pointing towards a potential loss for society. Thus, private equity funds serve as a means to mitigate information frictions.

A Appendix

A.1 Outside investors

We want to prove Lemma 5

Proof. Consider a contract with parameters α and C . We first observe that the contract must specify $C \leq U_{A_1} + U_{A_2} + (1 - \alpha)V_{A_1}$. If this is not the case, the manager will never return any cash-flow to the investor after stage 2, because he will always prefer to terminate the project. Assume $C \leq U_{A_1} + U_{A_2} + (1 - \alpha)V_{A_1}$, then we need to consider the manager's incentives if $s_A = 0$. Two kinds of contract are relevant, where either $C \geq U_{A_1} + U_{A_2}$ and the manager terminates at $s_A = 0$, or $C \leq U_{A_1} + U_{A_2}$ and the manager continues at $s_A = 0$. We analyze those two contracts in turn, before comparing them.

First, if $s_A = 0$, $C \geq U_{A_1} + U_{A_2}$ and the manager terminates the project, the outside investors' participation constraint is

$$p(\alpha(V_{A_1} - I_2) - (1 - p)C - \eta I_1) = 0. \quad (49)$$

The manager's ex ante expected utility is

$$\mathbb{E}U = p(U_{A_1} + U_{A_2} + (1 - \alpha)V_{A_1}) + (1 - p)C, \quad (50)$$

and by using (49)

$$= p(V_{A_1} - I_2 + U_{A_1} + U_{A_2}) - \eta I_1, \quad (51)$$

Thus, there is a degree of freedom in setting (α, C) . The manager chooses $C = U_{A_1} + U_{A_2}$ to most easily satisfy the PC-constraint (49). With this choice the outside investors' constraint is

$$p(\alpha V_{A_1} - I_2) \geq (1 - p)(U_{A_1} + U_{A_2}) + \eta I_1. \quad (52)$$

If the outside investors get no ownership, $\alpha = 0$, (52) can never be satisfied. This is intuitive, as they only have costs and no benefits of entering a contract with the manager. On the other hand, the outside investors cannot get more than full ownership, $\alpha = 1$, so if they do not break even in this case, they will never participate. Therefore, an OI-contract is only possible if

$$p(V_{A_1} - I_2) \geq (1 - p)(U_{A_1} + U_{A_2}) + \eta I_1,$$

or if

$$U_{A_1} + U_{A_2} \leq \frac{p(V_{A_1} - I_2) - \eta I_1}{1 - p} \stackrel{\text{def}}{=} \bar{\mathcal{B}}. \quad (53)$$

We observe that since $(1 - p)(U_{A_1} + U_{A_2}) + \eta I_1 > U_{A_1} + I_1$ the additional asset A_2 makes it more difficult for the manager to make a contract.¹⁶

Consider the possibility that $s_A = 0$ implies the manager continues. It follows readily from the previous calculations that an OI-contract is only possible if

$$pV_{A_1} \geq \eta I_1 + I_2, \quad (54)$$

in which case the manager gets the expected utility $U_{A_1} + U_{A_2} + p(1 - \alpha)V_{A_1} = U_{A_1} + U_{A_2} + pV_{A_1} - \eta I_1 - I_2$. It follows that the manager will prefer to terminate when $s = 0$ because $I_2 > U_{A_1} + U_{A_2}$. ■

A.2 Condition (29) holds

First, this requires that $s_Q^{\min} > 0$ because $U_{A_1} + U_{A_2} + U_Q < I_2 + I_2^{Q_2}$ by assumption. In this case the strategic buyer always invests. Suppose $U_{A_1} + U_{A_2} < \bar{\mathcal{B}}$, then the net gain from the takeover is

$$\begin{aligned} & V_{Q_1} + \left(p + (1 - p)s_Q^{\min}V_{Q_2} - I_2^{Q_2} \right) + pV_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_Q + pG_{SB} - \eta I_1 \\ & \quad - \left((V_{Q_1} + p(V_{Q_2} - I_2^{Q_2})) + (p(V_{A_1} - I_2 + U_{A_1} + U_{A_2}) - \eta I_1) \right) \\ & = (1 - p) \left(s_Q^{\min}V_{Q_2} + U_{A_1} + U_{A_2} - (I_2^{Q_2} + I_2) \right) + p(U_Q + G_{SB}). \end{aligned} \quad (55)$$

From (29) it follows that the net gain is positive.

The manager's fall-back option is to turn to the outside investors. This will provide him with

$$p(V_{A_1} - I_2 + U_{A_1} + U_{A_2}) - \eta I_1. \quad (56)$$

Suppose that the strategic buyer's bargaining power is φ_T^{SB} . Then the strategic buyer needs to offer at least

$$\begin{aligned} & p(V_{A_1} - I_2 + U_{A_1} + U_{A_2}) - \eta I_1 \\ & + (1 - \varphi_T^{SB}) \left((1 - p) \left(s_Q^{\min}V_{Q_2} + U_{A_1} + U_{A_2} - (I_2^{Q_2} + I_2) \right) + U_Q + pG_{SB} \right). \end{aligned} \quad (57)$$

¹⁶That is, α must be higher.

This serves as a lower bound for the manager when he negotiates a contract with the PE fund. Following the steps in Proposition 9 and assuming that the manager has full bargaining power, the manager requires that $\alpha < \bar{\alpha}_*$, where

$$\bar{\alpha}_* = 1 - \frac{p(V_{A_1} - I_2 + U_{A_1} + U_{A_2}) - \eta I_1 + (1-p) \left(s_Q^{\min} V_{Q_2} + U_{A_1} + U_{A_2} - (I_2^{Q_2} + I_2) + \frac{U_Q + pG_{SB}}{1-p} \right)}{p(V_{A_1} + G_{PE})}. \quad (58)$$

It is also required that $\alpha > \hat{\alpha}_{\min}$, and we have that

$$\hat{\alpha}_{\min} < \bar{\alpha}_* \quad (59)$$

$\Leftrightarrow$

$$(1+r)((1+r)I_1 + pI_2) < p(G_{PE} - G_{SB}) - (U_{A_1} + U_{A_2} + B^Q) - (1-p)(s_Q V_{Q_2} - I_2^{Q_2}) + \eta I_1 + I_2. \quad (60)$$

If $U_{A_1} + U_{A_2} \in (\bar{B}, I_2]$ the manager cannot obtain an OI-contract implying that the net gain is

$$\begin{aligned} & V_{Q_1} + \left(p + (1-p)s_Q^{\min} V_{Q_2} - I_2^{Q_2} \right) + pV_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_Q + pG_{SB} - \eta I_1 \\ & \quad - \left((V_{Q_1} + p(V_{Q_2} - I_2^{Q_2})) + (p(U_{A_1} + U_{A_2})) \right) \\ & = (1-p)(s_Q^{\min} V_{Q_2} - I_2^{Q_2}) + pV_{A_1} - I_2 + (1-d)(U_{A_1} + U_{A_2}) + U_Q + pG_{SB} - \eta I_1. \end{aligned} \quad (61)$$

From (29) and that $p > d$ it follows that the net gain is positive.

The manager's fall-back option is to keep the assets (he cannot turn to the outside investors), and this provides him with

$$d(U_{A_1} + U_{A_2}). \quad (62)$$

If the strategic buyer's bargaining power is φ_T^{SB} , the PE fund needs to offer at least a value of

$$d(U_{A_1} + U_{A_2}) + (1 - \varphi_T^{SB}) \left((1-p)(s_Q^{\min} V_{Q_2} - I_2^{Q_2}) + pV_{A_1} - I_2 + (1-d)(U_{A_1} + U_{A_2}) + U_Q + pG_{SB} - \eta I_1 \right) \quad (63)$$

implying that the highest acceptable share $\hat{\alpha}$ for the manager solves

$$\begin{aligned} (1 - \hat{\alpha})p(V_{A_1} + G_{PE}) & = d(U_{A_1} + U_{A_2}) \\ & + (1 - \varphi_T^{SB}) \left((1-p)(s_Q^{\min} V_{Q_2} - I_2^{Q_2}) + pV_{A_1} - I_2 + (1-d)(U_{A_1} + U_{A_2}) + U_Q + pG_{SB} - \eta I_1 \right), \end{aligned} \quad (64)$$

and if the manager has full bargaining power we obtain

$$\hat{\alpha}_* = 1 - \frac{pV_{A_1} - I_2 + (1-p)(s_Q^{\min}V_{Q_2} - I_2^{Q_2}) + U_{A_1} + U_{A_2} + U_Q + pG_{SB} - \eta I_1}{p(V_{A_1} + G_{PE})}. \quad (65)$$

Hence $\hat{\alpha}_* = \bar{\alpha}_*$; the two cases would differ if $\varphi_T^{SB} > 0$.

A.3 Proof of Lemma 14

Proof. The OI-contract must induce that the strategic buyer invests if $s_A = s_Q = 1$, and terminates if $s_A = s_Q = 0$. The contract (β, D) is offered to outside investors with $D \geq U_{A_1} + U_{A_2} + U_Q$. Consider first the case in which the strategic buyer invests if $s_A = s_Q = 1$, and terminates if $s_A = s_Q = 0$. The outside investors' participation constraint is

$$p(\underbrace{\beta(V_{Q_1} - I_2^{Q_2})}_{\text{cash left}} + V_{Q_2} + V_{A_1} + G_{SB}) - I_2 + (1-p)(V_{Q_1} - D) - \eta I_1 \geq 0. \quad (66)$$

The strategic buyer's (gross) expected utility is

$$p(U_{A_1} + U_{A_2} + U_Q + (1-\beta)(V_{Q_1} + V_{Q_2} - I_2^{Q_2} + V_{A_1} + G_{SB})) + (1-p)D, \quad (67)$$

and using the outside investors' participation constraint (66) we get

$$V_{Q_1} + p(U_{A_1} + U_{A_2} + U_Q + V_{Q_2} - I_2^{Q_2} + V_{A_1} - I_2 + G_{SB}) - \eta I_1. \quad (68)$$

We see that there is a degree of freedom in choosing (β, D) . Thus, let $D = U_{A_1} + U_{A_2} + U_Q$.

Using this we can rewrite (66) to

$$p\beta(V_{Q_1} - I_2^{Q_2} + V_{Q_2} + V_{A_1} + G_{SB}) + (1-p)V_{Q_1} \geq \eta I_1 + pI_2 + (1-p)(U_{A_1} + U_{A_2} + U_Q) \quad (69)$$

i.e.

$$\beta \geq \frac{\eta I_1 + pI_2 + (1-p)(U_{A_1} + U_{A_2} + U_Q - V_{Q_1})}{p(V_{Q_1} - I_2^{Q_2} + V_{Q_2} + V_{A_1} + G_{SB})}. \quad (70)$$

Hence, an OI-contract is only feasible if

$$U_{A_1} + U_{A_2} + U_Q \leq \frac{V_{Q_1} + p(V_{Q_2} - I_2^{Q_2} + V_{A_1} + G_{SB}) - (\eta I_1 + pI_2)}{1-p}. \quad (71)$$

Consider next the case in which the strategic buyer invests also if $s_A = s_Q = 0$ (i.e., always invest and never terminates). The outside investors' participation constraint is

$$\beta(V_{Q_1} - I_2^{Q_2} + p(V_{Q_2} + V_{A_1} + G_{SB}) + (1-p)s_Q^{\min}V_{Q_2}) = \eta I_1 + I_2. \quad (72)$$

The strategic buyers' (gross) expected utility is

$$U_{A_1} + U_{A_2} + U_Q + (1-\beta)(V_{Q_1} - I_2^{Q_2} + p(V_{Q_2} + V_{A_1} + G_{SB}) + (1-p)s_Q^{\min}V_{Q_2}), \quad (73)$$

and using the outside investors' participation constraint we get

$$U_{A_1} + U_{A_2} + U_Q + V_{Q_1} - I_2^{Q_2} + p(V_{Q_2} + V_{A_1} + G_{SB}) + (1-p)s_Q^{\min}V_{Q_2} - \eta I_1 - I_2. \quad (74)$$

To see that the strategic buyer will follow the desired strategy, we compare the (gross) expected utility from the two strategies. From the first strategy (with termination) (68) we have

$$p(U_{A_1} + U_{A_2} + U_Q + V_{Q_2} - I_2^{Q_2} + V_{A_1} - I_2 + G_{SB}) + V_{Q_1} - \eta I_1 > \quad (75)$$

$$p(U_{A_1} + U_{A_2} + U_Q - I_2 - I_2^{Q_2}) + pV_{Q_2} + (1-p)(s_Q^{\min}V_{Q_2} - I_2^{Q_2}) + p(V_{A_1} + G_{SB}) + V_{Q_1} - \eta I_1 > \quad (76)$$

$$U_{A_1} + U_{A_2} + U_Q - I_2 - I_2^{Q_2} + p(V_{Q_2} - I_2^{Q_2}) + (1-p)(s_Q^{\min}V_{Q_2} - I_2^{Q_2}) + p(V_{A_1} + G_{SB}) + V_{Q_1} - \eta I_1, \quad (77)$$

where the first inequality follows from the assumption that $s_Q^{\min}V_{Q_2} < I_2^{Q_2}$, and the second inequality follows from the assumption that $U_{A_1} + U_{A_2} + U_Q < I_2 + I_2^{Q_2}$. The last expression is the same as (74). Hence, the strategic buyer prefers the first strategy, i.e., invest only if $s_A = s_Q = 1$. ■

A.4 Proof of Lemma 6

Proof. The PE fund requires at least to break even, i.e., it must obtain at least an expected utility of 0. From (13) we get that

$$\frac{p[\alpha(V_{A_1} + G_{PE}) - (1+r)I_2] - (1+r)^2I_1 + \alpha V_F}{(1+r)^2} \geq \frac{V_F}{(1+r)^2} \quad (78)$$

iff

$$p[\alpha(V_{A_1} + G_{PE}) - (1+r)I_2] + \alpha V_F \geq (1+r)^2I_1 + V_F \quad (79)$$

iff

$$\alpha \geq \frac{p(1+r)I_2 + (1+r)^2I_1 + V_F}{p(V_{A_1} + G_{PE}) + V_F}. \quad (80)$$

The private equity fund's return only enters in the numerator and since p , I_1 , and I_2 are all positive, a higher r increases $\hat{\alpha}_{\min}$. The denominator is clearly positive, and we consider the condition

$$\hat{\alpha}_{\min} \leq 1, \quad (81)$$

iff

$$\frac{p(1+r)I_2 + (1+r)^2I_1 + V_F}{p(V_{A_1} + G_{PE}) + V_F} \leq 1, \quad (82)$$

iff

$$p(1+r)I_2 + (1+r)^2I_1 \leq p(V_{A_1} + G_{PE}), \quad (83)$$

and for $r = 0$ we have

$$pI_2 + I_1 \leq p(V_{A_1} + G_{PE}), \quad (84)$$

which holds by assumption because $G_{PE} \geq 0$. Thus, the assertion follows by continuity in r . ■

A.5 Proof of Corollary 7

Proof. We consider the derivative:

$$\frac{d\hat{\alpha}_{\min}}{dV_F} = \frac{p(V_{A_1} + G_{PE}) + V_F - (p(1+r)I_2 + (1+r)^2I_1 + V_F)}{(p(V_{A_1} + G_{PE}) + V_F)^2} \quad (85)$$

$$= \frac{1 - \hat{\alpha}_{\min}}{p(V_{A_1} + G_{PE}) + V_F} > 0, \quad (86)$$

by the assumption that $\hat{\alpha}_{\min} < 1$. Since V_F does not enter the numerator in (85) (after reducing terms), and since it increases the denominator, it follows trivially that the second order derivative is decreasing in V_F . Furthermore, rewriting $\hat{\alpha}_{\min}$ shows that the limit is one:

$$\hat{\alpha}_{\min} = \frac{p(1+r)I_2 + (1+r)^2I_1}{p(V_{A_1} + G_{PE}) + V_F} + \frac{V_F}{p(V_{A_1} + G_{PE}) + V_F} \longrightarrow 0 + 1 = 1, V_F \rightarrow \infty. \quad (87)$$

■

A.6 Proof of Lemma 8

Proof. Comparing (18) to (10) gives us

$$\begin{aligned} & (1-\alpha)(p(V_{A_1} + G_{PE}) + V_F) \\ & \geq p(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_Q + G_{SB}) - \eta I_1 + ((1-p)d_s(d(U_{A_1} + U_{A_2}) + U_Q)), \end{aligned} \quad (88)$$

equivalent to

$$\alpha \leq 1 - \frac{p(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_Q + G_{SB}) - \eta I_1 + (1-p)d_s(d(U_{A_1} + U_{A_2}) + U_Q)}{p(V_{A_1} + G_{PE}) + V_F} \stackrel{\text{def}}{=} \bar{\alpha}_*. \quad (89)$$

The assertions regarding the partial derivatives follow straightforward from this. The PE fund cannot have more than full ownership of the company. Therefore, we need to check that $\bar{\alpha}_* \leq 1$:

$$1 - \frac{p(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_Q + G_{SB}) - \eta I_1 + (1-p)d_s(d(U_{A_1} + U_{A_2}) + U_Q)}{p(V_{A_1} + G_{PE}) + V_F} \leq 1, \quad (90)$$

hence

$$p(V_{A_1} + U_{A_1} + U_{A_2} + U_Q + G_{SB}) + (1-p)d_s \cdot (d(U_{A_1} + U_{A_2}) + U_Q) \geq \eta I_1 + pI_2. \quad (91)$$

■

A.7 Proof of Proposition 9

Proof. The issue regarding $\bar{\alpha}_* \leq 1$ is handled before the proposition. Then, when $U_{A_1} + U_{A_2} \leq \min\{I_2, \mathcal{B}\}$, we have

$$\hat{\alpha}_{\min} < \bar{\alpha}_* \quad (92)$$

iff

$$\begin{aligned} & \frac{p(1+r)I_2 + (1+r)^2I_1 + V_F}{p(V_{A_1} + G_{PE}) + V_F} < \\ & 1 - \frac{p(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_Q + G_{SB}) - \eta I_1 + ((1-p)d_s \cdot (d(U_{A_1} + U_{A_2}) + U_Q))}{p(V_{A_1} + G_{PE}) + V_F} \end{aligned} \quad (93)$$

iff

$$(1+r)((1+r)I_1 + pI_2) + V_F < p(V_{A_1} + G_{PE}) + V_F - p(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_Q + G_{SB}) - \eta I_1 + (1-p)d_s(d(U_{A_1} + U_{A_2}) + U_Q) \quad (94)$$

iff

$$(1+r)((1+r)I_1 + pI_2) < p(G_{PE} - G_{SB}) - p(U_{A_1} + U_{A_2} + U_Q) - (1-p)d_s(d(U_{A_1} + U_{A_2}) + U_Q) + \eta I_1 + pI_2. \quad (95)$$

Assume $d_s = 0$. Then

$$\hat{\alpha}_{\min} < \bar{\alpha}_* \quad (96)$$

iff

$$(1+r)((1+r)I_1 + pI_2) < p(G_{PE} - G_{SB}) - p(U_{A_1} + U_{A_2} + U_Q) + \eta I_1 + pI_2. \quad (97)$$

If $U_{A_1} + U_{A_2} \in (\mathcal{B}, I_2]$, then similar calculations (using that the fall-back fall-back option for the manager is $d(U_{A_1} + U_{A_2})$) reveal that the cut-off level the same, if we also assume that $\varphi_T^{SB} = d_s = 0$. ■

A.8 Proof of Lemma 16

Proof. The PE fund requires at least to break even, i.e., it must obtain at least an expected utility of 0. Analogously to (13) we have that

$$\frac{p[\alpha(2V_{A_1} + G_{PE,2}) - (1+r)2I_2] - (1+r)^2 2I_1}{(1+r)^2} \geq 0 \quad (98)$$

iff

$$p[\alpha(2V_{A_1} + G_{PE,2}) - (1+r)I_2] \geq (1+r)^2 2I_1 \quad (99)$$

iff

$$\alpha \geq \frac{2(p(1+r)I_2 + (1+r)^2 I_1)}{p(2V_{A_1} + 2(G_{PE,1} + \frac{1}{2}G_{PE,syn}))}. \quad (100)$$

The private equity fund's return only enters in the numerator and since p , I_1 , and I_2 are all positive, a higher r increases $\hat{\alpha}_{\min}$. The denominator is clearly positive, and we consider the condition

$$\hat{\alpha}_{\min} \leq 1, \quad (101)$$

iff

$$p(1+r)I_2 + (1+r)^2 I_1 \leq p(V_{A_1} + G_{PE,1} + \frac{1}{2}G_{PE,syn}), \quad (102)$$

and for $r = 0$ we have

$$pI_2 + I_1 \leq p(V_{A_1} + G_{PE,1} + \frac{1}{2}G_{PE,syn}), \quad (103)$$

which holds by assumption because $G_{PE,1} + \frac{1}{2}G_{PE,syn} \geq 0$. Thus, the assertion follows by continuity in r . ■

A.9 Proof of Lemma 17

Proof. The A -manager requires at least a share α_A satisfying

$$\alpha_A p(2V_{A_1} + G_{PE,2}) \geq p(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_{SB,A} + G_{SB}) - \eta I_1 \quad (104)$$

$$\alpha \geq \frac{p(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_{SB,A} + G_{SB}) - \eta I_1}{2p(V_{A_1} + G_{PE,1} + \frac{1}{2}G_{PE,syn})} \stackrel{\text{def}}{=} \alpha_{A,\min}. \quad (105)$$

Similar the B -manager requires at least a share

$$\alpha \geq \frac{p(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_{SB,A} + G_{SB}) - \eta I_1}{2p(V_{A_1} + G_{PE,1} + \frac{1}{2}G_{PE,syn})} \stackrel{\text{def}}{=} \alpha_{B,\min}. \quad (106)$$

Hence, the coalition of the A -manager and the B -manager requires at least $\alpha_{A,\min} + \alpha_{B,\min}$. From this it follows that the maximum share the private equity fund can get is

$$\alpha \leq 1 - (\alpha_{A,\min} + \alpha_{B,\min})$$

thus

$$\bar{\alpha}_* \stackrel{\text{def}}{=} 1 - \frac{p(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_{SB,A} + G_{SB}) - \eta I_1}{p(V_{A_1} + G_{PE,1} + \frac{1}{2}G_{PE,syn})}. \quad (107)$$

We need to check that $\bar{\alpha}_* \leq 1$:

$$1 - \frac{p(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_{SB,A} + G_{SB}) - \eta I_1}{p(V_{A_1} + G_{PE,1} + \frac{1}{2}G_{PE,syn})} \leq 1, \quad (108)$$

hence

$$p(V_{A_1} + U_{A_1} + U_{A_2} + U_{SB,A} + G_{SB}) \geq \eta I_1 + pI_2. \quad (109)$$

■

A.10 Proof of Proposition 18

Proof. The issue regarding $\bar{\alpha}_* \leq 1$ is handled before the proposition. Then, when $U_{A_1} + U_{A_2} \leq \min\{I_2, \mathcal{B}\}$, we have

$$\hat{\alpha}_{\min} < \bar{\alpha}_* \quad (110)$$

iff

$$\frac{p(1+r)I_2 + (1+r)^2I_1}{p(V_{A_1} + G_{PE,1} + \frac{1}{2}G_{PE,syn})} < 1 - \frac{p(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_{SB,A} + G_{SB}) - \eta I_1}{p(V_{A_1} + G_{PE,1} + \frac{1}{2}G_{PE,syn})} \quad (111)$$

iff

$$\begin{aligned} & (1+r)((1+r)I_1 + pI_2) \\ & < p(V_{A_1} + G_{PE,1} + \frac{1}{2}G_{PE,syn}) - p(V_{A_1} - I_2 + U_{A_1} + U_{A_2} + U_{SB,A} + G_{SB}) + \eta I_1 \end{aligned} \quad (112)$$

iff

$$\begin{aligned} & (1+r)((1+r)I_1 + pI_2) \\ & < p(G_{PE,1} + \frac{1}{2}G_{PE,syn} - G_{SB}) - p(U_{A_1} + U_{A_2} + U_{SB,A}) + \eta I_1 + pI_2, \end{aligned} \quad (113)$$

as asserted. Using (109) in (36) proves the last assertion. ■

A.11 Proof of Corollary 12

Proof. From Proposition 9 condition (22) we have that a takeover by the private equity fund is possible if

$$(1+r)((1+r)I_1 + pI_2) \leq p(G_{PE} - G_{SB}) - p(U_{A_1} + U_{A_2} + U_Q) + \eta I_1 + pI_2, \quad (114)$$

iff

$$p(G_{PE} - G_{SB}) - p(U_{A_1} + U_{A_2} + U_Q) + (\eta - (1 + r)^2)I_1 - prI_2 \geq 0. \quad (115)$$

1. Suppose $r = 0$. Then (115) becomes

$$p(G_{PE} - G_{SB}) - p(U_{A_1} + U_{A_2} + U_Q) + (\eta - 1)I_1 \geq 0 \quad (116)$$

Comparing this to (48) for $d_s = 0$ proves the assertion as $u \in [0, 1]$.

2. Let $u = 0$ in (48), then society's excess value of the private equity fund's takeover is

$$p(G_{PE} - G_{SB}) + (\eta - 1)I_1. \quad (117)$$

Rewriting condition (115) for a private equity takeover being possible we have

$$p(G_{PE} - G_{SB}) + (\eta - 1)I_1 \geq p(U_{A_1} + U_{A_2} + U_Q) + r(r + 2)I_1 + prI_2 > 0, \quad (118)$$

that is, the left-hand side is society's excess value. It follows that if the excess value is between 0 and $p(U_{A_1} + U_{A_2} + U_Q) + r((r + 2)I_1 + pI_2)$, then there is an efficiency loss to society.

3. Rewriting (48) with $d_s = 0$ we have that a private equity fund's takeover is efficient if

$$p(G_{PE} - G_{SB}) + (\eta - 1)I_1 - up(U_{A_1} + U_{A_2} + U_Q) > 0. \quad (119)$$

If a takeover by the private equity fund is just possible, then we have from (118)

$$p(G_{PE} - G_{SB}) + (\eta - 1)I_1 = p(U_{A_1} + U_{A_2} + U_Q) + r(r + 2)I_1 + prI_2, \quad (120)$$

implying that a private equity fund takeover is efficient if

$$(1 - u)p(U_{A_1} + U_{A_2} + U_Q) + r(r + 2)I_1 + prI_2 > 0, \quad (121)$$

which is always the case. Thus, if a private equity fund takeover is possible, it is always efficient from society's point of view.

■

A.12 Proofs in the case with two firms

The structure of the proofs follow similar to the derivations above. The full proofs will be added in a future version of the paper.

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(a) Return premium, r . Variation with low information costs, $\eta = 1.5$

(b) Private benefits to SB, U_Q . Variation with high return premium, $r = 0.15$.

(c) Informational cost, η . Variation with high return premium, $r = 0.15$.

(d) Direct gain of private equity acquisition, G_{PE} . Variation with high return premium, $r = 0.15$.

Figure 2: Condition for a private equity takeover. The figure illustrates when a takeover by a private equity fund takes place; this happens when the value is above 0 (all terms collected on the right-hand side in (22)). The blue line depicts the base case, the purple line depicts a variation with low information costs or high return premium. The base case parametrization is $p = 0.5$, $I_1 = 4$, $I_2 = I_2^{Q_2} = 14$, $U_{A_1} = U_{A_2} = U_Q = 3$, $G_{SB} = G_{PE} = 10$, $\eta = 5/3$, $V_{A_1} = V_{Q_2} = 30$, and $r = 0.10$.

(a) Investment cost of first stage, I_1 . Variation with low information costs, $\eta = 1.5$.

(b) Investment cost of second stage, I_2 . Variation with low information costs, $\eta = 1.5$.

(c) Investment cost of first stage, I_1 , for fixed NPV. Variation with low information costs, $\eta = 1.5$.

Figure 3: Condition for a private equity takeover. The figure illustrates when a takeover by a private equity fund takes place; this happens when the value is above 0 (all terms collected on the right-hand side in (22)). The blue line depicts the base case, the purple line depicts a variation with low information costs. The base case parametrization is $p = 0.5$, $I_1 = 4$, $I_2 = I_2^{Q_2} = 14$, $U_{A_1} = U_{A_2} = U_Q = 3$, $G_{SB} = G_{PE} = 10$, $\eta = 5/3$, $V_{A_1} = V_{Q_2} = 30$, and $r = 0.10$.

(a) Information cost, η . Variation with society's weight on private benefits, $u \in \{0.25, 0.50, 0.75\}$.

(b) Society's weight on private benefits, u . Variation with low information cost, $\eta = 1.5$.

(c) Prob. of good signal, p . Variation of society's weight on private benefits, $u \in \{0.25, 0.50, 0.75\}$.

(d) Society's weight on private benefits, u . Variation of prob. of good signal, $p \in \{0.1, 0.5, 0.9\}$, for low information cost $\eta = 1.5$ and gain of private equity takeover $G_{PE} = 5$.

(e) Return premium, r . Variation of information cost, $\eta = 1.5$. The vertical dashed line shows the cut-off for the PE fund's acquisition.

Figure 4: **Gain to society of a private equity takeover**, (48). The base case parametrization is $p = 0.5$, $I_1 = 4$, $I_2 = I_2^{Q_2} = 14$, $U_{A_1} = U_{A_2} = U_Q = 3$, $G_{SB} = G_{PE} = 10$, $\eta = 5/3$, $V_{A_1} = V_{Q_2} = 30$, and $r = 0.10$.

(a) Stylized picture of payoff functions

Figure 5: **Value of Information.** Red lines are payoff to PE fund as function of posterior belief, with the green circle showing the expected payoff to a fully informed fund at prior belief $p = 0.5$. Black lines are payoff to a less informed player with private benefits, with the purple circle indicating the expected payoff.